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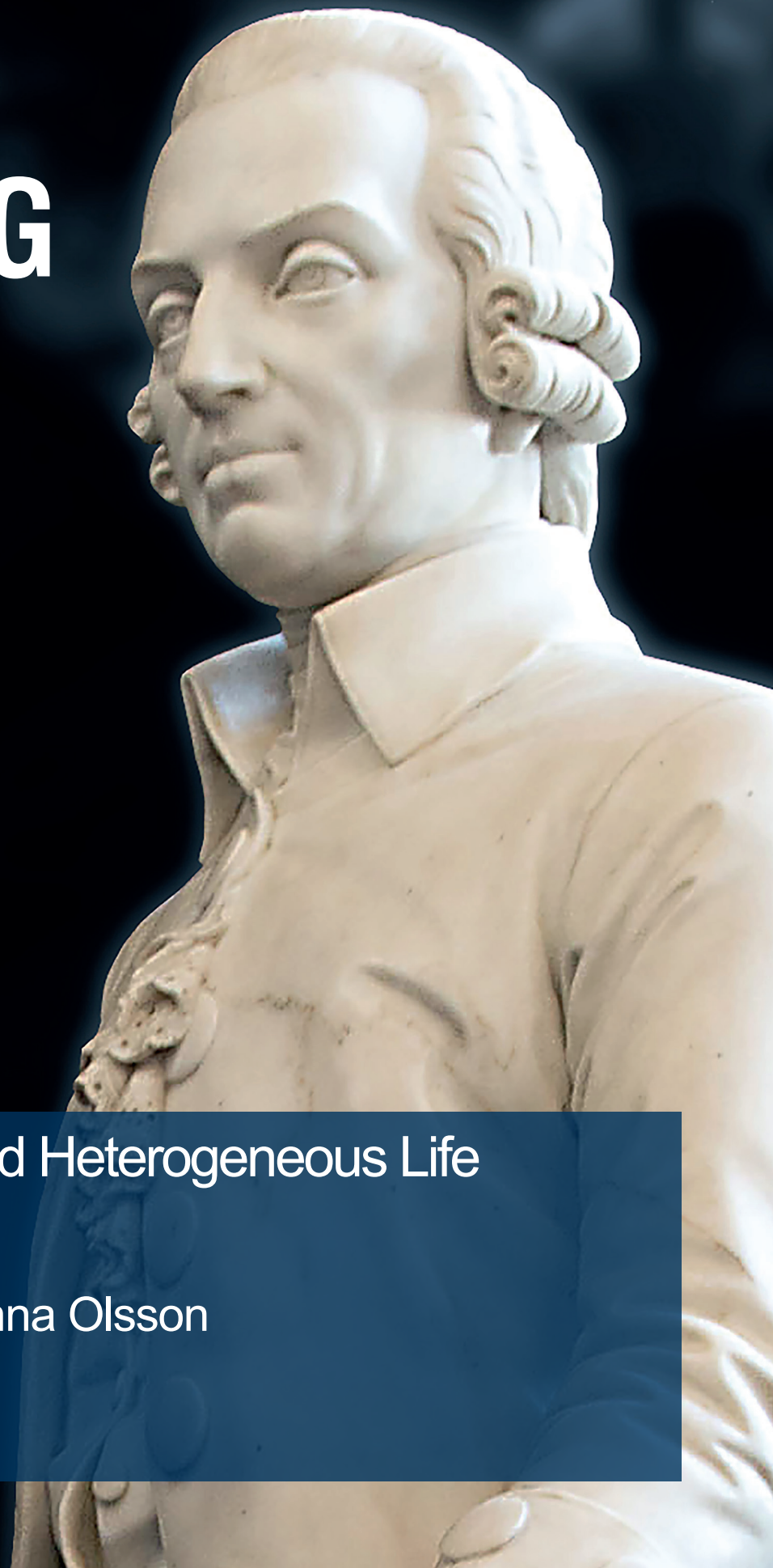

## Health Dynamics and Heterogeneous Life Expectancies

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# Health Dynamics and Heterogeneous Life Expectancies

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Using biennial data from the Health and Retirement Study, we estimate age-dependent health dynamics and survival probabilities at annual frequency conditional on race, sex, and health. The health gradient in life expectancy is steep and persists after controlling for socioeconomic status. Moreover, even conditional on health and socioeconomic status, the racial gap in life expectancy remains large. Simulations show that this gap affects savings rates but does not play a major role in explaining the racial wealth gap. However, differences in mortality imply that black individuals on average can expect to receive 15% less in Social Security benefits in present value terms.

**JEL Classification:** C23, E21, I14, J14
**Keywords:** Life expectancy; health dynamics; racial life expectancy gap

## 1 Introduction

Health shocks and uncertain survival are major sources of risk over the life cycle. A negative health shock can result in large medical expenditures (De Nardi, French, and Jones 2010; Kopecky and Koreshkova 2014), which affects the incentives to accumulate assets and could also affect the earnings potential (French 2005; Coile, Milligan, and Wise 2016). The survival probability directly affects the effective discount factor, a mechanism present in any life cycle model with uncertain life span. According to Finkelstein, Luttmer, and Notowidigdo (2013), health directly influences the marginal

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utility from consumption. In order to quantify the risk individuals face, model their choices in the presence of such risk, or evaluate the economic implications of health inequality, a realistic health and survival process is therefore crucial.

In this paper, we make three contributions: first, we provide improved estimates of health and survival dynamics at an *annual* frequency which are better suited for life cycle models than existing biennial estimates obtained from standard data sets, as such models are usually calibrated to one-year periods. Second, we use our estimates to quantify heterogeneity in life expectancy along the health gradient conditional on race, sex and socioeconomic status. Third, using a life cycle model, we examine the economic implications of these differences, in particular for savings and wealth accumulation as well as for Social Security wealth, where in addition to health we focus on differences by race as these are the most pronounced.

Our first contribution is methodological: existing papers estimating stochastic processes of health and survival dynamics are usually based on the Health and Retirement Study (HRS), a biennial panel representative for the elderly in the US, and their mortality estimates inherit this two-year frequency (Pijoan-Mas and Ríos-Rull (2014), Amengual, Bueren, and Crego (n.d.), Hosseini, Kopecky, and Zhao (2021b)). Our method instead directly estimates annual transition dynamics from the HRS and is moreover able to deal with varying transition lengths (only about 84% of observations in the HRS are best described as spanning two years) and periods of nonresponse. Additionally, in contrast to the above studies, we also report results for the black subsample in the HRS.[1]

Next, we use these estimates to compute life expectancy by health, race, and sex (our main specification), as well as by socioeconomic status. The estimated longevity health gradient is steep: for example, a 50-year-old nonblack man has a 80% chance of turning 70 if he is in excellent health, whereas this probability drops by 20 percentage points if he instead were in poor health. Furthermore, we show that even conditional on health, differences by race are substantial: a 50-year-old nonblack woman in excellent health has a 3.5 years higher life expectancy than a black woman of the same age and in the same health state. This racial gap is the result of two factors: the health distribution at a given age (which on average is worse within the black group), and the estimated survival dynamics going forward (which again are worse, i.e., blacks are more likely to experience a deterioration in health and have higher mortality). We show that only approximately one tenth of the difference in life expectancy at the age of 50 is due to

[1] We make the health-to-health (conditional on survival) and survival probabilities for different demographic groups and health classifications available online so they can be directly incorporated into standard life cycle models with minimal effort. See https://github.com/richardfoltyn/health-process.

worse initial health conditions, and nine tenths are due to adverse health dynamics and higher mortality in the following years. The latter are therefore much more important than initial health to understand the differences in longevity.

The relationship between socioeconomic status and life expectancy is well established (see, e.g., Chetty et al. (2016) and the references therein). In a second set of results, we therefore extend our analysis to incorporate two different measures of socioeconomic status: education level and permanent income. Our model predicts large differences across socioeconomic groups, in line with previous studies. We show that even conditional on socioeconomic group and health, there is a significant racial gap in life expectancy. For example, a nonblack woman with high school education and in best health is expected to live 2.4 more years than a black woman with the same education and health.

This racial gap gives rise to substantial differences in welfare between black and nonblack individuals due to the additional years of life enjoyed by the latter group (see Brouillette, Jones, and Klenow (2021) for one quantification). Our third contribution is to examine the importance for health and survival heterogeneity for additional economic outcomes in a standard overlapping-generations model. Already Smith (1995) conjectured that differences in life expectancy could contribute to lower savings rates and thus lower wealth accumulation among the black population. We show that while the differences in life expectancy indeed lead to differences in savings rates and wealth trajectories over the life cycle, the magnitudes are way too small to explain the observed wealth gap. In our model, nonblack individuals on average accumulate only 25% more wealth at the time of retirement compared to an otherwise identical black group, a gap that is about an order of magnitude smaller than in the data. Consequently, factors other than differences in discount rates due to differences in mortality are quantitatively more important.

Another economic measure that is potentially strongly effected by heterogeneity in life expectancy is the present value of expected Social Security benefits, so-called Social Security wealth. We find that the racial gap in Social Security wealth that is due to differences in life expectancy can be substantial, on average around 15% at the age of 50 and approximately 8% at the time of retirement. The welfare implications of such disparities are large: if a black man with median wealth in excellent health were given this difference as a one-time lump sum payment at retirement, he would perceive this transfer as being equivalent to a permanent consumption increase of 6.5% during his remaining lifetime.

This paper relates to two main strands of literature: first, papers that document heterogeneity in life expectancy, for instance across race, education and behavioral health

conditions such as smoking (Meara, Richards, and Cutler 2008), race and geographic region (Chang et al. 2015), or income and geographic region (Chetty et al. 2016). Compared to these studies, we provide estimates of life expectancy heterogeneity not only conditional on race (and different measures of socioeconomic status) but also current health, taking into account future health dynamics as suggested in the seminal work by Pijoan-Mas and Ríos-Rull (2014). In comparison to the latter, we extend their estimation methodology as described above, and also report results for the racial gap in life expectancy.

The other strand are papers estimating health and survival processes that can be used in life cycle models that study the effects of health and mortality. The most common approach is to use self-reported health, which can be thought of as letting the respondents themselves aggregate the multidimensional information about their health (that is potentially unobservable to the econometrician) into a single categorical variable. This measure has been shown to be surprisingly informative, see for instance Idler and Benyamini (1997) for an early overview, or more recent contributions by DeSalvo et al. (2006) and Latham and Peek (2013). Many studies using this data further aggregate the five health categories recorded in the HRS into two coarser groups, good or bad health (French 2005; De Nardi, French, and Jones 2010; De Nardi, Pashchenko, and Porapakkarm 2017). We show that using all five values is useful for two reasons: it trivially captures more of the heterogeneity in the population, and the finer measure is able to better capture the persistence and duration dependence of bad health.

An alternative method is to let the econometrician aggregate numerous physical and mental health indicators into a single index. For example, Poterba, Venti, and Wise (2017) use the first principal component extracted from 27 different health indicators, while Hosseini, Kopecky, and Zhao (2021b) construct an index based on the number of deficits accumulated over life. A related approach is taken by Amengual, Bueren, and Crego (n.d.) who assign individuals probabilities to fall into one of four latent health groups based on whether they are able to perform activities of daily life or cognitive tasks. Whereas these methods perform somewhat better in certain scenarios (for example predicting nursing home entry), they come with added complexity compared to the five-state Markov process presented here which makes their inclusion in standard life cycle models more challenging.

In the next section, we describe the HRS data and our estimation method. Section 3 presents the results for our main specification, while section 4 extends the analysis to include socioeconomic indicators. In section 5, we quantify the economic importance of racial inequality in life expectancy. The last section concludes.

# 2 Estimation

## 2.1 Data

We use the Health and Retirement Study (HRS), a representative panel of US households in older ages, to investigate longevity and health dynamics in the later stages of life. The survey includes questions about self-reported health and records the date of death, if applicable.

Our analysis is based on the survey years 1992–2014 taken from the HRS data compiled by RAND, version 2018 (V1) (Health and Retirement Study (2018)).[2,3] The first cohort included in the survey was between 51 and 61 years old in 1992, and thereafter new (older and younger) cohorts have been included. Many of the respondents have died over the sample period, making it an ideal data set for studying survival.

As can be seen from Figure 1, the survey was administered biennially for most cohorts and time periods. However, in practice, there is a some variation in the time elapsed between interviews. Each survey round is conducted over a period of time, so the actual time elapsed between interviews in consecutive waves varies between one and three years. For respondents missing one or more interviews, the time interval between two interviews or the time elapsed between the last interview and the date of death is more than three years. All in all, the time elapsed between two records is approximately two years for slightly more than 80% of the observations, while one- and three-year periods make up most of the remainder. Detailed statistics are shown in Table A.3 in the appendix.

For the remainder of the paper, we report statistics split along the dimensions of race and sex for black/nonblack as well as male/female subpopulations. The "black" sample consists of respondents who identify as black or African-American, while "nonblack" is the complementary group which also includes Hispanics. The HRS sample is not large enough to disaggregate the nonblack group further, since the (unweighted) sample of person-year observations is approximately 72.7% white, 15.7% black/African-American,

[2] Up until RAND version O (covering waves until 2012), the survey was complemented with death dates taken directly from the National Death Index (NDI), but this data was later removed from the public files. Our analysis of death dates in the releases following version O shows that without the NDI data, death dates are sometimes recorded with considerable lag. Using the RAND 2018 (V1) files but including only the years up to 2012 produces almost identical results to the ones obtained with the original version O data that included the NDI death dates. However, for later years we suspect that not all death dates have been recorded yet, which we believe gives rise to the nonresponse patterns documented in appendix section A.1. Based on these nonresponse patterns, we decided to only include waves up to and including the year 2014.

[3] The HRS (Health and Retirement Study) is sponsored by the National Institute on Aging (grant number NIA U01AG009740) and is conducted by the University of Michigan.

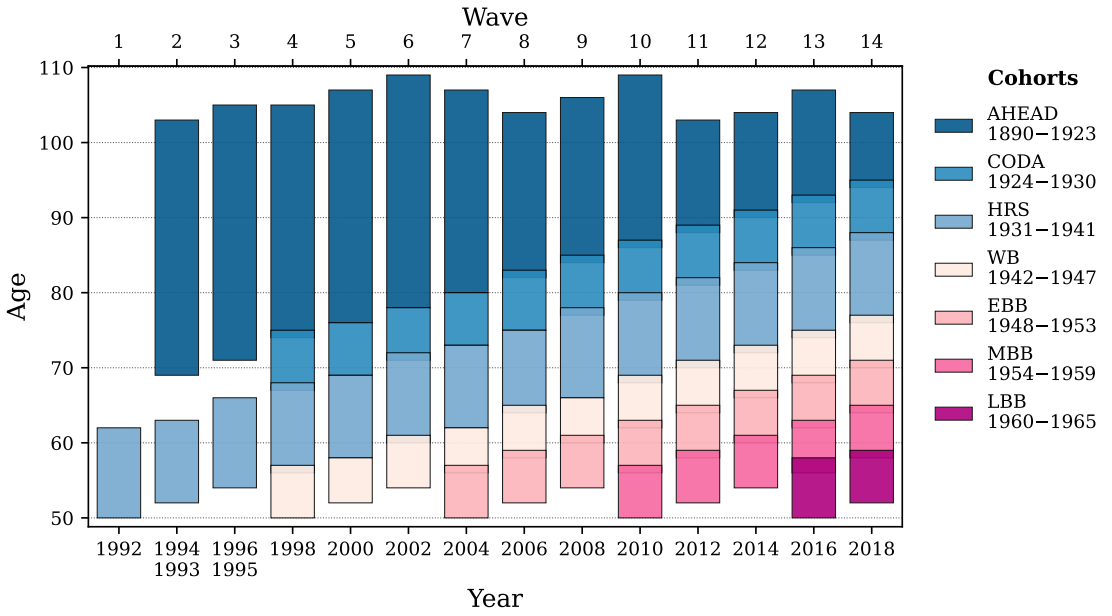


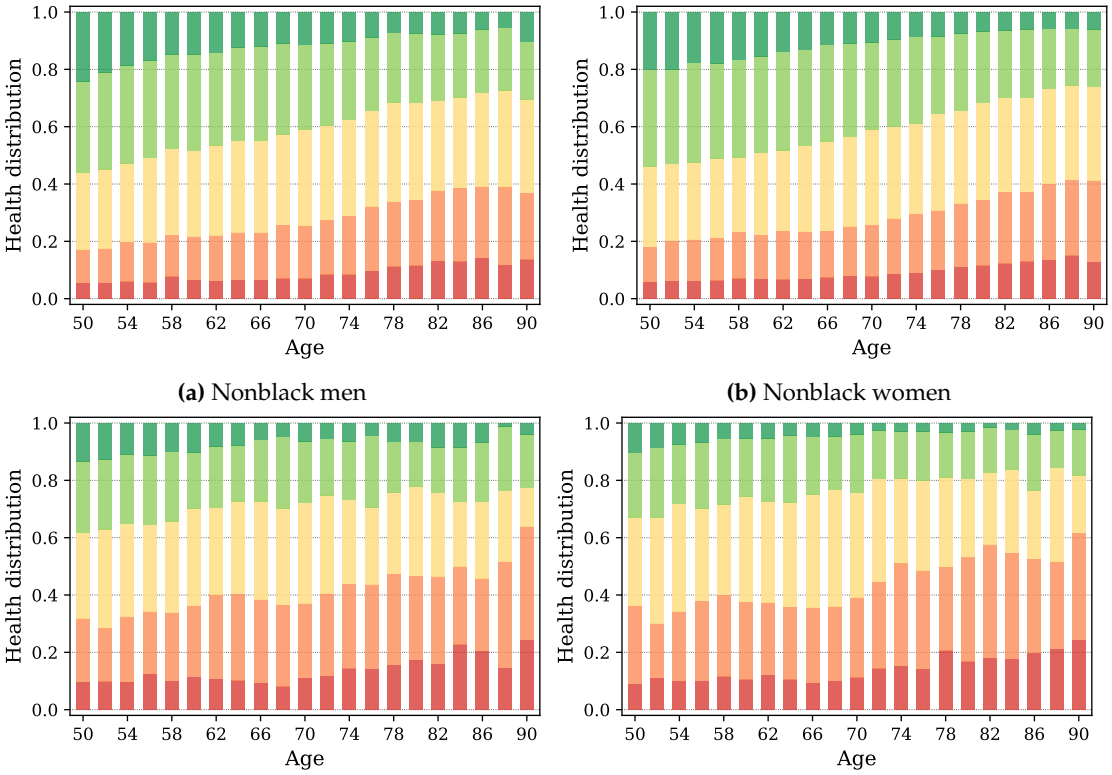

**Figure 1:** Longitudinal survey design of the HRS. The y-axis shows respondents' age by cohort and wave, ignoring spouses who are not age eligible. The legend lists all birth cohorts included in the HRS (using their official acronyms) as well as their birth years. AHEAD was initially a separate survey conducted in 1993 and 1995.

9.4% Hispanic, with other ethnicities together contributing the remaining 2.3%.[4]

The two key variables we use are the date of death and self-reported health. The latter is simply the respondent's answer to the question "Would you say your health is excellent, very good, good, fair, or poor?" The answers are coded on a scale from one to five, with one being "excellent," and we follow this convention throughout the paper. Self-reported health can be interpreted as a one-dimensional variable capturing high-dimensional information, letting the respondent aggregate this information him- or herself.

Figure 2 shows the distribution over health states for different demographic groups and ages. In general, the health distribution for black individuals is slightly worse than for nonblack individuals. Overall, health is declining in age, but the health distribution among 50-year-old individuals is not *that* much better than among 90-year-olds. This suggests that the aggregation of underlying health measures done by respondents also takes into account the relative health within their cohort. A 90-year-old respondent

[4]This partition is also in line with the US life tables, where the Hispanic subpopulation was added as a separate group only in 2006, while we use earlier data for some comparisons (see Arias (2014) for technical details of the life table program). Other groups than white and black (and later Hispanics) are not reported separately by the NVSS.




**(a)** Nonblack men

**(b)** Nonblack women

**(c)** Black men

**(d)** Black women

**Figure 2:** Distribution of health states by age. Dark green indicates best ("excellent") while red indicates worst ("poor") health. Observations are grouped into two-year age bins.

who reports "excellent" health might feel worse than he or she did 40 years earlier, but "excellent" in comparison to what the person perceives could be expected as a 90-year-old. Since all our estimates condition on age, this is taken into account.[5]

**Self-reported health vs. other health measures.** Using self-reported health has several advantages: first, a very similar question is asked in many other surveys, both in the US (e.g., the Panel Study of Income Dynamics (PSID) and the Medical Expenditure Panel Survey (MEPS)) and also globally (for instance the Survey of Health, Aging and Retirement in Europe (SHARE)). Hence, the insights into the dynamics of self-reported health and life expectancy conditional on this measure can be used for analyses based on many other data sets.

Second, a number of studies have shown that self-reported health is highly correlated

[5] Another interpretation is that the relevant measure is not self-reported health by itself, but self-reported health by age. In this sense, the variable takes on not five but $5 \times (99 - 50 + 1) = 250$ distinct values for the age range of 50–99 considered in our estimation.

with other subjective and objective measures of health and is also a good predictor for future mortality (see e.g. Idler and Benyamini (1997), DeSalvo et al. (2006), Latham and Peek (2013) and Pijoan-Mas and Ríos-Rull (2014)).

More recent papers propose alternative measures of health which are based on numerous indicators such as having problems performing daily activities of life or suffering from cognitive impairments. These are then aggregated by the researcher (as opposed to the respondent) into a single index. For example, Amengual, Bueren, and Crego (n.d.) combine 12 such indicators, and Hosseini, Kopecky, and Zhao (2021b) construct a frailty index from 28 underlying deficits. As one would expect, these more granular measures perform better than self-reported health, e.g., when predicting nursing home entry. However, the gain from this substantially more complex approach is often small.[6] Which approach to use therefore comes down to the research question and computational considerations if the health and survival dynamics are to be included in a life cycle model.[7,8] In the latter case, it is unclear whether a sufficiently simplified variant of these more elaborate health measures retains the improved predictive properties documented in these papers. On the other hand, no further simplifications are needed for self-reported health, since a discrete variable with five values following a Markov process can directly be added to any model: what you see is what you get.

**Estimation sample.** We exclude all observations with missing age, race, sex, education or self-reported health. Because our goal is to estimate probabilities of surviving to the next period and observing a particular health state, the fundamental input into our estimation procedure is a transition which consists of a set of observables at date $t$ and either a health state or death record at some future $t + n$. For this reason, we drop all individuals with only a single observation since the individual's state at the end of

[6] For example, for a probit model of nursing home entry, Hosseini, Kopecky, and Zhao (2021b) report a pseudo $R^2$ of 0.236 for the model with self-reported health, whereas a model using their frailty index has a pseudo $R^2$ of 0.264 (see their online appendix, Table 67). Amengual, Bueren, and Crego (n.d.) report an $R^2$ of 0.112 for a linear probability model of survival to the next HRS wave based on self-reported health which increases by about 0.01 when using their health measure (see their Table 4).

[7] The frailty index in Hosseini, Kopecky, and Zhao (2021b, 2021a) is continuous and consists of three components: a randomly drawn permanent effect, a transitory shock and an AR(1) term. The latter is commonly discretized using the Rouwenhorst method with at least five grid points, but no guidance is given about the other two components. The size of the discrete state space required to represent the estimated cross-sectional and time-series moments is thus not obvious.

[8] The four health groups in Amengual, Bueren, and Crego (n.d.) are latent, so one either has to add a three-dimensional simplex to the set of state variables to represent the probability distribution of belonging to any of these groups, or assign each individual the modal group.

| | All | Nonblack | | Black | |
|---|---|---|---|---|---|
| | | Male | Female | Male | Female |
| *Sample size* | | | | | |
| N. of individuals | 34,179 | 12,737 | 15,455 | 2,421 | 3,566 |
| N. of observations | 219,530 | 81,248 | 103,879 | 13,247 | 21,156 |
| Avg. observations/indiv. | 6.4 | 6.4 | 6.7 | 5.5 | 5.9 |
| *Age distribution* | | | | | |
| [50, 60) | 34.4% | 36.0% | 32.3% | 40.7% | 36.8% |
| [60, 70) | 30.6% | 31.8% | 29.3% | 33.0% | 31.2% |
| [70, 80) | 21.7% | 21.4% | 22.4% | 18.4% | 19.9% |
| [80, 90) | 11.3% | 9.5% | 13.3% | 7.0% | 10.0% |
| 90+ | 2.0% | 1.3% | 2.8% | 1.0% | 2.1% |
| Mean | 66.1 | 65.3 | 67.0 | 64.0 | 65.4 |
| *Self-reported health (all ages)* | | | | | |
| (1) Excellent | 12.5% | 13.6% | 12.7% | 8.8% | 5.1% |
| (2) Very good | 30.0% | 30.9% | 31.0% | 22.1% | 20.3% |
| (3) Good | 30.6% | 30.9% | 29.9% | 31.6% | 34.0% |
| (4) Fair | 18.6% | 17.1% | 18.1% | 25.9% | 28.1% |
| (5) Poor | 8.4% | 7.6% | 8.3% | 11.6% | 12.5% |

**Table 1:** Descriptive statistics for HRS estimation sample. The distribution of self-reported health is reported for samples pooled across all age groups. Mean age and population shares are weighted using HRS respondent-level weights.

the transition is not known. We only consider individuals aged 50 or older,[9] and we restrict the sample to a maximum age of 99 at transition start, even though individuals can be older when we observe them at the end of a transition. We estimate the health and survival process separately for the four subsamples of males/females and the nonblack/black, since it is well known that the life expectancies for these groups are very different. Table 1 shows the final number of individuals and person-year observations by subgroup included in the sample. Unsurprisingly, the black subsample is substantially smaller, which is reflected in the confidence intervals reported for estimates for the black subpopulation.

## 2.2 Estimation of transition probabilities

Our goal is to estimate a first-order Markov process for *annual* survival probabilities and health-to-health transitions conditional on survival. We use Pijoan-Mas and Ríos-Rull (2014) as a starting point, who estimate health and survival outcomes at a fixed two-year horizon using a multinomial logit model. We extend their analysis to account for some

[9] Each incoming HRS cohort is aged 51–56, but the survey contains younger individuals who are spouses of age-eligible respondents.

shortcomings of the HRS data:

1. The majority of life cycle models in macroeconomics and household finance where health and survival are of interest are calibrated to annual frequencies, but HRS waves are biennial. Due to variation in interview dates, we effectively observe transitions over one, two, three or more years, with about 80% of transitions being best described as two-year transitions.

2. Additionally, because our estimator keeps track of the distribution over latent health states for every year in which an individual is *not* observed, we can easily handle transitions which span an arbitrary number of periods of nonresponse. As a consequence, we can use recorded deaths that occur years after an individual stopped responding in the HRS as part of our estimation sample.

This is in contrast to a simpler approach which ignores variation in transition lengths in the HRS to estimate transition probabilities over a two-year horizon, and discards any transitions that include periods of nonresponse. In appendix section C, we compare the results from both approaches and find that when evaluated at *two-year horizons*, both yield similar results. Nevertheless, our estimates are more useful to researchers performing analyses at annual frequencies.[10]

While the HRS itself is organized into individual-year observations, for the purpose of estimating transition probabilities, we reinterpret the sample such that one *transition* constitutes one observation. Let $s_t$ be a binary indicator for whether a person is alive at date $t$,

$$s_t = \begin{cases} 1 & \text{if alive at } t \\ 0 & \text{else} \end{cases} \tag{1}$$

We assume that the one-period-ahead probability of survival is given by the binary-outcome logit model

$$p^s_{t+1} \equiv \Pr\left( s_{t+1} = 1 \mid h_t, \boldsymbol{x}_t \right) = \frac{1}{1 + e^{-g(h_t, \boldsymbol{x}_t | \gamma)}} \tag{2}$$

where $g(\bullet)$ is a function of current health $h_t$ and a vector $\boldsymbol{x}_t$ which contains any other

[10] As a by-product of their study of the savings behavior of the elderly, De Nardi, French, and Jones (2010) also estimate two-year transition and survival probabilities from the HRS and then use an approximation to recover annual transition probabilities. However, their method does not take into account varying transition lengths and uses a health classification with only two health states. While this approximation may be sufficient in some cases, it exhibits a downward bias in survival rates and has numerically undesirable properties if all five health states are used (see section D in the appendix for a detailed discussion).

variable of interest, in particular age, sex and race. Survival probabilities are governed by the parameter vector $\gamma$ which is to be estimated. Similarly, conditional on survival, the probability that health state $j$ is realized next period is given by the multinomial logit formula

$$p_{t+1|s}^{h,j} \equiv \Pr\left( h_{t+1} = j \,\middle|\, s_{t+1} = 1, h_t, \boldsymbol{x}_t \right) = \frac{e^{f_j(h_t, \boldsymbol{x}_t | \boldsymbol{\beta}_j)}}{\sum_\ell e^{f_\ell(h_t, \boldsymbol{x}_t | \boldsymbol{\beta}_\ell)}} \tag{3}$$

where each outcome-$j$-specific function $f_j$ is parametrized by the vector $\boldsymbol{\beta}_j$.[11] We normalize the parameter vector for the first outcome ("excellent" health) to $\boldsymbol{\beta}_1 = \mathbf{0}$ as the model is otherwise not identified.

This general setup makes it possible to assume different functional forms for the health-to-health and survival transitions. For example, one could impose that $f_j(\bullet)$ is linear in age whereas $g(\bullet)$ is quadratic.[12] While we experimented with richer models, adding higher-order terms in age turned out not to affect our results much. In our main specification, we therefore impose the same functional form for health-to-health and survival transitions, which are both assumed to be linear in age:[13]

$$g\left(h_{it}, m_i, b_i, a_{it} \,|\, \gamma\right) = \gamma_{0,hmb} + \gamma_{1,hmb} \cdot a_{it} \tag{4}$$

$$f_j\left(h_{it}, m_i, b_i, a_{it} \,|\, \gamma\right) = \beta_{0,jhmb} + \beta_{1,jhmb} \cdot a_{it} \qquad j = 2, \ldots, 5 \tag{5}$$

where $h_{it} = 1, \ldots, 5$ is individuals $i$'s health state at time $t$, $m_i$ and $b_i$ are indicator variables for *male* and *black*, and $a_{it}$ is an individual's age at the start of a transition. The vector of covariates is therefore $\boldsymbol{x}_{it} = (m_i, b_i, a_{it})$. Since we estimate all transitions for the male/female and black/nonblack groups separately, the estimated coefficients depend on the demographic group, the initial health state $h_{it}$ as well as the outcome (health state conditional on survival, or death).

We discuss the technical details of deriving the likelihood function and some pitfalls that arise due to variable transition lengths in the appendix section B. However, the intuition how our estimator handles variable transition lengths is straightforward: For

[11] We assume that all parameters in $\boldsymbol{\beta}_j$ are specific to outcome $j$ and there are no "common" parameters shared across all outcomes. This is due to the fact that we have no outcome-specific regressors and thus any common parameters would cancel out in (3), leaving these parameters unidentified.

[12] This allows for more parsimonious specifications, since adding one additional term to the multinomial logit in (3) adds $2 \times 2 \times 5 \times 4 = 80$ parameters to the model, whereas only $2 \times 2 \times 5 = 20$ more parameters are needed for the survival process (separate parameters have to be estimated for each race/sex combination!).

[13] When $f_j(\bullet)$ and $g(\bullet)$ are identical, the MLE simplifies to a multinomial logit on a pooled set of outcomes which includes both health conditional on survival, and death. However, unlike the multinomial logit estimators implemented in standard statistical software, our estimator still allows for variable transition lengths and periods of nonresponse.

any conjectured parameter vector, we obtain the conditional health-to-health and survival transition probabilities from (2) and (3). Given some initial health state $h_{it}$, we can then compute the the distribution of the latent $h_{it+n}$ over the states $\{1, 2, \ldots, 5\}$ as well as the probability of being alive for any year $t + n$ in which the individual is not observed. This allows us to bridge periods of nonresponse. In the appendix, we show that the log-likelihood function (and its gradient) can be computed in a recursive fashion, making the technical implementation relatively tractable.

In the next section, we present estimation results and use these to compute life expectancies for each demographic group. Our estimates are reported with bootstrapped confidence intervals which are based on the Rao-Wu rescaling bootstrap (Rao and Wu 1988) that takes into account the stratified cluster sampling of the HRS survey design. We provide details in section B.3 in the appendix.

In the appendix section C, we contrast the main specification with one that includes a quadratic term in age. The resulting transition probabilities are very similar except for some health and age combinations for the black subpopulation, which is due to the relatively smaller sample size.

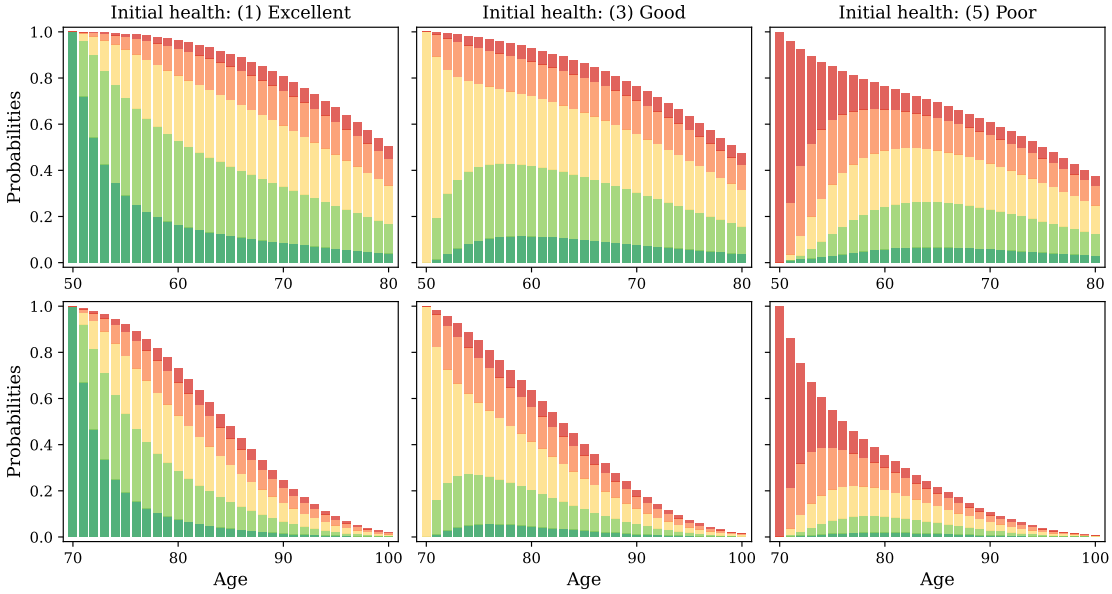
# 3 Estimation results

In this section, we present several types of model estimates: first, we report the model-predicted health and survival transition probabilities and contrast them with the raw data. Next, we compute the implied life expectancy by demographic group and quantify its health gradient. Last, we examine the persistence of health dynamics.

## 3.1 Health transitions and survival probabilities

In Figure 3, we plot the predicted health and survival probabilities for nonblack men. The figure shows the distribution over health states and the probability of being dead conditional on an initial health state and age over a forecast horizon of 30 years. As can be seen, the survival probability differs substantially depending on the initial health state: for a man aged 70 in excellent health, the predicted probability of surviving an additional 10 years is around 75%, but if he is in poor health, the probability is below 40%. The corresponding graphs for nonblack women, black men, and black women can be found in Figure A.10 in the appendix.

These distributions are obtained by repeatedly applying annual age-specific health-to-health transition and survival probabilities which are visualized in Figure A.11 and




**(a)** Nonblack men

**Figure 3:** Predicted distribution over health states and death conditional on initial health for a 50-year-old (upper row) and a 70-year-old (lower row). The colors indicate probability per health state (dark green being the best health state, red the worst). The white area represents the probability of being dead.

Figure A.12 in the appendix. As the figures show, health is persistent: for 70-year-olds in poor health, the probability of remaining in the same poor health state next year is around 75%. The probability of improving to anything better than the second-worst health state is low, below 5%.

The same pattern holds true for all health states: to remain in the current health state is the most likely outcome, and to improve or deteriorate one step is the second most likely outcome. For 50-year-olds in the best health state, the probability of remaining in excellent health is around 70%, but transitioning to the second-best state becomes more likely as they age.

The survival probabilities are unsurprisingly decreasing in age, but there is also a clear health gradient. The probability of surviving one additional year for a 70-year-old in the best health state (excellent) is almost 100% while for an otherwise identical individual in the worst health state it is closer to 90%. As is well known, the survival probability conditional on age is higher for women than for men.

### 3.1.1 Comparing model predictions and data

To compare model predictions to raw data moments, we compute the two-year transition probabilities implied by our annual model. We then plot these together with the fraction of individuals with a particular outcome in the subsample restricted to two-year transitions, which is the large majority of observations (84% of the full sample). Figure 4 and Figure 5 show the results for the nonblack and black subpopulations, respectively. Despite the rigid functional form assumption imposed by multinomial logit with linear functions (4) and (5), the estimated probabilities and the data are remarkably close for the nonblack groups. On the other hand, the data for the black population are more noisy due to the smaller number of observations, with unsurprising consequences for model fit and confidence intervals.

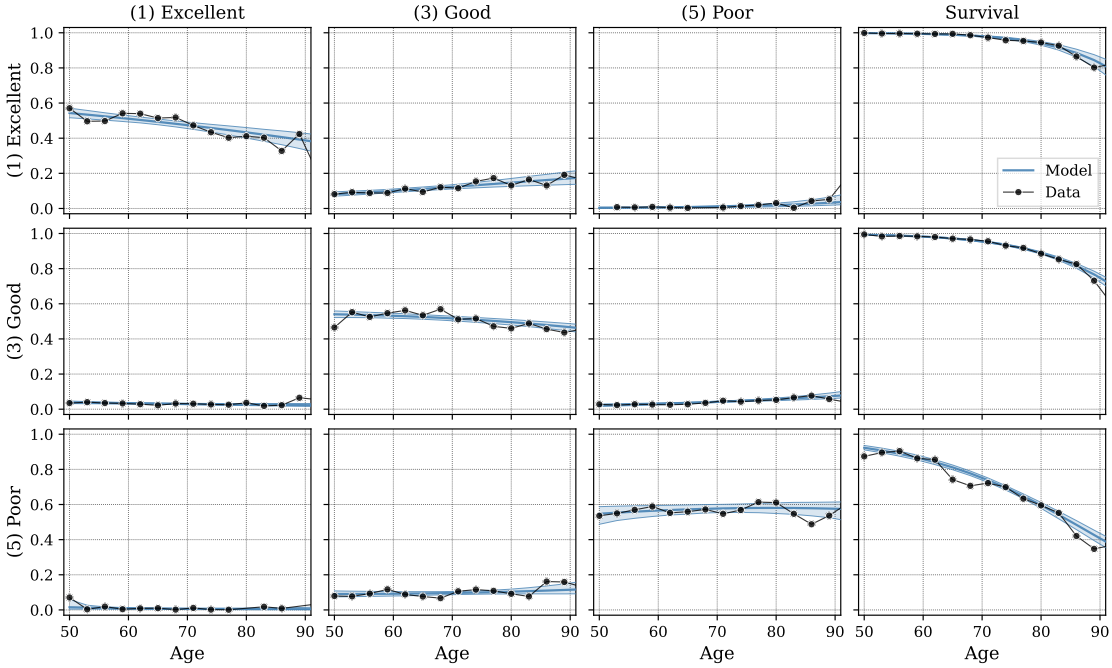
To assess how well our model predicts long-run outcomes, we compare actual survival rates as observed in the HRS with model predictions over a time horizon of up to 22 years. Figure 6 plots the model-predicted survival probabilities for all individuals observed in the survey in 1994 against the fraction actually surviving until 2014.[14] Each dot represents a two-year age bin, and we discard age bins with less than 20 observations. As can be seen, the estimated model captures the long-term survival probabilities well. In section E.3 in the appendix, we show analogous graphs for survival from each of the first ten survey waves until the year 2014, the last year in our sample. The overall message is that the model does well in predicting survival at both shorter and longer horizons, with a somewhat larger dispersion for the black subpopulation.

## 3.2 Life expectancy conditional on health

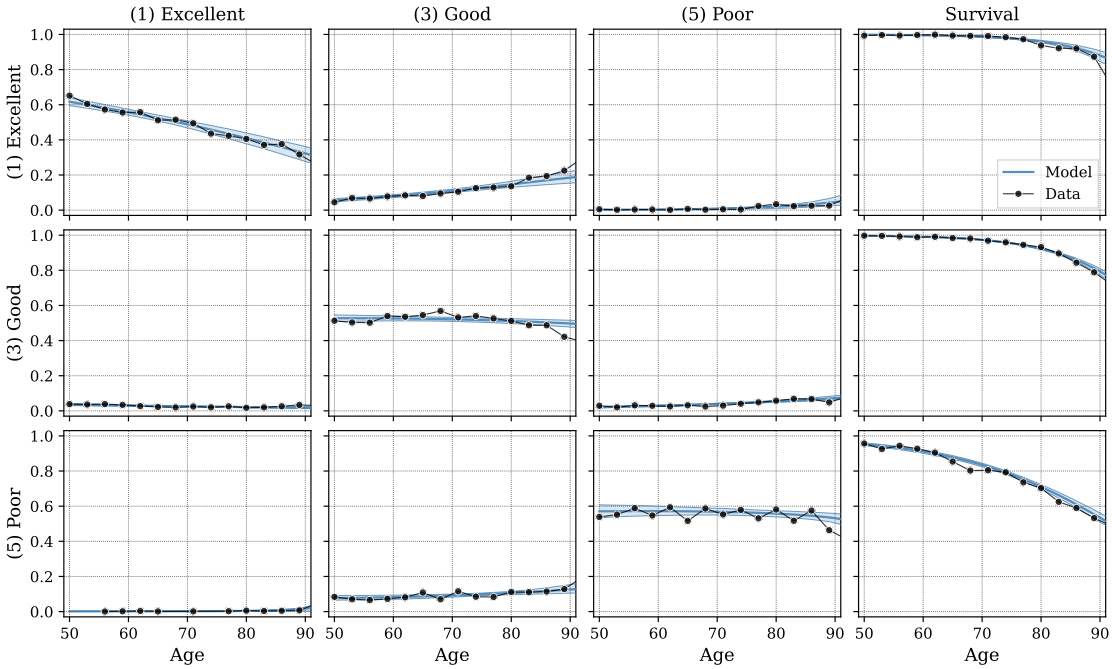
To calculate the life expectancy conditional on health, we need to take into account all future health-to-health transition probabilities. We follow Pijoan-Mas and Ríos-Rull (2014) and compute life expectancy at age $a$ conditional on initial health $h$ as

$$e_{a,h} = \left[ \sum_{\tilde{a}=a}^{a_{\max}} \sum_{k=1}^{H} \tilde{a} \left(1 - p^{s}_{\tilde{a}+1|k}\right) \mu_{k,\tilde{a}} \right] + \frac{1}{2}$$

[14] We show survival from 1994 (wave 2) onward instead of 1992 (wave 1) since the former includes both the HRS and AHEAD cohorts.

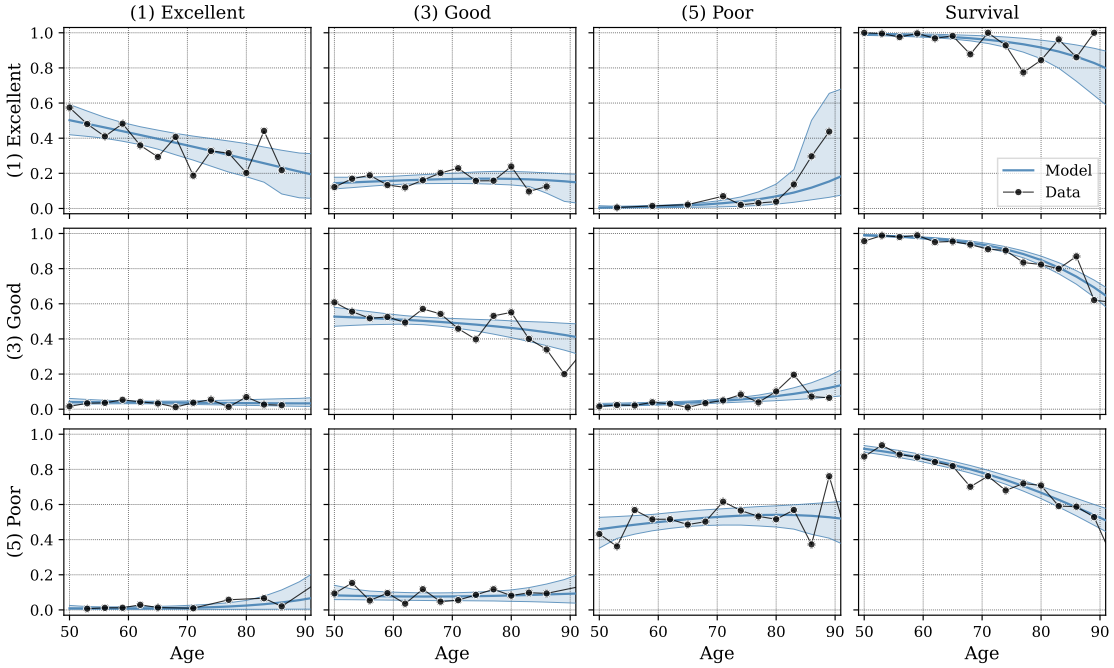



**(a)** Nonblack men

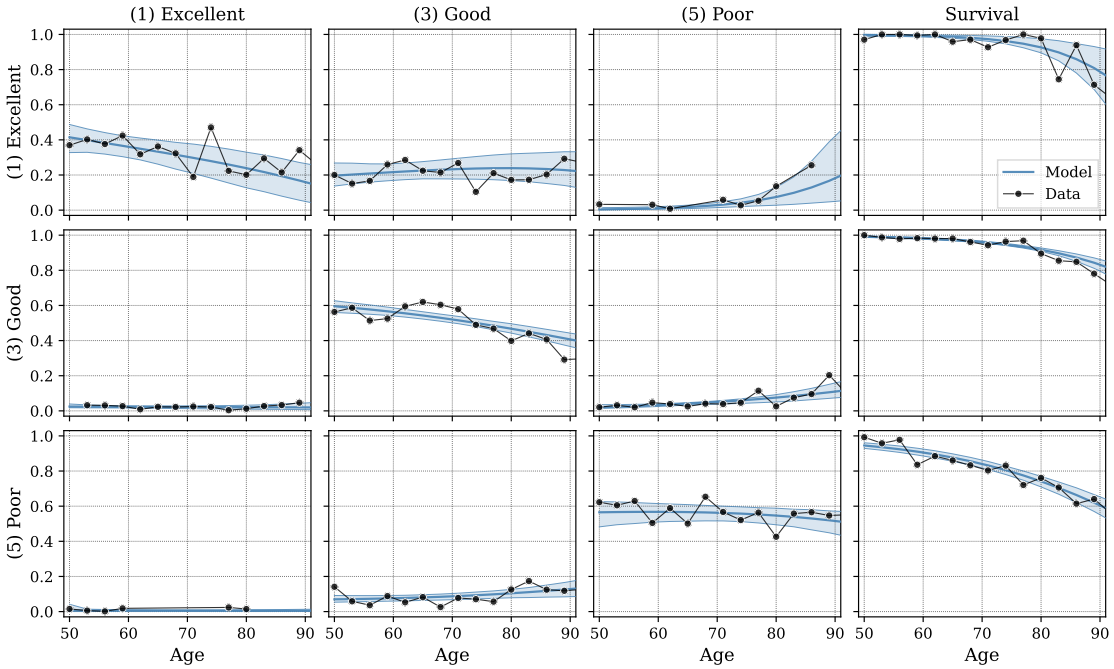



**(b)** Nonblack women

**Figure 4:** Two-year transition probabilities for nonblack groups. Graphs show the best ("excellent"), middle ("good") and worst ("poor") health states. Health transition probabilities are conditional on survival. Right-most column shows survival probabilities. Missing dots indicate that some transitions are not observed in the data. Shaded areas indicate bootstrapped 95% confidence intervals.




**(a)** Black men




**(b)** Black women

**Figure 5:** Two-year transition probabilities for black groups. Graphs show the best ("excellent"), middle ("good") and worst ("poor") health states. Health transition probabilities are conditional on survival. Right-most column shows survival probabilities. Missing dots indicate that some transitions are not observed in the data. Shaded areas indicate bootstrapped 95% confidence intervals.

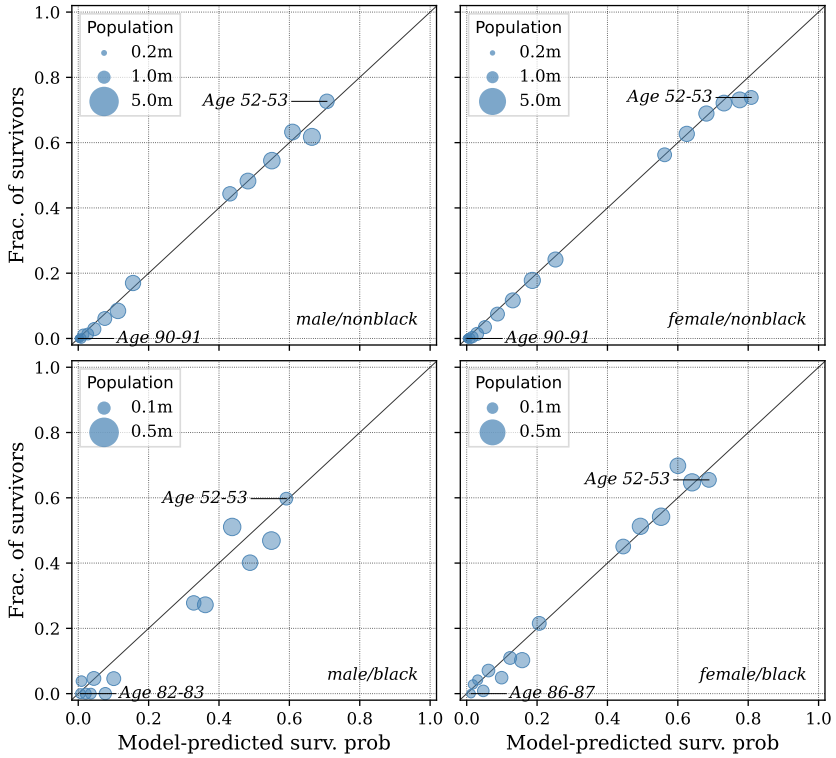



**Figure 6:** Model-predicted survival probabilities (on the x-axis) against the fraction of survivors (on the y-axis) for individuals observed in 1994. Each dot represents the fraction of survivors in 2014. Dots are grouped into two-year age bins based on age in 1994.

where

$$\mu_{j,\tilde{a}+1} = \sum_{k=1}^{H} p^{h,j}_{\tilde{a}+1|k,s} \times p^{s}_{\tilde{a}+1|k} \times \mu_{k,\tilde{a}}$$

$$\mu_{j,a} = \begin{cases} 1 & \text{if } j \text{ is initial health state} \\ 0 & \text{otherwise} \end{cases}$$

The transition probabilities are those defined in (2) and (3), and $\mu_{j,a}$ is the probability of being in health state $j$ at age $a$. The addition of the half year is to correct for the fact that people do not die exactly on their birthday, but deaths are instead approximately uniformly spread out over the year.

Figure 7 plots the resulting life expectancies conditional on the initial health state at the age of 50 and 70. As can be seen, the health gradient is substantial: the difference in expected life length between a 50-year-old nonblack man in the best and in the worst health state is 6.1 years. The figure also shows that the life expectancy is lower for the

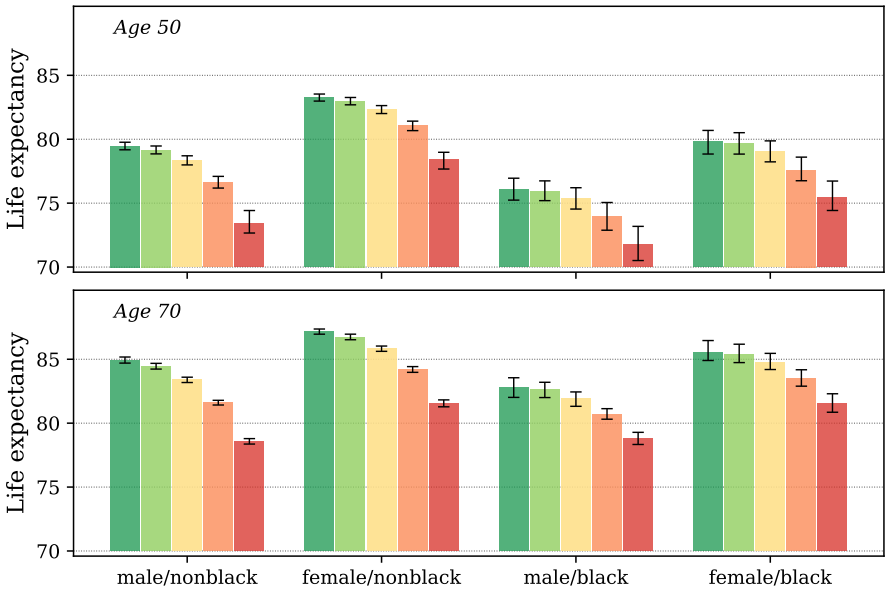



**Figure 7:** Life expectancy by race, sex and health state for at age 50 and 70. Error bars indicate bootstrapped 95% confidence intervals.

black subpopulation, even conditional on health. A 50-year-old black man in excellent health can expect to live another 26.2 years, while a nonblack man in the same excellent health can expect to live 3.4 years on top. Moreover, the health gradient is steeper for the nonblack population, both for males and (albeit less pronounced) for females, hence the the difference between the black and nonblack subpopulation increases for healthier individuals.

Table 2 shows the life expectancies by race, sex, and health and also the differences along the race and sex dimensions.[15] The "Average" row is calculated using the observed age-specific health distribution for each subgroup. The average life expectancy for 50-year-old nonblack men is 78.4 years, while it is only 74.9 years for black men. The difference of 3.5 years is the result of two factors. First, black men have a worse distribution over self-reported health at the age of 50 (see Figure 2). Second, conditional on health, their health dynamics and survival probabilities are worse from this age and onward. To disentangle these two effects we make the following experiment: we take the health and survival process of black men but use the initial health distribution of nonblack men. The life expectancy for this hypothetical group of 50-year-olds rises from 74.9 to 75.3 years. Hence, approximately 10% of the difference in life expectancies of black vs. nonblack 50-year-old men (0.4 of out 4 years) is due to worse initial health, and

[15] In the appendix section E.4, Figure A.17 shows graphs of life expectancies for the four demographic subgroups and all health states for all ages.

| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| Average | 78.4 | 82.4 | 74.9 | 78.5 | 3.5 | 3.8 | −4.0 | −3.6 |
| | [78.0, 78.8] | [82.0, 82.7] | [74.1, 75.9] | [77.7, 79.4] | [2.4, 4.5] | [3.0, 4.7] | [−4.3, −3.6] | [−4.9, −2.4] |
| (1) Excellent | 79.5 | 83.3 | 76.1 | 79.8 | 3.4 | 3.5 | −3.8 | −3.7 |
| | [79.2, 79.8] | [83.0, 83.5] | [75.2, 77.0] | [78.8, 80.7] | [2.4, 4.3] | [2.5, 4.5] | [−4.1, −3.5] | [−5.1, −2.3] |
| (3) Good | 78.3 | 82.3 | 75.3 | 79.0 | 3.0 | 3.3 | −4.0 | −3.7 |
| | [78.0, 78.7] | [82.0, 82.6] | [74.5, 76.2] | [78.2, 79.9] | [2.0, 4.0] | [2.4, 4.1] | [−4.4, −3.6] | [−5.0, −2.5] |
| (5) Poor | 73.4 | 78.4 | 71.8 | 75.4 | 1.7 | 3.0 | −5.0 | −3.7 |
| | [72.7, 74.4] | [77.7, 79.0] | [70.5, 73.2] | [74.4, 76.7] | [−0.0, 3.4] | [1.5, 4.2] | [−6.0, −3.7] | [−5.3, −2.0] |
| **Age 70** | | | | | | | | |
| Average | 83.2 | 85.6 | 81.5 | 84.2 | 1.7 | 1.4 | −2.4 | −2.7 |
| | [83.0, 83.4] | [85.4, 85.9] | [81.0, 81.9] | [83.6, 84.9] | [1.2, 2.3] | [0.7, 2.1] | [−2.6, −2.2] | [−3.8, −1.9] |
| (1) Excellent | 84.9 | 87.1 | 82.8 | 85.5 | 2.2 | 1.6 | −2.2 | −2.8 |
| | [84.7, 85.2] | [87.0, 87.4] | [82.0, 83.6] | [84.9, 86.5] | [1.4, 2.9] | [0.7, 2.3] | [−2.5, −2.0] | [−4.3, −1.6] |
| (3) Good | 83.4 | 85.8 | 81.9 | 84.8 | 1.5 | 1.1 | −2.4 | −2.9 |
| | [83.2, 83.6] | [85.6, 86.0] | [81.3, 82.4] | [84.2, 85.5] | [0.9, 2.1] | [0.3, 1.7] | [−2.7, −2.2] | [−4.0, −2.0] |
| (5) Poor | 78.6 | 81.5 | 78.8 | 81.5 | −0.2 | 0.0 | −3.0 | −2.7 |
| | [78.4, 78.8] | [81.3, 81.8] | [78.3, 79.3] | [80.9, 82.3] | [−0.8, 0.3] | [−0.7, 0.8] | [−3.3, −2.6] | [−3.7, −1.8] |

**Table 2:** Life expectancy by race, sex and initial health. Average life expectancy is computed as the weighted mean over health states at ages 50–51 (top) or 70–71 (bottom). Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Brackets indicate bootstrapped 95% confidence intervals.

the remainder results from worse health trajectories after that age. Table 3 shows the full set of combinations of health and survival dynamics and initial health distributions. These estimates suggest that the health and survival dynamics after the age of 50 (or 70) have a much larger effect on the average life expectancy than the health distribution at that age.

### 3.3 Comparing to life tables

The HRS data we use is from the period 1992 to 2014. With a substantially longer panel dimension, we could have computed cohort-specific health and survival probabilities by age. However, the sample is not large enough to permit this. Instead, the survival probabilities we calculate should be viewed as *period* life expectancies for the sample period as a whole and correspond to a weighted average of what is reported in the period life tables by the National Vital Statistics System (NVSS) during those years.[16]

[16]There are two types of life tables: period (or current) life tables and cohort (or generation) life tables. The (more common) period life table presents what would happen to a hypothetical cohort if it experienced the mortality conditions of a particular period in time throughout its entire life. The cohort life table, on

| | Nonblack | | Black | |
|---|---|---|---|---|
| | Male | Female | Male | Female |
| **Age 50** | | | | |
| *Initial health distribution* | | | | |
| Nonblack | 78.4 | 82.4 | 75.3 | 79.0 |
| Black | 77.8 | 81.9 | 74.9 | 78.5 |
| **Age 70** | | | | |
| *Initial health distribution* | | | | |
| Nonblack | 83.2 | 85.6 | 81.8 | 84.5 |
| Black | 82.7 | 85.1 | 81.5 | 84.2 |

**Table 3:** Life expectancies for actual and counterfactual health distributions. Each row reports average life expectancy using the indicated nonblack or black initial health distribution (of the same sex).

Our model gives an average life expectancy of 78.4 years for 50-year-old nonblack men and 82.4 for nonblack women. This is well in line with what is reported by the NVSS during this period.[17] For white men the NVSS life expectancy at the age of 50 is between 77.0 and 79.9 during the sample period, while for white women it ranges from 81.7 to 83.4. For black men, our model predicts 74.9 years, while NVSS reports between 72.8 and 77.2 for the period. For black women, our model predicts 78.5, while the NVSS reports between 78.3 and 81.5. Thus, in general the model predictions are well within what is reported by NVSS, even though the prediction for life expectancy for black women is on the lower end.[18]

The conclusions are similar for life expectancy at 70. Our model predicts a life expectancy of 83.2 years for nonblack men and 85.6 for nonblack women. The corresponding life expectancies reported by NVSS during the period 1992 to 2014 range between 82.3 and 84.5 for white men, and between 85.3 and 86.6 for white women. For black men the model predicts 81.5, while the NVSS estimates range from 80.8 and 83.3, and for the black women the model prediction is 84.2, while the NVSS estimates range from 83.9 to 86.1.

We also document changes in life expectancy over the sample period of 1992–2012. To this end, we augment the main model with a linear time trend. Again, our estimates

the other hand, presents the mortality experience of a particular birth cohort from the moment of birth through consecutive ages.

[17] All life tables can be found at https://www.cdc.gov/nchs/products/life_tables.htm.

[18] As pointed out by Pijoan-Mas and Ríos-Rull (2014), life expectancies computed from the HRS should differ slightly from the national average, since the HRS does not include institutionalized individuals. Note that in our analysis, individuals who moved to nursing homes do not count as institutionalized and are included in the sample. See appendix section A.2 for details.

are close to the NVSS figures: for nonblack 50-year-old men, we estimate a period life expectancy in 1992 of 76.9 years, and an increase to 79.1 in 2010, i.e., by 2.2 years. The NVSS reports an increase of 2.6 years for white men over the same period. For nonblack women, we estimate an increase of 1.1, while the NVSS reports 1.4 years.

Our estimated increase over time is slightly lower than the data from NVSS for the black population (but also with larger standard errors). For black males, we estimate an increase between 1992 and 2010 of 2.5 years (NVSS: 3.6 years). For black females, we estimate an increase of 2.0 years (NVSS: 2.6 years). In appendix section E.7, we provide detailed results for the specification with a time trend.

### 3.4 Duration dependency

Our estimated process is highly persistent, especially for the worst health state. Once there, the probability of remaining in the worst health state another period is above 75%.[19] The importance of health persistence is stressed by, e.g., Contoyannis, Jones, and Rice (2004).

It is common in the literature to aggregate the health states into two coarser categories: *good* (covering excellent, very good, and good health) and *bad* (covering fair and poor) (French 2005; French and Jones 2011; De Nardi, Pashchenko, and Porapakkarm 2017).[20] There are two benefits from using all five self-reported health states: First, trivially, a larger state space captures more of the heterogeneity in the population. Second, while a process estimated on a cruder two-state measure of health is appealing from a computational point, it struggles to capture some of the dynamics observed in the data.

For example, De Nardi, Pashchenko, and Porapakkarm (2017) document that the probability of transitioning from the coarser *bad* health state to the *good* health state decreases with time. The longer an individual has been unhealthy, the less likely he/she is to become healthy again. To address this issue, De Nardi, Pashchenko, and Porapakkarm (2017) use a higher-order Markov chain which also includes the lagged health states, thus effectively creating a first-order Markov process on $4 = 2 \times 2$ states. However, using a five-state process and following the literature by classifying the two worst health states as *bad* also partly captures this duration dependency.

To illustrate, let $\mathcal{G} = \{1, 2, 3\}$ and $\mathcal{B} = \{4, 5\}$ be the coarse *good* and *bad* health states,

[19] On a two-year horizon, the persistence of self-reported health is very similar to what Hosseini, Kopecky, and Zhao (2021b) find for their frailty index: using HRS data they conclude that "the difference in persistence [...] is small" (p. 72 online appendix).

[20] One reason is that to estimate yearly transitions, authors have resorted to using PSID, which until 1997 was a yearly survey. However, the number of individuals there is relatively small and therefore it is necessary to combine data into coarser health states.

respectively, and consider the following health-to-health transition matrix for the true model with five health states:

$$\mathbf{\Pi}^h = \begin{bmatrix} 3/4 & 1/4 & 0 & 0 & 0 \\ 1/4 & 1/2 & 1/4 & 0 & 0 \\ 0 & 1/4 & 1/2 & 1/4 & 0 \\ 0 & 0 & 1/4 & 1/2 & 1/4 \\ 0 & 0 & 0 & 1/4 & 3/4 \end{bmatrix}$$

Assume that all individuals start out at time $t = 0$ in health state 3, i.e., they start in $\mathcal{G}$. We are interested in the individuals in $\mathcal{B}$ at time $t = 2$ and their probabilities of transitioning back to $\mathcal{G}$, depending on whether they were in bad health for one or two periods.

After two periods, 31.25% of individuals are in $\mathcal{B}$; 18.75% have been in bad health for two periods, and two thirds of these are in health state 4 while one third are in health state 5. Hence, the probability of transitioning back to $\mathcal{G}$, conditional on having been in bad health for two periods, is 16.7%.

However, the probability of transitioning back to good health for the individuals who have only been in bad health for one period is 25% (this follows immediately since the unhealthy who were in good health in period $t = 1$ can, by construction, only be in health state 4 in this stylized example).

We now apply this reasoning to our estimated model. Formally, we define the age-dependent probability of recovering from the bad health state as a function of the number of periods $j$ already spent in bad health as

$$r_a(j) = \Pr\Big(h_{t+1} \in \mathcal{G} \;\Big|\; h_{t-k} \in \mathcal{B} \;\; \forall\, 0 \leq k < j,\; h_{t-j} = 3\Big)\,.$$

For simplicity, we assume that the individual was in the middle health state (3) prior to entering the bad health state $j$ periods ago. This make little difference as the probability of transitioning from states 1 or 2 directly into 4 or 5 is quite low, as shown in Figure A.11 and Figure A.12 in the appendix.

The recovery probabilities $r_a(j)$ for 50- and 70-year-olds are shown in Figure 8. As predicted by the stylized example, these probabilities are decreasing in the number of years spent in bad health. For example, a 50-year-old nonblack man who has spent just one year in bad health has a 24% probability of recovering, but if he had spent the last five years in bad health the probability is down to 18%. Even though the magnitude of the effect is smaller than the one reported in De Nardi, Pashchenko, and Porapakkarm

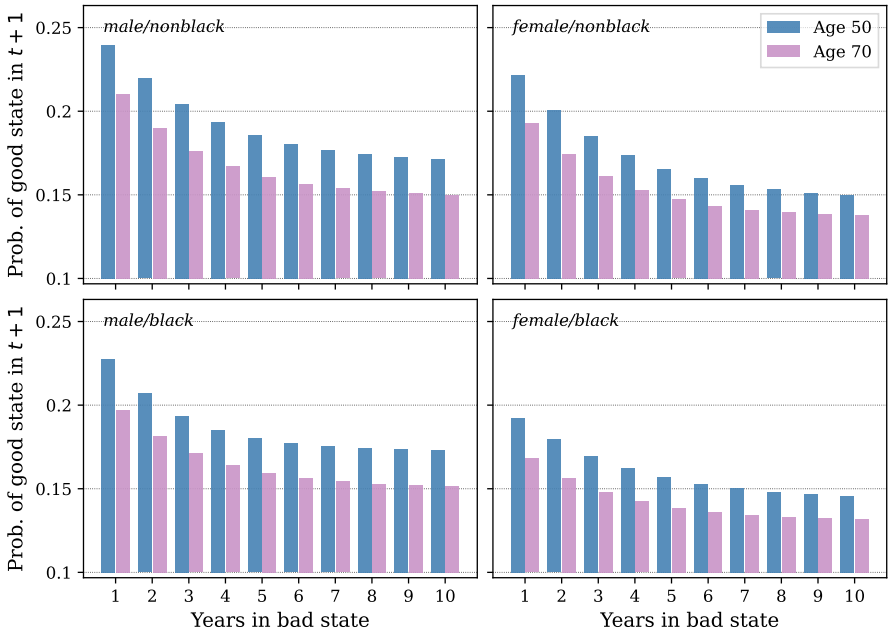



**Figure 8:** Probability of recovering from the bad health state $\mathcal{B}$ as a function of the number of years spent in bad health.

(2017) for the PSID, it is a substantial improvement over a first-order Markov chain with two states.

# 4 Life expectancy conditional on education or income

We now extend our main specification to include two indicators of socioeconomic status, education and income level, and we report life expectancy and its gradient with respect to health within each socioeconomic group.

## 4.1 Life expectancy and education level

We first extend (4) and (5) with education which we fully interact with age and health. We create three education groups defined as: 1) less than high school, 2) high school (broadly defined), and 3) a college degree or higher.[21]

Table 4 shows the life expectancy for 50- and 70-year-olds conditional on education level. The rows labeled "Average" report the life expectancy for each education group computed as the weighted average over the health distribution observed in the HRS

[21] Table A.2 in the appendix contains the distribution of individuals and person-year observations by education and demographic subgroup.

for that particular group, race, sex and age. As is well known, life expectancy is higher for individuals with higher levels of education: a 50-year-old nonblack male with a college degree has a life expectancy that is 7.3 years higher than one with no high school education. The estimated magnitudes are well in line with what is reported by, e.g., Rostron, Boies, and Arias (2010), who find a difference of 8.6 years in life expectancy for 45-year-old males and 4.5 years for 65-year-old males comparing no high school to college educated using data from 2005, i.e., towards the end of our sample.[22] In line with findings by Hummer and Hernandez (2013), we estimate a flatter education gradient for the black population: for example, the difference for black males at age 50 between college vs. high school educated is only 4.8 years (vs. 7.3 years for nonblack).

Table 4 also reports the life expectancy additionally broken down by health state. For the purpose of this analysis, we collapse the data into three health groups, merging health states 1 (excellent) and 2 (very good) into "best" health, and health states 4 (fair) and 5 (poor) into "worst" health, as otherwise sample sizes in some of the covariate cells become too small.

The first observation is that a health gradient exists even after additionally conditioning on education level. The life expectancy of a nonblack man with a college degree in best health is 4.5 years higher than an otherwise identical man in worst health. Moreover, the point estimates indicate that the health gradient is slightly stronger for the high-educated group than the low-educated group, confirming findings by Dowd and Zajacova (2007) and Burström and Fredlund (2001).

The second observation is that even conditional on education level and health, black individuals have a lower life expectancy, and this is true for both men and women. However, the estimates are noisy, especially for the black college educated population since there are few such individuals in our sample.

### 4.2 Life expectancy and income level

We now examine the impact of socioeconomic status as defined by income level. To this end, we sum up non-financial income at the household level and adjust for household size. Thereafter, we compute six-year averages (i.e., three waves) counted from the first time we observe the individual and interpret this measure as a proxy for permanent income. For most cohorts, this means that we use an income measure from their 50s, i.e.,

[22] The consensus in the literature is that the life expectancy gradient in education has increased over time, at least partly due to a stronger negative selection into the no-high-school group, see National Academies of Sciences and Medicine (2015) for further references.

| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| *No high school* | | | | | | | | |
| Average | 74.6 | 78.7 | 72.5 | 76.0 | 2.1 | 2.7 | −4.0 | −3.5 |
| | [73.9, 75.4] | [78.1, 79.2] | [71.1, 74.3] | [74.9, 77.1] | [0.2, 3.7] | [1.5, 3.9] | [−5.0, −3.0] | [−5.1, −1.6] |
| (1) Best health | 75.8 | 79.5 | 73.4 | 76.9 | 2.4 | 2.5 | −3.7 | −3.5 |
| | [75.2, 76.4] | [79.0, 80.0] | [72.2, 74.9] | [76.0, 77.9] | [0.8, 3.9] | [1.4, 3.7] | [−4.6, −2.8] | [−5.0, −2.0] |
| (3) Worst health | 73.7 | 77.9 | 71.6 | 75.4 | 2.0 | 2.5 | −4.3 | −3.8 |
| | [72.8, 74.6] | [77.3, 78.6] | [70.1, 73.6] | [74.4, 76.6] | [−0.2, 3.8] | [1.2, 3.9] | [−5.4, −3.0] | [−5.5, −1.9] |
| *High school* | | | | | | | | |
| Average | 77.8 | 82.6 | 75.4 | 79.5 | 2.4 | 3.1 | −4.8 | −4.0 |
| | [77.3, 78.4] | [82.1, 83.0] | [74.1, 76.9] | [78.2, 81.0] | [0.8, 3.9] | [1.5, 4.5] | [−5.4, −4.1] | [−6.3, −2.0] |
| (1) Best health | 78.6 | 83.2 | 76.5 | 80.8 | 2.1 | 2.4 | −4.6 | −4.3 |
| | [78.1, 79.0] | [82.7, 83.6] | [75.2, 77.8] | [79.6, 82.2] | [0.6, 3.6] | [0.9, 3.7] | [−5.2, −3.9] | [−6.4, −2.3] |
| (3) Worst health | 75.8 | 81.0 | 74.0 | 77.9 | 1.8 | 3.1 | −5.2 | −3.9 |
| | [75.2, 76.5] | [80.4, 81.6] | [72.5, 75.5] | [76.5, 79.6] | [0.1, 3.6] | [1.4, 4.6] | [−6.0, −4.2] | [−6.3, −1.7] |
| *College* | | | | | | | | |
| Average | 81.9 | 84.9 | 77.3 | 81.4 | 4.6 | 3.5 | −3.0 | −4.1 |
| | [81.1, 82.7] | [84.0, 85.8] | [74.2, 79.9] | [77.3, 83.9] | [2.0, 7.8] | [0.9, 7.8] | [−4.2, −1.7] | [−7.8, 1.0] |
| (1) Best health | 82.4 | 85.3 | 78.2 | 81.7 | 4.1 | 3.6 | −2.9 | −3.4 |
| | [81.6, 83.2] | [84.4, 86.2] | [76.1, 80.6] | [75.2, 84.4] | [1.7, 6.5] | [0.8, 9.9] | [−4.1, −1.7] | [−7.1, 3.9] |
| (3) Worst health | 77.9 | 82.7 | 74.2 | 80.6 | 3.7 | 2.1 | −4.7 | −6.4 |
| | [76.4, 79.5] | [81.0, 84.2] | [67.9, 78.2] | [78.4, 83.2] | [−1.2, 10.5] | [−1.0, 4.8] | [−6.8, −2.7] | [−13.9, −1.8] |
| **Age 70** | | | | | | | | |
| *No high school* | | | | | | | | |
| Average | 81.6 | 83.8 | 80.7 | 83.0 | 0.9 | 0.8 | −2.2 | −2.3 |
| | [81.3, 81.8] | [83.5, 84.1] | [80.1, 81.2] | [82.3, 83.7] | [0.2, 1.6] | [0.1, 1.6] | [−2.6, −1.8] | [−3.4, −1.2] |
| (1) Best health | 82.9 | 85.0 | 81.7 | 84.0 | 1.2 | 1.1 | −2.1 | −2.3 |
| | [82.6, 83.3] | [84.7, 85.4] | [81.0, 82.3] | [83.1, 84.8] | [0.4, 2.0] | [0.2, 2.0] | [−2.6, −1.7] | [−3.5, −1.0] |
| (3) Worst health | 80.4 | 82.9 | 79.7 | 82.4 | 0.6 | 0.5 | −2.5 | −2.6 |
| | [80.1, 80.6] | [82.5, 83.2] | [79.2, 80.3] | [81.8, 83.1] | [0.0, 1.2] | [−0.3, 1.2] | [−2.9, −2.1] | [−3.6, −1.6] |
| *High school* | | | | | | | | |
| Average | 83.0 | 86.0 | 82.2 | 85.6 | 0.8 | 0.3 | −2.9 | −3.5 |
| | [82.7, 83.3] | [85.7, 86.2] | [81.1, 83.4] | [84.7, 86.7] | [−0.4, 2.0] | [−0.8, 1.3] | [−3.3, −2.6] | [−5.1, −1.9] |
| (1) Best health | 84.3 | 87.0 | 83.3 | 86.8 | 1.0 | 0.2 | −2.7 | −3.5 |
| | [83.9, 84.6] | [86.7, 87.3] | [82.0, 84.7] | [85.8, 88.0] | [−0.4, 2.2] | [−1.0, 1.2] | [−3.2, −2.3] | [−5.4, −1.8] |
| (3) Worst health | 80.9 | 83.8 | 80.9 | 83.9 | 0.0 | −0.2 | −2.9 | −3.1 |
| | [80.6, 81.2] | [83.4, 84.1] | [79.8, 82.0] | [82.9, 85.1] | [−1.1, 1.1] | [−1.3, 0.9] | [−3.3, −2.5] | [−4.7, −1.6] |
| *College* | | | | | | | | |
| Average | 85.4 | 87.0 | 81.9 | 85.1 | 3.5 | 1.9 | −1.6 | −3.2 |
| | [84.9, 86.0] | [86.3, 87.8] | [79.6, 84.7] | [83.2, 88.0] | [0.7, 6.1] | [−0.9, 4.0] | [−2.7, −0.6] | [−6.4, −0.3] |
| (1) Best health | 86.4 | 87.9 | 83.2 | 86.9 | 3.2 | 1.0 | −1.5 | −3.7 |
| | [85.9, 87.1] | [87.2, 88.6] | [80.8, 86.0] | [84.8, 89.9] | [0.4, 5.9] | [−2.0, 3.2] | [−2.3, −0.5] | [−7.3, −0.5] |
| (3) Worst health | 81.8 | 84.1 | 79.5 | 82.6 | 2.3 | 1.5 | −2.2 | −3.0 |
| | [81.2, 82.5] | [83.1, 85.1] | [77.2, 82.5] | [80.5, 85.0] | [−0.7, 4.8] | [−1.2, 4.0] | [−3.5, −1.0] | [−6.0, 0.2] |

**Table 4:** Life expectancy by race, sex and education for model with three health states. Average life expectancy is computed as the weighted mean over health states at ages 50–51 (top) or 70–71 (bottom). Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Brackets indicate bootstrapped 95% confidence intervals.

during their prime working age.[23] To take into account life cycle effects, we then rank respondents' permanent income within the distribution of individuals of the same age, and we group these ranks into terciles. The tercile dummies are then included in (4) and (5), fully interacted with age and health.[24]

The motivation for creating a time-invariant permanent income measure is that we are unable to estimate transitions within the income distribution on top of age-specific health transitions for the black subsample due to the small sample size. With a larger sample one could estimate a joint transition matrix for health and income group (as is done in Pijoan-Mas and Ríos-Rull (2014) for whites). Our income definition is closer to the mid-career earnings measures used by Bosworth and Burke (2014) (who use age 41–50) and Waldron (2007) (who uses age 45–55).

In our analysis, we prefer household rather than individual income. The household level better captures the socioeconomic differences that have implications for health and mortality, especially for women: a housewife with no own income who is married to a high-earner is better classified as rich rather than poor.

The upper half of Table 5 shows the life expectancy for individuals at the age of 50 conditional on their income tercile. The rows labeled "Average" report the life expectancy for each income group computed as the weighted average over the health distribution observed in the HRS for that particular group, race, sex and age. The income gradient in life expectancy is large, and larger for black than for nonblack individuals. For a black male in the third tercile, the life expectancy is 8.7 years higher than for one in the first tercile, while the corresponding figure for nonblack males is 6.5 years. Using a different method, the National Academies of Sciences and Medicine (2015) documents a difference of 5.1 years between the upper and lower income quintile for males in the 1930 cohort, which is slightly lower than our period estimates for 1992 to 2014.

The bottom half of Table 5 shows the corresponding figures for individuals at the age of 70. The difference in life expectancy between the first and the third tercile for nonblack males is 2.5 years, higher than the 1.3 years Waldron (2007) estimates as the difference between first and fourth quartile for 70-year-olds based on data from 1999–2001. The difference in estimates is partly due to different income definitions and estimation methods, but also points to the increase in the life expectancy income gradient over time

[23] For the AHEAD and CODA cohorts the classification is based on retirement income at the age of 70 (see Figure 1), but given the high correlation between retirement income and earnings during working life this is not a major concern.

[24] See section A.2 in the appendix for a detailed description of how the permanent income measure is constructed. Table A.2 in the appendix reports the distribution of individuals and person-year observations by permanent income tercile and demographic subgroup.

(see National Academies of Sciences and Medicine 2015).

Table 5 additionally breaks down the differences by health. As with education, we collapse the data to three health states, and again the health gradient in life expectancy is present even after conditioning on income. The point estimates suggest that there is a racial gap even within health and income bins, however, the precision of these estimates varies considerably. The estimated differences are larger for the lower terciles, and for the better health groups: the largest differences at the age of 50 can be found among the poorest tercile in good health, where a nonblack man has a life expectancy that is 3.7 years higher than for a black man, and a nonblack woman can expected to live 4.4 more years compared to a black woman. The estimated differences at the age of 70 are smaller (and for some subgroups even reversed), but also less precise.

## 5 Economic implications

In this final section, we examine the implications of differences in health dynamics and survival for economic outcomes. We do this through the lens of a quantitative models, since this permits us to shut down any other differences between individuals across race and sex observed in the data, which would confound the analysis. Moreover, we are also able to quantify the welfare implications of the inequality in life expectancy.

We use the same framework as in Foltyn and Olsson (2021), which can be thought of as an overlapping-generations version of an Aiyagari (1994)-type economy and additionally features survival risk which varies by health, inelastic labor supply during working age, an exogenous retirement age, and a US-style Social Security system financed by payroll taxes. Households can choose to save in risk-free capital to insure themselves against income fluctuations as well as for consumption in old age (on top of retirement benefits). We do not describe the model in any technical detail here but instead refer interested reader to the exposition in Foltyn and Olsson (2021).

We use the health and survival processes estimated above and solve the household problem for all four demographic groups.[25] Using the model, we first examine the effects of life expectancy on savings and wealth accumulation. In a second step, we quantify the impact of life expectancy on Social Security wealth (the present value of retirement benefits a person is expected to receive) and uncover substantial differences between black and nonblack individuals which have sizable welfare implications.

[25] We compute the general equilibrium using the health and survival process for nonblack males and find the partial-equilibrium solution to the household problem for the remaining groups, taking as given the prices from the nonblack/male economy. In Foltyn and Olsson (2021), we show that aggregate prices are not very sensitive to assumptions about life expectancy, so we view this as an acceptable shortcut.

| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| *1st tercile* | | | | | | | | |
| Average | 75.0 | 79.8 | 71.2 | 75.1 | 3.8 | 4.7 | −4.8 | −3.9 |
| | [74.2, 75.8] | [79.3, 80.2] | [69.8, 72.7] | [74.0, 76.4] | [1.9, 5.5] | [3.4, 5.8] | [−5.6, −4.0] | [−5.8, −2.0] |
| (1) Best health | 76.2 | 80.8 | 72.5 | 76.4 | 3.7 | 4.4 | −4.6 | −3.9 |
| | [75.4, 76.9] | [80.4, 81.3] | [71.3, 73.8] | [75.2, 77.7] | [2.1, 5.1] | [3.0, 5.6] | [−5.5, −3.8] | [−5.8, −2.0] |
| (3) Worst health | 73.5 | 78.5 | 70.2 | 74.2 | 3.3 | 4.3 | −5.0 | −4.0 |
| | [72.6, 74.4] | [78.0, 79.0] | [68.7, 72.0] | [73.1, 75.5] | [1.2, 5.2] | [3.0, 5.4] | [−6.0, −4.0] | [−5.9, −1.9] |
| *2nd tercile* | | | | | | | | |
| Average | 78.1 | 82.6 | 76.0 | 81.6 | 2.1 | 1.0 | −4.5 | −5.6 |
| | [77.5, 78.7] | [82.0, 83.1] | [75.0, 77.0] | [80.1, 83.2] | [0.9, 3.3] | [−0.7, 2.7] | [−5.2, −3.6] | [−7.8, −3.6] |
| (1) Best health | 78.7 | 83.0 | 76.5 | 82.3 | 2.3 | 0.7 | −4.3 | −5.8 |
| | [78.2, 79.3] | [82.5, 83.5] | [75.4, 77.5] | [80.9, 83.8] | [1.1, 3.5] | [−0.8, 2.2] | [−4.9, −3.5] | [−7.7, −4.0] |
| (3) Worst health | 76.1 | 81.3 | 74.7 | 80.5 | 1.4 | 0.8 | −5.1 | −5.7 |
| | [75.2, 77.1] | [80.6, 81.9] | [73.4, 75.9] | [78.7, 82.3] | [−0.2, 3.0] | [−1.2, 2.6] | [−6.3, −3.9] | [−8.1, −3.4] |
| *3rd tercile* | | | | | | | | |
| Average | 81.4 | 85.4 | 79.9 | 83.2 | 1.5 | 2.2 | −4.0 | −3.3 |
| | [80.9, 82.0] | [84.8, 86.1] | [78.0, 81.8] | [81.0, 85.2] | [−0.5, 3.7] | [0.1, 4.4] | [−4.8, −3.2] | [−5.7, −0.7] |
| (1) Best health | 81.8 | 85.7 | 80.2 | 83.3 | 1.5 | 2.4 | −3.9 | −3.1 |
| | [81.2, 82.3] | [85.1, 86.4] | [78.3, 82.1] | [80.6, 85.3] | [−0.5, 3.6] | [0.4, 5.0] | [−4.7, −3.2] | [−5.5, 0.0] |
| (3) Worst health | 78.9 | 83.6 | 78.7 | 82.7 | 0.2 | 0.9 | −4.7 | −4.0 |
| | [77.8, 79.9] | [82.6, 84.6] | [75.9, 81.0] | [80.4, 85.2] | [−2.5, 3.3] | [−1.6, 3.3] | [−6.0, −3.4] | [−7.3, −1.0] |
| **Age 70** | | | | | | | | |
| *1st tercile* | | | | | | | | |
| Average | 81.9 | 84.3 | 79.9 | 82.7 | 2.0 | 1.6 | −2.4 | −2.8 |
| | [81.5, 82.3] | [84.1, 84.7] | [79.4, 80.6] | [82.0, 83.7] | [1.2, 2.6] | [0.7, 2.5] | [−2.9, −2.0] | [−4.0, −1.7] |
| (1) Best health | 83.4 | 85.8 | 81.2 | 83.9 | 2.2 | 1.9 | −2.3 | −2.6 |
| | [83.0, 83.9] | [85.5, 86.1] | [80.6, 82.0] | [82.9, 85.0] | [1.3, 2.9] | [0.8, 3.0] | [−2.8, −1.8] | [−4.0, −1.3] |
| (3) Worst health | 80.4 | 82.9 | 79.1 | 82.0 | 1.3 | 0.9 | −2.6 | −3.0 |
| | [80.1, 80.7] | [82.6, 83.3] | [78.6, 79.6] | [81.3, 82.9] | [0.7, 1.8] | [−0.0, 1.8] | [−3.0, −2.2] | [−4.0, −1.8] |
| *2nd tercile* | | | | | | | | |
| Average | 83.1 | 85.8 | 81.3 | 85.8 | 1.8 | −0.0 | −2.7 | −4.5 |
| | [82.8, 83.4] | [85.5, 86.1] | [80.4, 82.1] | [84.9, 86.7] | [0.9, 2.7] | [−1.0, 1.0] | [−3.1, −2.3] | [−5.9, −3.3] |
| (1) Best health | 84.3 | 86.7 | 82.1 | 86.9 | 2.2 | −0.1 | −2.5 | −4.7 |
| | [84.0, 84.6] | [86.4, 87.0] | [81.1, 83.1] | [85.9, 87.8] | [1.2, 3.2] | [−1.1, 0.9] | [−2.8, −2.0] | [−6.2, −3.4] |
| (3) Worst health | 81.0 | 83.9 | 80.1 | 84.3 | 0.9 | −0.4 | −2.8 | −4.2 |
| | [80.7, 81.3] | [83.5, 84.3] | [79.4, 80.9] | [83.4, 85.3] | [0.1, 1.7] | [−1.5, 0.6] | [−3.3, −2.4] | [−5.5, −3.0] |
| *3rd tercile* | | | | | | | | |
| Average | 84.4 | 87.4 | 84.3 | 86.5 | 0.1 | 0.9 | −2.9 | −2.2 |
| | [84.1, 84.8] | [86.9, 87.9] | [82.6, 86.1] | [85.1, 88.4] | [−1.7, 2.0] | [−0.9, 2.3] | [−3.6, −2.3] | [−4.5, −0.1] |
| (1) Best health | 85.4 | 88.1 | 85.2 | 87.4 | 0.2 | 0.7 | −2.7 | −2.2 |
| | [85.0, 85.8] | [87.7, 88.6] | [83.3, 87.1] | [86.1, 89.2] | [−1.8, 2.2] | [−1.1, 2.0] | [−3.3, −2.1] | [−4.5, −0.1] |
| (3) Worst health | 81.5 | 84.5 | 83.0 | 84.7 | −1.5 | −0.2 | −3.0 | −1.7 |
| | [81.1, 82.0] | [83.9, 85.1] | [81.2, 84.7] | [82.8, 86.9] | [−3.3, 0.5] | [−2.4, 1.7] | [−3.7, −2.2] | [−4.1, 0.5] |

**Table 5:** Life expectancy by race, sex and permanent income tercile for model with three health states. Average life expectancy is computed as the weighted mean over health states at ages 50–51 (top) or 70–71 (bottom). Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Brackets indicate bootstrapped 95% confidence intervals.

### 5.1 Savings rates and wealth accumulation

The racial wealth gap is large, much larger than the income gap. How much of it can be explained by income differences and demographic variables is debated, see, e.g., Barsky et al. (2002) and Altonji and Doraszelski (2005). One factor that has been identified to contribute to the wealth gap is the difference in savings rates between black and white households. As suggested already by Smith (1995), life expectancy could play a role in this: lower life expectancy for blacks should reduce savings incentives.

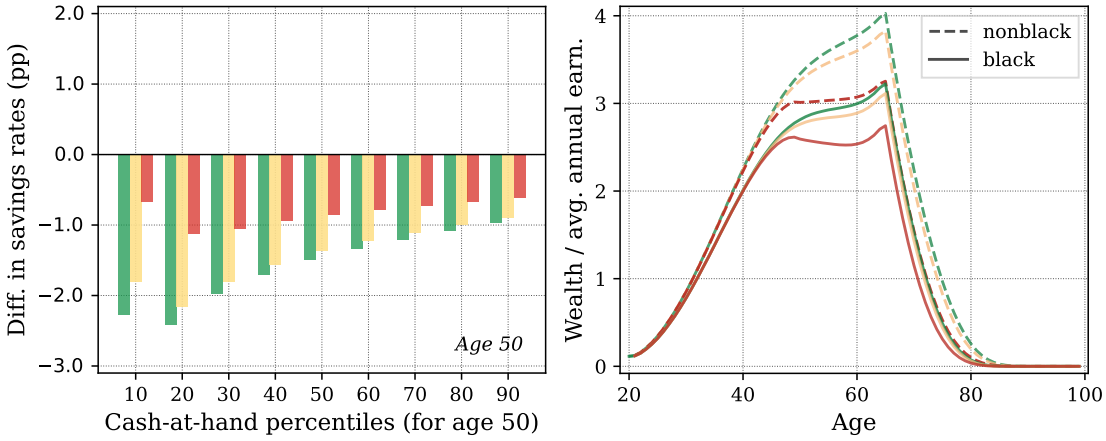
We use our model to evaluate this hypothesis quantitatively. To focus on the importance of life expectancy in isolation, we let the black and the nonblack agents in our model be identical in every respect except for their health and survival prospects.

Figure 9(a) shows the differences in total savings rates for black and nonblack men, respectively. We define the total savings rate as the fraction of current cash-at-hand an agent saves for the next period, where cash-at-hand is the sum of beginning-of-period wealth and income (both financial and non-financial) received in the period.[26] As the figure shows, a black man in excellent health at the age of 50 who is at the median of the cash-at-hand distribution has a 1.5 percentage points lower savings rate than a nonblack man with the same health, age and wealth. The reason for this behavior is straightforward: a nonblack man in excellent health expects to live another 29.5 years, while the corresponding black man can only expect another 26.1 – a difference of 3.4 years (as can be seen in Table 2). The difference in savings rates for 50-year-olds in poor health is smaller since the difference in remaining life time is smaller: 23.4 years (for nonblack) vs. 21.8 years (for black) – a difference of "only" 1.7 years. Another way to understand these differences is through the lens of discount rates. In the appendix section F.1, we show that the effective discount rates for black men are generally lower than those of nonblack men. Because periods farther into the future are more heavily discounted, the black population has lower incentives to save for old age.

To gauge the quantitative implication of these differences in savings rates, we simulate the model over the life cycle and compare the wealth accumulation of black and nonblack individuals. As Figure 9(b) shows, the differences in life expectancy contribute to differences in wealth accumulation. Just prior to retirement (which occurs at age 65), nonblack individuals in excellent health have on average accumulated 25% more wealth than their black counterparts.

However, a difference of 25% is a far cry from the actual wealth gap between white and black documented for the US. For example, Blau and Graham (1990) estimate the

[26] In this class of models, cash-at-hand, not wealth, is the state variable that is relevant for household optimization.




**(a)** Savings rates

**(b)** Lifecycle profile of wealth

**Figure 9:** Effect of health and survival on savings behavior and wealth accumulation. The left panel shows the difference in savings rates between a black and a nonblack man in percentage points. The right panel reports the wealth accumulation profiles over the life cycle. The best (green), middle (yellow) and worst (red) health states are shown.

average wealth by white households to exceed that of black households by a factor of 5.5 for young families in the 1970s, while Altonji and Doraszelski (2005) estimate a factor of around 4 using PSID data, and Derenoncourt et al. (2021) report a factor of 6. Our results therefore corroborate the conjecture in Altonji and Doraszelski (2005), who "doubt that it [the difference in life expectancy] plays a major role" in explaining the racial wealth gap.

## 5.2 Social security wealth

The US Social Security system not only redistributes from high- to low-income earners due to its regressive replacement rates, but also from individuals with short life spans to those with long life expectancy who continue receiving benefits for more years. The interplay between those two channels has been extensively evaluated (see, e.g., National Academies of Sciences and Medicine (2015), Auerbach et al. (2017), Sánchez-Romero and Prskawetz (2017), Sanchez-Romero, Lee, and Prskawetz (2020), Haan, Kemptner, and Lüthen (2020), and the references therein), and some papers have looked at the average outcomes for different racial groups (Liu and Rettenmaier 2003).

In this section, we proceed in two steps: we first report the differences in the present value of expected Social Security benefits ("Social Security wealth") between black and nonblack groups. In a second step, we ask how much black individuals would value this difference in consumption terms if they received it as a one-time lump-sum payment when they retire.

The Social Security system in our model closely mimics the one in the US, in particular, it uses the same bend-point formula to produce a regressive replacement rate and the same maximum amount subject to payroll taxes. In this sense the model results should be informative about differences observed in the real world.

Intuitively, two individuals with identical life cycle profiles of labor earnings would have paid the same amount of Social Security contributions and should thus be entitled to the same Social Security benefits once retired. However, if these individuals have different life expectancies, the present value of their expected Social Security wealth will in fact be different. To formalize our comparison between black and nonblack individuals, assume that retirement happens exogenously at age $a_R = 65$, and once retired, a person is entitled to annual Social Security benefits $y_R$ which are constant for the remaining life. The present value of expected Social Security wealth at retirement is then defined as

$$W_{a_R}(m, b, h_{a_R}, y_R) = \sum_{\tilde{a}=a_R}^{a_{\max}} \frac{1}{(1+r)^{\tilde{a}-a_R}} \Pr(s_{\tilde{a}} = 1 \mid m, b, h, a_R)\, y_R \tag{6}$$

where $m$ and $b$ are indicators for male and black, $h$ is the initial health state and $\Pr(s_{\tilde{a}} = 1 \mid m, b, h, a)$ is the probability of being alive at age $\tilde{a} \geq a_R$ given the initial state $(m, b, h, a_R)$. Individuals are assumed to survive to a maximum age $a_{\max}$ which we set to 99. Future cash flows are discounted using a fixed interest rate $r$. For any age prior to retirement, our model features persistent income risk, so the present value of Social Security wealth at some age $a < a_R$ is the discounted expected value of (6),

$$W_a(m, b, h_a, y_a) = \frac{1}{(1+r)^{a_R-a}} \Pr(s_{a_R} = 1 \mid m, b, h_a, a)\, \mathbf{E}\left[ W_{a_R}(m, b, h_{a_R}, y_R) \,\middle|\, h_a, y_a, a \right]$$

where expectations are taken over income and health at the time of retirement, and survival until retirement is uncertain.

We use the above formulas to quantify differences in Social Security wealth between the black and nonblack groups conditioning on sex, health and age. Specifically, we compute the relative difference $\Delta$ as

$$\Delta_a(m, h) = \frac{W_a(m, b = 1, h, y)}{W_a(m, b = 0, h, y)} - 1 \tag{7}$$

for ages $a \in \{50, 65, 70\}$, males and females, and the best, middle and worst health states. Since benefits are constant in retirement and labor income $y$ is uncorrelated with

health, sex or race in the model, $y$ cancels out in (7).[27] Figure 10 plots the results for three discount rates used to compute the present value: a low interest rate scenario with $r = 1\%$, a high interest rate scenario with $r = 4\%$ and the equilibrium interest rate arising in the model if we assume that all individuals have the health and survival process of nonblack males, $r = 2.4\%$.

As the figure shows, the differences in expected Social Security wealth are substantial. At the age of 50, a black man in excellent health has an expected Social Security wealth which is 17% lower than that of a nonblack man in excellent health. This again is a consequence of the life expectancy gap of 3.4 years for this group reported in Table 2.

By the age of 65, the life expectancy gap across races shrinks, which is reflected in the smaller differences in Social Security wealth. Moreover, for men in poor health this gap is negligible, as indicated by the very small difference in Social Security wealth.[28] The picture remains overall unchanged when comparing black to nonblack women as shown in the bottom panel of Figure 10.

Lastly, in Table 6 we report the relative differences in Social Security wealth averaged over health states using the empirical health distributions for black individuals observed in the HRS at each age (note that these statistics show the expected differences, not the difference of expected values between black and nonblack groups). Thus, the table quantifies how much the black population on average loses out in Social Security wealth given their lower life expectancy, holding everything else constant. At the age of retirement, this loss amounts to 7–8% for both males and females, and is even double that at age 50.[29]

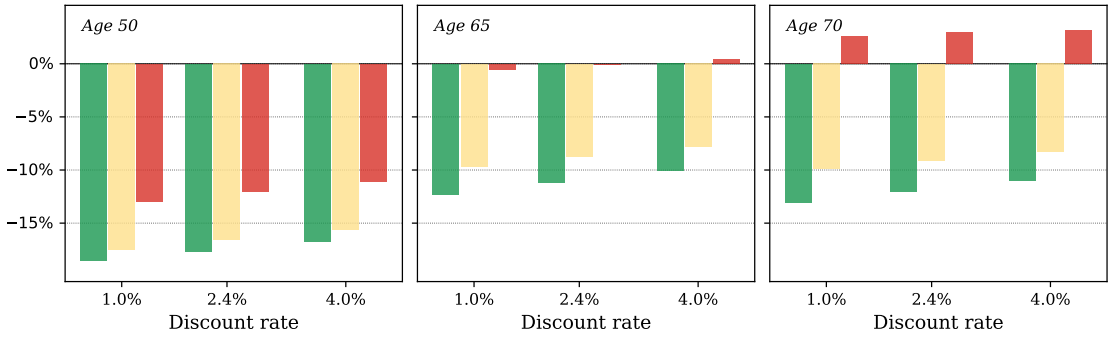
In the remainder of this section, we assess the welfare implications of these substantial differences in Social Security wealth. To this end, we compute the increase in consumption that is required in every period to make a black individual equally well off as giving him or her the difference in Social Security wealth as a one-time lump-sum payment at the time of retirement. Specifically, denote by

$$\Delta^W_{a_R}(m,h,y) = W_{a_R}(m,b=0,h,y) - W_{a_R}(m,b=1,h,y)$$

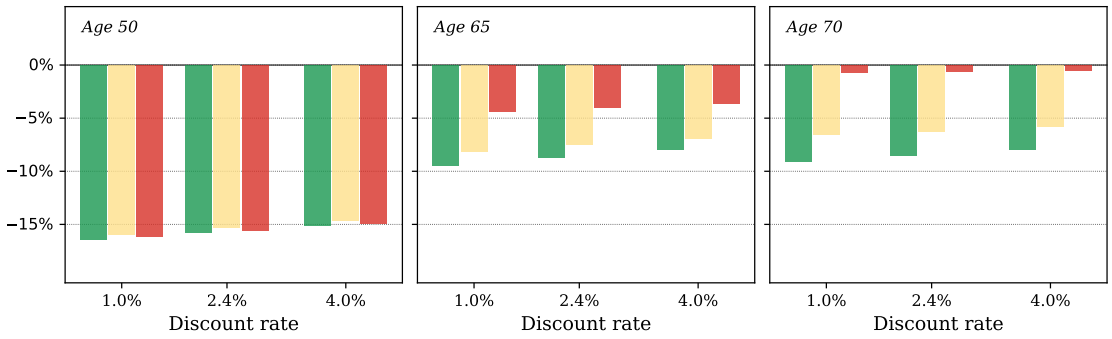
[27] Of course the *level* of Social Security wealth depends on income and retirement benefits. We plot Social Security wealth in levels in the appendix in Figure A.19 and Figure A.20 for nonblack and black men, respectively.

[28] To be precise, it is not only the life expectancy but the whole time path of survival probabilities that matters for these calculations. This is the reason for the difference being slightly below 0 for the low interest rate and slightly above zero for the high interest rate when comparing 65-year-old males in poor health.

[29] We include only the middle interest rate in the table since the magnitudes are similar across all three scenarios.




**(a)** Black vs. nonblack men




**(b)** Black vs. nonblack women

**Figure 10:** Relative difference in present discounted value of expected Social Security wealth between black and nonblack males.

the absolute difference at the onset of retirement for two individuals who are identical except for their race, where the definition of Social Security wealth is the same as in (6). Let

$$V_{a_R}(x_{a_R}, m, b, h_{a_R}, y_R) = \mathbf{E}\left[\sum_{\tilde{a}=a_R}^{a_{\max}} \beta^{\tilde{a}-a_R} u\Big(C\left(x_{\tilde{a}}, m, b, h_{\tilde{a}}, y_R\right)\Big) \,\middle|\, m, b, h_{a_R}, a_R\right]$$

be an individual's value function which is defined on cash-at-hand $x$ as an additional state, and denote by $C(\bullet)$ the consumption policy function which characterizes optimal consumption. We are interested in finding the value $\Delta_c$ which represents a permanent relative increase in consumption such that

$$V_{a_R}\left(x_{a_R} + \Delta^W_{a_R}(m, b, h_{a_R}), m, b, h_{a_R}, y_R\right) = \mathbf{E}\left[\sum_{\tilde{a}=a_R}^{a_{\max}} \beta^{\tilde{a}-a} u\Big(C\left(x_{\tilde{a}}, m, b, h_{\tilde{a}}, y_R\right) \cdot (1+\Delta_c)\Big) \,\middle|\, m, b, h_{a_R}, a_R\right]$$

| | Male | Female |
|---|---|---|
| Age 50 | −16.3% | −15.6% |
| Age 65 | −7.7% | −7.2% |
| Age 70 | −7.5% | −5.6% |

**Table 6:** Model-predicted loss in Social Security wealth due to life expectancy differences of black subpopulation relative to the nonblack group of the same sex. The loss is averaged over the health distribution at each age as observed in the HRS. Interest rate is set to 2.4% in the calculation.

In words, $\Delta_c$ represents the permanent relative increase in consumption required to give an individual the same utility as the one-time lump sum payment $\Delta^W_{a_R}$. Our calibration uses log preferences which makes computing $\Delta_c$ particularly easy as it can be separated from the expected utility term:

$$\begin{aligned} &V_{a_R}\left(x_{a_R}+\Delta^W_{a_R}(m,b,h_{a_R}),m,b,h_{a_R},y_R\right) \\ &\quad=\mathbf{E}\left[\sum_{\tilde{a}=a_R}^{a_{\max}}\beta^{\tilde{a}-a}\Big(\log C\left(x_{\tilde{a}},m,b,h_{\tilde{a}},y_R\right)+\log(1+\Delta_c)\Big)\,\middle|\, m,b,h_{a_R},a_R\right] \\ &\quad=V_{a_R}\left(x_{a_R},m,b,h_{a_R},y_R\right)+\mathbf{E}\left[\sum_{\tilde{a}=a_R}^{a_{\max}}\beta^{\tilde{a}-a}\log(1+\Delta_c)\,\middle|\, m,b,h_{a_R},a_R\right] \end{aligned}$$

The expectation in the last term is taken only with respect to survival, and therefore

$$\mathbf{E}\left[\sum_{\tilde{a}=a_R}^{a_{\max}}\beta^{\tilde{a}-a}\log(1+\Delta_c)\,\middle|\, m,b,h_{a_R},a_R\right]=\log(1+\Delta_c)\sum_{\tilde{a}=a}^{a_{\max}}\beta^{\tilde{a}-a}\Pr\left(s_{\tilde{a}}=1\,|\,m,b,h_{a_R},a_R\right)$$

Consequently, with log preferences we can recover $\Delta_c$ by interpolating the individual's value function at $x$ and $x+\Delta^W$ and rescaling the difference by the sum of discounted survival probabilities.

Figure 11 plots the resulting consumption equivalent variation for the age of 65. As the figure shows, the resulting welfare differences are large for most combinations of health and financial resources.[30] For example, a black individual with median cash-at-hand in excellent health on average perceives the lump-sum transfer of unrealized Social Security wealth as being equivalent to a permanent increase in consumption by 6.5% during his remaining life time. Again, the only exception are black men in poor health who can expect to receive almost the same Social Security wealth as their nonblack peers, and hence their CEVs are almost exactly zero. Overall, the average CEV is 4.2% for black

[30] Any differences in income are averaged out in these graphs. See Figure A.21 in the appendix for CEVs disaggregated by income level.

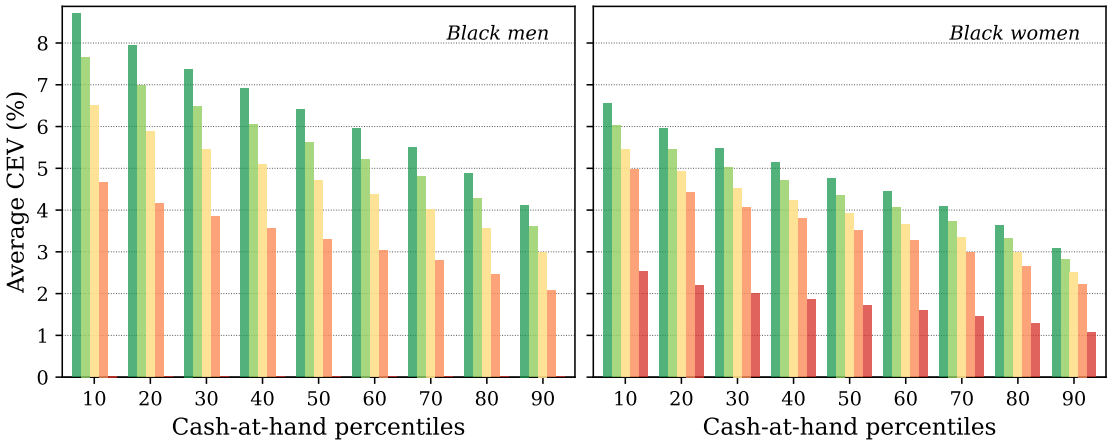



**Figure 11:** CEV implied by the difference in Social Security wealth between black and otherwise identical nonblack individuals at the age of 65. CEV values are averaged over labor productivity. The worst health state for black men is not visible because the CEV is almost zero.

men and 3.7% for black women (using their respective equilibrium distributions over health, labor productivity and cash-at-hand at the age of 65) .

# 6 Conclusion

Health dynamics and uncertain survival are major risks facing individuals. To incorporate these risks in life cycle models, a health and survival process that captures the main features of the data while being sufficiently parsimonious is required. In this paper, we provide such estimates for annual age-dependent health transitions and survival probabilities for different demographic groups of the US population.

These health and survival probabilities can be used to compute life expectancies by race, sex and socioeconomic status. The race gap in life expectancy is well known and large. We show that even conditioning on health and different measures for socioeconomic status, this gap persists. Moreover, we are able to disentangle the importance of initial health, say at age 50, and the health and survival dynamics individuals face beyond that age. We document that the latter explain about 90% of the racial gap in life expectancy at age 50, while the initial health distribution plays a minor role.

The racial life expectancy gap has substantial welfare implications even beyond the mechanical effect of being able to enjoy a few additional years of life. We illustrate this by showing that mortality alone creates disparities in Social Security wealth of approximately 15% on average at the age of 50. Using model simulations, we show

that this gap is substantial in welfare terms: at the age of retirement, it is on average equivalent to a permanent increase in consumption of about 4% for black men, and slightly lower for black women.

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# Health Dynamics and Heterogeneous Life Expectancies

**Online appendix**

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# Table of contents

# A Detailed data description

## A.1 HRS waves used for the analysis

The RAND HRS Longitudinal File 2018 (V1) consists of 14 waves administered over the years 1992–2018. The first cohort included in the survey was between 51 and 61 years old in 1992, and thereafter new (older and younger) cohorts have been added, as illustrated in Figure 1 in the main text. Figure A.1 shows the number of respondents with positive sampling weights by wave and cohort. At the time of this writing, the sampling weights for wave 14 are not yet publicly available.

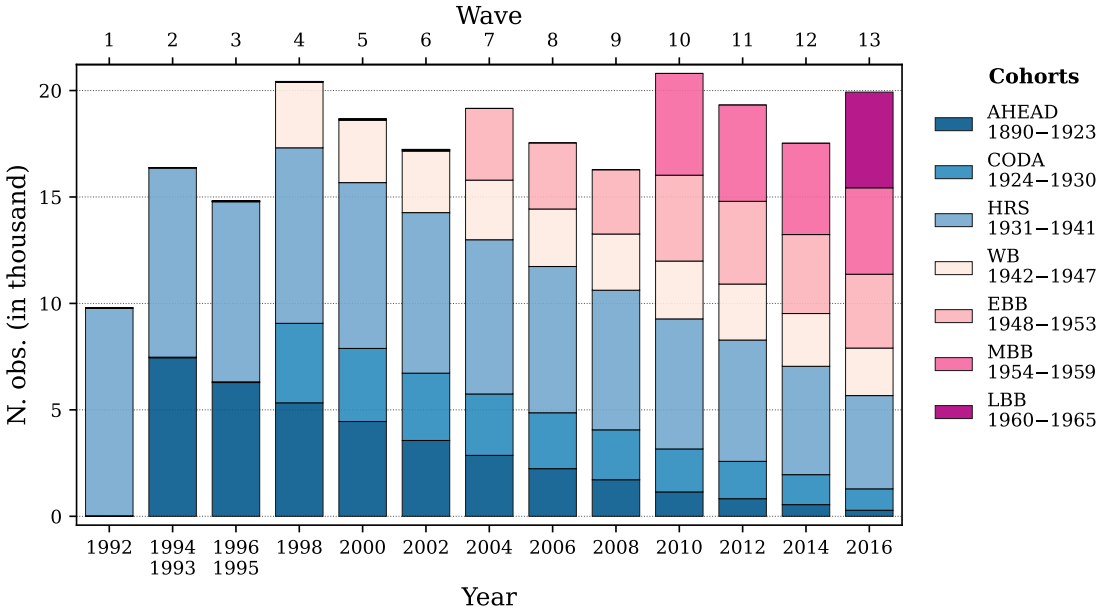



**Figure A.1:** Number of observations by wave and cohort. Only observations with positive weight are included. Sampling weights for wave 14 are not available in RAND HRS 2018 (V1). AHEAD was initially a separate survey conducted in 1993 and 1995.

Figure A.2 shows the fraction of respondents in each wave who are marked as non-respondents in all subsequent waves but do not have a death date on record. For example, in wave 11 (administered in 2012), approximately four percent of participants did not respond to any of the later waves 12–14. Since no death date is recorded for these individuals, we cannot use these observations to estimate survival probabilities. However, since death dates are sometimes recorded with considerable lag, we suspect that some of these individuals are already deceased, but their death dates will be updated only in future waves. So as to not bias our survival probability estimates, we opt to drop the last two waves from the estimation sample, since these exhibit unusually high

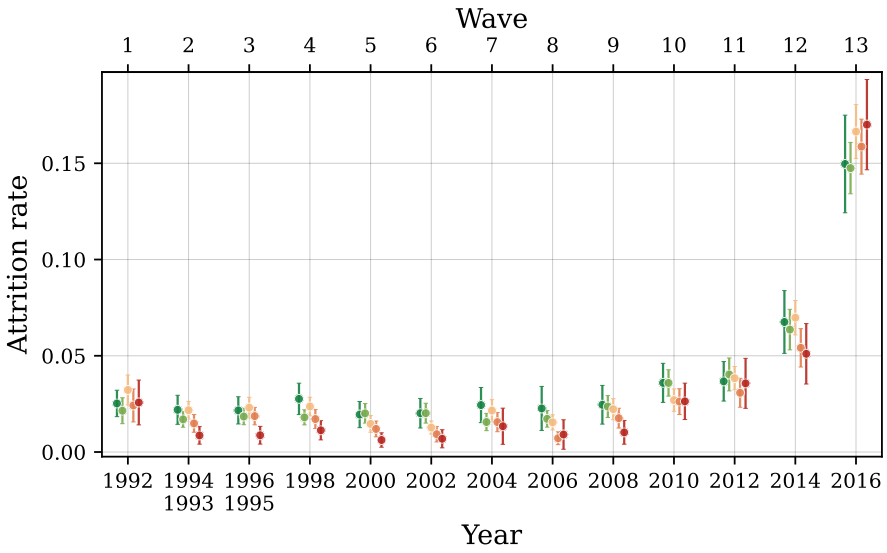



**Figure A.2:** HRS attrition by wave: fraction of participants who do not respond to any of the subsequent survey waves. Dark green indicates best ("excellent") while red indicates worst ("poor") health. Error bars indicate bootstrapped 95% confidence intervals.

nonresponse rates compared to the historical averages.

## A.2 Description of main variables

We list the central variables used in our analysis in Table A.1. Some of them are computed from several variables in the RAND HRS version, a process we detail below.

**Race.** The variable `RARACEM` takes on three values: *White/Caucasian*, *Black/African-American* or *Other*. For our purposes, we combine the first and third groups to obtain the indicator variable $black \in \{0, 1\}$.

**Education.** The HRS variable `RAEDUC` records five categories: *less than high school*, *GED*, *high school graduate*, *some college* and *college and above*. We combine the three middle groups into one and use the classification *less than high school*, *high school* and *college*.

Table A.2 shows the distribution of individuals and person-year observations by education. As can be seen, only 10.9% of the black males and 12.6% of the black females have a college degree in our sample, while the corresponding figures for the nonblack population is 23.5% (males) and 16.2% (females).

| Variable | Description | RAND HRS variable(s) |
|---|---|---|
| Health | Self-reported health with possible answers (1) excellent, (2) very good, (3) good, (4) fair, or (4) poor | RwSHLT |
| Age | Respondent's age at time of interview end date | RwAGEY_E |
| Birth date | Respondent's date of birth (year, month) | RABYEAR, RABMONTH |
| Death date | Respondent's date of death, if applicable (year, month) | RADYEAR, RADMONTH |
| Race | Respondent's race, recoded to *black* indicator variable | RARACEM |
| Sex | Respondent's sex, recoded to *male* indicator variable | RAGENDER |
| Interview date | Interview end date (year, month) | RwIWENDY, RwIWENDM |
| Education | Educational attainment, recoded to three groups | RAEDUC |
| Income | Respondent's income computed as the sum of several non-financial income and transfer variables | RwIEARN, RwIUNWC, RwIPENA, RwISRET, RwISSDI, RwIGXFR |
| Weight | Respondent-level sampling weights | RwWTREP, RwWTR_NH, RwWTCRNH |
| Stratum ID | Stratum identifier used for stratified cluster bootstrapping of confidence intervals | RAESTRAT |
| PSU ID | Primary stage unit (cluster) identifier used for stratified cluster bootstrapping of confidence intervals | RAEHSAMP |

**Table A.1:** Variables from RAND HRS version 2018v1 used in the analysis

**Income.** Following Pijoan-Mas and Ríos-Rull (2014), we compute a respondent's non-financial income as the sum of several variables: earnings (RwIEARN), unemployment benefits and worker's compensation (RwIUNWC), pension and annuity income (RwIPENA), Social Security retirement income (RwISRET), income from Social Security disability (SDI) and Supplemental Security income (SSI) (RwISSDI), and any other government transfers (RwIGXFR). All variables are deflated to year 2000 dollars using the CPI.

In the analysis, we assign households into household-level permanent income terciles which are computed as follows:

1. We aggregate the individual incomes defined above to the household level and assign each respondent the corresponding share of this pooled income (i.e., for couple household each individual gets one half of the household income). The motivation behind this is intra-household risk sharing: to the extent that health outcomes depend on financial resources, the relevant measure of such resources is at the household level.

2. To obtain a proxy for permanent income, we then compute the average income over a window of 3 waves (approximately six years).

3. Using this income measure, we find a respondent's position in the income distri-

bution of a reference population defined in terms of age in order to control for the life cycle effects. For example, for a 50-year-old respondent, "permanent" income is computed as the average income reported at ages 50–55 (three waves), and the individual's income rank is determined by comparing his or her income against the permanent income of other 50-year-olds.

4. Since we do not want to model transitions between income terciles, we assign each respondent a *fixed* income tercile based on the first tercile computed with the above method. For example, a 50-year-old individual from the HRS cohort who enters the survey in 1992 will be assigned the income tercile we computed at age 50. A 75-year-old who enters the survey as part of the AHEAD cohort in 1994 will be assigned the income tercile we observed for this person at age 75.

Table A.2 reports the fraction of individuals and person-year observations in each income bin. Note that the above procedure does not guarantee that individuals are evenly spread across terciles, mainly because the table shows the unweighted shares, but also because the reference population is age-dependent and thus different across individuals. A larger fraction of the nonblack population are classified as belonging to third tercile.

**Sampling weights.** We use the HRS respondent-level sampling weights. These weights are time varying, so for each transition we use a respondent's weight at transition start. Prior to wave 5, the HRS did not provide weights for individuals who moved to nursing homes, in which case the variable `RwWTRESP` was zero. From wave 5 onward, nursing home weights are provided in `RwWTR_NR`, and the combined weight is stored in `RwWTCRNH` (only one of `RwWTRESP` or `RwWTR_NR` is non-zero, depending on the nursing home status). For individuals who moved to nursing homes prior to wave 5, we back-fill their nursing-home weight from wave 5 to earlier waves if they were alive in wave 5. For respondents who moved to nursing homes prior to wave 5 but died before wave 5 and thus were never assigned a nursing-home weight, we forward-fill any missing weights using the last non-zero value of `RwWTRESP` as long as the respondent is alive.

**Transition length.** For each transition, we compute the transition length as the difference between two consecutive interview dates (if the respondent is alive at transition end), or the difference between the last interview date and the date of death. We round the transition length to the nearest full year. The resulting distribution of transition lengths is shown in Table A.3.

| | All | Nonblack | | Black | |
|---|---|---|---|---|---|
| | | Male | Female | Male | Female |
| **Distribution of individuals** | | | | | |
| *By education* | | | | | |
| No high school | 27.2% | 24.6% | 25.5% | 39.0% | 36.1% |
| High school | 54.6% | 51.9% | 58.3% | 50.1% | 51.3% |
| College | 18.2% | 23.5% | 16.2% | 10.9% | 12.6% |
| *By income tercile* | | | | | |
| 1st tercile | 39.9% | 33.7% | 40.8% | 45.6% | 54.0% |
| 2nd tercile | 35.0% | 35.7% | 35.8% | 33.2% | 30.6% |
| 3rd tercile | 25.1% | 30.5% | 23.5% | 21.2% | 15.4% |
| **Distribution of observations** | | | | | |
| *By education* | | | | | |
| No high school | 25.9% | 22.7% | 24.0% | 41.8% | 37.8% |
| High school | 55.6% | 52.5% | 60.0% | 48.0% | 50.3% |
| College | 18.5% | 24.8% | 16.0% | 10.1% | 12.0% |
| *By income tercile* | | | | | |
| 1st tercile | 37.2% | 30.7% | 38.7% | 42.2% | 51.8% |
| 2nd tercile | 36.6% | 37.0% | 37.3% | 35.2% | 32.5% |
| 3rd tercile | 26.2% | 32.3% | 24.1% | 22.6% | 15.8% |

**Table A.2:** Distribution of individuals and person-year observations by education and permanent income tercile in the estimation sample (unweighted).

# B Details on the maximum-likelihood estimator

In this section, we describe the MLE approach that jointly estimates the parameters governing both health-to-health and survival transitions.

## B.1 An illustrative example

Before deriving the probabilities that enter the log-likelihood function, it is worthwhile to work through an illustrative example. We consider a simplified setup with only two health states and assume a two-year transition, as illustrated in Figure A.3.

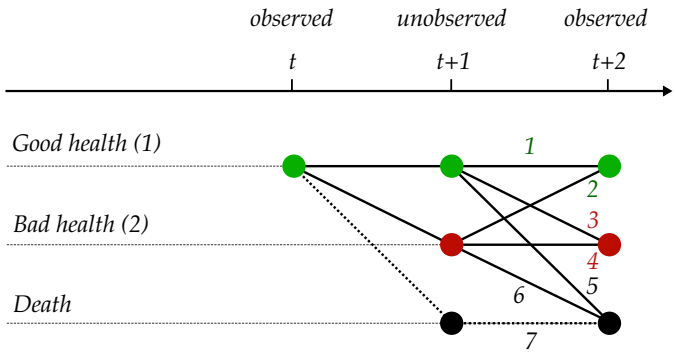



**Figure A.3:** Simplified transition paths

At $t+2$ there are three possible outcomes, but seven distinct paths via which these outcomes can be realized. The probability distribution that should enter the likelihood function is one over paths, not outcomes. To make this point, first consider a PMF over outcomes in $t+2$, which is given by the three probabilities

$$\begin{aligned}
&\Pr\left(h_{t+2}=1 \,|\, h_t\right)\\
&\Pr\left(h_{t+2}=2 \,|\, h_t\right)\\
&\Pr\left(s_{t+2}=0 \,|\, h_t\right)
\end{aligned}$$

where $h_{t+2}$ is the future health state, and $s_{t+2}$ is an indicator which is one if the individual is alive at $t+2$. For either health state $j=1,2$ these can be computed as follows:

$$\Pr\left(h_{t+2}=j \,|\, h_t\right)=\sum_{\ell=1}^{2}\Pr\left(h_{t+2}=j \,|\, h_{t+1}=\ell, h_t\right)\Pr\left(h_{t+1}=\ell \,|\, h_t\right)$$

On the other hand, the probability of observing death in $t+2$ can be written as

$$\Pr(s_{t+2}=0 \mid h_t) = \sum_{\ell=1}^{2} \Pr(s_{t+2} \mid h_{t+1}=\ell, h_t) \Pr(h_{t+1}=\ell \mid h_t) + \Pr(s_{t+1}=0 \mid h_t)$$

The issue with this formulation is that whenever death in $t+2$ is observed, the probability of this outcome includes the case that the individual already died in $t+1$, which corresponds to path *7* in Figure A.3. However, due to how the transition data is constructed this is impossible, as a one-period observation would have been recorded if an individual had already died in $t+1$ (the date of death is recorded independently of the wave structure in the HRS). Hence, the probability associated with path *7* should never enter the likelihood function. This issue becomes even more pronounced for longer transitions, since the probability of ending up in the absorbing death state in the penultimate period is strictly increasing in the transition length.

To properly address this issue, the distribution that enters the likelihood needs to be over paths, not outcomes at transition end. Naturally, in the above example we do not know whether path *1* or *2* was realized when we observe the outcome $h_{t+2}=1$, so the probabilities of both will have to be included in that case, and analogously for the remaining outcomes.

To shut down all paths leading to "premature" death before the terminal period (which is only path *7* in the above example), we want to evaluate the probabilities of the events

$$\Pr(h_{t+2}=1 \wedge s_{t+1}=1 \mid h_t)$$
$$\Pr(h_{t+2}=2 \wedge s_{t+1}=1 \mid h_t)$$
$$\Pr(s_{t+2}=0 \wedge s_{t+1}=1 \mid h_t)$$

For either health outcome $j=1,2$ we find that

$$\begin{aligned}\Pr(h_{t+2}=j \wedge s_{t+1}=1 \mid h_t) &= \Pr(s_{t+1}=1 \mid h_{t+2}=j, h_t) \times \Pr(h_{t+2}=j \mid h_t) \\ &= \Pr(h_{t+2}=j \mid h_t)\end{aligned}$$

which follows since

$$\Pr(s_{t+1}=1 \mid h_{t+2}=j, h_t) = 1$$

An individual who is in health state $j$ at $t+2$ must have been alive at $t+1$, so the

additional restriction that $s_{t+1} = 1$ is redundant for health outcomes. However, this is not the case for the probability of being dead in $t + 2$:

$$\begin{aligned}\Pr\left(s_{t+2} = 0 \wedge s_{t+1} = 1 \,|\, h_t\right) &= \sum_{\ell=1}^{2} \Pr\left(s_{t+2} = 0 \wedge s_{t+1} = 1 \,|\, h_{t+1} = \ell, h_t\right) \\ &\qquad \times \Pr\left(h_{t+1} = \ell \,|\, h_t\right) \\ &= \sum_{\ell=1}^{2} \Pr\left(s_{t+2} = 0 \,|\, h_{t+1} = \ell, h_t\right) \Pr\left(h_{t+1} = \ell \,|\, h_t\right)\end{aligned}$$

The second line follows since conditional on $h_{t+1} = \ell$, we necessarily have $s_{t+1} = 1$. This formulation shuts down any paths with $s_{t+1} = 0$.

### B.2 Full model

We refer to a transition's starting date as $\underline{t}$, to its length (in years) as $T$, and to its end date as $\bar{t} = \underline{t} + T$. We denote by the tuple $(h_t, \boldsymbol{x}_t)$ the information available at time $t$, $t \in \{\underline{t}, \ldots, \underline{t} + T\}$, where $h_t \in \mathcal{H} \equiv \{1, \ldots, H\}$ is an individual's self-reported health state, with 1 representing the best and $H$ the worst realization. The vector $\boldsymbol{x}_t$ contains any other variables of interest, in particular age. We allow for time-invariant characteristics such as birth year, sex, race or education level to be included in $\boldsymbol{x}_t$, but restrict the time-varying variables to age and potentially calendar year. This restriction is necessary as we need to compute the evolution of $\boldsymbol{x}_t$ over $\underline{t} + 1, \underline{t} + 2, \ldots$ for multi-year transitions, which is not possible in general except for variables that follow a deterministic path (such as age and calendar year).[1]

Our goal is to characterize the one-year-ahead survival probability

$$p^{s}_{t+1} \equiv \Pr\left(s_{t+1} = 1 \,|\, h_t, \boldsymbol{x}_t\right)$$

defined in (2) in the main text and the health-to-health transition probabilities conditional on survival,

$$p^{h,j}_{t+1|s} \equiv \Pr\left(h_{t+1} = j \,\middle|\, s_{t+1} = 1, h_t, \boldsymbol{x}_t\right)$$

which were defined in (3). Here we use the notation $p^{h}_{\bullet|s}$ to indicate that this probability is conditional on survival. We can then compute the unconditional probability of being

[1] We could of course incorporate a time-varying discrete variable $z \in \mathcal{Z}$ by extending the state space from $\mathcal{H}$ to $\mathcal{Z} \times \mathcal{H}$ as in Pijoan-Mas and Ríos-Rull (2014). However, this introduces too many new parameters (parameters are outcome specific) to estimate such a model on the small black subsample.

in health state $j$ in the next period as

$$p_{t+1}^{h,j} = p_{t+1|s}^{h,j} \times p_{t+1}^{s} \tag{A.1}$$

Below we will frequently want to emphasize that we condition on a particular health state $h_t = k$, and hence we will use the expressions

$$p_{t+1|k}^{s} \equiv \Pr\left( s_{t+1} = 1 \mid h_t = k, \boldsymbol{x}_t \right) \tag{A.2}$$

$$p_{t+1|k,s}^{h,j} \equiv \Pr\left( h_{t+1} = j \,\middle|\, s_{t+1} = 1, h_t = k, \boldsymbol{x}_t \right) \tag{A.3}$$

In the remainder of this section, we lay out the estimation strategy to determine the parameter vectors $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}_j$ for all outcomes $j$, which we collect in the vector $\boldsymbol{\theta}$,

$$\boldsymbol{\theta} \equiv \left( \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_j, \ldots, \boldsymbol{\beta}_H, \boldsymbol{\gamma} \right) \in \mathbb{R}^K$$

where $K = (H-1)K_h + K_s$, $\boldsymbol{\beta}_j \in \mathbb{R}^{K_h}$ for each $j$ and $\boldsymbol{\gamma} \in \mathbb{R}^{K_s}$. We omit the normalized base outcome parameter vector $\boldsymbol{\beta}_1 = \mathbf{0}$ for health state 1. From any transition bracketed by the dates $\underline{t}$ and $\bar{t}$ we obtain one observation, a PMF over health states "augmented" by the state of death. We call this vector $\boldsymbol{\mu}_t \in \mathbb{R}^{H+1}$. In $\underline{t}$ we impose the degenerate initial distribution

$$\boldsymbol{\mu}_{\underline{t}} = (\underbrace{0, \ldots, 0, 1, 0, \ldots, 0}_{H \text{ elements}}, 0)^\top \tag{A.4}$$

with unity in the position corresponding to the initial health state $h_{\underline{t}}$.

The one-year health-to-health transition matrix conditional on survival is given by

$$\boldsymbol{\Pi}_t^h(\boldsymbol{x}_t|\boldsymbol{\beta}) = \begin{bmatrix} p_{t+1|1,s}^{h,1} & \cdots & p_{t+1|1,s}^{h,H} \\ \vdots & \ddots & \vdots \\ p_{t+1|H,s}^{h,1} & \cdots & p_{t+1|H,s}^{h,H} \end{bmatrix} \tag{A.5}$$

where the conditional probabilities $p_{t+1|k,s}^{h,j}$ are defined in the same way as in (A.3). This transition matrix is a function of the covariate vector $\boldsymbol{x}_t$ but not of the current health state $h_t$ as it contains transitions for all $h_t$.

Let $\boldsymbol{\pi}_t^s$ be the vector of survival probabilities between periods $t$ and $t+1$ for each health state $k \in \{1, \ldots, H\}$ today,

$$\boldsymbol{\pi}_t^s\left(\boldsymbol{x}_t|\boldsymbol{\gamma}\right) = \left( p_{t+1|1}^{s}, \ldots, p_{t+1|k}^{s}, \ldots, p_{t+1|H}^{s} \right)^\top \tag{A.6}$$

where any element $p^s_{t+1|k}$ is obtained as stated in (2). Given the distribution over health states *conditional* on being alive in $t$, $\boldsymbol{\mu}^h_t$, the probability of being alive in $t+1$ is therefore

$$p^s_{t+1}(\boldsymbol{\gamma}) = \boldsymbol{\pi}^s_t(\boldsymbol{\gamma})^\top \boldsymbol{\mu}^h_t \tag{A.7}$$

We can now write down the joint health/survival transition matrix, given by

$$\boldsymbol{\Pi}_t(\boldsymbol{x}_t|\boldsymbol{\theta}) = \begin{bmatrix} p^{h,1}_{t+1|1,s} p^s_{t+1|1} & \cdots & p^{h,H}_{t+1|1,s} p^s_{t+1|1} & \left(1 - p^s_{t+1|1}\right) \\ \vdots & \ddots & \vdots & \vdots \\ p^{h,1}_{t+1|H,s} p^s_{t+1|H} & \cdots & p^{h,H}_{t+1|H,s} p^s_{t+1|H} & \left(1 - p^s_{t+1|H}\right) \\ 0 & \dots & 0 & 1 \end{bmatrix}$$

We can then generate the distribution $\boldsymbol{\mu}_t$ over health/death states for any $t$ by repeatedly applying the transition matrix, starting with the degenerate initial distribution (A.4). The law of motion for $\boldsymbol{\mu}_t$ is therefore

$$\boldsymbol{\mu}_{t+1}(\boldsymbol{\theta})^\top = \boldsymbol{\mu}_t(\boldsymbol{\theta})^\top \boldsymbol{\Pi}_t(\boldsymbol{x}_t|\boldsymbol{\theta}) \tag{A.8}$$

In line with the initial discussion on computing PMFs over outcomes versus realizations of complete paths, we need to discard any paths that pass through the state $s_{\bar{t}-1} = 0$. This can be achieved by computing the PMF $\boldsymbol{\mu}_{\bar{t}-1}$ according to (A.8) and then defining the "pseudo" PMF

$$\widetilde{\boldsymbol{\mu}}_{\bar{t}-1} = (\mu_{1,\bar{t}-1}, \dots, \mu_{H,\bar{t}-1}, 0)$$

Note that $\sum_j \widetilde{\mu}_{j,\bar{t}-1} = \Pr\left(s_{\bar{t}-1} = 1 \,|\, h_{\underline{t}}, \boldsymbol{x}_{\underline{t}}\right)$, the probability of being alive in $\bar{t}-1$. The terminal distribution of interest can then be computed as before, i.e.,

$$\boldsymbol{\mu}_{\bar{t}}(\boldsymbol{\theta}) = \widetilde{\boldsymbol{\mu}}_{\bar{t}-1}(\boldsymbol{\theta})^\top \boldsymbol{\Pi}_{\bar{t}-1}(\boldsymbol{x}_{\bar{t}-1}|\boldsymbol{\theta}) \tag{A.9}$$

**Log-likelihood.** We are now ready to write down the likelihood function for observation $i$. Let $\delta^{h,j}_t$ be an indicator variable defined as

$$\delta^{h,j}_t = \begin{cases} 1 & \text{if } h_t = j \\ 0 & \text{else} \end{cases} \tag{A.10}$$

and $s_{\bar{t}}$ be the indicator for being alive in $\bar{t}$, analogous to (1). Then the likelihood function for transition $i$ is given by

$$\mathcal{L}_i(\boldsymbol{\theta}) = s_{\bar{t}} \left( \sum_{j=1}^{H} \delta_{\bar{t}}^{h,j} \log \mu_{j,\bar{t}}(\boldsymbol{\theta}) \right) + (1 - s_{\bar{t}}) \log \mu_{H+1,\bar{t}}(\boldsymbol{\theta}) \tag{A.11}$$

The estimated parameter vector $\widehat{\boldsymbol{\theta}}$ is the vector that maximizes the weighted sum of the log-likelihoods over all observations.[2]

### B.3 Confidence intervals

We compute confidence intervals nonparametrically using the Rao-Wu rescaling bootstrap method proposed for stratified cluster sampling survey designs (see Rao and Wu (1988), and Rust and Rao (1996) and Heeringa, West, and Berglund (2017) for textbook-style treatments).

The RAND HRS variable `RAESTRAT` records the stratum IDs, and within each stratum, `RAEHSAMP` identifies exactly two primary stage units (PSU) or clusters. The HRS is divided into 56 strata and thus contains a total of 112 distinct clusters.

A bootstrap algorithm which takes into account this stratified cluster sampling is implemented as follows:

1. Strata are held fixed, so each bootstrapped sample contains all strata. Within each stratum, one of the two PSUs is randomly (and independently) selected for inclusion in the bootstrap sample $b = 1, 2, \ldots, B$.

2. Let $k$, $j$, $i$ and $t$ index strata, clusters, respondents and time, respectively. Then each respondent-level sampling weight in bootstrap sample $b$ is rescaled according to the following formula:

$$w_{kjit}^{b} = \begin{cases} 2 \cdot w_{kjit} & \text{if cluster } j \text{ is included in } b \\ 0 & \text{else} \end{cases}$$

   This rescaling formula is a simplification of the general case in Rao and Wu (1988) for a setting in which each stratum has exactly two clusters.

   Note that the assignment of households to strata and clusters does not change over time, so if a PSU is selected to be in the sample, a household, its respondents and

[2] Even though it is conceptually a standard log-likelihood estimation, the implementation is non-standard and not included in any existing software, but specifically implemented for the problem at hand.

all of their transitions are selected.

3. The model is re-estimated for all race/sex groups on the bootstrapped sample.

4. Any statistics of interest are computed, e.g., the life expectancy of a particular subpopulation, the difference in life expectancies between black and nonblack groups, the probability of survival at age 50 for a black women in poor health, etc. Denote such a statistic by $\widehat{\theta}_b$.

5. The procedure is repeated for all $b = 1, 2, \ldots, B$ bootstrap samples, where we set $B = 1,001$ for computational reasons (estimating the main specification with five health states for all four demographic groups 1,001 times takes almost 2 hours on a 12-core machine).

6. We obtain the $(1 - \alpha)$ confidence interval for $\theta$ using the empirical CDF $\widehat{F}_\theta$ constructed from $\left\{\widehat{\theta}_1, \widehat{\theta}_2, \ldots, \widehat{\theta}_B\right\}$ and applying the percentile method. Thus the confidence interval is

$$CI_{1-\alpha} = \left[\widehat{F}_\theta^{-1}(\alpha/2), \widehat{F}_\theta^{-1}(1 - \alpha/2)\right]$$

# C Alternative MLE specifications

In this section, we compare our main specification which uses a linear age term in (4) and (5) to two alternative models: one with a quadratic term in age, and one which imposes a fixed two-year transition length and discards all transitions that occur between non-consecutive waves. Under the first alternative, the transitions are governed by the functions

$$\begin{aligned} g\left(h_{it}, m_i, b_i, a_{it} \mid \gamma\right) &= \gamma_{0,hmb} + \gamma_{1,hmb} \cdot a_{it} + \gamma_{2,hmb} \cdot a_{it}^2 \\ f_j\left(h_{it}, m_i, b_i, a_{it} \mid \gamma\right) &= \beta_{0,hmb} + \beta_{1,jhmb} \cdot a_{it} + \beta_{2,jhmb} \cdot a_{it}^2 \qquad j = 2, \ldots, 5\,, \end{aligned}$$

which is a straightforward extension of the main model. The second variant merits a more detailed discussion.

## C.1 Fixed two-year transition length

Table A.3 shows the relative frequencies of transition lengths observed in our HRS data. In the pooled sample, 84% of transitions are best characterized as two-year transitions (after rounding the number of months between consecutive interviews or between the last interview and a recorded death date). Treating the remainder as two-year transitions therefore potentially introduces measurement error.

| | | All | Nonblack | | Black | |
|---|---|---|---|---|---|---|
| | | | Male | Female | Male | Female |
| N. of years | 1 | 6.8% | 7.1% | 6.3% | 8.7% | 7.3% |
| | 2 | 84.0% | 83.9% | 84.6% | 80.7% | 83.3% |
| | 3 | 6.4% | 6.2% | 6.6% | 6.0% | 6.2% |
| | $\geq 4$ | 2.8% | 2.8% | 2.6% | 4.6% | 3.3% |
| N. of waves | 1 | 96.9% | 97.0% | 97.2% | 95.0% | 96.4% |
| | 2 | 2.1% | 2.0% | 1.9% | 3.3% | 2.4% |
| | $\geq 3$ | 1.0% | 1.0% | 0.9% | 1.7% | 1.2% |

**Table A.3:** Transition lengths in the HRS. Table reports the fraction of transitions in each category. The number of years are computed as the number of months between consecutive interviews (or a recorded death date), rounded to the nearest full year.

Moreover, the bottom part of the table reports the fraction of transitions which span one, two, and three or more survey waves.

Figure A.4 illustrates how such differences in transitions lengths arise. First, consider *person 1* who does not respond in survey wave 7: this gives rise to a 4-year transition from wave 6 to wave 8, an observation which cannot be used when requiring transitions

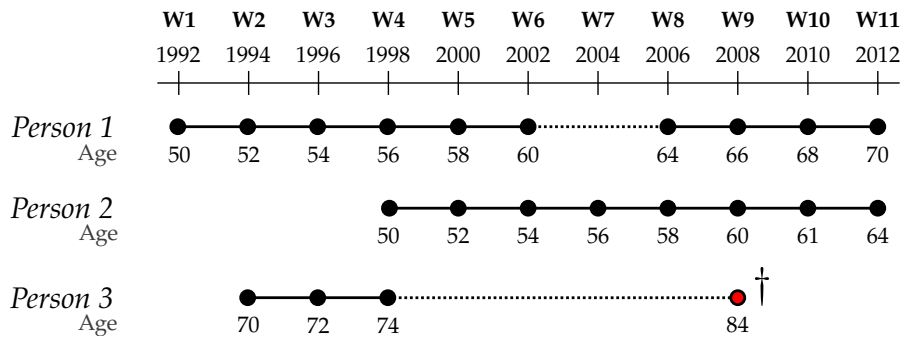



**Figure A.4:** Illustration of irregular and missing observations in the HRS. Waves and calendar years represent the survey's biennial structure, and a respondent's age is recorded at the date of an interview or death.

to be at 2-year frequencies between adjacent waves. Next, *person 2* illustrates that even if a respondent is present in consecutive waves, due to randomness of when an interview is administered in the field, the transition between waves 9 and 10 is best described as a 1-year transition between ages 60 and 61, whereas the next transition occurs over three years. Lastly, *person 3* shows a case where an individual permanently stops responding to the survey from wave 5 onward, but the death at age 84 is nevertheless recorded in the HRS. This gives rise to a 10-year transition that ends in death.

One approach (e.g., the one used in Pijoan-Mas and Ríos-Rull (2014)) is to drop all transitions over two or more waves, which amounts to five percent of the sample for black men, a group which already has relatively few observations in the HRS. An estimator using *all* the transitions is therefore more efficient in the sense that it does not discard these data points.

In the remainder of this section, we compare the differences between the main specification (linear in age and flexible transition length) with the alternative of either imposing a fixed two-year transition length, or employing a specification that is quadratic in age. First, Figure A.8 plots the life expectancy at ages 50 and 70 when we collapse health into a single state, thus eliminating any health heterogeneity. As shown, all models have very similar predictions. The exact life expectancies are also listed in Table A.4, and they never differ by more than half a year, which is usually within the 95% confidence interval of the main estimates. The table additionally includes the differences in race and sex, which are again very similar across the three specifications.

Next, we reintroduce health heterogeneity and repeat the above exercise. In Figure A.6 and Figure A.7, we compare the transition probabilities across the three specifications for the nonblack and black subsamples, respectively, and also contrast the model predictions with the raw data. To facilitate this comparison, we use two-year transition and survival

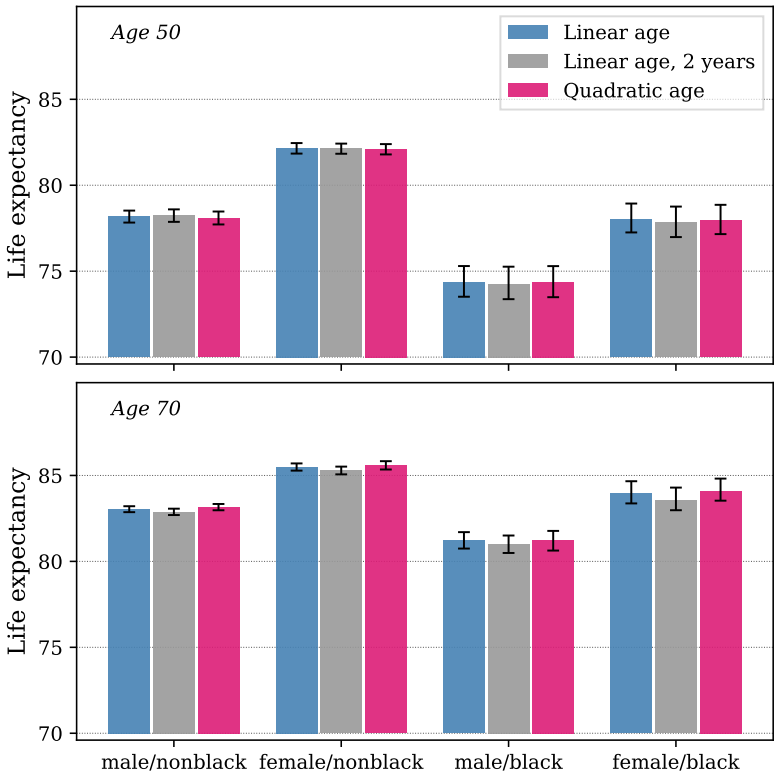



**Figure A.5:** Life expectancy by race and sex for alternative model specifications without health heterogeneity. Error bars indicate bootstrapped 95% confidence intervals.

probabilities since these are most frequently observed in the HRS, and since it's not possible to back out one-year probabilities from the fixed two-year model (see section D). As in the main text, we restrict the figures to three health states, even though all models are estimated on the full set of five self-reported health states. The transition probabilities are very similar across models in most cases, with some differences emerging predominantly for the black subsample that arise due to the smaller sample size.

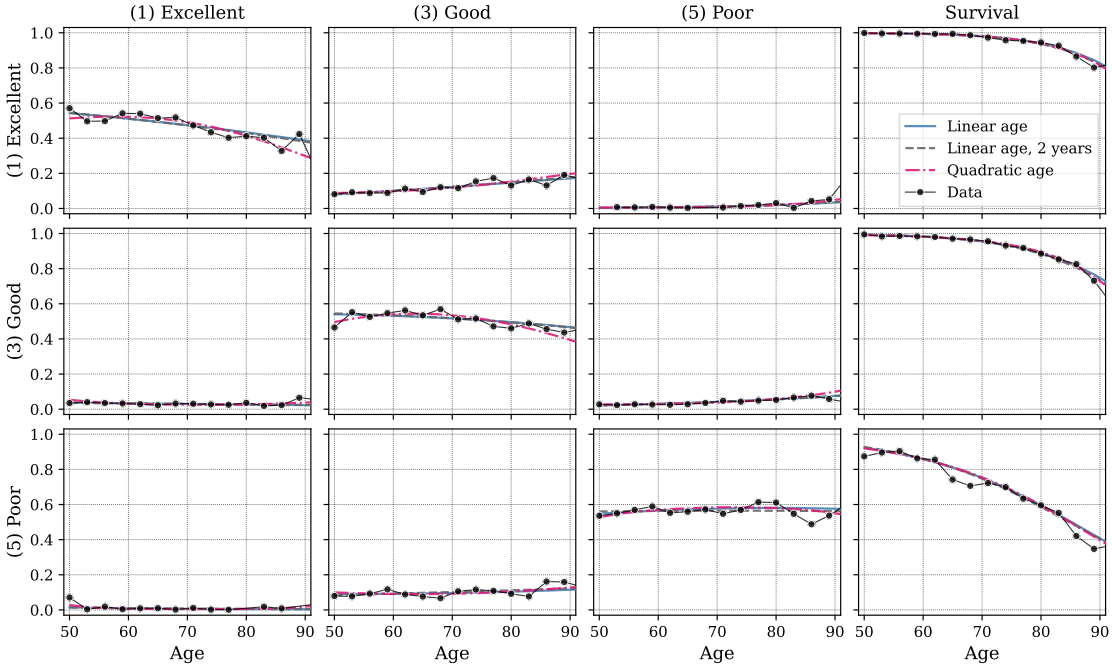
Turning to life expectancies, Figure A.8 plots these values disaggregated by race, sex and health, and Table A.5 reports the point estimates and confidence intervals as well as race and sex differences. As before, the three specifications give similar estimates, differing by at most 0.7 years for the black subsample.

To summarize, a parsimonious model with a linear age term for the most part yields results that are quite close to a quadratic age specification with substantially more parameters (more specifically, $100 = 2 \times 2 \times 5 \times 5$ additional parameters for a model that includes race, sex and five health states). For most groups, the model with fixed two-year transition periods comes very close to the more flexible but also more complex estimator

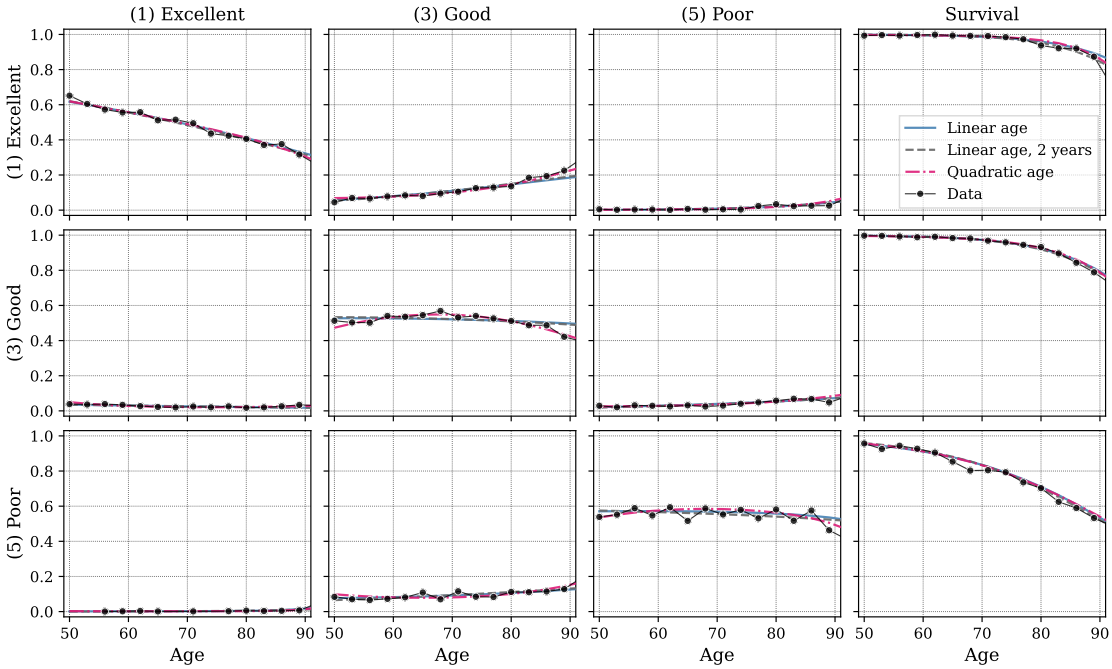
| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| Linear age | 78.2 | 82.2 | 74.4 | 78.0 | 3.8 | 4.1 | −4.0 | −3.7 |
| | [77.8, 78.5] | [81.8, 82.5] | [73.5, 75.3] | [77.3, 78.9] | [2.8, 4.8] | [3.2, 5.0] | [−4.4, −3.6] | [−4.9, −2.3] |
| Linear age, 2 years | 78.2 | 82.1 | 74.3 | 77.8 | 4.0 | 4.3 | −3.9 | −3.6 |
| | [77.9, 78.6] | [81.8, 82.4] | [73.4, 75.3] | [77.0, 78.8] | [2.9, 5.0] | [3.3, 5.2] | [−4.3, −3.5] | [−4.8, −2.3] |
| Quadratic age | 78.1 | 82.1 | 74.4 | 77.9 | 3.7 | 4.2 | −4.0 | −3.6 |
| | [77.7, 78.5] | [81.8, 82.4] | [73.5, 75.3] | [77.2, 78.9] | [2.6, 4.7] | [3.2, 5.0] | [−4.4, −3.5] | [−4.8, −2.2] |
| **Age 70** | | | | | | | | |
| Linear age | 83.0 | 85.5 | 81.2 | 84.0 | 1.8 | 1.5 | −2.5 | −2.7 |
| | [82.9, 83.2] | [85.3, 85.7] | [80.7, 81.7] | [83.4, 84.7] | [1.3, 2.4] | [0.8, 2.2] | [−2.7, −2.2] | [−3.7, −1.9] |
| Linear age, 2 years | 82.9 | 85.3 | 81.0 | 83.6 | 1.9 | 1.7 | −2.4 | −2.6 |
| | [82.7, 83.1] | [85.1, 85.5] | [80.5, 81.5] | [83.0, 84.3] | [1.3, 2.5] | [1.0, 2.4] | [−2.6, −2.2] | [−3.5, −1.7] |
| Quadratic age | 83.2 | 85.6 | 81.2 | 84.1 | 1.9 | 1.5 | −2.4 | −2.9 |
| | [83.0, 83.3] | [85.3, 85.8] | [80.6, 81.8] | [83.5, 84.8] | [1.3, 2.6] | [0.7, 2.1] | [−2.7, −2.2] | [−4.0, −2.0] |

**Table A.4:** Life expectancy by race and sex for alternative model specifications without health heterogeneity. Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Brackets indicate bootstrapped 95% confidence intervals.

underlying our main results. However, the former exhibits a tendency to estimate lower survival chances in old age, in particular among the black subpopulation, which is also evident in the right-most column of Figure A.7, where the survival probability is generally slightly below the other lines. This might be due to the fact that with a fixed two-year transition length, it does not matter whether a person died in the first or second year, whereas surviving the first year results in higher estimated survival probabilities in the flexible model. This distinction can be particularly potent among the very old who already have a low survival probability. Even though the fixed two-year approach performs well for the most part, ultimately its usefulness comes down to whether one is willing to calibrate an economic model where such estimates are to be incorporated to a biennial frequency.

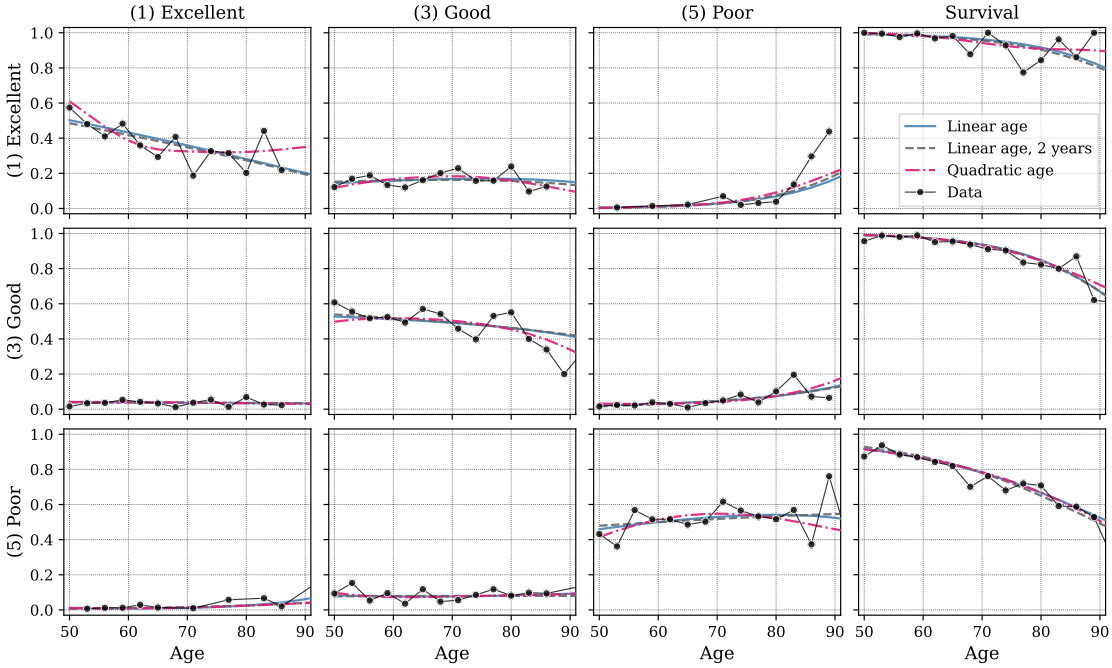



**(a)** Nonblack men

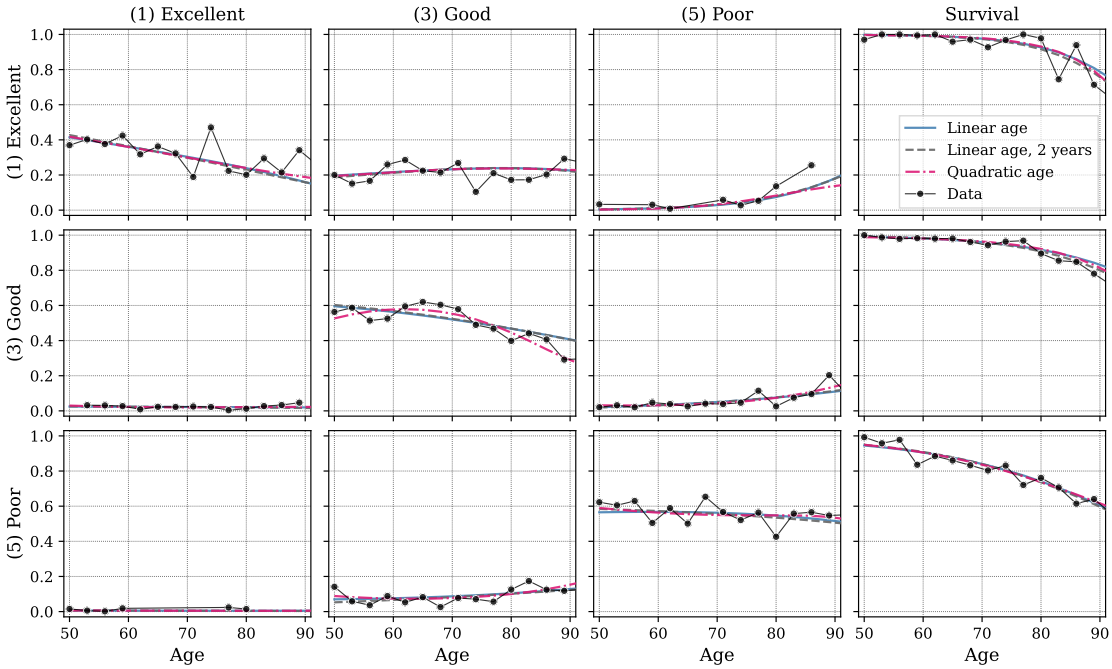



**(b)** Nonblack women

**Figure A.6:** Two-year transition probabilities for nonblack groups for alternative model specifications. Graphs show the best ("excellent"), middle ("good") and worst ("poor") health states. Health transition probabilities are conditional on survival. Right-most column shows survival probabilities. Missing dots indicate that some transitions are not observed in the data.

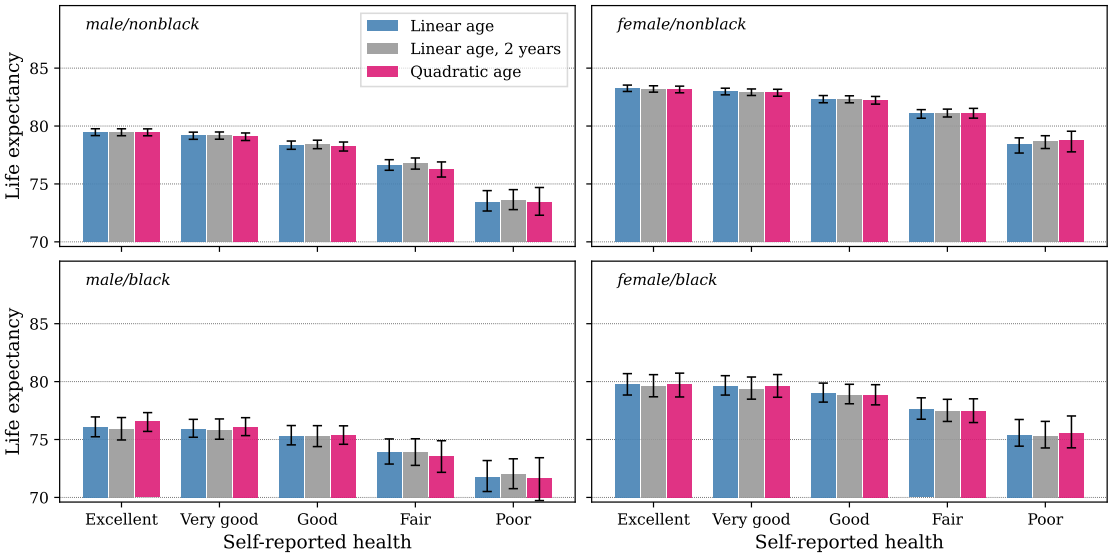



**(a)** Black men

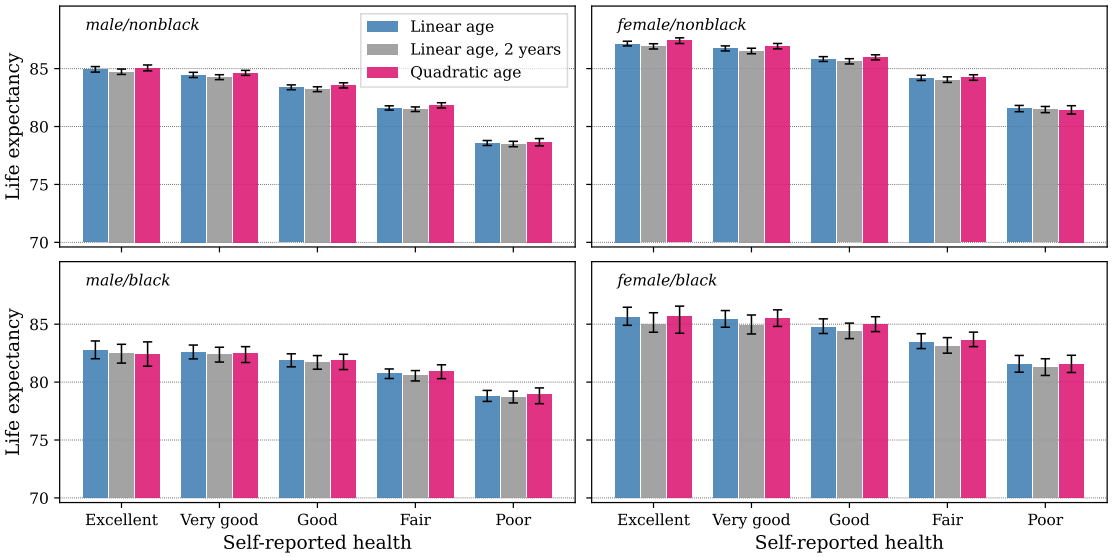



**(b)** Black women

**Figure A.7:** Two-year transition probabilities for black groups for alternative model specifications. Graphs show the best ("excellent"), middle ("good") and worst ("poor") health states. Health transition probabilities are conditional on survival. Right-most column shows survival probabilities. Missing dots indicate that some transitions are not observed in the data.




**(a)** Age 50




**(b)** Age 70

**Figure A.8:** Life expectancy by race, sex and health state for alternative model specifications. Error bars indicate bootstrapped 95% confidence intervals.

| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| *Health: (1) Excellent* | | | | | | | | |
| Linear age | 79.5 | 83.3 | 76.1 | 79.8 | 3.4 | 3.5 | −3.8 | −3.7 |
| | [79.2, 79.8] | [83.0, 83.5] | [75.2, 77.0] | [78.8, 80.7] | [2.4, 4.3] | [2.5, 4.5] | [−4.1, −3.5] | [−5.1, −2.3] |
| Linear age, 2 years | 79.5 | 83.2 | 75.9 | 79.6 | 3.6 | 3.6 | −3.7 | −3.7 |
| | [79.2, 79.8] | [82.9, 83.5] | [75.0, 76.9] | [78.7, 80.6] | [2.5, 4.6] | [2.5, 4.6] | [−4.1, −3.4] | [−5.2, −2.2] |
| Quadratic age | 79.5 | 83.2 | 76.6 | 79.8 | 2.9 | 3.3 | −3.7 | −3.3 |
| | [79.2, 79.8] | [82.9, 83.4] | [75.7, 77.3] | [78.7, 80.7] | [2.0, 3.8] | [2.3, 4.5] | [−4.0, −3.4] | [−4.7, −1.9] |
| *Health: (3) Good* | | | | | | | | |
| Linear age | 78.3 | 82.3 | 75.3 | 79.0 | 3.0 | 3.3 | −4.0 | −3.7 |
| | [78.0, 78.7] | [82.0, 82.6] | [74.5, 76.2] | [78.2, 79.9] | [2.0, 4.0] | [2.4, 4.1] | [−4.4, −3.6] | [−5.0, −2.5] |
| Linear age, 2 years | 78.4 | 82.3 | 75.3 | 78.9 | 3.1 | 3.4 | −3.9 | −3.6 |
| | [78.0, 78.8] | [82.0, 82.6] | [74.4, 76.2] | [78.1, 79.8] | [2.0, 4.2] | [2.5, 4.3] | [−4.3, −3.5] | [−4.9, −2.3] |
| Quadratic age | 78.2 | 82.2 | 75.4 | 78.8 | 2.8 | 3.4 | −4.0 | −3.4 |
| | [77.8, 78.6] | [81.9, 82.6] | [74.6, 76.2] | [78.0, 79.7] | [1.9, 3.8] | [2.4, 4.3] | [−4.5, −3.5] | [−4.7, −2.2] |
| *Health: (5) Poor* | | | | | | | | |
| Linear age | 73.4 | 78.4 | 71.8 | 75.4 | 1.7 | 3.0 | −5.0 | −3.7 |
| | [72.7, 74.4] | [77.7, 79.0] | [70.5, 73.2] | [74.4, 76.7] | [−0.0, 3.4] | [1.5, 4.2] | [−6.0, −3.7] | [−5.3, −2.0] |
| Linear age, 2 years | 73.6 | 78.7 | 72.0 | 75.3 | 1.6 | 3.4 | −5.1 | −3.3 |
| | [72.8, 74.5] | [78.1, 79.2] | [70.8, 73.3] | [74.3, 76.6] | [−0.0, 3.2] | [2.0, 4.6] | [−6.1, −3.9] | [−4.8, −1.6] |
| Quadratic age | 73.4 | 78.8 | 71.6 | 75.6 | 1.8 | 3.2 | −5.3 | −3.9 |
| | [72.3, 74.7] | [77.8, 79.6] | [69.7, 73.4] | [74.3, 77.0] | [−0.3, 4.1] | [1.5, 4.7] | [−6.8, −3.6] | [−6.3, −1.8] |
| **Age 70** | | | | | | | | |
| *Health: (1) Excellent* | | | | | | | | |
| Linear age | 84.9 | 87.1 | 82.8 | 85.5 | 2.2 | 1.6 | −2.2 | −2.8 |
| | [84.7, 85.2] | [87.0, 87.4] | [82.0, 83.6] | [84.9, 86.5] | [1.4, 2.9] | [0.7, 2.3] | [−2.5, −2.0] | [−4.3, −1.6] |
| Linear age, 2 years | 84.7 | 86.9 | 82.5 | 85.0 | 2.3 | 1.9 | −2.2 | −2.6 |
| | [84.5, 85.0] | [86.7, 87.1] | [81.6, 83.3] | [84.3, 86.0] | [1.5, 3.1] | [0.9, 2.7] | [−2.5, −1.9] | [−4.1, −1.3] |
| Quadratic age | 85.1 | 87.4 | 82.4 | 85.7 | 2.7 | 1.8 | −2.4 | −3.3 |
| | [84.8, 85.3] | [87.2, 87.7] | [81.4, 83.5] | [84.2, 86.6] | [1.6, 3.7] | [0.8, 3.2] | [−2.7, −2.1] | [−4.9, −1.3] |
| *Health: (3) Good* | | | | | | | | |
| Linear age | 83.4 | 85.8 | 81.9 | 84.8 | 1.5 | 1.1 | −2.4 | −2.9 |
| | [83.2, 83.6] | [85.6, 86.0] | [81.3, 82.4] | [84.2, 85.5] | [0.9, 2.1] | [0.3, 1.7] | [−2.7, −2.2] | [−4.0, −2.0] |
| Linear age, 2 years | 83.2 | 85.6 | 81.7 | 84.3 | 1.5 | 1.3 | −2.4 | −2.6 |
| | [83.0, 83.4] | [85.4, 85.9] | [81.1, 82.3] | [83.8, 85.1] | [0.9, 2.2] | [0.5, 1.9] | [−2.7, −2.1] | [−3.7, −1.7] |
| Quadratic age | 83.6 | 86.0 | 81.9 | 85.0 | 1.7 | 1.0 | −2.4 | −3.1 |
| | [83.3, 83.8] | [85.8, 86.2] | [81.1, 82.4] | [84.4, 85.6] | [1.1, 2.5] | [0.3, 1.7] | [−2.7, −2.2] | [−4.4, −2.2] |
| *Health: (5) Poor* | | | | | | | | |
| Linear age | 78.6 | 81.5 | 78.8 | 81.5 | −0.2 | 0.0 | −3.0 | −2.7 |
| | [78.4, 78.8] | [81.3, 81.8] | [78.3, 79.3] | [80.9, 82.3] | [−0.8, 0.3] | [−0.7, 0.8] | [−3.3, −2.6] | [−3.7, −1.8] |
| Linear age, 2 years | 78.5 | 81.5 | 78.7 | 81.2 | −0.2 | 0.2 | −3.0 | −2.5 |
| | [78.3, 78.7] | [81.2, 81.7] | [78.2, 79.2] | [80.6, 82.0] | [−0.8, 0.4] | [−0.6, 0.9] | [−3.4, −2.6] | [−3.6, −1.6] |
| Quadratic age | 78.7 | 81.4 | 78.9 | 81.5 | −0.2 | −0.1 | −2.8 | −2.6 |
| | [78.3, 79.0] | [81.1, 81.8] | [78.1, 79.5] | [80.8, 82.3] | [−0.8, 0.5] | [−0.9, 0.8] | [−3.3, −2.3] | [−3.8, −1.7] |

**Table A.5:** Life expectancy by race, sex and health for alternative model specifications. Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Brackets indicate bootstrapped 95% confidence intervals.

# D Alternative method to obtain one-year transitions

## D.1 The general problem

In this section, we demonstrate that in general, it is not possible to recover correct one-year transition probabilities from transition probabilities which were estimated over a two-year horizon.

**Survival.** Take survival first, and assume that we have estimated

$$\Pr\left( s_{a+2} = 1 \mid h_a, a \right) ,$$

the probability of surviving to the next wave for a respondent at age $a$ in health state $h_a$, where $s_a$ is a dummy indicating survival at age $a$. One simple approach to obtain one-year survival probabilities is to assume that they are the same at $a$ and $a+1$, and therefore

$$\begin{aligned} \Pr\left( s_{a+1} = 1 \mid h_a, a \right) &= \Pr\left( s_{a+2} = 1 \mid h_{a+1} = ?, a+1 \right) \\ &= \sqrt{\Pr\left( s_{a+2} = 1 \mid h_a, a \right)} \end{aligned} \tag{A.12}$$

Comparing this to the correct expression, given by

$$\Pr\left( s_{a+2} = 1 \mid h_a, a \right) = \sum_{j=1}^{H} \Pr\left( s_{a+2} = 1 \mid h_{a+1} = j, a+1 \right) \times \Pr\left( h_{a+1} = j \mid h_a, a \right)$$

highlights two problems with this reasoning: first, because health itself does not necessarily remain unchanged between $a$ and $a+1$, there is no reason that the survival probability between $a$ and $a+1$ should be the same. For example, an individual who suffered from a severe health condition at age $a$ but recovered within a year would likely face a lower mortality risk at $a+1$.

Second, even if we are willing to accept (A.12) and additionally assume that the health state is fixed at $h_a = j$ for all periods, we could alternatively compute the one-year survival probability $\Pr\left( s_{a+2} = 1 \mid h_{a+1} = j, a+1 \right)$ at age $a+1$ as the square root of the estimated two-year probability between ages $a+1$ and $a+3$,

$$\Pr\left( s_{a+2} = 1 \mid h_{a+1} = j, a+1 \right) = \sqrt{\Pr\left( s_{a+3} = 1 \mid h_{a+1} = j, a+1 \right)} \tag{A.13}$$

Combining (A.12) and (A.13) implies that

$$\sqrt{\Pr\left(\, s_{a+2} = 1 \mid h_a = j, a \,\right)} = \sqrt{\Pr\left(\, s_{a+3} = 1 \mid h_{a+1} = j, a+1 \,\right)}$$

but there is no reason why this equality should hold in general. In fact, it will not hold if survival chances are decreasing in age. To summarize, the assumption that mortality is constant over two-year periods yields contradictory one-year survival probabilities, and the fact that health itself can change every year makes this assumption unreasonable in the first place.

**Health transitions.** The problem becomes even more severe with health transitions, since for a five-state health process the option to "take the square root" disappears. Even with only two health states there is not enough information in two-year transitions to identify one-year transition probabilities.

To illustrate, assume that we have estimated the two-year transition matrix at age $a$ for a two-state health process which takes on the values *good* and *bad*:

$$\mathbf{\Xi}_a = \begin{bmatrix} \xi_a^g & 1-\xi_a^g \\ 1-\xi_a^b & \xi_a^b \end{bmatrix}$$

where $\xi_a^j$ denotes the probability to remain in health state $j$ over the next two years. Let $\pi_a^j$ be the probability to stay in the same health state for one year, and let $\mathbf{\Pi}_a$ be the one-year transition matrix,

$$\mathbf{\Pi}_a = \begin{bmatrix} \pi_a^g & 1-\pi_a^g \\ 1-\pi_a^b & \pi_a^b \end{bmatrix}$$

As argued above, one cannot assume that the transition probabilities are constant over the two-year period, because one would get different transition probabilities depending on whether the underlying two-year transition starts at $a-1$ or $a$. Hence, two different transition matrices $\mathbf{\Pi}_a$ and $\mathbf{\Pi}_{a+1}$ are required, such that conditional on survival,

$$\mathbf{\Xi}_a = \mathbf{\Pi}_a \mathbf{\Pi}_{a+1} \tag{A.14}$$

Writing out the nonlinear equation system explicitly,

$$\begin{aligned} \xi_a^g &= \pi_a^g \pi_{a+1}^g + (1-\pi_a^g)(1-\pi_{a+1}^b) \\ \xi_a^b &= (1-\pi_a^b)(1-\pi_{a+1}^g) + \pi_a^b \pi_{a+1}^b \end{aligned}$$

we see that there are two equations but four unknowns $(\pi_a^g, \pi_a^b, \pi_{a+1}^g, \pi_{a+1}^b)$. Hence there is no unique solution even for two health states, and with more health states, the number of unknowns increases considerably faster than the number of equations.

### D.2 The approximation method of De Nardi, French, and Jones (2010)

De Nardi, French, and Jones (2010), henceforth DFJ, use the following approximation to obtain one-year transition probabilities from two-year estimates.[3] First, they assume that one-year survival probabilities are given by

$$\Pr_{\text{DFJ}}\left( s_{a+1} = 1 \mid h, a \right) = \sqrt{\Pr\left( s_{a+2} = 1 \mid h, a \right)} \quad \forall\, a, h$$

ignoring the resulting inconsistencies discussed in the previous section. Second, for health transitions conditional on survival, they assume that health transition probabilities are constant over the two-year period. In this special case, (A.14) becomes

$$\boldsymbol{\Xi}_a = \boldsymbol{\Pi}_a \boldsymbol{\Pi}_a \tag{A.15}$$

which gives rise to a a system of two equations and two unknowns (they only have a *good* and *bad* health state).

We replicate their approximation using our fixed two-year estimates from section C. With all five health states, (A.15) becomes a nonlinear equation system in $5 \times 4 = 20$ unknowns which needs to be solved using a numerical root-finder.[4]

We first contrast the two methods using the model without health heterogeneity, as then any differences are due to mortality. In Figure A.9, we plot the survival probabilities for all groups, and the DFJ approximation generates higher mortality rates at all ages. While the magnitudes shown in the figure are not large, they accumulate to a drop in life expectancy of about half a year, as shown in Table A.6.

The reason for this downward bias is straightforward: since the chances of survival

[3] The method is not explicitly discussed in their paper or the supplemental material available on the journal website. However, details can be found in the code package provided on Eric French's website.

[4] There is no guarantee that this nonlinear equation system has a unique root, or that a numerical root finder is able to find one in a reliable manner. We used the `HYBR` and `LM` methods from the Fortran `MINPACK` library, which struggled to solve (A.15) for the smaller black samples and for high ages. We view this as another reason to prefer our maximum likelihood estimation method.

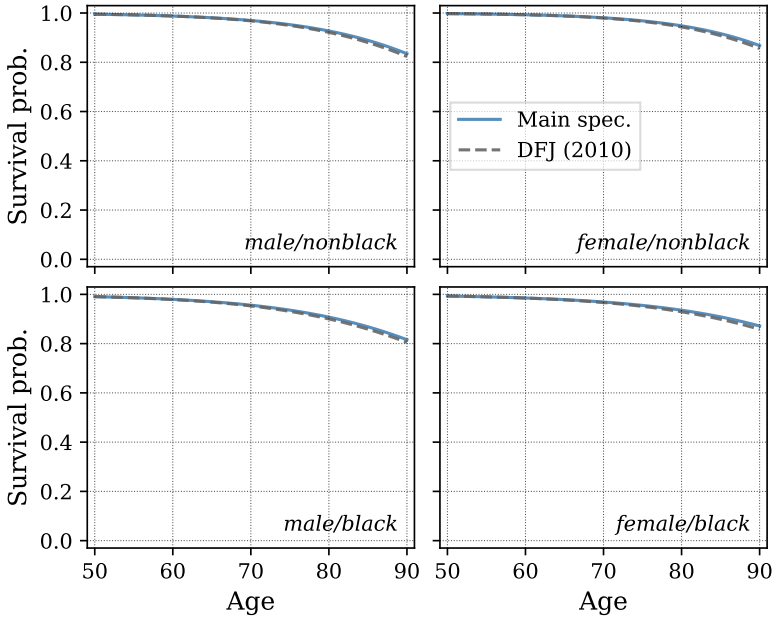



**Figure A.9:** One-year survival probabilities for model without health heterogeneity: main specification vs. the approximation from De Nardi, French, and Jones (2010).

| | Nonblack | | Black | |
|---|---|---|---|---|
| | Male | Female | Male | Female |
| **Age 50** | | | | |
| Main spec. | 78.2 | 82.2 | 74.4 | 78.0 |
| | [77.8, 78.5] | [81.8, 82.5] | [73.5, 75.3] | [77.3, 78.9] |
| DFJ (2010) | 77.8 | 81.7 | 73.8 | 77.4 |
| | – | – | – | – |
| **Age 70** | | | | |
| Main spec. | 83.0 | 85.5 | 81.2 | 84.0 |
| | [82.9, 83.2] | [85.3, 85.7] | [80.7, 81.7] | [83.4, 84.7] |
| DFJ (2010) | 82.6 | 84.9 | 80.7 | 83.3 |
| | – | – | – | – |

**Table A.6:** Life expectancy by race and sex for model without health heterogeneity: main specification vs. the approximation from De Nardi, French, and Jones (2010). Brackets indicate bootstrapped 95% confidence intervals.

drop with age, it follows that

$$\begin{aligned}
\Pr_{\text{DFJ}}\left( s_{a+1} = 1 \mid a \right) &= \sqrt{\Pr\left( s_{a+2} = 1 \mid a \right)} \\
&= \sqrt{\Pr\left( s_{a+2} = 1 \mid a+1 \right) \Pr\left( s_{a+1} = 1 \mid a \right)} \\
&< \sqrt{\Pr\left( s_{a+1} = 1 \mid a \right) \Pr\left( s_{a+1} = 1 \mid a \right)} \\
&= \Pr\left( s_{a+1} = 1 \mid a \right)
\end{aligned}$$

This effect carries over to the full model with health heterogeneity, but is additionally accompanied by changes in health transitions that don't seem to follow any salient pattern. In Table A.7, we contrast the life expectancies resulting from the DFJ approximation with our main results (the rows labeled "main spec." are the same as in Table 2). As the table illustrates, the downward bias remains since almost all life expectancies are (weakly) lower than our main estimates. For example, for a nonblack man at the age of 50, the approximation predicts a life expectancy that is 0.6 years lower. The disparities are similar for other demographic groups as well as for the life expectancy at the age of 70. Note that most of these differences are statistically significant. The only exception are individuals in poor health, where the point estimates are almost identical across both methods.

| | Nonblack | | Black | |
|---|---|---|---|---|
| | Male | Female | Male | Female |
| **Age 50** | | | | |
| *Health: (1) Excellent* | | | | |
| Main spec. | 79.5 | 83.3 | 76.1 | 79.8 |
| | [79.2, 79.8] | [83.0, 83.5] | [75.2, 77.0] | [78.8, 80.7] |
| DFJ (2010) | 78.9 | 82.6 | 75.4 | 79.0 |
| | – | – | – | – |
| *Health: (3) Good* | | | | |
| Main spec. | 78.3 | 82.3 | 75.3 | 79.0 |
| | [78.0, 78.7] | [82.0, 82.6] | [74.5, 76.2] | [78.2, 79.9] |
| DFJ (2010) | 77.8 | 81.7 | 74.8 | 78.3 |
| | – | – | – | – |
| *Health: (5) Poor* | | | | |
| Main spec. | 73.4 | 78.4 | 71.8 | 75.4 |
| | [72.7, 74.4] | [77.7, 79.0] | [70.5, 73.2] | [74.4, 76.7] |
| DFJ (2010) | 73.5 | 78.4 | 71.9 | 75.1 |
| | – | – | – | – |
| **Age 70** | | | | |
| *Health: (1) Excellent* | | | | |
| Main spec. | 84.9 | 87.1 | 82.8 | 85.5 |
| | [84.7, 85.2] | [87.0, 87.4] | [82.0, 83.6] | [84.9, 86.5] |
| DFJ (2010) | 84.2 | 86.4 | 82.0 | 84.5 |
| | – | – | – | – |
| *Health: (3) Good* | | | | |
| Main spec. | 83.4 | 85.8 | 81.9 | 84.8 |
| | [83.2, 83.6] | [85.6, 86.0] | [81.3, 82.4] | [84.2, 85.5] |
| DFJ (2010) | 82.8 | 85.1 | 81.3 | 83.9 |
| | – | – | – | – |
| *Health: (5) Poor* | | | | |
| Main spec. | 78.6 | 81.5 | 78.8 | 81.5 |
| | [78.4, 78.8] | [81.3, 81.8] | [78.3, 79.3] | [80.9, 82.3] |
| DFJ (2010) | 78.6 | 81.5 | 78.7 | 81.2 |
| | – | – | – | – |

**Table A.7:** Life expectancy by race, sex and health: main specification vs. the approximation from De Nardi, French, and Jones (2010). Brackets indicate bootstrapped 95% confidence intervals.

# E Additional estimation results

## E.1 Predicted health distributions

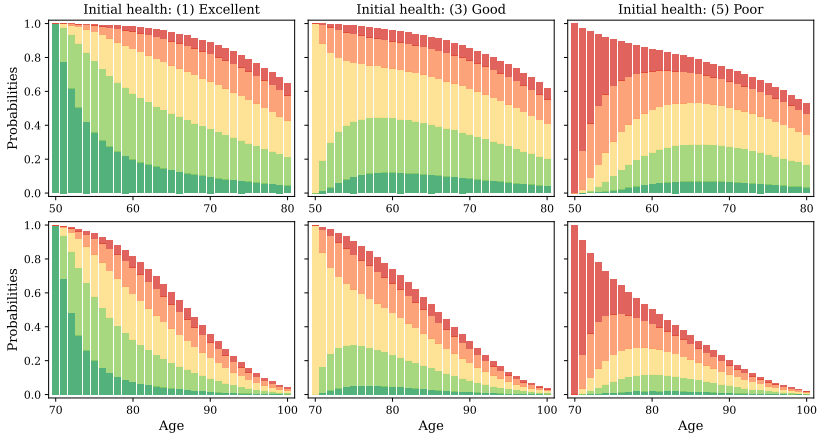
In Figure A.10, we plot the predicted distribution over health states and death for the remaining demographic groups: nonblack women, black men, and black women (the corresponding Figure 3 can be found in the main text).

## E.2 Transition matrices (one-year horizon)

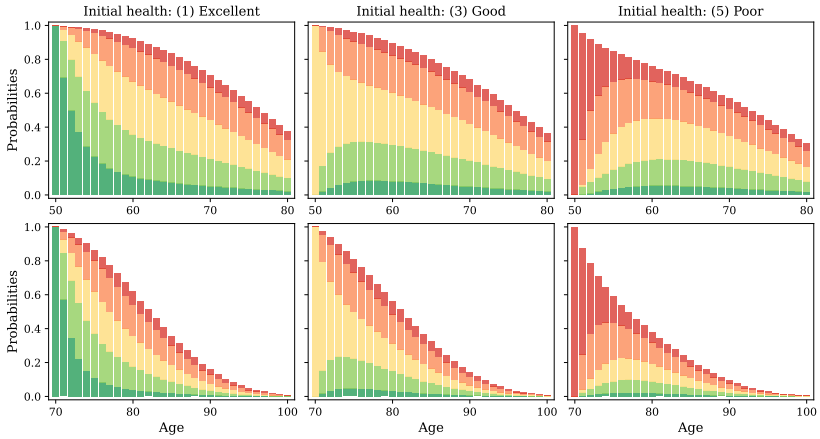
The one-year health-to-health transition and survival probabilities are shown in Figure A.11 and Figure A.12 for the nonblack and black subpopulations, respectively.

## E.3 Long-run survival outcomes

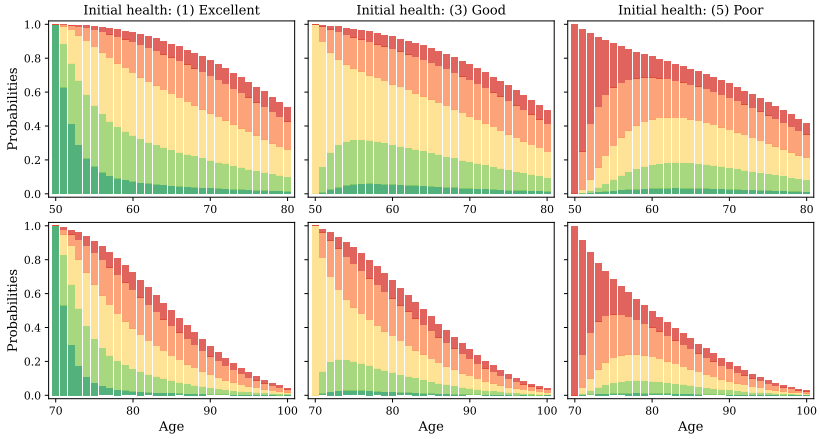
In Figure A.13, Figure A.14, Figure A.15 and Figure A.16, we show how the survival predictions from the estimated model line up with actual fractions of survivors in the HRS for all race and sex combinations. These graphs expand Figure 6 in the main text to include the first ten waves of our sample and can thus be used to assess the quality of short- and long-run predictions. See the main text for a detailed description of how the graphs are constructed.




**(a)** Nonblack women




**(b)** Black men




**(c)** Black women

**Figure A.10:** Predicted distribution over health states and death conditional on initial health for a 50-year-old (upper row) and a 70-year-old (lower row). The colors indicate probability per health state (dark green being the best health state, red the worst). The white area represents the probability of being dead.

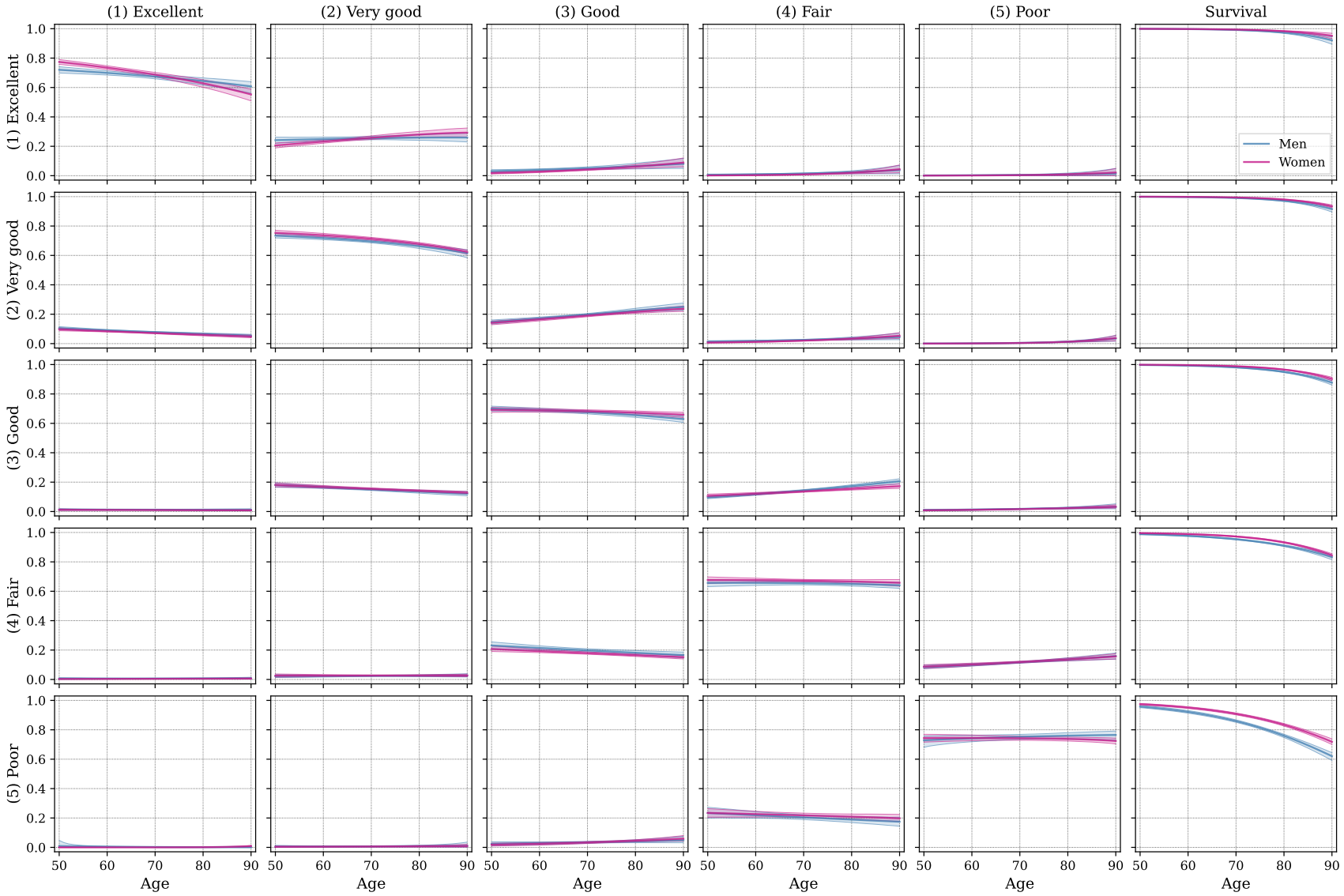



**Figure A.11:** One-year transition probabilities for the nonblack subpopulation. Shaded areas indicate bootstrapped 95% confidence intervals.

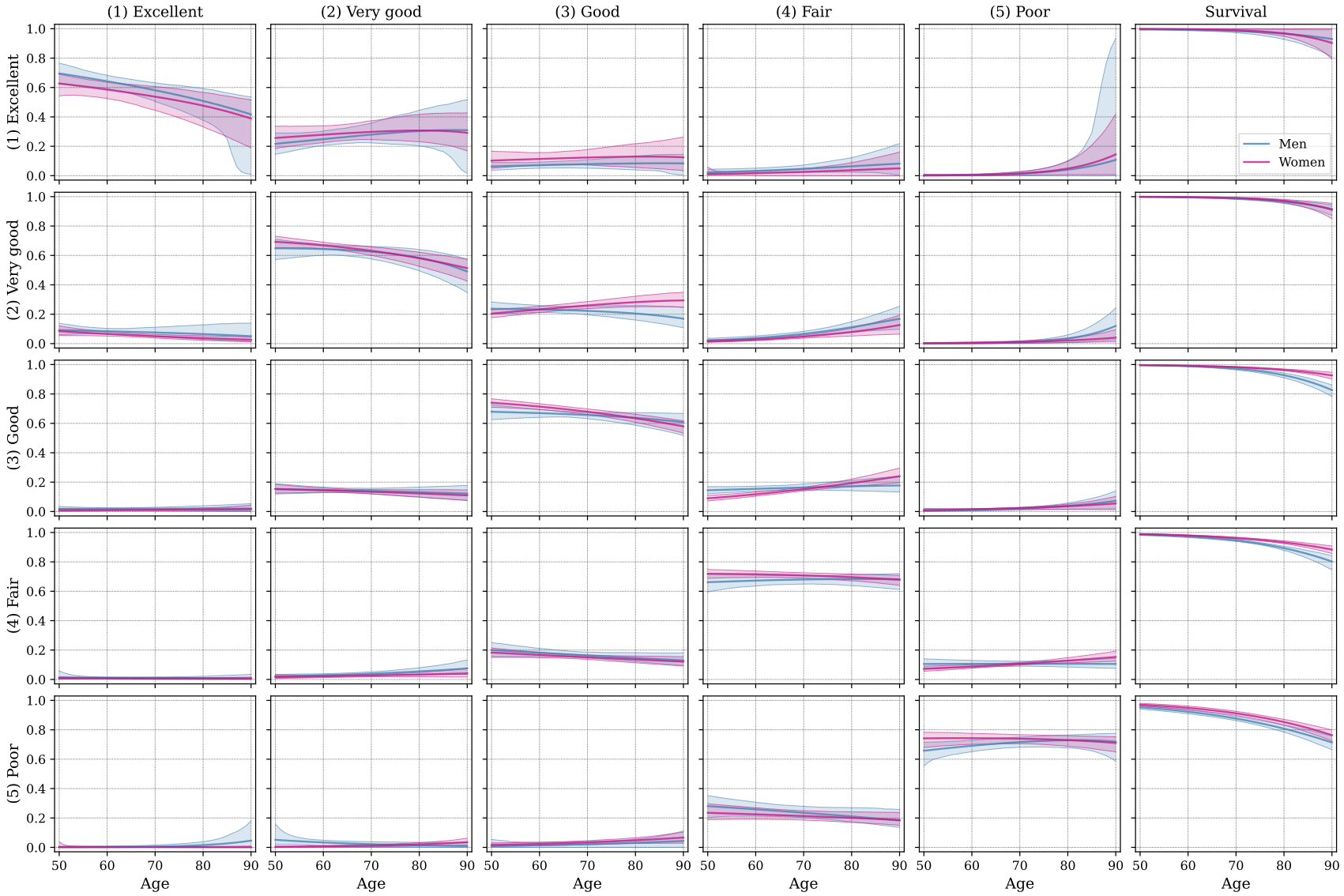



**Figure A.12:** One-year transition probabilities for the black subpopulation. Shaded areas indicate bootstrapped 95% confidence intervals.

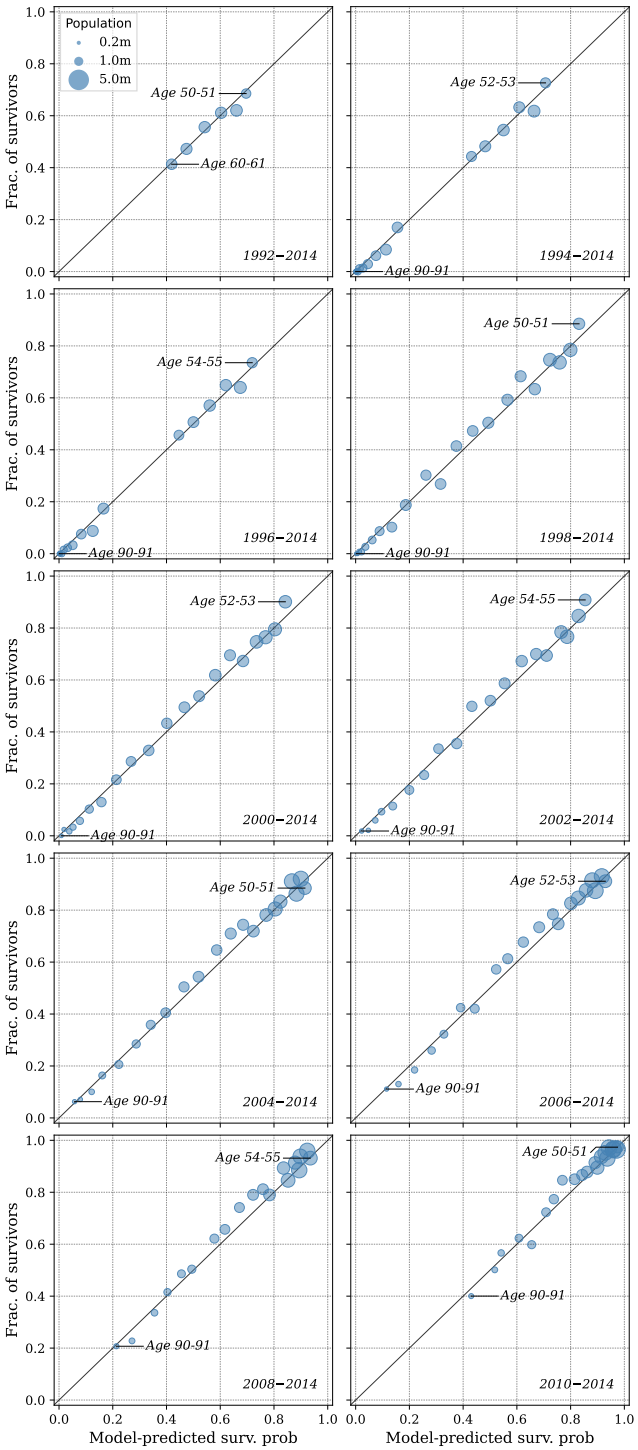



**Figure A.13:** Model-predicted survival probabilities (on the x-axis) against the fraction of survivors (on the y-axis) for *nonblack men*, plotted for ten different time periods. The top left graph represents the time period between the first wave (1992) and the last wave (2014). Each dot represents a two-year age bin.

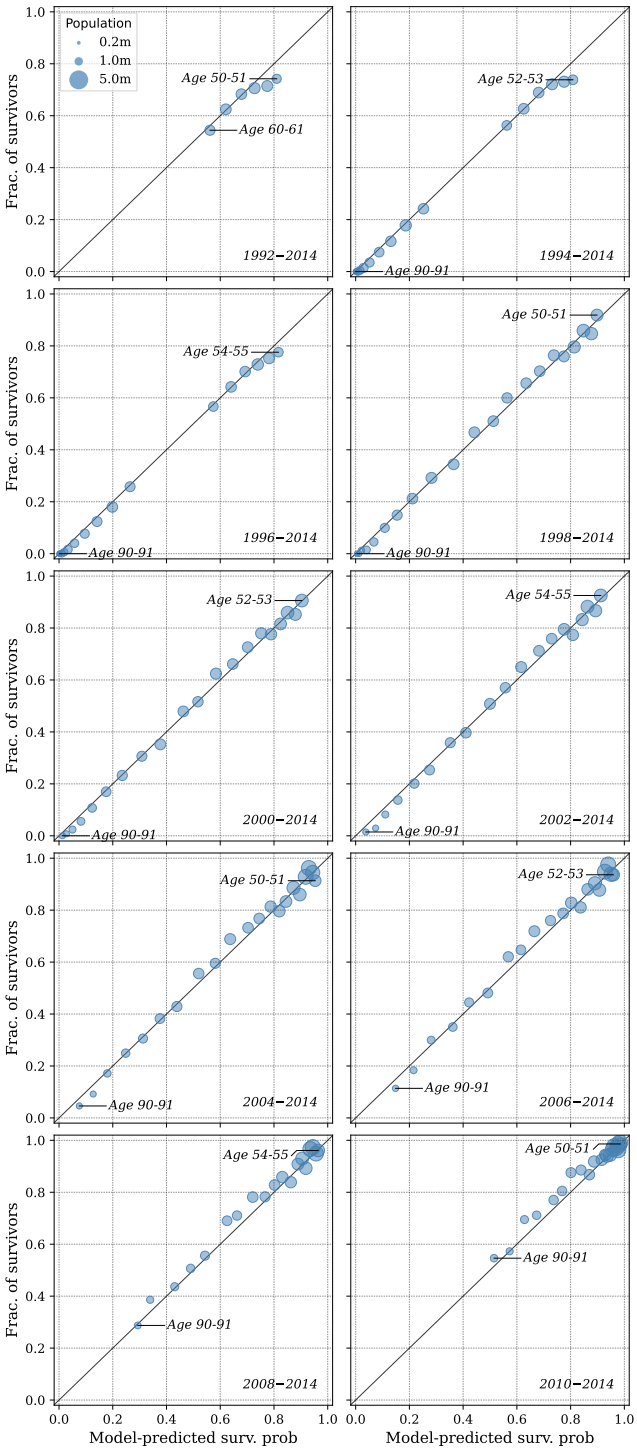



**Figure A.14:** Model-predicted survival probabilities (on the x-axis) against the fraction of survivors (on the y-axis) for *nonblack women*, plotted for ten different time periods. The top left graph represents the time period between the first wave (1992) and the last wave (2014). Each dot represents a two-year age bin.

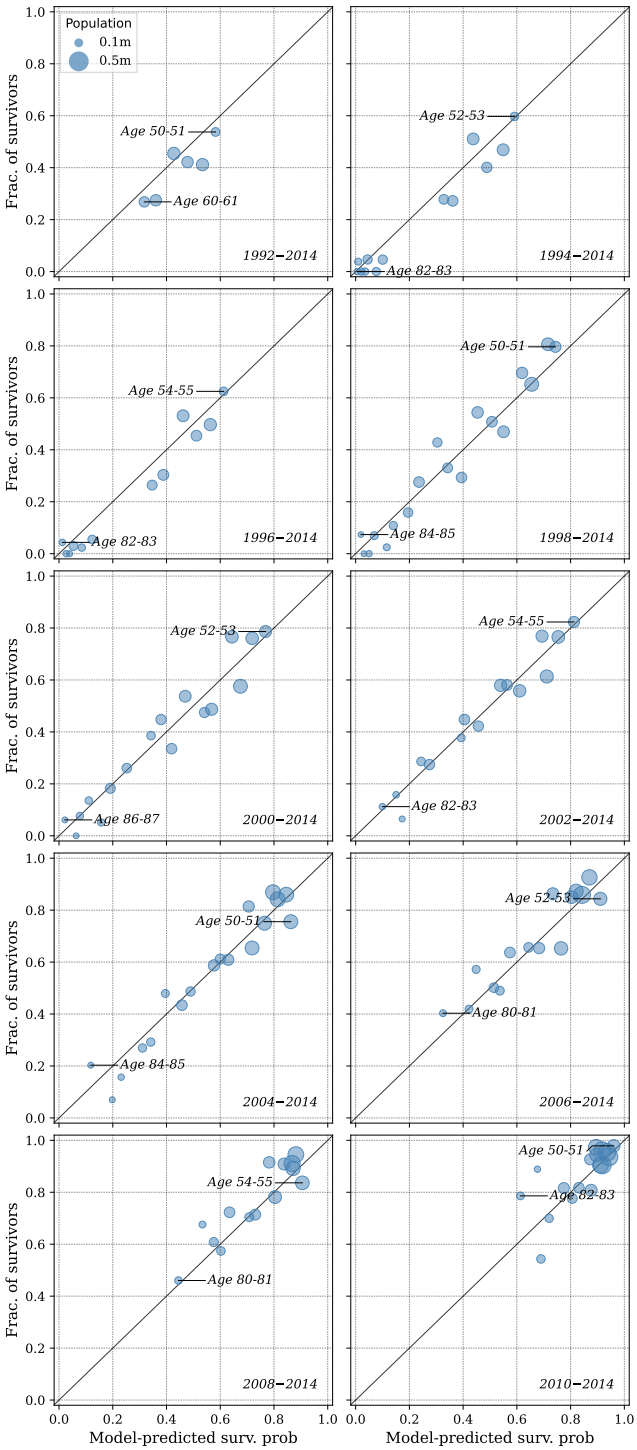



**Figure A.15:** Model-predicted survival probabilities (on the x-axis) against the fraction of survivors (on the y-axis) for *black men*, plotted for ten different time periods. The top left graph represents the time period between the first wave (1992) and the last wave (2014). Each dot represents a two-year age bin.

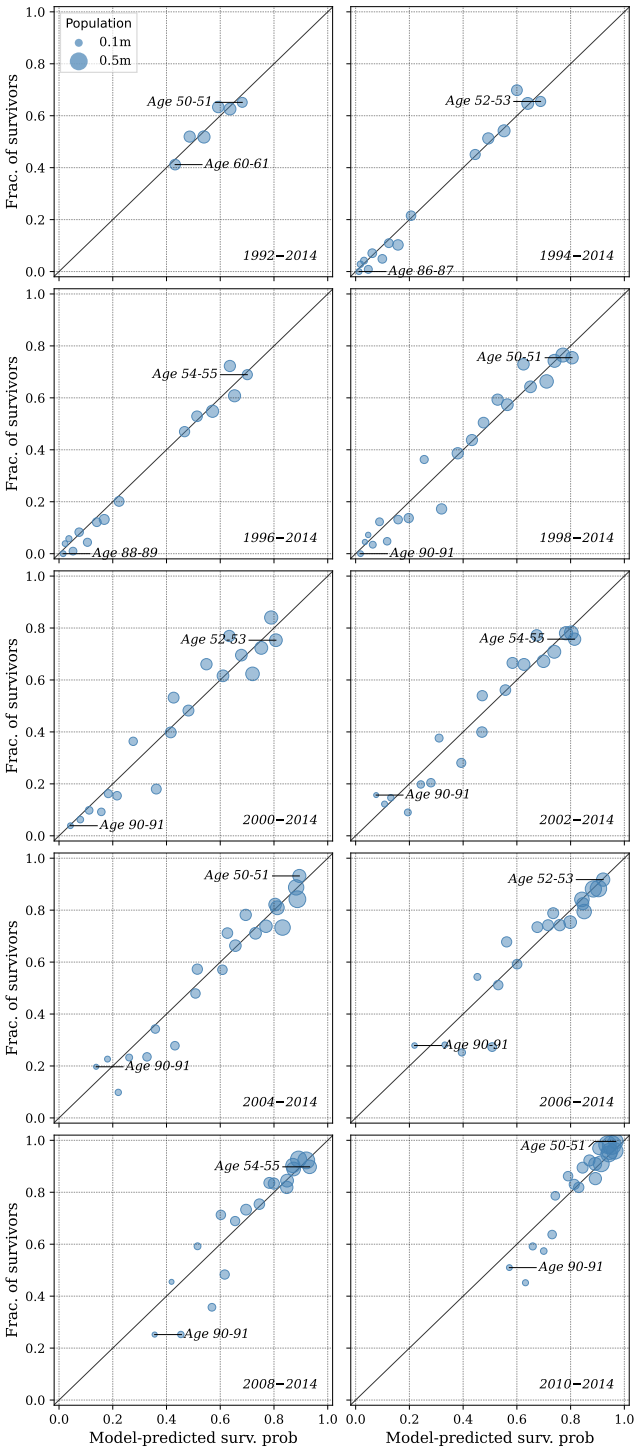



**Figure A.16:** Model-predicted survival probabilities (on the x-axis) against the fraction of survivors (on the y-axis) for *black women*, plotted for ten different time periods. The top left graph represents the time period between the first wave (1992) and the last wave (2014). Each dot represents a two-year age bin.

## E.4 Life expectancies by age

In the main text, we report life expectancy at ages 50 and 70. Figure A.17 shows life expectancy conditional on health for the whole age range of 50–90 for all four demographic subgroups.

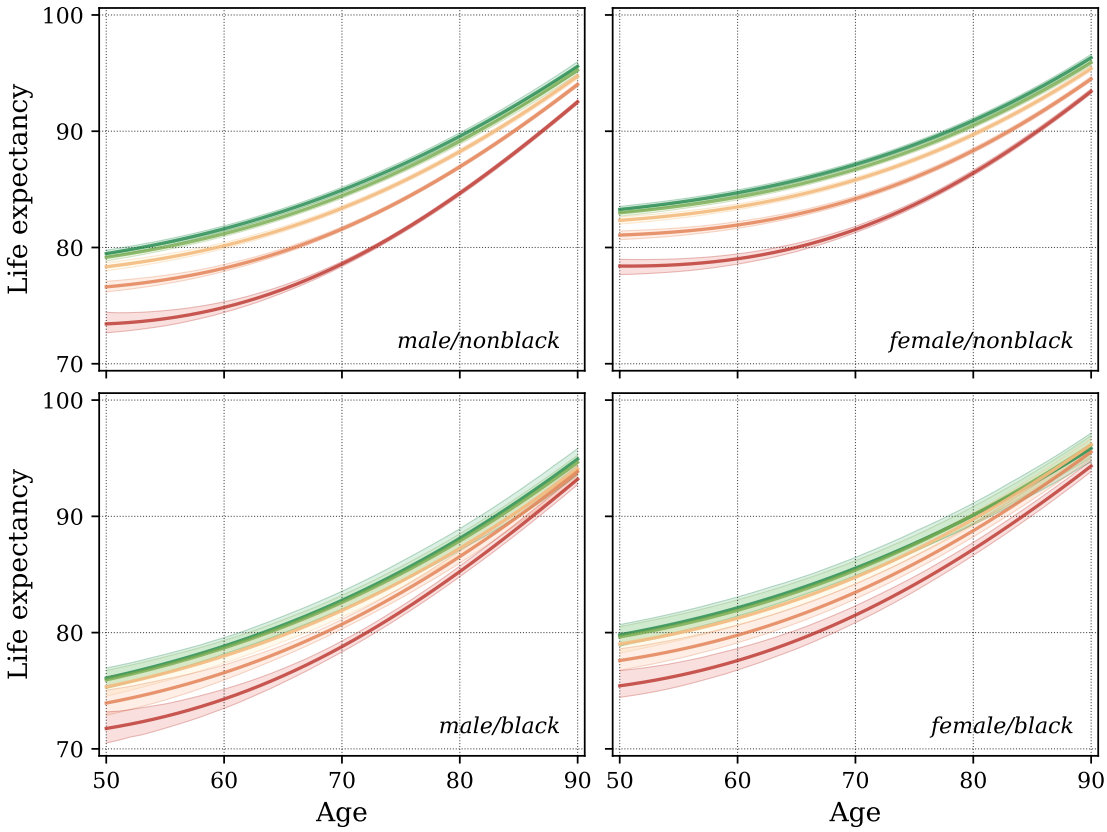



**Figure A.17:** Life expectancy by age and health state. Dark green indicates best ("excellent") while red indicates worst ("poor") health. Shaded areas indicate bootstrapped 95% confidence intervals.

## E.5 Life expectancies by education group

Table A.8 shows life expectancy by race, sex and education for the model without health heterogeneity.

| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| No high school | 75.0 | 78.8 | 72.5 | 75.9 | 2.5 | 2.9 | −3.8 | −3.4 |
| | [74.3, 75.7] | [78.3, 79.4] | [71.0, 74.1] | [75.1, 77.0] | [0.7, 4.1] | [1.7, 4.0] | [−4.9, −2.8] | [−5.2, −1.7] |
| High school | 77.9 | 82.7 | 75.4 | 79.3 | 2.5 | 3.4 | −4.8 | −3.9 |
| | [77.4, 78.3] | [82.2, 83.1] | [74.2, 76.6] | [77.9, 80.9] | [1.2, 3.8] | [1.7, 4.8] | [−5.4, −4.1] | [−6.1, −1.9] |
| College | 81.7 | 84.9 | 77.4 | 81.6 | 4.3 | 3.3 | −3.2 | −4.2 |
| | [80.9, 82.6] | [83.9, 85.9] | [75.0, 80.2] | [79.5, 83.9] | [1.3, 6.9] | [0.8, 5.7] | [−4.5, −1.9] | [−7.9, −0.3] |
| **Age 70** | | | | | | | | |
| No high school | 81.5 | 83.9 | 80.5 | 82.9 | 1.0 | 1.0 | −2.3 | −2.3 |
| | [81.3, 81.8] | [83.5, 84.2] | [80.0, 81.1] | [82.3, 83.5] | [0.4, 1.6] | [0.2, 1.7] | [−2.7, −1.9] | [−3.4, −1.3] |
| High school | 82.9 | 85.9 | 82.1 | 85.4 | 0.8 | 0.5 | −3.0 | −3.3 |
| | [82.6, 83.2] | [85.6, 86.2] | [80.9, 83.3] | [84.5, 86.6] | [−0.4, 2.0] | [−0.8, 1.5] | [−3.3, −2.6] | [−5.1, −1.7] |
| College | 85.3 | 87.0 | 82.5 | 85.6 | 2.9 | 1.3 | −1.6 | −3.1 |
| | [84.8, 85.9] | [86.2, 87.7] | [80.2, 86.4] | [83.7, 88.0] | [−1.0, 5.1] | [−1.1, 3.4] | [−2.6, −0.7] | [−6.3, 0.9] |

**Table A.8:** Life expectancy by race, sex and education for model without health heterogeneity. Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Brackets indicate bootstrapped 95% confidence intervals.

## E.6 Life expectancies by income group

Table A.9 shows life expectancies by race, sex and permanent income tercile for the model without health heterogeneity.

| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| 1st tercile | 75.0 | 79.7 | 70.9 | 74.8 | 4.1 | 4.9 | −4.7 | −3.9 |
| | [74.4, 75.8] | [79.2, 80.1] | [69.5, 72.4] | [73.8, 76.0] | [2.3, 5.7] | [3.6, 6.1] | [−5.4, −3.9] | [−5.7, −1.9] |
| 2nd tercile | 78.1 | 82.6 | 75.9 | 81.3 | 2.2 | 1.3 | −4.6 | −5.4 |
| | [77.4, 78.7] | [82.1, 83.2] | [74.9, 76.9] | [79.9, 82.9] | [1.0, 3.4] | [−0.4, 2.9] | [−5.3, −3.8] | [−7.3, −3.5] |
| 3rd tercile | 81.2 | 85.3 | 79.8 | 83.5 | 1.5 | 1.8 | −4.1 | −3.7 |
| | [80.6, 81.8] | [84.7, 86.0] | [77.6, 81.9] | [81.4, 85.5] | [−0.9, 3.8] | [−0.4, 3.9] | [−4.9, −3.2] | [−6.3, −1.2] |
| **Age 70** | | | | | | | | |
| 1st tercile | 81.8 | 84.3 | 79.8 | 82.6 | 2.0 | 1.6 | −2.5 | −2.9 |
| | [81.4, 82.2] | [84.0, 84.6] | [79.3, 80.3] | [81.9, 83.5] | [1.4, 2.6] | [0.7, 2.5] | [−2.9, −2.0] | [−4.0, −1.8] |
| 2nd tercile | 83.0 | 85.8 | 81.3 | 85.5 | 1.7 | 0.3 | −2.7 | −4.2 |
| | [82.8, 83.3] | [85.4, 86.1] | [80.5, 82.1] | [84.6, 86.4] | [1.0, 2.5] | [−0.8, 1.3] | [−3.1, −2.3] | [−5.5, −2.9] |
| 3rd tercile | 84.3 | 87.2 | 84.4 | 86.7 | −0.0 | 0.4 | −2.9 | −2.4 |
| | [83.9, 84.7] | [86.7, 87.7] | [82.2, 86.4] | [85.3, 88.5] | [−2.2, 2.2] | [−1.3, 1.9] | [−3.4, −2.3] | [−4.8, −0.4] |

**Table A.9:** Life expectancy by race, sex and permanent income tercile for model without health heterogeneity. Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Brackets indicate bootstrapped 95% confidence intervals.

## E.7 Life expectancies by calendar year

In this section, we document changes in life expectancy over the sample period of 1992–2012. To this end we augment the main specification in (4) and (5) with a term that is linear in the calendar year $t$:

$$g\left(h_{it}, m_i, b_i, a_{it} \mid \gamma\right) = \gamma_{0,hmb} + \gamma_{1,hmb} \cdot a_{it} + \gamma_{2,hmb} \cdot t$$
$$f_j\left(h_{it}, m_i, b_i, a_{it} \mid \gamma\right) = \beta_{0,hmb} + \beta_{1,jhmb} \cdot a_{it} + \beta_{2,jhmb} \cdot t \qquad j = 2, \ldots, 5\,,$$

In Table A.10, we report the estimated increase in life expectancies over the 20-year period using the model without health heterogeneity. In Table A.12 we repeat the exercise, now allowing for differences in health. To make the model manageable we again re-categorize health into three groups instead of the five reported in the HRS. For life expectancy at the age of 50, we report the results for the year 2010 as there were hardly any 50-year-olds in the survey in 2012, and we therefore cannot compute the distribution over health for this age group which is required for our estimates averaged over all health states. For analogous reasons, we report the life expectancy at age 70 for the year 1994 instead of 1992. In Table A.11, we contrast our estimates with the NVSS figures for the corresponding calendar years. As can be seen, the model with health heterogeneity is generally slightly closer to the NVSS figures. The largest discrepancies between the average estimates from the health model and the NVSS figures can be found for 50-year-old black women.

| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| Year: 1992 | 76.6 | 81.4 | 72.6 | 77.0 | 4.0 | 4.4 | −4.8 | −4.4 |
| | [76.0, 77.2] | [80.8, 82.0] | [71.1, 74.1] | [75.9, 78.2] | [2.3, 5.7] | [3.2, 5.6] | [−5.6, −4.0] | [−6.0, −2.9] |
| Year: 2012 | 79.2 | 82.7 | 75.5 | 78.7 | 3.7 | 4.0 | −3.4 | −3.1 |
| | [78.7, 79.9] | [82.1, 83.2] | [74.4, 76.8] | [77.5, 80.0] | [2.3, 5.1] | [2.5, 5.3] | [−4.1, −2.7] | [−5.0, −1.5] |
| **Age 70** | | | | | | | | |
| Year: 1992 | 81.9 | 84.9 | 80.1 | 83.2 | 1.8 | 1.7 | −3.0 | −3.2 |
| | [81.5, 82.3] | [84.5, 85.4] | [79.3, 80.9] | [82.5, 84.0] | [1.0, 2.7] | [0.8, 2.5] | [−3.6, −2.4] | [−4.1, −2.3] |
| Year: 2012 | 83.8 | 85.9 | 82.0 | 84.4 | 1.8 | 1.4 | −2.1 | −2.4 |
| | [83.4, 84.2] | [85.5, 86.3] | [81.1, 82.9] | [83.6, 85.5] | [0.8, 2.7] | [0.3, 2.4] | [−2.6, −1.6] | [−3.9, −1.1] |

**Table A.10:** Life expectancy by race and sex at the beginning and end of the sample period for model without health heterogeneity. Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Brackets indicate bootstrapped 95% confidence intervals.

| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| Year: 1992 | 77.1 | 81.9 | 73.0 | 78.5 | 4.1 | 3.4 | −4.8 | −5.5 |
| Year: 1994 | 77.2 | 81.7 | 73.1 | 78.5 | 4.1 | 3.2 | −4.5 | −5.4 |
| Year: 2010 | 79.7 | 83.3 | 76.6 | 81.1 | 3.1 | 2.2 | −3.4 | −4.5 |
| Year: 2012 | 79.9 | 83.4 | 77.0 | 81.5 | 2.9 | 1.9 | −3.5 | −4.5 |
| **Age 70** | | | | | | | | |
| Year: 1992 | 82.4 | 85.6 | 81.0 | 84.3 | 1.4 | 1.3 | −3.2 | −3.3 |
| Year: 1994 | 82.5 | 85.4 | 81.0 | 84.1 | 1.5 | 1.3 | −2.9 | −3.1 |
| Year: 2010 | 84.2 | 86.4 | 82.9 | 85.8 | 1.3 | 0.6 | −2.2 | −2.9 |
| Year: 2012 | 84.4 | 86.5 | 83.2 | 85.9 | 1.2 | 0.6 | −2.1 | −2.7 |

**Table A.11:** Life expectancy by race and sex at the beginning and end of the sample period taken from the NVSS. Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Data source: https://www.cdc.gov/nchs/products/life_tables.htm

| | Nonblack | | Black | | Diff. in race | | Diff. in sex | |
|---|---|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female | Nonblack | Black |
| **Age 50** | | | | | | | | |
| *Average over health* | | | | | | | | |
| Year: 1992 | 76.9 | 81.6 | 73.0 | 77.1 | 3.9 | 4.5 | −4.7 | −4.1 |
| | [76.3, 77.5] | [81.0, 82.2] | [71.4, 74.7] | [75.9, 78.3] | [2.2, 5.7] | [3.4, 5.8] | [−5.5, −3.9] | [−5.6, −2.4] |
| Year: 2010 | 79.1 | 82.7 | 75.5 | 79.1 | 3.6 | 3.6 | −3.6 | −3.6 |
| | [78.5, 79.6] | [82.2, 83.1] | [74.4, 76.5] | [78.0, 80.4] | [2.3, 4.9] | [2.2, 4.7] | [−4.2, −2.9] | [−5.2, −2.0] |
| *(1) Best health* | | | | | | | | |
| Year: 1992 | 77.6 | 82.1 | 74.1 | 78.1 | 3.5 | 4.0 | −4.6 | −4.1 |
| | [77.0, 78.1] | [81.5, 82.7] | [72.6, 75.6] | [77.0, 79.2] | [2.0, 5.0] | [2.9, 5.2] | [−5.3, −3.8] | [−5.5, −2.4] |
| Year: 2010 | 79.9 | 83.3 | 76.4 | 80.1 | 3.5 | 3.2 | −3.4 | −3.7 |
| | [79.4, 80.3] | [82.8, 83.8] | [75.4, 77.5] | [79.0, 81.4] | [2.3, 4.7] | [1.9, 4.5] | [−4.0, −2.9] | [−5.4, −2.1] |
| *(3) Worst health* | | | | | | | | |
| Year: 1992 | 74.4 | 79.9 | 71.4 | 75.5 | 3.1 | 4.4 | −5.5 | −4.2 |
| | [73.7, 75.2] | [79.2, 80.6] | [69.5, 73.4] | [74.4, 76.8] | [1.0, 5.2] | [3.0, 5.7] | [−6.4, −4.5] | [−5.9, −2.2] |
| Year: 2010 | 76.6 | 80.9 | 73.9 | 77.7 | 2.7 | 3.1 | −4.2 | −3.8 |
| | [75.9, 77.3] | [80.3, 81.4] | [72.6, 75.2] | [76.6, 79.2] | [1.2, 4.2] | [1.7, 4.3] | [−5.0, −3.4] | [−5.6, −2.2] |
| **Age 70** | | | | | | | | |
| *Average over health* | | | | | | | | |
| Year: 1994 | 82.3 | 85.1 | 80.3 | 83.2 | 2.0 | 1.9 | −2.8 | −2.9 |
| | [81.9, 82.7] | [84.7, 85.5] | [79.6, 81.0] | [82.6, 83.9] | [1.2, 2.7] | [1.1, 2.6] | [−3.3, −2.3] | [−3.7, −2.0] |
| Year: 2012 | 83.8 | 86.0 | 82.3 | 84.8 | 1.6 | 1.2 | −2.1 | −2.5 |
| | [83.5, 84.2] | [85.6, 86.4] | [81.3, 83.1] | [83.8, 85.9] | [0.6, 2.7] | [0.1, 2.2] | [−2.6, −1.6] | [−4.0, −1.2] |
| *(1) Best health* | | | | | | | | |
| Year: 1994 | 83.4 | 86.1 | 81.7 | 84.5 | 1.8 | 1.7 | −2.7 | −2.8 |
| | [83.0, 83.8] | [85.8, 86.6] | [81.0, 82.4] | [83.7, 85.2] | [1.0, 2.6] | [0.8, 2.5] | [−3.3, −2.2] | [−3.7, −1.8] |
| Year: 2012 | 85.2 | 87.2 | 83.2 | 85.8 | 2.0 | 1.4 | −1.9 | −2.6 |
| | [84.9, 85.6] | [86.7, 87.5] | [82.2, 84.2] | [84.8, 87.0] | [0.9, 3.1] | [0.1, 2.5] | [−2.4, −1.4] | [−4.3, −1.1] |
| *(3) Worst health* | | | | | | | | |
| Year: 1994 | 80.2 | 83.2 | 79.3 | 82.2 | 0.8 | 1.0 | −3.0 | −2.9 |
| | [79.8, 80.6] | [82.8, 83.6] | [78.7, 80.2] | [81.5, 83.0] | [0.0, 1.6] | [0.2, 1.8] | [−3.5, −2.6] | [−3.7, −2.0] |
| Year: 2012 | 81.5 | 83.8 | 80.8 | 83.4 | 0.7 | 0.3 | −2.3 | −2.7 |
| | [81.1, 81.9] | [83.4, 84.2] | [79.9, 81.6] | [82.5, 84.5] | [−0.3, 1.7] | [−0.7, 1.3] | [−2.8, −1.8] | [−4.0, −1.5] |

**Table A.12:** Life expectancy by race and sex at the beginning and end of the sample period for model with three health states. Right columns show differences in race (holding sex fixed) and sex (holding race fixed). Brackets indicate bootstrapped 95% confidence intervals.

# F Additional results from the OLG model

## F.1 Effective discount rates

An individual's effective discount rate (determined by the common discount factor $\beta$ and future survival probabilities) is time-varying and depends on the planning horizon. After a bad health shock, the discount rate immediately rises since it implies a shorter expected life span, while the opposite happens in the event of a good health shock.

The fact that the effective discount rate depends on the whole sequence of future age- and health-dependent transition and survival probabilities makes it difficult to compare across different individuals. We therefore use the following measure of effective average discounting to make such comparisons possible. For an individual of age $a$ in health $h$, we implicitly define the effective average discount rate $\varrho$ at a horizon of $T$ years as

$$\beta^T \cdot \Pr\left( s_{a+T} = 1 \,|\, m, b, h, a \right) = \left( \frac{1}{1+\varrho} \right)^T$$

This measure additionally relates the discounting of future states in our framework to the discount rate of a standard infinite-horizon model without survival risk, where the geometric mean discount rate is simply given by $\varrho = \beta^{-1} - 1$ irrespective of the forecast horizon.

Using a specific numerical example, in our OLG model the calibrated discount factor is $\beta = 0.9805$, and the survival probability of a nonblack male in excellent health at the age of 50 to age 100 is 1.12%. This implies that events 50 years in the future are effectively discounted at an average geometric rate of

$$\varrho = \Pr\left( s_{a+T} = 1 \,|\, m, b, h, a \right)^{-1/T} \beta^{-1} - 1 \approx 11.6\%$$

per year. Figure A.18 plots these rates for different time horizons for black and nonblack males. As the figure illustrates, the effective discount rate varies substantially in the population. As individuals age, the discount rate generally increases (due to a lower probability of survival), except for the worst health state which is associated with a markedly lower chance of survival even at middle ages. Moreover, with the exception of 70-year-olds in bad health, nonblack individuals have a higher effective discount rate because of their lower life expectancy.

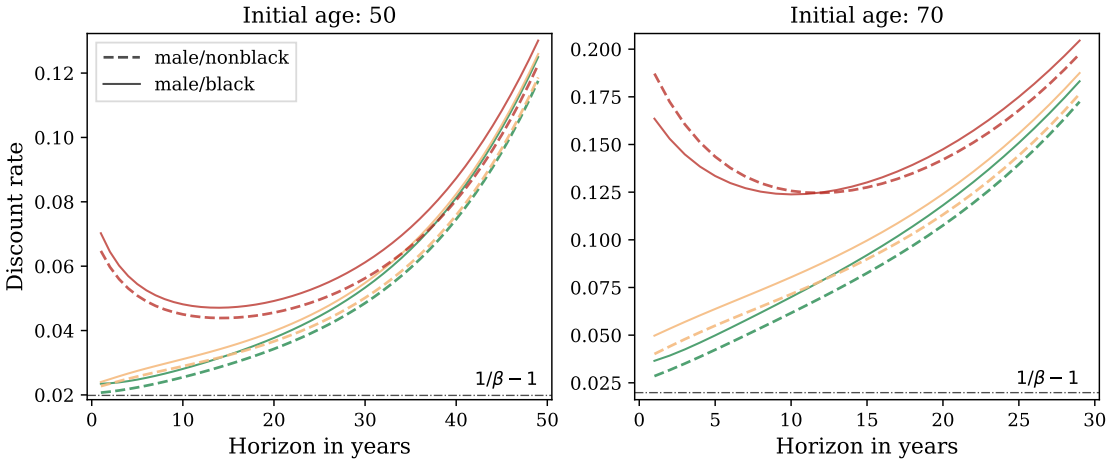



**Figure A.18:** Effective discount rate as a function of current age (50 or 70) and forecast horizon. The thin black line indicates the discount rate without survival risk, $\beta^{-1} - 1$. Dark green indicates best ("excellent") while red indicates worst ("poor") health.

## F.2 Social Security wealth

### F.2.1 Social Security wealth in levels

Figure A.19 and Figure A.20 show the model-predicted Social Security wealth for nonblack and black males, respectively, for different interest rate scenarios (in rows), health, and labor productivity. The columns show Social Security wealth computed at the age of 50, 65 (the exogenous retirement age) and 70.

Unsurprisingly, the present value of Social Security benefits in *levels* is strongly inversely related to the interest rate, whereas the *relative* difference between black and nonblack groups is not very sensitive to which interest rate is applied. Intuitively, Social Security wealth increases with labor income (which is tied to labor productivity), except at the very top of the income distribution due to the cap on earnings subject to payroll taxes (the so-called contribution and benefit base), so that higher earnings do not translate into higher Social Security entitlements. In this model, the gradient with respect to health arises solely from the fact that less healthy individuals are less likely to receive benefits for many years, thus decreasing their Social Security wealth.

### F.2.2 Disaggregated welfare calculations

In Figure A.21, we show the CEV welfare measure disaggregated by pre-retirement labor productivity which is directly tied to labor and retirement income. Higher labor productivity translates into higher CEV values ceteris paribus since the amount of foregone benefits increases compared to a nonblack individual with a longer life expectancy.

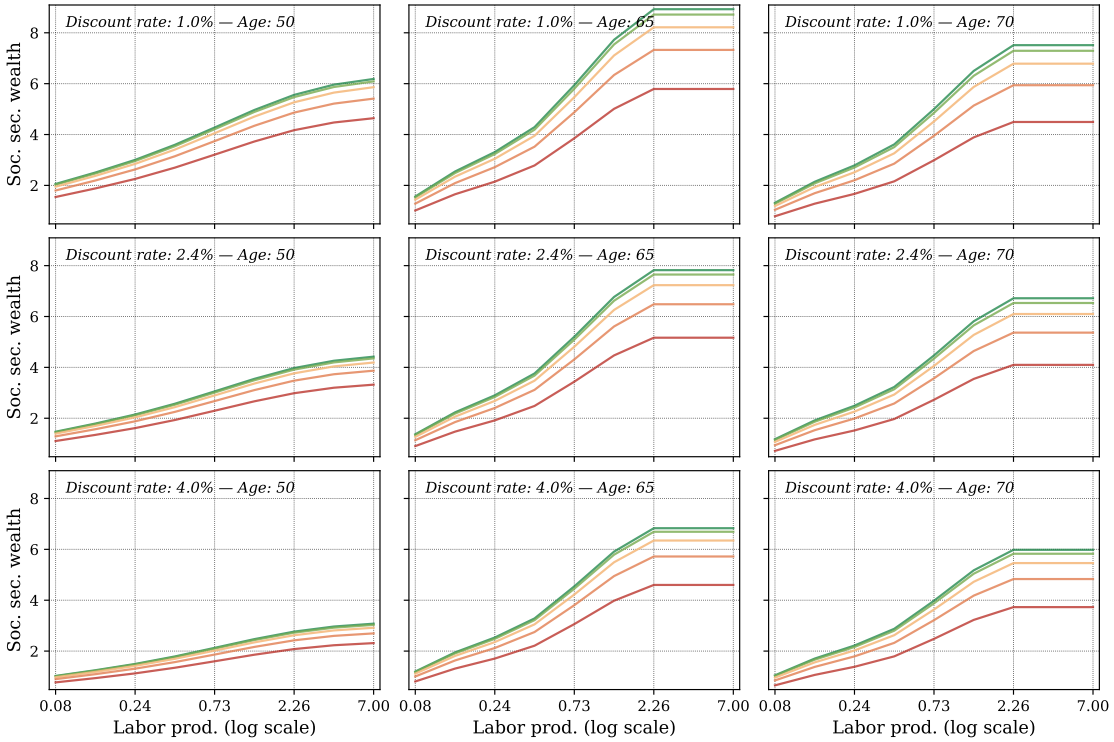



**Figure A.19:** Discounted expected Social Security wealth for *nonblack* males. Values on the y-axis are expressed in terms of average pre-tax annual earnings of working-age individuals. Average labor productivity is normalized to unity.

The figure should not be interpreted as saying that the highest losses are incurred by black individuals with the highest income and lowest cash-at-hand, since in equilibrium the probability of observing such individuals is close to zero. This is taken into account when computing the weighted average CEVs reported in the main text.

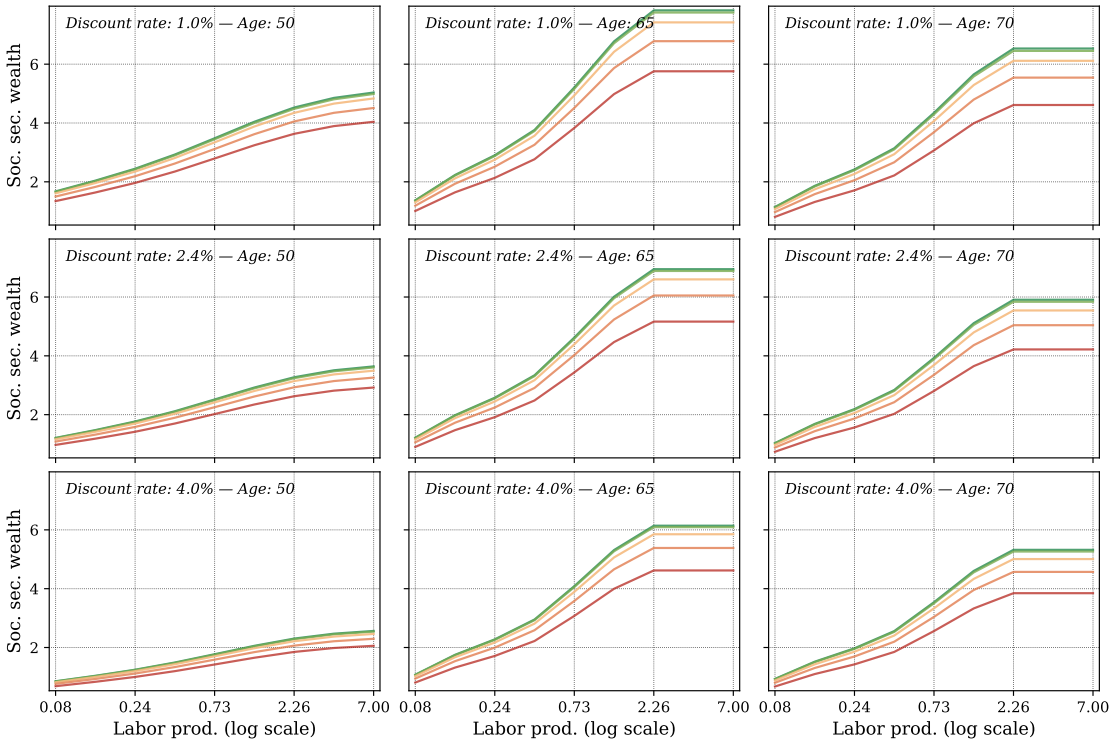
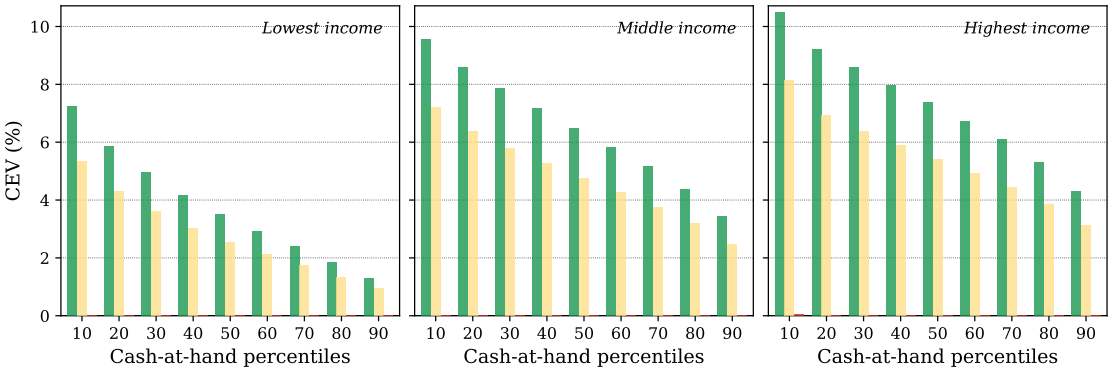
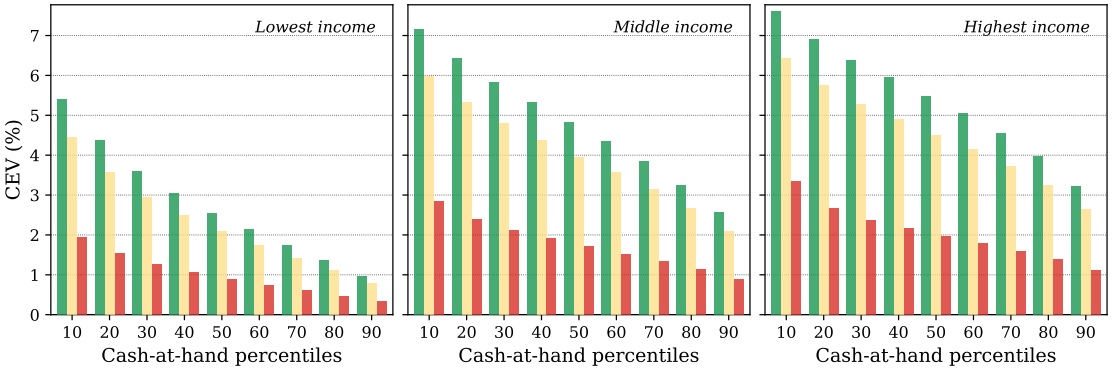



**Figure A.20:** Discounted expected Social Security wealth for *black* males. Values on the y-axis are expressed in terms of average pre-tax annual earnings of working-age individuals. Average labor productivity is normalized to unity.




**(a)** Black men




**(b)** Black women

**Figure A.21:** CEV implied by the difference in Social Security wealth between black and otherwise identical nonblack individuals at the age of 65. CEVs are shown for the lowest, middle and highest labor productivity, where productivities are obtained by discretizing an AR(1) process into a Markov chain with nine states. The worst health state for black men is not visible because the CEV is almost zero.

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