Wealth inequality and externalities from *ex ante* skill heterogeneity*

Konstantinos Angelopoulos
University of Glasgow and CESifo

Spyridon Lazarakis
University of Glasgow

James Malley
University of Glasgow and CESifo

July 13, 2017

Abstract

This paper develops an incomplete markets model with state dependent (Markovian) stochastic earnings processes and *ex ante* skill heterogeneity corresponding to being university educated or not. Using the Wealth and Assets Survey for Great Britain, we find that the university educated group has higher average wealth, higher earnings risk but lower within group wealth inequality. Using estimates of the earnings processes for each group to calibrate the model, we find wealth inequality within and between the groups that is consistent with the data. Moreover, the predictions for overall wealth inequality are closer to the data, compared to the benchmark model with *ex ante* identical households. In this framework, *ex ante* skill heterogeneity generates a between-group pecuniary externality which in turn leads to the predicted differences in wealth inequality between the groups and works as an amplification mechanism to increase overall wealth inequality.

Keywords: incomplete markets, education differences, pecuniary externalities

JEL Classification: E21, E25, H23

*We are grateful for financial support from the Economic and Social Research Council and the Scottish Government. The views expressed here are solely our own.

1 Introduction

There is an extensive literature that has examined the importance of idiosyncratic earnings shocks in generating wealth inequality when agents cannot fully insure against uncertain income streams. In this class of models, agents are *ex post* heterogeneous, even if *ex ante* identical. Since the original contributions of Bewley (1986), Imrohoroglu (1989), Huggett (1993) and Aiyagari (1994), these models have evolved to allow for increased economic realism. The benchmark incomplete markets model featuring stochastic labour income, one asset and *ex ante* identical agents, can capture qualitative properties of the wealth distribution. However, quantitatively it underpredicts the extent of inequality, both overall (e.g. as captured by measures such as the Gini index) and at the top end of the wealth distribution. Motivated by this, the literature has explored several extensions aimed at improving the model's predictions relating to wealth inequality (see e.g. the reviews in Quadrini and Rios-Rull (2014) and Krueger *et al.* (2016)).

In the benchmark incomplete markets model, wealth differences in the stationary equilibrium are attributed to exogenous conditions that change over time (given the underlying market failures). In this framework, differences in the experience of idiosyncratic shocks across individuals imply different choices for wealth accumulation, since the latter is state dependent. Naturally, then, the importance of increased uncertainty in creating higher inequality has long been noted in the literature (see e.g. Castaneda *et al.* (2003), De Nardi *et al.* (2010), Benhabib *et al.* (2011)).[1] Hence, the need to use high quality earnings data is critical in evaluating the role of individual risk in generating inequality.[2]

In addition to luck that is associated with uninsured idiosyncratic earnings shocks, other factors that have an impact on choices contribute to wealth inequality.[3] As a result, extensions to the benchmark model in the literature have also analysed factors relating to, amongst others, work effort and oc-

[1] In fact, since Castaneda *et al.* (2003), a common practice in the literature to help the model capture the wealth inequality is to exactly calibrate the characteristics of the stochastic earnings process. This has been a very useful approach when the interest of the analysis is not the evaluation of different sources of inequality, but rather the use of the model as vehicle to study properties of the economic system and counterfactuals (e.g. in the form of evaluating the inequality implications of economic policy, see e.g. Meh (2005) and Kitao (2008)).

[2] See e.g. Quadrini and Rios-Rull (2014) on the importance of earnings and wealth data for the calibration and evaluation of incomplete markets models.

[3] For instance, Frank Knight (1935, p. 48) writes: "The ownership of personal or material productive capacity is based upon a complex mixture of inheritance, luck and effort, probably in that order of relative importance".

cupational choice (e.g. Quadrini (2000)), bequests and inheritance motives (e.g. De Nardi (2004)), and differences in preferences (e.g. Krusell and Smith (1998)). While *ex ante*, permanent differences between the agents regarding their productive capacity have also been considered (see e.g. Castaneda *et al.* (1998), Quadrini (2000), and Guvenen (2006)), these have not always been shown to help the model improve its predictions regarding inequality (see e.g. De Nardi (2015) and Krueger *et al.* (2016)).[4] Nevertheless, ability and skills related to the level of education at the beginning of work life, which are in effect permanent differences between workers, are fundamental determinants of an individual's level of earnings and exposure to earnings risk.[5]

In this paper, we examine the role of *ex ante* skill differences corresponding to being university educated or not in generating wealth inequality between groups, within groups, and for the whole economy. We examine both the magnitude of the contribution of this heterogeneity to generate wealth inequality and the mechanism by which this takes place. In particular, we show that key to model's predictions on wealth inequality is that the *ex ante* skill heterogeneity generates a pecuniary externality that works via the interest rate. To highlight the working and importance of this channel, we add this type of *ex ante* heterogeneity to the benchmark incomplete markets model of Aiyagari (1994). In this framework, these *ex ante* and permanent skill differences interact with luck and choices, to create *ex post* inequality.

1.1 Skill heterogeneity and wealth inequality

We specify an otherwise standard Aiyagari (1994) model with state-dependent (Markovian) stochastic earnings processes and allow the two groups of households to differ in their earnings processes, both in the state-space and in the transition matrix for idiosyncratic earnings shocks. Using recent advances in theoretical research in Acikgoz (2016), this model can be shown to have a well-defined stationary equilibrium with a unique invariant wealth distribu-

[4]Differences in initial conditions on assets do not affect inequality in this class of models in a stationary equilibrium that is characterised by a unique invariant distribution. However, *ex ante* differences in e.g. skill, preferences, or access to markets, which capture predetermined, but permanent differences between the agents, do affect the equilibrium and thus potentially can affect inequality.

[5]On the importance of skills and education for inequality in a historical context see e.g. Goldin and Katz (2008). Several studies have also documented differences in earnings risk between skilled and unskilled groups associated with university education (see e.g. Castro and Coen-Pirani (2008) and Hagedorn *et al.* (2016) for the US, as well as Angelopoulos *et al.* (2017) for Great Britain).

tion for each type of household.[6]

We calibrate the model to Great Britain (GB) and use earnings and wealth data from the Wealth and Asset Survey (WAS). The WAS is an extensive survey that, to our knowledge, has not been used previously to calibrate heterogeneous agent, incomplete markets models. Using the WAS survey presents several advantages that are important for the aim of the analysis.

First, WAS has an extensive coverage of 41,000 individuals (23,000 households) on average over four waves. It is carefully designed so that it does not under-sample from the wealthy. Moreover, it provides sufficient detail to allow us to construct measures of earnings net of taxes and inclusive of benefits. This measure coheres well with labour income in the model on which households base their consumption and savings decisions.

Second, it allows us to split the sample based on whether individuals have a university degree or not at the cutoff age of 25. Given that the transition from the non-university to the university group is effectively zero past that age in the data, we can treat university education as a proxy of an *ex ante* and permanent labour market skill. University education is related to higher mean earnings and also implies differences in idiosyncratic shock processes. We estimate both of these from the data and find that university educated have higher earnings uncertainty.

Finally, WAS allows us to calculate the wealth distribution for both groups of the university and non-university educated, as well as the whole sample, consistently and for the same households for which we estimated earnings processes. We find that university educated have more than two times higher wealth than non-university educated across the four waves. In addition, within group wealth inequality is consistently higher for the non-university educated group, and the wealth inequality Gini index over the whole sample is about 0.7 across the four waves.

Using the WAS data to estimate the earnings processes we find that the model predicts wealth inequality both within and between the university and non-university educated groups that is consistent with the data. In particular, the university educated group has significantly lower within group wealth inequality than the non-university educated group, despite having more persistent and volatile stochastic earnings processes. Moreover, mean wealth of university educated is more than twice as high as mean wealth for the non-university educated. The model also predicts a Gini wealth inequality index of about 0.64 and, although it still under-predicts the extent of income

[6] Acemoglu and Jensen (2015) have also provided a general proof of existence of equilibrium for this class of models.

inequality at the very top end (top 1%), it nearly matches wealth ownership of the very high percentiles, i.e. for the top 90-95% and for the top 95-99%.

To investigate the contribution of *ex ante* skill heterogeneity to the improved performance of the model, we consider a benchmark model without *ex ante* skill heterogeneity. We find that this model generates a Gini index for wealth inequality about 0.59. Therefore, *ex ante* skill heterogeneity contributes to an increased overall Gini index, in addition to allowing the model to produce between and within group inequality. *Ex ante* skill heterogeneity also improves the model's predictions regarding the wealth concentration in the top percentiles. Given that the potential for *ex ante* skill heterogeneity to generate wealth inequality has not been fully analysed in the literature to date, it is worth briefly discussing the channel by which it takes place.

1.2 Pecuniary externality

The main reason why *ex ante* skill heterogeneity matters for wealth inequality is that the differences in the processes for earnings between the groups create an externality that affects within-group inequality via aggregate savings and the interest rate. In particular, earnings differences, both in terms of mean earnings and idiosyncratic uncertainty, suggest different asset supply functions. The equilibrium interest rate is determined by the aggregate asset supply function, which is higher (lower) than the asset supply functions for university and non-university groups respectively. In other words, the savings of each group move the market interest rate away from the level that would be the equilibrium outcome consistent with the asset supply of each group. Consequently, households in the non-university and university educated groups lower and raise their savings respectively, which in turn implies that within group wealth inequality is increased for the non-university and decreased for the university educated.

The above suggests that *ex ante* skill heterogeneity in this framework generates a between-group pecuniary externality.[7] In fact, we find that without between-group pecuniary externalities, the model with *ex ante* identical non-university population produces lower wealth inequality than a model with *ex ante* identical university-educated population, which is contrary to the data. In turn, the correct ranking of within group inequalities is critical in improving (i.e. in this case, in increasing) the model's inequality predictions at

[7] The importance of pecuniary externalities implicit in the benchmark model with *ex ante* identical agents for the efficiency implications of general equilibrium has been pointed out in the literature (see e.g. Greewald and Stiglitz (1986) and Davilla *et al.* (2012)). Here we examine pecuniary externalities arising from *ex ante* skill heterogeneity, and focus on their implications for wealth inequality, as opposed to efficiency.

the aggregate level, compared with the model with *ex ante* identical agents. This is explained by the higher proportion of non-university educated in the population.

Therefore, the pecuniary externality embodied in the model with *ex ante* skill heterogeneity is critical in generating the correct ranking of within group wealth inequality and works as an amplification mechanism increase aggregate wealth inequality. This pecuniary externality is due to both factors defining *ex ante* heterogeneity, i.e. the differences in mean earnings, as well as in the transition matrices. We further show that both factors are important quantitatively and need to work in the right direction, for the pecuniary externality to improve the model's predictions on wealth inequality.

Since inequality in this framework is to some extent a result of an externality, we consider a Pigouvian-type policy to reduce overall inequality. In particular, we examine a form of Pigouvian-type incentivisation policy that aims to encourage savings for the group of low skilled and discourage them for the group of high skilled. Our general finding is that this scheme reduces overall inequality, inequality within the group of the non-university educated, as well as between group inequality. While the effect on inequality is substantial, the effects on aggregate quantities are trivial. On the other hand, such policies increase inequality within the group of university educated, and the main redistribution is from the university poor to the non-university poor.

The rest of the paper is organised as follows. We first present the model and data/calibration in Sections 2 and 3 respectively. The model is discussed in some detail to formally introduce the economic environment and clarify the economic quantities used later. We also consider the data and estimation of the earnings processes, given that the dataset is important in facilitating the analysis undertaken. We then examine the quantitative implications of the model. We first evaluate the predictions of the model with respect to inequality in Section 4. In Section 5 we then analyse the pecuniary externality mechanism which is at the heart of the nexus between *ex ante* skill heterogeneity and wealth inequality. Following this, we analyse the effects of Pigouvian-type intervention in Section 6 and present our conclusions in Section 7. Finally, we provide Appendices including details relating to the data, computational algorithms and the stationary recursive equilibrium with policy.

2 Economic environment

We compute the long-run *stationary equilibrium* of an economy that is populated by a continuum of infinitely lived agents (households) distributed on the interval $I = [0, 1]$ with measure ϕ. Time is discrete and denoted by $t = 0, 1, 2, ...$There are two types of households, university educated households, which belong to a set $I^u \subset I$ and households that have a level of education below university, which belong to a set $I^b \subset I$, such that $I^u \cup I^b = I$ and $I^u \cap I^b = \emptyset$. The proportions of university and non-university educated households are given respectively by $n^u \equiv \int_{I^u} i\phi\,(di)$ and $n^b \equiv \int_{I^b} i\phi\,(di) = 1 - n^u$. Therefore, there is *ex ante* heterogeneity in the population determined by the education level of the household, which is assumed to be given.[8]

All households derive utility from consuming one good that can be acquired by spending either labour income or accumulated savings. Households are identical in their preferences. However, their labour income depends on their education level, because it determines their productivity. In particular, while labour supply is exogenous and fixed to unity, households' predictable earnings component differ, reflecting their different education, skill level and participation in production, so that the two groups of households face different wage rates. In addition, each household is subject to idiosyncratic shocks, which affect labour income, by determining residual labour productivity. Households draw idiosyncratic shocks independently from a Markov chain which differs for university and non-university educated households. In particular, both the state-space and the transition matrix differ across the two household types, implying that both the level of labour income and the size and persistence of productivity shocks differ for each household type, reflecting different opportunities and earnings risk.

There are incomplete financial markets which implies that households cannot insure against shocks to labour income. In particular, there is a single asset in the economy. In a stationary equilibrium, aggregate quantities are constant. In what follows we present the problem for a "typical" university educated household, denoted by the superscript u, and the problem for a "typical" below-university educated household, denoted by the superscript b.

2.1 Idiosyncratic labour productivity

Denote the idiosyncratic component of labour productivity of a typical household $h = u, b$ at time t by s_t^h. At the beginning of period t, the household observes the realisation of its idiosyncratic labour productivity shock, which follows a Markov chain with state-space S^h and transition matrix Q^h. We

[8]The earnings and wealth data in the U.K. refer to households whose head is University educated or not. At the age of 25, which is the minimum age for heads of households in our sample, the education level is predetermined.

follow Acikgoz (2016) and assume that the stochastic process $\left(s_t^h\right)_{t=0}^{\infty}$ satisfies `Assumption 1`.[9]

`Assumption 1`

(a) There are finitely many possible realisations of labour productivity, $[1, ..., m]$ and it evolves according to an m-state Markov chain with $m \times m$ transition matrix $Q_{ss'}^h = \Pr\left(s_{t+1}^h = s' | s_t^h = s\right)$, and $S^h = [\overline{s}_1^h, \overline{s}_2^h, ..., \overline{s}_m^h]$, $\overline{s}_1^h \geq 0$, $\overline{s}_{j+1}^h > \overline{s}_j^h$, $j = 1, ..., m-1$ is the state-space with the σ-algebra $\mathcal{S}^h$ that is the power set of S^h. Denote by π_{ij}^h the elements of $Q_{ss'}^h$.

(b) There exists n_0 such that $\left(\pi_{ij}^h\right)^n > 0$, $\forall (i,j)$, for all $n > n_0$, where $n \in \mathbb{N}_+$. Moreover, $\pi_{11}^h > 0$.

The transition matrix $Q_{ss'}^h$ provides the conditional probability that the household will be in state s' in period $t+1$, given that it is in state s in period t. Part (b) implies that the Markov chain is *irreducible* and *aperiodic* and guarantees that it has a unique invariant distribution. We denote the unique invariant distribution by ξ^h.

2.2 Households

Households have a perfectly inelastic labour supply (normalised to 1), different skill levels ζ^h, $h = u, b$, and do not care for leisure. They derive utility from consuming one good that can be acquired by either spending labour income or accumulated savings of a single asset. They receive a productivity shock s_t^h which is observed at the beginning of period t. Thus, each period, households receive labour income $w\zeta^h s_t^h$ and interest income from accumulated assets ra_t^h, and use their income for consumption and to invest in future assets, subject to the budget constraint for each $h = u, b$:

$$c_t^h + a_{t+1}^h = (1+r)\, a_t^h + w\zeta^h s_t^h, \tag{1}$$

where $c^h \geq 0$, $a_t^h \geq -\phi^h$ and $-\phi^h < 0$ denotes a borrowing limit on the household. The set comprising a_t^h is defined as $\mathcal{A}^h = [-\phi^h, +\infty)$. The prices (interest rate and wage rates) are assumed to be fixed and non-random quantities. This holds if the household's actions take place in a stationary equilibrium, which is defined below. A household chooses how much of its income to

[9] On notation. For any set D in some n-dimensional Euclidean space $\mathbb{R}^n$, $\mathcal{B}(D)$ denotes the Borel $\sigma-$algebra of D. The set of probability measures on the measurable space $(D, \mathcal{B}(D))$, is denoted by $\mathcal{P}(D)$.

consume and how much to invest by accumulating assets. Households assess consumption streams with an intertemporal discount factor $\beta \in (0,1)$, using a per period utility function $u(c_t^h)$, which satisfies the following assumption:

`Assumption 2`

The function $u : [0, +\infty) \rightarrow \mathbb{R}$ is bounded, twice continuously differentiable, strictly increasing and strictly concave. Furthermore, it satisfies the conditions $\lim_{c \rightarrow 0} u_c(c) = +\infty$, $\lim_{c \rightarrow \infty} u_c(c) = 0$ and $\lim_{c \rightarrow \infty} \inf -\frac{u_{cc}(c)}{u_c(c)} = 0$.

The assumptions imposed on the utility function are typically employed in the literature of partial equilibrium income fluctuation problems (see e.g. Miao (2014, ch. 8)) and in the literature relating to incomplete markets with heterogeneous agents in general equilibrium ((see e.g. Aiyagari (1994) and Acikgoz (2016))). The assumption that $\lim_{c \rightarrow \infty} \inf -\frac{u_{cc}(c)}{u_c(c)} = 0$ implies that the degree of absolute risk aversion tends to zero as consumption tends to infinity.[10]

The interest rate and wage rate are taken as given and satisfy $r > -1$ and $w > 0$. Moreover, as has been shown (see e.g. Aiyagari (1994), Miao (2014, ch. 8) and Acikgoz (2016)), a necessary condition for an equilibrium with finite assets at the household level in this class of models is that $\beta(1+r) < 1$. Borrowing limits are imposed following e.g. Aiyagari (1994), i.e. assets must satisfy:

$$\begin{aligned} &a_t^h \geq -\phi^h, \text{ where} \\ &\phi^h = \min\left[\gamma, \frac{\overline{s}_1^h \zeta^h w}{r}\right], \text{ if } r > 0 \text{ or} \\ &\phi^h = \gamma, \text{ if } r \leq 0 \end{aligned} \tag{2}$$

and $\gamma > 0$ is arbitrary parameter, capturing an *ad hoc* debt limit. This restriction implies that even if the financial markets have the power to confiscate all of the income of the household, they would never lend so much that the household reaches an asset position where its lifetime labour income (assuming the worst productivity shock always realised) was not sufficient to repay debt. This requires that $-r\phi^h + w\zeta^h \overline{s}_1^h > 0$. Hence, if the household is at the borrowing limit and receives the worst case labour income shock, it always has at least one option to have non-negative consumption, by borrowing again the maximum possible. The various assumptions on prices and tax rates are summarised below.

[10] Boundedness is not needed for equilibrium (see Acikgoz (2016)). In the calibration and computation below we will use a CRRA utility function which is not bounded below. However, we will work there with a compact set for assets, needed for computation, which, given the continuity of the utility function, implies boundedness.

`Assumption 3`

Assume that $(1+r)>0$, $w>0$, $\beta(1+r)<1$ and $-r\phi^h + w\zeta^h \overline{s}_1^h \geq 0$.

The problem of the typical household $h=u,b$ is summarised as follows. For given values of (w,r) that satisfy `Assumption 3` and given initial values $(a_0^h, s_0^h) \in \mathcal{A}^h \times S^h$, the household chooses plans $\left(c_t^h\right)_{t=0}^{\infty}$ and $\left(a_{t+1}^h\right)_{t=0}^{\infty}$ that solve the maximisation problem:

$$V^h(a_0,s_0) = \sup_{\left(c_t^h, a_{t+1}^h\right)_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^h), \tag{3}$$

subject to (2), where $\beta \in (0,1)$, $c_t^h \geq 0$ is given by (1), $u(c_t^h)$ satisfies `Assumption 2` and s_t^h satisfies `Assumption 1`. To obtain the dynamic programming formulation of the household's problem, let $v^h\left(a_t^h, s_t^h; w, r\right)$ denote the optimal value of the objective function starting from asset-productivity state $\left(a_t^h, s_t^h\right)$ and given the interest and wage rate. The Bellman equation is:

$$v^h\left(a_t^h, s_t^h; w, r\right) =$$

$$= \max_{\substack{a_{t+1}^h \geq -\phi^h \\ c_t^h \geq 0}} \{u(c_t^i) + \beta \textstyle\sum_{s_{t+1}^h \in S^h} v^h\left(a_{t+1}^h, s_{t+1}^h; w, r\right) Q^h_{s_t^h, s_{t+1}^h}\}. \tag{4}$$

In this case, we aim to find the value function $v^h\left(a_t^h, s_t^h; w, r\right)$ and the policy functions $a_{t+1}^h = g^h\left(a_t^h, s_t^h\right)$ and $c_t^h = q^h\left(a_t^h, s_t^h\right)$, which generate the optimal sequences $\left(a_{t+1}^{*h}\right)_{t=0}^{\infty}$ and $\left(c_t^{*h}\right)_{t=0}^{\infty}$ that solve (3). Standard dynamic programming results imply that the policy functions exist, are unique and continuous.

Following e.g. Stokey *et al.* (1989, ch. 9), we define $\Lambda^h\left[(a,s), A\times B\right] : \left(\mathcal{A}^h \times S^h\right) \times \left(\mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right) \to [0,1]$, for all $a \times s \in \mathcal{A}^h \times S^h$, $A \times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h$, to be the transition functions on $\left(\mathcal{A}^h \times S^h\right)$, induced by the Markov processes $\left(s_t^h\right)_{t=0}^{\infty}$ and the optimal policies $g^h\left(a_t^h, s_t^h\right)$. In particular, the transition function is given by:

$$\Lambda^h\left[(a,s), A\times B\right] = \begin{Bmatrix} Q^h\left(s,B\right), \text{ if } g^h\left(a,s\right) \in A^h \\ 0, \text{ if } g^h\left(a,s\right) \notin A^h \end{Bmatrix}. \tag{5}$$

Given `Assumptions 1-3`, Proposition 5 in Acikgoz (2016) implies that the Markov process on the joint state-space $\left(\mathcal{A}^h \times S^h\right)$ with transition matrix Λ^h has, for each $h=u,b$, a unique invariant distribution denoted by $\lambda^h\left(A\times B\right)$. Furthermore, Proposition 6 in Acikgoz (2016) implies that assets for the typical household tend to infinity when $\beta(1+r) \to 1$. Moreover, Theorem 1 in Acikgoz (2016) implies that the expected value of assets using the invariant distribution is continuous in the net interest rate, r.

2.3 Firm

A single firm operates the technology to transform accumulated assets from the households to capital to be used in production and an aggregate constant returns to scale production function, using as inputs the average (per capita) levels of capital K and employment L. The production function is given by $F(K, L)$ and is assumed to satisfy the usual Inada conditions. In particular, F is continuously differentiable in the interior of its domain, strictly increasing, strictly concave and satisfies: $F(0, L) = 0$, $F_{KL} > 0$, $F_L > 0$, $\lim_{K \to 0} F_K(K, L) \to +\infty$ and $\lim_{K \to \infty} F_K(K, L) \to 0$. The capital stock depreciates at a constant rate $\delta \in (0, 1)$. The firm takes the interest and wage rate as given and chooses capital and employment to maximise profits, which gives the standard first order conditions, defining factor input prices equal to the relevant marginal products:

$$w = \partial F(K, L)/\partial L, \tag{6}$$

$$r = \partial F(K, L)/\partial K - \delta. \tag{7}$$

2.4 General equilibrium

We define a stationary recursive equilibrium following e.g. Ljungqvist and Sargent (2012, ch. 18), Miao (2014, ch. 17) and Acikgoz (2016). Aggregation over the households can be obtained by using the methods discussed e.g. in Acemoglu and Jensen (2015). The versions of the Strong Law of Large Numbers delivered by these methods (see e.g. Uhlig (1996) and Al-Najjar (2004)) imply that: (i) at the aggregate level idiosyncratic uncertainty is cancelled out, so that aggregate outcomes are fixed (non-random) quantities; and (ii) the invariant distribution at the household level also gives the proportion of households at the cross-sectional level. Aggregation implies the following market clearing conditions:

$$\begin{array}{l} K = \int_I a_t^i \phi\,(di) \\ L = \int_{I^u} \zeta^u s_t^i \phi\,(di) + \int_{I^b} \zeta^b s_t^i \phi\,(di) = \\ \quad = n^u \zeta^u \sum_{j \in S^u} \overline{s}_j^u \xi^u\left(\overline{s}_j^u\right) + n^b \zeta^b \sum_{j \in S^b} \overline{s}_j^b \xi^b\left(\overline{s}_j^b\right). \end{array} \tag{8}$$

We define the distribution of households over the joint state-space, for $h = u, b$. Given individual asset holdings a_t^i and exogenous shocks s_t^i at period t by household, $i \in I^h$, the joint distribution over asset accumulation and shocks across households for each household type, $\overline{\lambda}_t^h \in \mathcal{P}\left(\mathcal{A}^h \times S^h\right)$ is given by:

$$\overline{\lambda}_t^h (A \times B) = \varphi\left(i \in I^h : \left(a_t^i, s_t^i\right) \in A \times B, A \times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right). \quad (9)$$

The measure $\overline{\lambda}_t^h (A \times B)$ gives the fraction of households whose asset holdings and shocks at period t lie in the set $A \times B$. Using this, we can define the stationary recursive equilibrium as follows.

`Definition of Stationary Recursive Equilibrium`

For $h = u, b$, a *Stationary Recursive Equilibrium*, is aggregate stationary distributions $\overline{\lambda}^h (A \times B)$, policy functions $a_{t+1}^h = g^h\left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathcal{A}^h$, $c_t^h = q^h\left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathbb{R}_+$, value functions $v^h\left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathbb{R}$, and positive real numbers $K, w(K), r(K)$ such that:

1. The firm maximises its profits given prices, so that $(w(K), r(K))$ satisfy

$$w(K) = \partial F(K, L)/\partial L, \quad (10)$$
$$r(K) = \partial F(K, L)/\partial K - \delta. \quad (11)$$

2. The policy functions $a_{t+1}^h = g^h\left(a_t^h, s_t^h\right)$ and $c_t^h = q^h\left(a_t^h, s_t^h\right)$ solve the households' optimum problems in (4) given prices and aggregate quantities, and the value functions $v^h\left(a_t^h, s_t^h\right)$ solve equations (4).

3. $\lambda^h (A \times B)$ is a stationary distribution

$$\lambda^h (A \times B) = \int_{\mathcal{A}^h \times S^h} \Lambda^h [(a, s), A \times B]\, \lambda^h (da, ds), \quad (12)$$

for all $A \times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h$, where $\Lambda^h [(a, s), A \times B] : \left(\mathcal{A}^h \times S^h\right) \times \left(\mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right) \to [0, 1]$ are transition functions on $\left(\mathcal{A}^h \times S^h\right)$ induced by the Markov process $\left(s_t^h\right)_{t=0}^{\infty}$ and the optimal policy $g^h\left(a_t^h, s_t^h\right)$.

4. When $\lambda^h (A \times B)$ describe the cross-section of households at each date, i.e. $\overline{\lambda}^h (A \times B) = \lambda^h (A \times B)$, markets clear. In particular, asset market clears, that is the cross-section average value of K is equal to the average of the households' decisions

$$\begin{aligned} K = n^u \textstyle\int_{\mathcal{A}^u \times S^u} g^u (a, s)\, \overline{\lambda}^u (da, ds) + \\ + n^b \textstyle\int_{\mathcal{A}^b \times S^b} g^b (a, s)\, \overline{\lambda}^b (da, ds). \end{aligned} \quad (13)$$

Secondly, the labour market clears

$$\begin{aligned} L = n^u \zeta^u \int_{\mathcal{A}^u \times S^u} s^u (a, s) \overline{\lambda}^u (da, ds) + \\ + n^b \zeta^b \int_{\mathcal{A}^b \times S^b} s^b \overline{\lambda}^b (da, ds) = 1, \end{aligned} \tag{14}$$

and the goods market clears, which, using factor input market clearing, implies

$$\begin{aligned} & F(K, 1) - \delta K = \\ & = n^u \int_{\mathcal{A}^u \times S^u} q^u (a, s) \overline{\lambda}^u (da, ds) + n^b \int_{\mathcal{A}^b \times S^b} q^b (a, s) \overline{\lambda}^b (da, ds). \end{aligned} \tag{15}$$

Following standard arguments (commonly used in this class of models since Aiyagari (1994)), it is straight forward to show that continuity of the asset supply and demand functions at the aggregate level with respect to the interest rate as well as the limit properties of supply and demand for assets, imply that a general equilibrium exists.[11] A more general proof of existence of equilibrium for this class of models can be found in Acemoglu and Jensen (2015).

3 Calibration

We calibrate the model to British data, at an annual frequency, and estimate the parameters relating to the Markov processes for the idiosyncratic shocks for the university and non-university educated households using data from the Wealth and Assets Survey (WAS). Specifically, we make use of the 3rd and 4th waves for which we have earnings, income and wealth data at the household level.[12] These two waves cover the period between 2010 and 2014 (see Appendix A for details). We then evaluate the predictions of the model regarding wealth inequality against the data. In particular, we compute the general equilibrium solution of the model by implementing a standard numerical algorithm which is summarised in Appendix B.

There are two types of parameters in the model. The first refers to parameters that differ between households and capture their labour productivity,

[11] For details on a proof that can be applied here see Acikgoz (2016), Theorem 1, and note that continuity of mean assets with respect to the interest rate, for each type of household, implies continuity for the weighted average between households as well.

[12] We are using waves 3 and 4 since these are the only waves that contain measures of net disposable income at the household level. Whilst waves 1 and 2 provide the relevant earnings data they do not report social benefits, transfers and in general non-labour income.

income and idiosyncratic uncertainty. We calculate these parameters using individual and household earnings data from the Wealth and Asset Survey (WAS). The second category refers to parameters that are common to both types of households, in particular parameters relating to preferences, and production. We set or calibrate this group of parameters following common practice in general equilibrium models and GB data.

3.1 Wealth inequality

The WAS is a longitudinal survey reporting information on earnings, income, the ownership of assets (financial assets, physical assets and property), pensions, savings and debt in Great Britain (GB).[13] An important feature of WAS is that it uses a 'probability proportional to size' method of sampling cases. This means that the probability of an address being selected is proportional to the number of addresses within a given geographic area, with a higher number of addresses being selected from densely populated areas. The design of WAS recognizes the fact that wealth is highly skewed, with a small proportion of households owning a large share of the wealth. Thus, WAS over-samples addresses likely to be in the wealthiest 10% of households at a rate three times the average.

Another strength of WAS is the longitudinal nature of the survey. Each wave of the survey returns to households and individuals interviewed at the previous wave. From the third wave forward, new cohorts samples were introduced at each wave to take account of the reduction in the size of the existing sample due to the attrition that naturally occurs between waves. Longitudinal analysis of the survey allows changes in wealth to be tracked and the large overall sample size provides robust cross-sectional estimates. This ensures both good coverage of the very wealthy and more precise estimates of overall household wealth. However, as in similar surveys the very rich (e.g. Forbes 400) are not included and this can affect the estimates of the top 1%.

Households are defined as the family or group of individuals who are living in the same residence. The head is defined as the member of the household in whose name the accommodation is owned or rented, or is otherwise responsible for the accommodation. Following the quantitative literature using incomplete markets models, we select household heads between 25-59 years of age with non-zero income. We use household net worth as our measure for wealth. It is the sum of assets minus debt for all household members. Net worth also admits a substantial proportion of the population which have negative current wealth. Details on the wealth data are in Appendix A.

[13] The WAS does not provide information for Northern Ireland.

We summarise, in Table 1, the main characteristics of wealth inequality for the whole sample as well as separately for the groups of university and non-university educated, using WAS. In defining these two groups, we use the information provided in WAS on education achievement and classify those individuals who have degree level or above as belonging to university group, and otherwise to the below university educated group. Households are allocated to a group depending on the educational attainment of the head of the household. The four waves correspond, respectively, to the time periods 2006-08, 2008-10, 2010-12 and 2012-14.

Table 1: Wealth Inequality in Great Britain

	Gini Total	Gini Uni	Gini non-Uni	$\frac{\text{wealth Uni}}{\text{wealth non-Uni}}$
wave 1	0.6754	0.6239	0.6850	1.9816
wave 2	0.6852	0.6247	0.7040	2.0552
wave 3	0.7077	0.6474	0.7295	2.1617
wave 4	0.7372	0.6853	0.7488	2.4132

Table 1 shows that wealth inequality has increased between waves 1 and 4. For example: (i) the overall Gini index increases from 0.675 to 0.737; (ii) the respective Gini indices for the two sub-groups have increased; and (iii) between-group wealth inequality (i.e. the ratio of mean wealth for the two groups) has rise.[14]

Note, however, that the relative magnitude of within-group wealth inequality between the two groups (i.e. the relative Gini indices between the two groups) has not changed much over time, suggesting that this qualitative characteristic of wealth inequality have remained more stable. It is also worthwhile noting when looking at Table 1 that in GB, wealth inequality is lower overall compared with the US data (see e.g. Rios-Rull and Kuhn (2016)).

3.2 Markovian processes

Household net disposable income is our main measure of income that we use to estimate the extent and persistence of idiosyncratic income uncertainty since wealth inequality is measured using household-level data.[15] We estimate the parameters pertaining to idiosyncratic earnings uncertainty for the whole sample and also separately for the university and non-university educated groups.

[14] Moreover, the wealth share of the top percentiles has also generally increased over time.

[15] Note that we include the self-employed.

3.2.1 Household income process

Household income is composed of: (i) a component capturing aggregate conditions common to all individuals/households; (ii) predictable part at the individual/household level (capturing observed characteristics of the household); and (iii) an element capturing idiosyncratic shocks. We denote the natural logarithm of the measure of income in wave t as $y_{i,t}^h$, for $h = u, b$, and assume that it follows the process:

$$y_{i,t}^h = \beta_{0,t}^h + f_t^h(x_{i,t}) + \widehat{\varepsilon}_{i,t}^h \tag{16}$$

where $\beta_{0,t}^h$ captures effects that are common to all individuals/households; the function $f_t^h(x_{i,t})$ includes observable characteristics which may affect labour income, e.g. family composition, gender, education, experience, race, region of residence; and $\widehat{\varepsilon}_{i,t}^h$ is the unobserved idiosyncratic component. Note that the aggregate and predictable components depend on time, implying that the coefficients capturing the effect of observables are allowed to be time-varying. We are mainly interested in the statistical properties of $\widehat{\varepsilon}_{i,t}^h$, for the groups considered. Hence, we need to partial out the other effects. In line with the literature, (see e.g. Mincer (1974), Meghir and Pistaferri (2004), and Blundell and Etheridge (2010)) we specify the relationship in (16) as:

$$\begin{aligned} y_{i,t}^h = \beta_{0,t}^h + \textstyle\sum_{j=1}^{6} \beta_{j,t}^h AGE_i + \beta_{7,t}^h ED_i + \beta_{8,t}^h ASSORT_i + \\ + \beta_{9,t}^h HSIZE_i + \beta_{10,t}^h MAR_i + \beta_{11,t}^h GENDER_i + \widehat{\varepsilon}_{i,t}^h. \end{aligned} \tag{17}$$

where $t = 1, 2$ for waves 3 and 4 respectively. As discussed in more detail in Appendix A, the interviews of the individuals in waves 3 and 4 have a two year gap in between them.

AGE_i denotes age-group dummies referring to the age of the head. ED_i is an education level dummy and captures the income difference between agents with some qualifications and with those with no qualifications. It applies to non-university educated group only. $ASSORT_i$ is a dummy variable that aims to capture the effect on income variability of assortative mating i.e. if the head and the spouse of household belong both to the same educational group or not. $HSIZE_i$ is number of members living in the household, while, MAR_i, and $GENDER_i$ are dummies for the marital status and sex of the head of the household respectively. We run least squares regressions using equation (17) for each wave separately since we find that the parameters change over time. We retain the residuals $\widehat{\varepsilon}_{i,t}$ for $t = 1, 2$ as a proxy for the unobserved component of $y_{i,t}$ and, following the literature, assume that they are determined by an exogenous $AR(1)$ process:

$$\widehat{\varepsilon}_{i,2}^h = \rho^h \widehat{\varepsilon}_{i,1}^h + \mu_{i,2}^h, \tag{18}$$

where $\left|\rho^h\right| < 1$ and $\mu_{i,2}^h$ is a white noise process with variance $\left(\sigma_\mu^h\right)^2$. We further assume that the $AR(1)$ process is covariance-stationary with a zero mean and variance $\left(\sigma_{\widehat{\varepsilon}}^h\right)^2 = \frac{\left(\sigma_\mu^h\right)^2}{1-\left(\rho^h\right)^2}$.

3.2.2 Finite state Markov-chain approximation

To approximate (18) by a discrete state-space process, $\varepsilon_{i,2}^h$, we apply the Rouwenhorst (1995) method. This requires building a Markov chain with m-states incorporating: (i) a symmetric and equal-spaced state space, $\varepsilon_m^h = \{\overline{\varepsilon}_1^h, ..., \overline{\varepsilon}_m^h)$, where $\overline{\varepsilon}_1^h = -\psi^h$, $\overline{\varepsilon}_N^h = \psi^h$ and $\psi^h \neq 0$;[16] and (ii) a transition matrix Q^h which for $m \geq 2$ is determined by two parameters, $\upsilon^h, \nu^h \in (0, 1)$. Kopecky and Suen (2010) show that Markov chain for $\varepsilon_{i,2}^h$, computed by the Rouwenhorst method, converges to an invariant binomial distribution, $\xi^{h(m)}$. This distribution has the same unconditional mean of zero and unconditional variance $\left(\sigma_\varepsilon^h\right)^2 = \frac{\left(\sigma_\mu^h\right)^2}{1-\left(\rho^h\right)^2}$ as in the AR(1) case. Each element of $\xi^{h(m)}$ is given by:

$$\xi_j^{h(m)} = \begin{pmatrix} m-1 \\ j-1 \end{pmatrix} \left(\omega^h\right)^{m-j} \left(1-\omega^h\right)^{j-1}, \; for \; j = 1, 2, ..., m, \tag{19}$$

where m denotes the discrete number of states; and $\omega^h \equiv \frac{1-\nu^h}{2-(\upsilon^h+\nu^h)} \in (0, 1)$. Kopecky and Suen (2010) further show the conditional means and variances as well as the first-order autocorrelations are the same across processes.

Thus, for a given number of m states which we set to 7, to estimate (19) requires that we pin down υ^h, ν^h and ψ^h. In light of the analysis in Kopecky and Suen (2010), we first know that $\upsilon^h = \nu^h$ since the unconditional mean of the Markov chain:

$$E(\varepsilon_{i,2}^h) = \frac{(\nu^h - \upsilon^h)\psi^h}{2-(\upsilon^h+\nu^h)} \tag{20}$$

is equal to zero. We further know that the first-order autocorrelation of the Markov chain is given by:

$$\begin{aligned} &Corr(\varepsilon_{i,2}^h, \varepsilon_{i,1}^h) \equiv \rho^h = \upsilon^h + \nu^h - 1 \\ &\Rightarrow \upsilon^h = \nu^h = \tfrac{1+\rho^h}{2} \;\; \text{since } \upsilon^h = \nu^h. \end{aligned} \tag{21}$$

Finally, since the unconditional variance of the Markov and AR(1) processes

[16] Note that the elements of S^h defined in `Assumption 1` are equal to the exponential of ε_m^h.

are the same, we can calculate ψ^h as follows:

$$\begin{aligned} \left(\sigma^h_{\widehat{\varepsilon}}\right)^2 &= \left(\psi^h\right)^2 \left[1 - 4\omega^h(1-\omega^h) + \tfrac{4\omega^h(1-\omega^h)}{m-1}\right] \\ \Rightarrow \psi^h &= \sqrt{\left(\sigma^h_{\widehat{\varepsilon}}\right)^2 (m-1)} \;\; \text{since } \upsilon^h = \nu^h. \end{aligned} \tag{22}$$

Thus, to identify υ^h, ν^h and ψ^h requires that we estimate ρ^h and $\left(\sigma^h_{\widehat{\varepsilon}}\right)^2$ using the WAS data for the third and fourth waves.

The results for the university, non-university and the whole sample are presented in Table 2. As can be seen, the persistence and variance of the university educated group is higher than the other group while the parameters for the whole sample are closer to the below university group. We have also checked the cases of household total gross earnings plus benefits and household total gross earnings. In the first case, the relative magnitudes between the groups are more or less the same, however, the magnitudes of both correlation and variance for both groups are simply proportionally higher. In the second case, the below university group exhibits higher persistence but lower variance of earnings. Thus, it seems that government policy has an effect on the persistence and variance of the income variables.[17]

Table 2: Income Process parameters

	University	Non-University	Pooled
$\sigma_{\widehat{\varepsilon}}$	0.5216	0.4379	0.4710
ρ	0.6635	0.6043	0.6310

Since the correlations reported in Table 2 are calculated over a two-year period, they need to be transformed to an annual basis to to cohere with the annual calibration of the model. This is achieved by taking the square root each of the values reported in the last row of Table 2.

Finally, we make use of the non-idiosyncratic component of earnings, $\beta_{0,t} + f_t(x_{i,t})$, calculated in the data using (17), to calibrate the fixed skill parameters ζ^h, $h = u, b$. In particular, we normalise $\zeta^b = 1$ and set ζ^u to match the ratio of the predicted components $\widehat{\beta_{0,t}} + f_t(\widehat{x}_{i,t})$ between the two groups. For the household disposable income this gave a value around $\zeta^u = 1.5$.

3.3 Parameters common to all households

The parameters that are common to all households are summarised in Table 3. In particular, regarding preferences, we set $\beta = 0.97$ which implies

[17] The variance estimates in Table 2 are obtained using WAS wave 3. The ranking of variances between the two groups is the same using wave 4, but the difference is smaller. The total variance is also very similar.

an equilibrium interest rate of around 2 percent per year. Following the literature we use a CRRA utility function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{23}$$

and set $\sigma = 1.5$, which is the mid-point of values typically employed in calibration studies for the UK (see also Harrison and Oomen (2010) who econometrically estimate $\sigma = 1.52$). The *ad hoc* borrowing limit is calibrated to $\gamma = 0.85$, so that we match, in equilibrium, the percentage of indebted agents (i.e. those with negative net-worth) in WAS. These shares are approximately 19%, 10% and 23% for the pooled sample, university and non-university educated respectively. The annual depreciation rate is set to $\delta = 0.1$ (see, e.g. Faccini *et al.* (2011) and Harrison and Oomen (2010)). We use a Cobb-Douglas production function with constant returns to scale with respect to its inputs:

$$Y = AK^{\alpha}L^{1-\alpha}. \tag{24}$$

We normalise $A = 1$ and set α to 0.3 (see, e.g. Faccini *et al.* (2011) and Harrison and Oomen (2010)). The value of n_u is calculated as the share of university educated households to the total number of households in the WAS wave 3 sample.

Table 3: Parameters

σ	β	δ	α	n_u	ζ^u	γ
1.5	0.97	0.1	0.3	0.36	1.5	0.85

4 Wealth inequality

We next summarise the data and model predictions for key statistics of wealth inequality in Table 4 following the standard practice in the choice of these statistics, see e.g. Quadrini and Rios-Rull (2015) and Krueger *et al.* (2016). Wealth inequality in the data in Table 4 is obtained using WAS data for wave 3 (effectively the mid-point of wealth inequality in Table 1), consistent with the earnings data used to estimate the earnings process, as discussed in the previous Section. Households whose head is university educated have lower wealth inequality than households whose head is not university educated, despite the fact that the university educated group has higher earnings risk (see Table 2). Recall also from Table 1 that the first group also has higher wealth on average, compared with the second, with the ratio being 2.16.

The wealth inequality results from the model presented in Section 2 and calibrated in Section 3 are summarised in Table 4. Our model with *ex ante* skill heterogeneity is denoted as EHM. We also present results of the Aiyagari model with *ex ante* identical households, calibrated using the pooled earnings data, as explained in Section 3. These results are presented under the column named Aiyagari.

Regarding overall wealth inequality, the Gini index for the EHM model is 0.643 and is not very far from the Gini index in the data of 0.707. Similarly, the predictions for the share of wealth owned by the five quantiles are relatively close, as is the predicted share of wealth for the top 10%. It is only the top 1% wealth share that the model clearly misses, as is typically the case in this class of models. On the other hand, the model nearly matches wealth ownership of the very high, but not top, percentiles, i.e. for the top 90-95% and for the top 95-99%.

We next turn to comparisons of wealth inequality for the two groups. The average wealth ratio of university to non-university educated households predicted by the model is 2.75. Importantly, the model is consistent with key properties of within group wealth inequality, for both groups. Although the differences between the model and the data are somewhat higher than for the whole sample, the relative indicators between the two groups in the model are always similar to the data. Note, in particular, that when indicators are similar in the data for the two groups (the top 1% share in wealth), they are also similar in the model. When they are higher in the data for the university group (the Q1, Q2 and Q3 share), they are also higher in the model. Whereas, when the indicator is higher in the data for the non-university group (the remaining cases) they are also higher in the model.

Table 4: Wealth Distributions (Data vs Models)

		Total		Uni	Non-uni	Uni	Non-uni
	Data	Aiyagari	EHM	Data		EHM	
Gini	0.7077	0.5878	0.6433	0.6474	0.7295	0.5471	0.6569
Q1 share	-0.0115	-0.0231	-0.0261	-0.0049	-0.0161	-0.0057	-0.0502
Q2 share	0.0118	0.0508	0.0380	0.0408	0.0022	0.0597	0.0359
Q3 share	0.0910	0.1436	0.1238	0.1057	0.0765	0.1448	0.1334
Q4 share	0.2130	0.2730	0.2437	0.2087	0.2305	0.2640	0.2821
Q5 share	0.6957	0.5558	0.6206	0.6498	0.7068	0.5371	0.5987
T 90-95%	0.1475	0.1420	0.1534	0.1361	0.1552	0.1360	0.1540
T 95-99%	0.1986	0.1510	0.1824	0.1844	0.1961	0.1442	0.1614
T 1%	0.1594	0.0542	0.0742	0.1466	0.1496	0.0519	0.0592

The power of the model to capture the inequality patterns when comparing the two groups is a particularly interesting result. This is because the earnings process for the university educated implies more risk than that for the non-university educated (recall the discussion in Section 2) and thus, given the results in the literature (see e.g. Quadrini and Rios-Rull (2015) and Krueger *et al.* (2016)), one would expect that the model would predict higher inequality for the university educated group. We will come back to this point in the next sub-section, where we explain why this model generates the correct inequality predictions.

To summarise, the first general message that comes from Table 4, is that the model with *ex ante* skill heterogeneity, as presented in Section 2 and calibrated in Section 3, captures qualitatively and approximates quantitatively many features of between and within university/non-university wealth inequality in Great Britain. An additional important remark regarding the results in Table 4 is that the benchmark Aiyagari model with *ex ante* identical households provides weaker inequality predictions, compared with the model with *ex ante* skill heterogeneity. In fact, for nearly every statistic in Table 3, the improvements are sizeable (an exception is Q1 and the top 90-95% where both models effectively differ from the data similarly). In particular, these results show that *ex ante* skill heterogeneity increases the Gini index by about 5.5 points, and it further contributes to significant improvements in the top 5%.

The Aiyagari model with *ex ante* identical agents does predict higher inequality than typically found in the literature when the earnings processes are calibrated using earnings data,[18] which is the result of the higher earnings uncertainty implied using the estimates of the earnings processes in Section 3 and because we allow for borrowing. *Ex ante* skill heterogeneity adds to this, and improves the predictions of the model even further. The second general message from Table 4 is that *ex ante* skill heterogeneity matters for wealth inequality at the aggregate level. In the next Section we examine why this is the case.

5 Pecuniary externalities & wealth inequality

In this section we analyse the mechanism of pecuniary externalities. We first show how it works to amplify wealth inequality at the aggregate level, and to lead to correct predictions regarding the ranking of wealth inequality between the two groups. We then analyse the contribution of the two forms of *ex ante* heterogeneity to this amplification mechanism, effectively demonstrating that

[18] The predicted the Gini index of the Aiyagari (1994) model ranges roughly between 0.35 and 0.45 in the literature, depending on the earnings data used, and can be higher when the model allows for borrowing, as we do here (see e.g. the reviews in Quadrini and Rios-Rull (1997, 2015) and Krueger *et al.* (2016)).

it need not always increase aggregate wealth inequality.

5.1 A pecuniary externality under skill heterogeneity

In Figure 1, we plot the asset supply curves for both groups of university and non-university educated, as well as the aggregate asset supply and demand functions. While the asset supply curves for each group encapsulate their optimal policy functions and thus choices for savings given the market incompleteness, the general equilibrium is obtained at the intersection point of the aggregate supply curve with the aggregate demand curve. This general equilibrium gives an interest rate of $r^* = 0.019$ and capital stock of $a^* = 3.754$. Note that the efficient interest rate in this economy, defined as the interest rate under complete financial markets which allows the agents to eliminate idiosyncratic risk, is given by 0.031, implying, via the asset demand function, an efficient capital stock of 3.27. In the benchmark model with *ex ante* skill heterogeneity these quantities are reduced and increased, respectively, to r^* and a^*, implying inefficiently high asset accumulation, as has been shown since Aiyagari (1994).

The equilibrium interest rate implies mean assets for the university educated group that are equal to $a_s^u = 6.334$ and for the non-university educated group that are equal to $a_s^b = 2.302$ (see Figure 1). However, these mean assets, for both groups, are not the mean quantities that are consistent with their own asset supply curves if they faced the asset demand on their own. These latter quantities are given by the relevant intersection points of the two group-level asset supply curves with the aggregate demand curve, as a_n^u and a_n^b in Figure 1. Hence, compared with their own asset supply schedule, the asset supply of the other group lowers or increases (for the non-university and university educated groups, respectively), the interest rate. Thus, reducing or increasing, the incentives to save (note that $a_n^b > a_s^b$ and that $a_n^u < a_s^u$). In turn, this under- or over-accumulation works to increase or decrease inequality in each group, by increasing or decreasing the exposure to earnings variability. Therefore, the asset supply of each group creates a pecuniary externality which affects inequality in the other group.[19]

To analyse further this mechanism, we plot, in Figure 2, the asset supply and demand for each group, assuming that each particular group defines the entire population. Therefore, the group-specific asset supply curves in Figure 2 are obtained by setting the population shares of university and

[19] While under complete markets pecuniary externalities do not reduce efficiency, they may do so under incomplete markets (see e.g. Greewald and Stiglitz (1986) and Davilla *et al.* (2012)). Here we examine pecuniary externalities arising from *ex ante* skill heterogeneity and focus on their implications for wealth inequality, as opposed to efficiency.

Figure 1: General Equilibrium

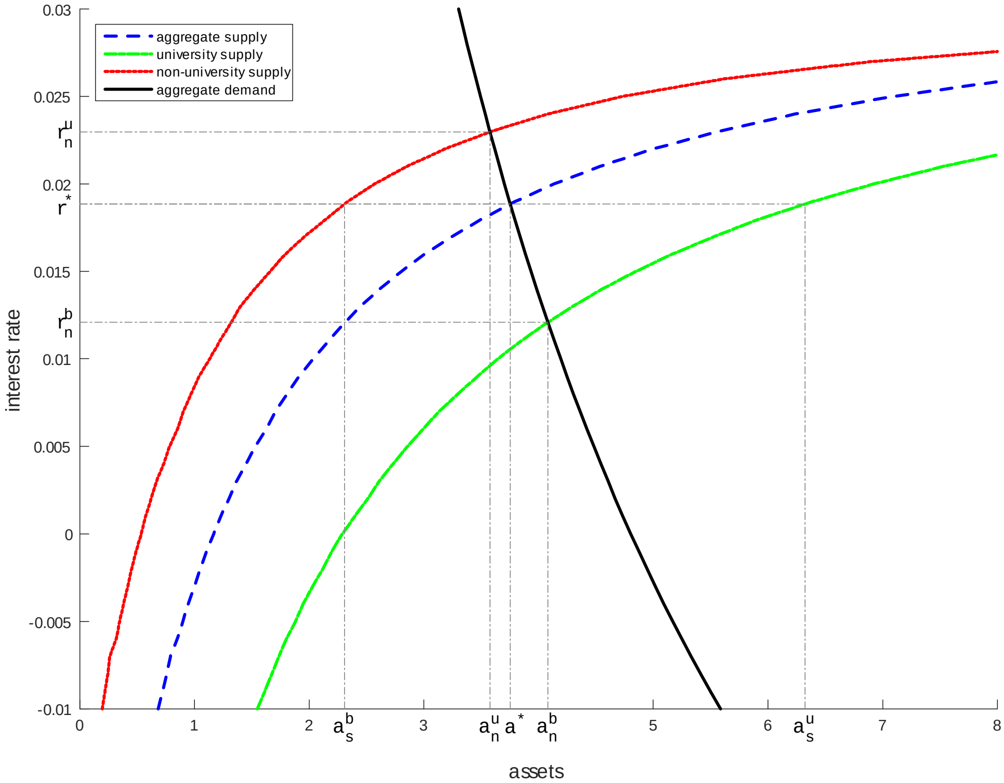
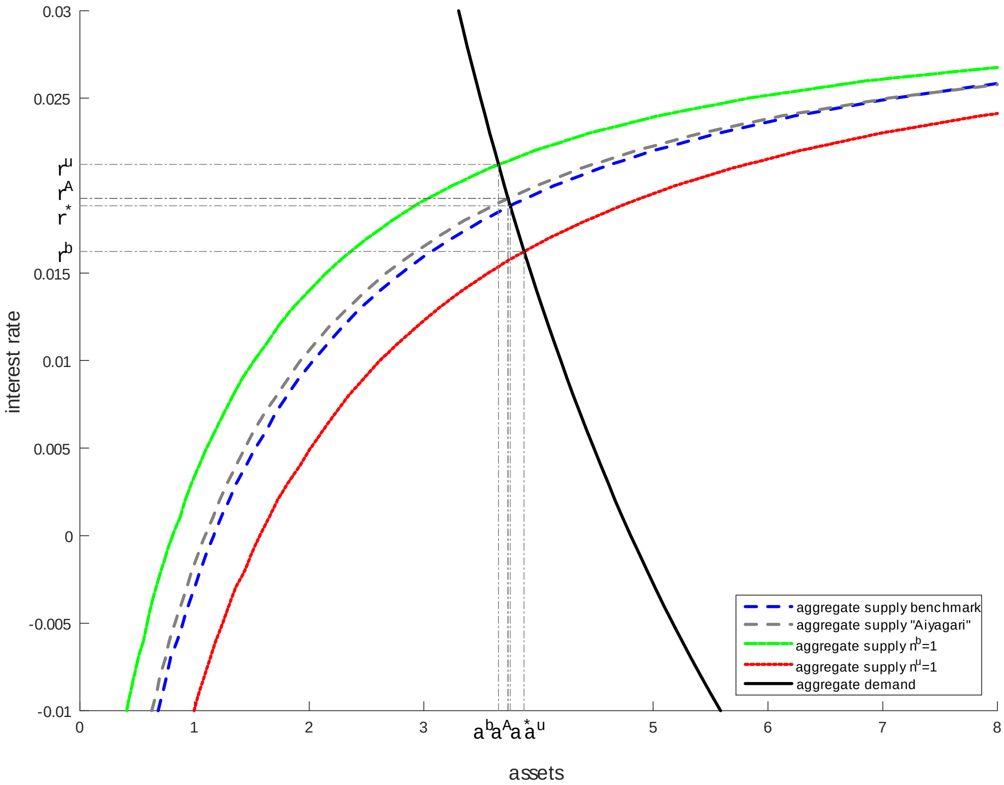


Figure 2: Externalities From Skill Heterogeneity



non-university to one and zero and vice-versa, without changing otherwise the calibration. We also plot the asset supply curve for the case where the sample is pooled, that is, the asset supply curve for the model with *ex ante* identical agents. This corresponds to model results reported in Table 4 under the column Aiyagari. For comparability, we also plot the aggregate supply for the benchmark model with *ex ante* skill heterogeneity, as in Figure 1. The asset demand curve is common in all these specifications, as in each case the mean labour input is normalised to be one. We denote the equilibrium points as follows: (i) for $n^b = 1$ we use r^b and a^b; (ii) for $n^u = 1$ we use r^b and a^b; (iii) for the Aiyagari model we use r^A and a^A; and (iv) for the EHM model we use r^* and a^*. By comparing the subplots in Figure 2, we can see that $a^u > a^*$, and $a^b < a^*$ and that that $a^u > a^A$, and $a^b < a^A$. Moreover, by comparing Figures 1 and 2, we can see that $a^u < a^u_s$ and that $a^b > a^b_s$.

These results confirm the previous intuition. In particular, in the economy with *ex ante* skill heterogeneity, the asset supply of each group creates a pecuniary externality to the other group which works to reduce or increase precautionary savings. In particular, the increased asset supply of the university-educated group reduces the interest rate, relative to the one that would be the equilibrium outcome had everyone in the population been non-university educated, and vice versa for the asset supply of the non-university educated. The implication for the non-university educated group is that this reduces the interest rate that they face, which implies that their assets are also reduced (i.e. $a^b > a^b_s$). In turn, this suggests that non-university educated, having reduced precautionary savings, are more exposed to earnings risk and thus there is increased within group wealth inequality. The effects are reversed for the group of university educated, for whom the pecuniary externality arising from the reduced asset supply of the non-university educated works to reduce within group wealth inequality.

To quantify the wealth inequality implications of this pecuniary externality, we summarise in Table 5 the wealth inequality measures for all model variants in Figure 2. Note that the first two columns are repeated from Table 4 for convenience. As can be seen by comparing columns for $n^b = 1$ and $n^u = 1$ in Table 5 to the last two columns in Table 4, wealth inequality for the non-university educated groups is decreased without pecuniary externalities, while it is increased for the university educated group. In this case, the higher earnings risk for the university educated determines the ranking of within group inequalities, so that wealth inequality is higher for the university educated group. Recall from the data in Table 4 that this is not realistic. The pecuniary externality is thus the critical factor so that in Table 4 the wealth inequality for the non-university educated in higher.

Table 5: Wealth Distributions and Pecuniary Externality

	EHM	Aiyagari	$n^u = 1$	$n^b = 1$
Gini	0.6433	0.5878	0.6054	0.5739
Q1 share	-0.0261	-0.0231	-0.0245	-0.0210
Q2 share	0.0380	0.0508	0.0443	0.0554
Q3 share	0.1238	0.1436	0.1380	0.1500
Q4 share	0.2437	0.2730	0.2671	0.2694
Q5 share	0.6206	0.5558	0.5751	0.5461
T 90-95%	0.1534	0.1420	0.1459	0.1383
T 95-99%	0.1824	0.1510	0.1572	0.1454
T 1%	0.0742	0.0542	0.0578	0.5230

In turn, the correct ranking of within group inequalities in Table 4 is important in improving (i.e. in this case, in increasing) the inequality predictions at the aggregate level, compared with the model with *ex ante* identical agents. This is because there is a higher proportion of non-university educated people in the data. Hence when this group is correctly predicted to have higher inequality, overall inequality at the aggregate level is also higher. Note that the model with *ex ante* identical agents produces a Gini inequality index that is in between the respective indices of the $n^b = 1$ and $n^u = 1$ cases, since the hypothetical "average" agent has an earnings process that also features "averaged" properties. In contrast, for the model with *ex ante* skill heterogeneity, the Gini inequality index that is in between the respective indices of the last two columns in Table 4. Therefore, the pecuniary externality embodied in the model with *ex ante* skill heterogeneity is critical in generating the correct ranking of within group wealth inequality and in increasing aggregate wealth inequality.

5.2 Differences in earnings processes

Ex ante skill heterogeneity creates pecuniary externalities which here are shown to help the model regarding its predictions on inequality. However, this is not a given outcome, as we show in this section. The first thing to note is that *ex ante* heterogeneity here takes two forms. First, it implies higher mean earnings for the university educated. Second, it implies higher earnings risk for the university educated. The differences between the two groups regarding mean earnings and earnings risk here both work to amplify inequality via the pecuniary externality channel.

To see this, we summarise in Table 6 wealth inequality indicators from two experiments where we nullify these two differences, one at a time. In particular, we first assume that both groups have equal mean earnings (but maintain the difference in the transition matrices) and then assume that both have the same transition matrix, associated with the pooled sample in Table 3 (but maintain the difference in mean earnings). As can be seen, in both cases aggregate inequality is reduced, as the pecuniary externality is reduced.

The pecuniary externality is higher (and thus aggregate inequality tends to increase more due to this channel), the bigger the difference is between the asset supplies of the two groups.[20] The two forms of *ex ante* heterogeneity have distinct (and potentially different) effects on the distance between the asset supply functions. In particular, a higher earnings difference tends to increase the change, since in general equilibrium assets increase with mean productivity (see e.g. Acemoglu and Jensen (2015).[21] Moreover, increased earnings uncertainty also increases savings, as agents increase precautionary wealth (see e.g. Acemoglu and Jensen (2015) for a theoretical analysis

[20] This follows from the discussion in the previous sub-section, and we confirm it below.

[21] Also, in a partial equilibrium context, savings are typically increasing in earnings and income in this class of models (see e.g. Miao (2002) for a theoretical result that savings increase in earnings, and Aiyagari (1994) that savings increase in disposable resources).

and Aiyagari (1994) for quantitative applications). Both of these effects are confirmed in the model, as the results in Table 6 confirm.

Table 6: Wealth Distributions (Counterfactuals)

	EHM	EHM: Equal earnings	EHM: Equal risk
Gini	0.6433	0.6017	0.6003
Q1 share	-0.0261	-0.0235	-0.0235
Q2 share	0.0380	0.0483	0.0488
Q3 share	0.1238	0.1393	0.1382
Q4 share	0.2437	0.2650	0.2654
Q5 share	0.6206	0.5710	0.5711
T 90-95%	0.1534	0.1443	0.1429
T 95-99%	0.1824	0.1586	0.1570
T 1%	0.0742	0.0608	0.0597

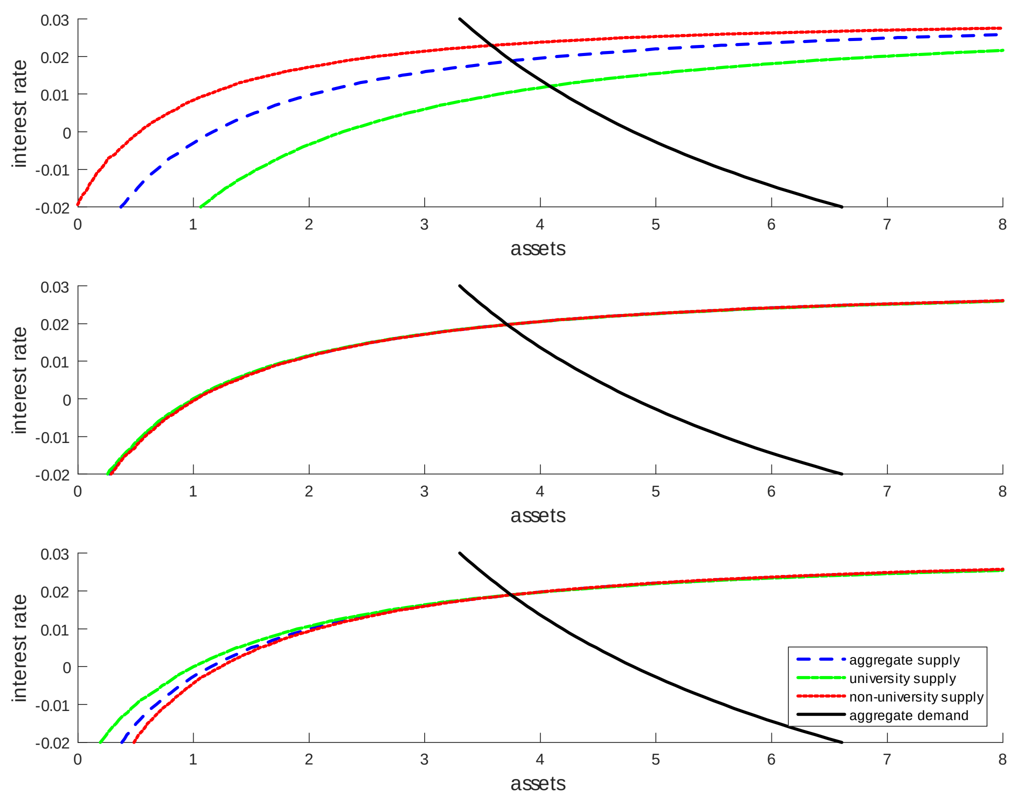
The important consequence of these points is that in the model with *ex ante* skill heterogeneity calibrated as in Section 3, both factors work in the same direction to increase the distance between the asset supply curves, and thus ultimately aggregate inequality. This happens because the university educated have both higher mean earnings and higher earnings uncertainty. In fact, in this model, the pecuniary externality channel is strong enough to create wealth inequality effects for each group that are ranked in the reverse order compared with the ranking implied by the earnings risk, as we saw in the previous sub-section. In other words, while we may expect the university group to have higher wealth inequality because they face higher earnings uncertainty, they are able to accumulate more precautionary wealth to self-insure and thus reduce within group wealth inequality. The effects are reversed for the non-university educated group.

5.3 Two counterfactuals

To illustrate the workings of these two components of *ex ante* heterogeneity, and, in particular, demonstrate that they can work to offset each other, we consider two counterfactuals in Figure 3. The first subplot of this Figure repeats Figure 1 to facilitate comparison.

In the second subplot, we leave the differences in the transition matrices between the university and non-university educated groups as they are in Table 1 and increase mean earnings for the non-university educated group so that its asset supply moves to the right. In particular, we calibrate the relative difference in mean earnings so that the asset supply curves become the same, which is obtained by effectively transposing the mean earnings

Figure 3: Changes in Earnings and Risk



premium in favour of the non-university educated. Thus, if mean earnings for the non-university educated group are increased sufficiently, relative to the university educated group, the increased incentive that the latter groups has for savings due to their more uncertain income is offset so that the two asset supply curves are the same.

In the third subplot, we examine the role of increased earnings uncertainty for the non-university educated and/or of reduced earnings uncertainty for the university educated, in the form of increases/decreases in the variances of the estimated earnings process. In particular, we calibrate the relative difference in the variances of the earnings processes between the two groups so that the asset supply curves become the same. This is obtained by effectively exchanging the earnings variances for the two groups.

In the two equilibria in the second and third subplot in Figure 3, the *ex ante* heterogeneity is such that there is no pecuniary externality, since the asset supply curves are effectively the same. In other words, one form of *ex ante* heterogeneity has offset the effects of the other, to eliminate the externality via the interest rate. In both these cases, aggregate inequality is reduced to about 0.58-0.59. Therefore, elimination of the pecuniary externality has eliminated the increase in the Gini index from the model with *ex ante* identical agents to the model with *ex ante* skill heterogeneity.

Hence, this analysis demonstrates that there can be *ex ante* skill heterogeneity without an increase in wealth inequality, which in turn may explain why the role of *ex ante* heterogeneity has not been explored much in the literature. In particular, consider the case where the lower earnings group also faces higher earnings uncertainty.[22] If the difference in the latter is sufficiently large relative to the former, then we can effectively be at a situation as those in Figure 3, where inequality is not increased as a result of allowing for *ex ante* heterogeneity.

6 Pigouvian-type policy

Ex ante skill heterogeneity creates a pecuniary externality that works in our analysis to increase inequality for the non-university educated and at the aggregate level. It is thus natural to consider whether Pigouvian-type policies, that alter the effective prices that agents face, can affect the externality and reduce inequality. We next revisit the base model presented in Section 2, and add policy instruments to implement Pigouvian-type policies. In particular, we add a subsidy to the income from savings for below-university educated households and a tax to income from savings for university educated households. Starting from the base model in Section 2, we can thus examine the effect of policy intervention aiming to increase incentives for those with lower skills and income to save, and, vice versa, to decrease incentives for those with higher skills and income to save. As we shall see below, this policy has little effects on general equilibrium prices and quantities. Our interest is on the effects of this type of policy on wealth inequality and not on aggregate efficiency.[23]

6.1 The model with Pigouvian policy

The setup is the same as in Section 2, except for changes in the budget constraint of the household, and the addition of a government budget constraint.

[22] For instance, this may be the case if earnings risk is approximated by unemployment risk, since job separation rates and unemployment are lower for less skilled workers (see e.g. Hagedorn *et al.* (2016)).

[23] Davilla *et al.* (2012) examine constrained efficiency in incomplete markets models and the role of pecuniary externalities, which however do not arise from *ex ante* skill heterogeneity as in this setup.

We summarise the changes here. First, the budget constraint is now given for each $h = u, b$ by:

$$c_t^h + a_{t+1}^h = \left[1 + \left(1 - \tau^{h,r}\right) r\right] a_t^h + w\zeta^h s_t^h, \tag{25}$$

where $c^h \geq 0$, $a_t^h \geq -\phi^h$ and $-\phi^h < 0$ denotes a borrowing limit on the household. The household receives an asset income subsidy or pays a tax, $\tau^{h,r}$, where $\tau^{u,r} \geq 0$ and $\tau^{b,r} \leq 0$. Define the net interest rate, $\widetilde{r}$ as:

$$\widetilde{r}^h = \left(1 - \tau^{h,r}\right) r, \tag{26}$$

so that (25) can be written as:

$$c_t^h + a_{t+1}^h = \left(1 + \widetilde{r}^h\right) a_t^h + w\zeta^h s_t^h. \tag{27}$$

The borrowing limit is now expressed in terms of net interest rate, and `Assumption 3` is modified to:

`Assumption 3P`

Assume that $\tau^{u,r} < 1$, $(1 + \widetilde{r}^h) > 0$, $w > 0$, $\beta(1 + \widetilde{r}^h) < 1$ and $-\widetilde{r}^h \phi^h + w\zeta^h \overline{s}_1^h \geq 0$.

The Bellman equation is:

$$v^h \left(a_t^h, s_t^h; w, \widetilde{r}^h\right) =$$

$$= \max_{\substack{a_{t+1}^h \geq -\phi^h \\ c_t^h \geq 0.}} \{u(c_t^i) + \beta \textstyle\sum_{s_{t+1}^h \in S^h} v^h \left(a_{t+1}^h, s_{t+1}^h; w, \widetilde{r}^h\right) Q_{s_t^h, s_{t+1}^h}^h\}. \tag{28}$$

Given `Assumptions 1,2,3P`, and the analysis in Acikgoz (2016), we can establish as before that the Markov process on the joint state-space $\left(\mathcal{A}^h \times S^h\right)$ with transition matrix Λ^h given in (5) has, for each $h = u, b$, a unique invariant distribution denoted by $\lambda^h (A \times B)$. Furthermore, assets for the typical household tend to infinity when $\beta(1+\widetilde{r}^h) \rightarrow 1$. Moreover, the expected value of assets using the invariant distribution is continuous in the net interest rate, $\widetilde{r}^h$, and thus, by (26), in r as well.

The government budget constraint is given by:

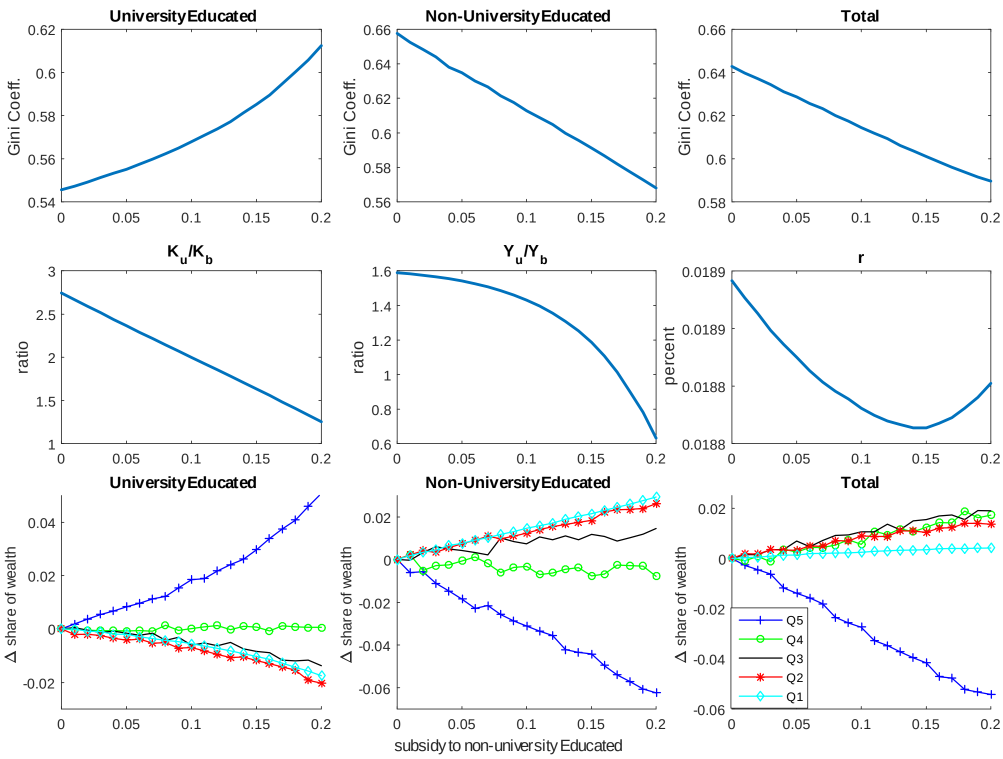
$$\int_{I^b} \tau^{b,r} r a_t^i \phi(di) = \int_{I^u} \tau^{u,r} r a_t^i \phi(di), \text{ or}$$

$$\tau^{b,r} K^b = \tau^{u,r} K^u \tag{29}$$

where $K^b = \int_{I^b} a_t^i \phi(di)$ and $K^u = \int_{I^u} a_t^i \phi(di)$.

We assume that $\tau^{b,r}$ is given exogenously and that $\tau^{u,r}$ follows residually to satisfy the government budget. We present the definition of the stationary recursive equilibrium with policy and the computation algorithm in Appendix C.

Figure 4: Tax and subsidy policy



6.2 Results

We plot the effects of this policy on wealth inequality and the aggregate economy in Figure 4. We start with the benchmark model where $\tau^{b,r} = \tau^{u,r} = 0$, and show general equilibrium results by increasing the subsidy to the non-university educated, $\tau^{b,r}$. In the first row of Figure 4 we plot the Gini indices for the two groups as well at the aggregate level. In the second row we plot two measures of between group wealth inequality, the wealth and the income ratio of the two groups, as well as the interest rate that pins down the aggregate quantities. Finally, in the third row, we plot the percentage change in the share of wealth owned by the five quantiles, as a result of increases in $\tau^{b,r}$.

The main thing to note is that this policy has substantial effects on wealth inequality, both within and between the groups. In particular, it decreases inequality between the two groups, bringing them closer in terms of mean wealth and income. Moreover, it decreases inequality within the group of non-university educated, as well as aggregate wealth inequality, while, on the other hand, it increases wealth inequality within the group of university educated. The Pigouvian-type policy analysed, increases savings from the non-university educated and decreases them for the university educated. Hence, the fall in the measures of between group wealth inequality in Figure 4. Overall, savings are increased, which is captured by the drop in the interest rate. The increase in wealth for the non-university educated implies that they are better insulated against idiosyncratic shocks. Thus, wealth inequality is reduced and vice versa for the university educated.

While this policy reduces overall inequality, it is decreasing wealth for the lower quantiles of the university educated, who find themselves with a smaller share (declining shares of wealth) of a smaller pie (lower wealth on average for the university educated). On the other hand, it increases wealth for the lower quantiles of the non-university educated households, who find themselves with a higher share (increased shares of wealth) of a bigger pie (higher wealth on average for the non-university educated). The main redistribution therefore is from the university to the non-university agents with little wealth.

It is also interesting to note that with respect to aggregate quantities, this policy has very small effects, as the interest rate changes very little as does aggregate capital. Hence, general equilibrium prices and quantities are not affected by this type of intervention. Davilla *et al.* (2012) have investigated tax-transfer schemes that can improve aggregate efficiency of the incomplete markets economy with *ex ante* identical agents, and implement a constrained efficiency equilibrium with potentially significant differences in general equilibrium prices and quantities. For the economy with *ex ante* skill heterogeneity that we analyse, we study instead a tax-subsidy Pigouvian policy which does not affect general equilibrium prices and quantities, but does affect inequality substantially.

7 Conclusions

This paper developed an incomplete markets model with state dependent (Markovian) stochastic earnings processes and *ex ante* skill heterogeneity corresponding to being university educated or not. We allowed the two groups to differ in their earnings processes, both in the state-space and in the transition matrix for idiosyncratic earnings shocks. Using the Wealth and Assets Survey for Great Britain to estimate the earnings processes, we found that this model predicted wealth inequality which was closer to that in the British data than the benchmark model with *ex ante* identical agents. Moreover, the model predicted wealth inequality both within and between the university and non-university educated groups that was also consistent with the data. Our analysis showed that *ex ante* skill heterogeneity in this framework generated a between-group pecuniary externality that is critical in improving the predictions of the model regarding wealth inequality.

In this framework, *ex ante* skill heterogeneity affects wealth inequality because the differences in the earning processes between the groups imply interest rate externalities. In particular, earnings differences, both in terms of mean earnings and idiosyncratic uncertainty, imply that the savings of each group move the market interest rate away from the level that would be the equilibrium outcome consistent with the asset supply of each group. The equilibrium interest rate is determined by the aggregate asset supply function, which is higher (lower) than the asset supply functions for university and non-university groups respectively. Consequently, households in the non-university and university educated groups lower and raise, respectively, their savings. Therefore, within group wealth inequality is increased and decreased, respectively, while overall wealth inequality is increased.

The between-group pecuniary externality is due to both factors defining *ex ante* heterogeneity, i.e. the differences in mean earnings and in the transition matrices, and we found both of these to be important quantitatively. For the pecuniary externality to improve the model's predictions on wealth inequality, both factors need to work in the right direction. In particular, the mechanism is stronger the bigger the difference in mean earnings is and the more uncertainty the higher earnings group faces. However, it is possible for the uncertainty channel to offset the mean earnings channel, thus eliminating the pecuniary externality and the gains it provides in terms of increasing the inequality predictions of the model. Therefore, ex ante skill heterogeneity is less likely to matter in situations where the lower earnings groups also have higher earnings risk. In turn, this may explain why the channel of ex ante skill differences has not been underlined previously in the literature as contributing to higher wealth inequality in the stationary equilibrium.

The analysis shows that inequality in this framework is to some extent a result of an externality. Hence, we considered a Pigouvian-type policy to reduce overall inequality. In particular, we examined a form of Pigouvian-type incentivisation policy that aimed to encourage savings for the group of low skilled and discourage savings for the group of high skilled. Our general finding was that this scheme reduced overall inequality, inequality within the group of the non-university educated, as well as between group inequality, but increased inequality within the group of university educated and disadvantaged the lower quantiles of the university educated. Therefore, the desired extent of intervention depends on the preferences of the policy maker.

8 Appendix A

8.1 Demographics

1. **Head of the Household:** the head is defined as the member of the household in whose name the accommodation is owned or rented, or is otherwise responsible for the accommodation. In households with a sole householder that person is the household reference person. In households with joint householders the person with the highest income is taken as the head. If both householders have exactly the same income, the older is taken as the head. (P_FLAG4W=1 or P_FLAG4W=3)

2. **Education level:** (i) degree level or above; (ii) has qualification, other level; and (iii) no qualifications. (EdLevelW)

3. **Age:** The WAS EUL (free edition) does not provide the exact age of the participants but only age bands. We make use of the 5 years bands. (DVAge17W)

4. **Marital Status:** *De facto* marital status of the head or his/her partner. (HRPDVMrDfW)

5. **Year:** The WAS provides the year of interview only in the fourth wave. However, since the interviews are conducted every two years around the same month for each respondent, we use the variable PresyrW4 to find the year of the interview in waves 1-3. For example, a household interviewed by the WAS in 2014, if present in previous waves, must have been interviewed again in 2012(wave 3), 2010(wave 2) and 2008(wave 1). We work similarly for those who were interviewed in 2013 and 2012. For the respondents for which we do not have the year of interview (e.g. those absent from wave 4), we set their year of the interview to be in the middle of the interview periods when most of interviews are conducted. Every wave spans three different years, for example, wave 1 starts in 2006 and finishes in 2008. Thus, we set the year for respondents with no information for date of interview to be 2007. We work in a similar fashion for the rest of the waves.

8.2 Definition of Wealth variables

1. **Net property wealth:**[24] is the sum of all property values minus the value of all mortgages and amounts owed as a result of equity release.

[24] All monetary values are expressed in 2012 prices as measured by CPIH.

(HPROPWW)

2. **Net financial wealth:** is the sum of the values of formal and informal financial assets, plus the value of certain assets held in the names of children, plus the value of endowments purchased to repay mortgages, less the value of non-mortgage debt. The informal financial assets exclude very small values (less than £250) and the financial liabilities are the sum of current account overdrafts plus amounts owed on credit cards, store cards, mail order, hire purchase and loans plus amounts owed in arrears. Finally, money held in Trusts, other than Child Trust Funds, is not included. (HFINWNTW_sum)

3. **Net Worth:** is the sum of the net property wealth and net financial wealth.

4. **Physical wealth:** is the sum of the values of household contents, collectibles and valuables, and vehicles (including personalised number plates). (HPHYSWW)

We follow the relevant literature and we focus on Net-Worth excluding physical wealth. For comparison, we show the results of wealth inequality if we include physical wealth in net-Worth. Comparing to Table 1, we observe that physical wealth has equalising properties. Moreover, we abstract from private pension wealth analysis.

Table A.1: Net-Worth plus physical wealth, 3rd wave

mean	sd	CV	mean/p50	Gini	top1%	top5%
University Educated						
310,070	539,750	1.741	1.688	0.582	0.127	0.295
Non-University Educated						
159,590	293,350	1.838	1.832	0.616	0.119	0.288
Total						
212,570	404,320	1.902	1.806	0.618	0.133	0.309

8.3 Definition of income variables

1. **Individual annual earnings**: measures annual labour income from all sources and is the sum of (i) main job gross annual earnings; (ii) main job self-employment income; (iii) annual gross bonus value; (iv) second job gross annual earnings; and (v) second job self-employment income. If the individual is self-employed (i.e. (ii) or (v)), we multiply by 0.7 the relevant components to reflect an average non-labour income share of 0.3 (see, e.g. Heathcote *et al.* (2010)).

2. **Minimum wage**: is the lowest wage per hour a worker is entitled to in the U.K. The minimum wage was introduced in the U.K. in 1999.

3. **Household total earnings**: are defined as the sum of individual annual earnings (see the definition above) within the household. Imputed values are included only if they refer to a respondent who is not the head of the household.

4. **Household earnings plus benefits**: is equal to household total earnings, plus social benefits, plus annual transfers income. In particular, transfers income can contain the following: (i) income from government training; (ii) educational grants; (iii) redundancies; and (iv) one-off income from relatives or friends.

5. **Household net disposable income**: is equal to household total earnings plus benefits but net of taxes and insurance contributions.

8.4 Sample selection

Households are defined as the family or group of individuals who are living in the same residence. We use the definition of head of the household as provided by the WAS. The head is defined as the member of the household in whose name the accommodation is owned or rented, or is otherwise responsible for the accommodation. We select households whose head is aged between 25-59 years old, implying that we retain 51,248 observations of households over the four waves. In Table A.2 we show the various steps of sample selection. To calculate the wealth inequality indices we use the sample up to step 2. Moreover, with respect to the estimation of the parameters of the income processes, we restrict further the sample to 10,290 observations since we have information for households income only in waves 3 and 4. Additionally, we keep only the household interviewed in both waves to calculate variance and correlation from common observations. Finally, we also discard households whose net disposable income is less than half of the product between the minimum legal hourly wage times 520 hours. This means that we use the sample after the selection step 6 in Table A.2.

Table A.2: Households and household members

selection step	households
1. whole sample	92,129
2. heads' age $\geq$25, $\leq$59	51,248
3. head's earnings>0	48,539
4. waves 3 & 4	20,205
5. net disposable income>threshold	19,804
6. keep if present in both waves 3 & 4	10,290

9 Appendix B

The computational algorithm described below is based on the "canonical" approach (see also Ljungqvist and Sargent (2012, ch. 18) and Miao (2014, ch. 17.1)).

`Computational algorithm`

1. Guess a value for $K = K_j$ from a domain $\left[K^{\min}, K^{\max}\right]$ and calculate $r(K_j)$, $w(K_j)$.

2. Solve the "typical" households' problem to obtain $g^h\left(a_t^h, s_t^h\right)$, for $h = u, b$.

3. Use $g^h\left(a_t^h, s_t^h\right)$ and the properties of the Markov processes $\left(s_t^h\right)$ to construct the transition functions $\Lambda_{K_j}^h$. Using $\Lambda_{K_j}^h$, calculate the stationary distributions λ^h.

4. Using λ^h, compute the average value of capital
$K_j^* = n^u \int_{\mathcal{A}^u \times S^u} g^u(a, s)\, \lambda^u(da, ds) + n^b \int_{\mathcal{A}^b \times S^b} g^b(a, s)\, \lambda^b(da, ds)$.

5. If $\left|K_j^* - K_j\right| < e$, where e is a pre-specified tolerance level, a stationary equilibrium has been found. If not, go back to step 1, update and repeat until convergence.

To implement this algorithm we first choose $K^{\min} = -0.85$. As discussed in more detail in the calibration section, we choose this value to match the percentage of indebted agents in WAS. We then let $K^{\max} = 50$, which implies that, in the solution, the probability of asset holdings greater than 40 is less than $3.1 * 10^{-4}$. We discretise $\left[K^{\min}, K^{\max}\right]$ by allowing for 1000 points. We have found that the obtained wealth distribution is robust to increasing $K^{\max}$

up to 100 and to decreasing in down to 40 (further decreases imply that the upper bound is binding with higher probability than $3.1 * 10^{-4}$).

An important theoretical result allowing the implementation of this algorithm is that λ^h is the unique invariant distribution for the typical household $h = u, b$. As discussed earlier, the computed stationary general equilibrium may not be unique. To check whether more than one equilibria exist, we solve the problem of the household in Step 2 and compute the invariant cross-sectional distribution and mean of asset supply in Steps 3-4, for a range of interest rates consistent with the model, and examine whether asset demand and supply intersect more than once.

10 Appendix C

We define the stationary recursive equilibrium with policy.

`Definition of Stationary Recursive Equilibrium (with policy)`
For $h = u, b$, a *Stationary Recursive Equilibrium*, is aggregate stationary distributions $\overline{\lambda}^h (A \times B)$, policy functions $a_{t+1}^h = g^h \left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \rightarrow \mathcal{A}^h$, $c_t^h = q^h \left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \rightarrow \mathbb{R}_+$, value functions $v^h \left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \rightarrow \mathbb{R}$, and positive real numbers $K, K^u, K^b, w(K), r(K), \tau^{u,r}\left(K^u, K^b\right)$, where $K = K^u + K^b$, such that:

1. The firm maximises its profits given prices, so that $(w(K), r(K))$ satisfy

$$w(K) = \partial F(K, L)/\partial L, \tag{30}$$
$$r(K) = \partial F(K, L)/\partial K - \delta. \tag{31}$$

2. Given values for $\tau^{b,r}$, $\tau^{u,r}\left(K^u, K^b\right)$ satisfies (29) and $\widetilde{r}^h \left(K, K^u, K^b\right) = \left(1 - \tau^{h,r}\right) r(K)$.

3. The policy functions $a_{t+1}^h = g^h \left(a_t^h, s_t^h\right)$ and $c_t^h = q^h \left(a_t^h, s_t^h\right)$ solve the households' optimum problems in (4) given prices and aggregate quantities, and the value functions $v^h \left(a_t^h, s_t^h\right)$ solve equations (4).

4. $\lambda^h (A \times B)$ is a stationary distribution

$$\lambda^h (A \times B) = \int_{\mathcal{A}^h \times S^h} \Lambda^h \left[(a, s), A \times B\right] \lambda^h (da, ds), \tag{32}$$

for all $A \times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h$, where $\Lambda^h\left[(a,s), A \times B\right] : \left(\mathcal{A}^h \times S^h\right) \times \left(\mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right) \rightarrow [0,1]$ are transition functions on $\left(\mathcal{A}^h \times S^h\right)$ induced by the Markov process $\left(s_t^h\right)_{t=0}^{\infty}$ and the optimal policy $g^h\left(a_t^h, s_t^h\right)$.

5. When $\lambda^h(A \times B)$ describe the cross-section of households at each date, i.e. $\overline{\lambda}^h(A \times B) = \lambda^h(A \times B)$, markets clear. In particular, asset market clears, that is the cross-section average value of K is equal to the average of the households' decisions

$$\begin{aligned} K = n^u \textstyle\int_{\mathcal{A}^u \times S^u} g^u(a,s)\, \overline{\lambda}^u(da, ds) + \\ + n^b \textstyle\int_{\mathcal{A}^b \times S^b} g^b(a,s)\, \overline{\lambda}^b(da, ds). \end{aligned} \tag{33}$$

Secondly, labour market clears

$$\begin{aligned} L = n^u \zeta^u \textstyle\int_{\mathcal{A}^u \times S^u} s^u(a,s)\, \overline{\lambda}^u(da, ds) + \\ + n^b \zeta^b \textstyle\int_{\mathcal{A}^b \times S^b} s^b \overline{\lambda}^b(da, ds) = 1. \end{aligned} \tag{34}$$

And goods market clears, which, using factor input market clearing, implies

$$\begin{aligned} & F(K,1) - \delta K = \\ & = n^u \textstyle\int_{\mathcal{A}^u \times S^u} q^u(a,s)\, \overline{\lambda}^u(da, ds) + n^b \textstyle\int_{\mathcal{A}^b \times S^b} q^b(a,s)\, \overline{\lambda}^b(da, ds). \end{aligned} \tag{35}$$

It can again be shown that continuity of the asset supply and demand functions at the aggregate level with respect to the interest rate as well as the limit properties of supply and demand for assets, imply that a general equilibrium exists. To compute this, we amend the algorithm in Appendix B as follows.

`Computational algorithm (with policy)`

1. Guess a value for $K = K_j$ from a domain $\left[K^{\min}, K^{\max}\right]$ and for $K^h = K_j^h$, or $h = u, b$, from a domain $\left[K^{\min}, K^{\max}\right]$ and calculate $r(K_j)$, $w(K_j)$, $\tau^{u,r}\left(K_j^h\right)$, $\widetilde{r}^h\left(K, K_j^h\right)$ and w. Check whether $\frac{-1}{1-\tau^{h,r}} < \widetilde{r}^h < \frac{1}{1-\tau^{h,r}}\left[\frac{1}{\beta} - 1\right]$.

2. Solve the "typical" households' problem to obtain $g^h\left(a_t^h, s_t^h\right)$.

3. Use $g^h\left(a_t^h, s_t^h\right)$ and the properties of the Markov processes $\left(s_t^h\right)$ to construct the transition functions $\Lambda_{K_j}^h$. Using $\Lambda_{K_j}^h$, calculate the stationary distributions λ^h.

4. Using λ^h, compute the average value of capital
   $K_j^* = n^u \int_{\mathcal{A}^u \times S^u} g^u(a,s)\, \lambda^u(da,ds) + n^b \int_{\mathcal{A}^b \times S^b} g^b(a,s)\, \lambda^b(da,ds)$
   and $K_j^{h*} = n^h \int_{\mathcal{A}^h \times S^h} g^h(a,s)\, \lambda^h(da,ds)$.

5. If $\left|K_j^* - K_j\right| < e$, where e is a pre-specified tolerance level, continue to Step 6. If not, go back to step 1, update K_j and repeat until convergence.

6. If $\left|K_j^{h*} - K_j^h\right| < e$, a stationary equilibrium has been found. If not, go back to step 1, update K_j^h and repeat until convergence.

The notes following the computational algorithm in Appendix B apply here as well.

References

[1] Acemoglu, D. and M. Jensen (2015). Robust comparative statics in large dynamic economies, *Journal of Political Economy*, 123, 587-641.

[2] Acikgoz, O. (2016). On the existence and uniqueness of stationary equilibrium in Bewley economies with production, MPRA Paper no. 71066 (forthcoming in *Journal of Economic Theory*).

[3] Al-Najjar, N. (2004). Aggregation and the law of large numbers in large economies, *Games and Economic Behavior*, 47, 1-35.

[4] Aiyagari, R. (1994). Uninsured idiosyncratic risk and aggregate saving, *Quarterly Journal of Economics*, 109, 659-684.

[5] Angelopoulos, A., Lazarakis, S. and J. Malley (2017). Asymmetries in earnings, employment and wage risk in Great Britain, CESifo Working Paper no. 6400.

[6] Benhabib, J., Bisin, A. and S. Zhu (2011). The distribution of wealth and fiscal policy in economies with finitely lived agents, *Econometrica*, 79, 123–157.

[7] Bewley, T. (1986). Stationary monetary equilibrium with a continuum of independently fluctuating consumers, In Werner Hildenbrand and Andreu Mas-Collel (editors), *Contributions to Mathematical Economics in Honor of Gerard Debreu*, NorthHolland.

[8] Blundell, R. and B. Etheridge (2010). Consumption, income and earnings inequality in Britain, *Review of Economic Dynamics*, 13, 76–102.

[9] Castaneda, A., Diaz-Gimenez, J. and J. Rios-Rull (1998). Exploring the income distribution business cycle dynamics, *Journal of Monetary Economics*, 42, 93–130.

[10] Castaneda, A., Diaz-Gimenez, J. and J. Rios-Rull (2003). Accounting for the U.S. earnings and wealth inequality, *Journal of Political Economy*, 111, 818–857.

[11] Castro R. and D. Coen-Pirani (2008). Why have aggregate skilled hours become so cyclical since the mid-1980s?, *International Economic Review*, 49, 135-184.

[12] Davila, J., Hong, J., Krusell, P. and J. Rios-Rull (2012). Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks, *Econometrica*,80, 2431-2467.

[13] De Nardi, M. (2004). Wealth inequality and intergenerational links, *Review of Economic Studies*, 71, 743-768.

[14] De Nardi, M., French, E. and J. Jones (2010). Why do the elderly save? The role of medical expenses, *Journal of Political Economy*, 118, 39–75.

[15] De Nardi, M. (2015). Quantitative models of wealth inequality: A survey, NBER Working Papers 21106, National Bureau of Economic Research, Inc.

[16] Faccini, R. Millard, S. and F. Zanetti (2011). Wage rigidities in an estimated DSGE model of the UK labour market, Bank of England Working Paper No. 408.

[17] Floden, M. and J. Linde (2001). Idiosyncratic risk in the United States and Sweden: Is there a role for government insurance?, *Review of Economic Dynamics*, 4, 406-437.

[18] Goldin, C. and L. Katz (2008). *The Race between Education and Technology*, Harvard University Press.

[19] Greewald, B. and J. Stiglitz (1986). Externalities in economies with imperfect information and incomplete markets, *The Quarterly Journal of Economics,* 101, 229-264.

[20] Guvenen, F. (2006). Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective, *Journal of Monetary Economics*, 53, 1451-1472.

[21] Hagedorn, M., Manovskii, I. and S. Stetsenko (2016). Taxation and Unemployment in Models with Heterogeneous Workers, *Review of Economic Dynamics*, 19, 161-189.

[22] Harrison, R. and O. Oomen (2010). Evaluating and estimating a DSGE model for the United Kingdom, Bank of England Working Paper No. 380.

[23] Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies, *Journal of Economic Dynamics and Control*, 17, 953–969.

[24] Imrohoruglu, A. (1989). Cost of business cycles with indivisibilities and liquidity constraints, *Journal of Political Economy*, 97, 1364-1383.

[25] Imrohoroglu, A. (1992). The welfare cost of inflation under imperfect insurance, *Journal of Economic Dynamics and Control*, 16, 79-91.

[26] Kitao, S. (2008). Entrepreneurship, taxation, and capital investment, *Review of Economic Dynamics*, 11, 44–69.

[27] Knight, F. (1935). *The Ethics of Competition and Other Essays*, Harper & Bros. and George Allen & Unwin.

[28] Krueger, D. and F. Perri (2006). Does income inequality lead to consumption inequality? Evidence and theory, *Review of Economic Studies*, 163-193.

[29] Dirk K., Mitman K. and F. Perri (2016). Macroeconomics and household heterogeneity, NBER Working Papers 22319, National Bureau of Economic Research, Inc.

[30] Kopecky, K. and R. Suen (2010). Finite state Markov-chain approximations to highly persistent processes, *Review of Economic Dynamics*, 13, 701-714.

[31] Krusell, P., and A. Smith (1998). Income and wealth heterogeneity in the macroeconomy, *Journal of Political Economy*, 106, 867–896.

[32] Ljungqvist, L. and T. Sargent (2012). *Recursive Macroeconomic Theory*, 3rd edition, The MIT Press.

[33] Meghir, C., and L. Pistaferri (2004). Income variance dynamics and heterogeneity, *Econometrica*, 72, 1-32.

[34] Meh, C. (2005). Entrepreneurship, wealth inequality and taxation, *Review of Economic Dynamics*, 8, 688-719.

[35] Miao, J. (2014). *Economic Dynamics in Discrete Time*, The MIT Press.

[36] Mincer, J. (1974). *Schooling, Experience and Earnings*, Columbia University Press.

[37] Quadrini, V. and J. Rios-Rull (1997). Models of the distribution of wealth, *Federal Reserve Bank of Minneapolis Quarterly Review*, 21,1–21.

[38] Quadrini, V. (2000). Entrepreneurship, saving and social mobility, *Review of Economic Dynamics*, 3, 1-40.

[39] Quadrini, V. and J. Rios-Rull (2014). Inequality in macroeconomics, In A.B. Atkinson and F. Bourguignon, editors, *Handbook of Income Distribution*, vol. 2B, Elsevier.

[40] Rios-Rull, J. V., and M. Kuhn (2016). 2013 Update on the US Earnings, Income, and Wealth Distributional Facts: A View from Macroeconomics, *Quarterly Review*, (April), 1-75.

[41] Rouwenhorst, K. (1995). Asset pricing implications of equilibrium business cycle models, In: Cooley, T.F. (Ed.), *Frontiers of Business Cycle Research*, Princeton University Press.

[42] Stokey, N., Lucas, R. and E. Prescott (1989). *Recursive Methods in Economic Dynamics*, Harvard University Press.

[43] Uhlig, H. (1996). A law of large numbers for large economies, *Economic Theory*, 8, 41-50.