A Re-examination of the link between Real Exchange Rates and Real Interest Rate Differentials[1]

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Abstract

Although the real exchange rate - real interest rate (RERI) relationship is central to most open economy macroeconomic models, empirical support for the relationship is generally found to be rather weak. In this paper we reinvestigate the RERI relationship using bilateral real exchange rate data spanning the period 1978 to 1997. We propose an alternative way of investigating the relationship using the present value VAR-based test of Campbell and Shiller (1987). Our empirical results provide robust evidence that the RERI relationship is economically significant and that the real interest rate differential is a reasonable approximation of the expected rate of depreciation over longer horizons. Although we report a statistical rejection of cross equation restrictions, this can largely be ascribed to the fact that excess returns on a currency have a significant degree of medium-run predictability, rather than to a rejection of the RERI. Our findings corroborate Baxter's (1994) substantive conclusion that there is an important link between real exchange rates and real interest rates at business cycle frequencies.

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1   Introduction

Many well-known exchange rate models highlight the role of the real interest rate differential as a key determinant of real exchange rates. For example, sticky price models (see Dornbusch (1976) and Mussa (1984)) and optimising models (see, for example, Grilli and Roubini (1992) and Obstfeld and Rogoff (1996)) emphasize the effect of liquidity impulses on real interest rates and consequently the real exchange rate. This relationship is often summarised in the form of the real exchange rate - real interest rate (RERI) relationship.

However, despite its centrality to many open economy macro models, the empirical evidence on the RERI relationship is rather mixed. In this paper we revisit the RERI relationship and suggest a new way of testing the relationship, based on the VAR-method of Campbell and Shiller (1987) for testing present value models. Our results indicate that the real interest rate differential is a reasonable proxy for the expected real depreciation of the US dollar and can be interpreted as the transitory part of the real exchange rate. This empirical finding provides strong support for the results of Baxter (1994) and also of Edison and Pauls (1993) who have emphasized that the link between real exchange rates and real interest differentials is to be found in the business cycle domain, instead of lower frequencies.

Our way of casting the RERI relationship into an empirical model rests on the idea that the real interest rate differential is the sum of expected period-to-period changes in real exchange rates. In this context, the real interest rate differential can be interpreted as the spread variable in a present value model in which the discount factor is known and equal to one.[1] This interpretation allows us to proxy expected real exchange rate changes from a bivariate VAR that includes real exchange rate changes and the real interest rate differential.

In our analysis we use bilateral real exchange rates for the G7 countries. The sample period is 1978 quarter 2 to 1997, quarter 4. In common with most other applications of the VAR-based present value approach, we find that the cross-equation restrictions of the present-value model are statistically rejected. However, we note that this can be attributed to the time variability of the discount factor, rather than to a rejection of the RERI model *per se*. Indeed, we present graphical evidence which indicates that the RERI is strongly supported and is an economically significant relationship, in the sense that expected real exchange rate changes are highly correlated with real interest rate differentials and that this correlation is correctly signed

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[1] See Engel and West (2004) for a discussion of the implications of a unitary discount rate in a present value variant of the monetary exchange rate model.

throughout.

We further illustrate the empirical relevance of the RERI by investigating how various structural shocks affect the relationship: under the null of the RERI, shocks to the real interest rate differential should only have a transitory impact on the real exchange rate, whereas shocks that do not affect the real interest rate differential should be associated with the permanent component. We find that these hypothesised relationships are in fact in the data. Furthermore, we also find that a positive interest rate shock leads to a temporary decline (appreciation) in the real exchange rate that is then gradually offset as relative prices and nominal interest rates adjust. This, again, is very much in line with theoretical predictions. We examine the robustness of this conclusion using an adaptation of the method suggested by King and Watson (1997), which involves examining the robustness of the response of the two variables to the choice of identification scheme. Interestingly, it turns out that our structural conclusions are independent of the particular approach to identification that we choose: the same pattern arises based on long-run identification schemes in the spirit of Blanchard and Quah (1989), more conventional short-run Choleski decompositions and, in fact, based on most other possible identifications.

The outline of the remainder of this paper is as follows. In the next section we consider the RERI relationship in some detail and discuss how the VAR-based method of Campbell and Shiller (1987) can be adapted to explore the RERI link. We then go on to outline how the model may be estimated using the projections from a simple VAR model. In section 3 we present our empirical results, while in section 4 we examine the impact of structural shocks on the long-run relationship between real exchange rates and the real interest differential. Section 5 provides a further discussion of our results and concludes.

2   The RERI as a present value relationship

The standard derivation of the RERI (see, for example, Meese and Rogoff (1988)) has as its starting point the familiar risk adjusted uncovered interest parity condition:

$$\mathbf{E}_t(s_{t+1} - s_t) = (i_t - i_t^*) + \sigma_t, \tag{1}$$

where s_t is the log of the spot exchange rate (home currency price of a unit of foreign exchange), i_t is the one period domestic interest rate, $\mathbf{E}_t$ is the conditional expectations operator, an asterisk denotes a foreign magnitude and σ_t is a stationary (time-varying) risk premium. The latter term is often

alternatively referred to as an excess return and we shall consider it in more detail below. Assuming rational expectations, equation (1) may be rewritten as:

$$s_{t+1} - s_t = (i_t - i_t^*) + \sigma_t + \epsilon_t, \tag{2}$$

where is ϵ_t is an *iid* random error.

The nominal exchange rate is usually thought of as an I(1) process and it therefore follows that the left hand side variable in (2), $s_{t+1} - s_t$, must be I(0). Since $\sigma_t + \epsilon_t$ is stationary, by assumption, it follows that the interest differential, $i_t - i_t^*$, must also be stationary - the domestic interest rate must be cointegrated with the foreign interest rate. The balanced nature of this expression, in terms of the orders of integration, is a standard feature of arbitrage conditions and is the starting point of the cointegration testing methods first proposed by Campbell and Shiller (1987) for present value models. It turns out that translating (2) into the equivalent real interest parity condition produces a similar balance in terms of the integratedness of the right and left hand side variables. For example, by subtracting the expected inflation differential, $\mathbf{E}_t(p_{t+1} - p_t) - \mathbf{E}_t(p_{t+1}^* - p_t^*)$, from both sides of (2), where p_t denotes the log of the domestic price level, and assuming rational expectations the following expression may be obtained:

$$q_{t+1} - q_t = (r_t - r_t^*) + \sigma_t + \epsilon_{t+1} + u_{t+1}, \tag{3}$$

where $q_t = s_t + p_t^* - p_t$, r_t denotes the domestic real interest rate, defined as $r_t = i_t - \mathbf{E}_t(p_{t+1} - p_t)$, and u_{t+1} is an *iid* inflation forecast error. Since the two disturbance terms – ϵ_{t+1} and u_{t+1} – and the excess-return (or risk premium) are stationary, it must follow, as in equation (2), that $q_{t+1} - q_t$ and $r_t - r_t^*$ are integrated of the same order. Since the real exchange rate is usually thought to be I(1), or close to I(1), $q_{t+1} - q_t$ must be I(0) and therefore so too must $r_t - r_t^*$. However, it follows from this that q_t and $r_t - r_t^*$ cannot be cointegrated (see Baxter (1994)).

On using the UIP condition at horizon $k - \mathbf{E}_t(s_{t+k} - s_t) = (i_t(k) - i_t^*(k)) -$ where $i_t(k)$ represents the nominal interest rates at time t on k-period bonds and on subtracting expected k-horizon relative inflation rates we obtain the k-period version of the real interest parity relationship, (3), as:

$$(\mathbf{E}_t q_{t+k} - q_t) = r_t(k) -_k r_t^*(k), \tag{4}$$

where $r_t(k) = i_t(k) - (E_t(p_{t+k} - p_t))$ and we have suppressed the risk premium.

Expression (4) is useful because it indicates that the current real interest rate differential contains sufficient information for forecasting the expected

long-run change in the real exchange rate. Hence, while an econometrician may not have access to the information set used by economic agents to form expectations, equation (4) states that current real interest differentials embody all of that information. This is a familiar insight that was first proposed by Campbell and Shiller (1987) in the context of present value models, but has not, to our knowledge, been used in the literature on the RERI relation. In particular, equation (4) indicates that past levels of the real interest rate differential should be included in the forecasting equation for real exchange rate changes. To obtain such a forecasting equation, we rewrite the expected long-run change in q as the sum of period-to-period changes:

$$q_{t+k} - q_t = \sum_{l=1}^{k} \mathbf{E}\left(\Delta q_{t+l}\right). \tag{5}$$

A straightforward way to proxy the expectations in equation (5) is to use a forecast from a VAR that includes past levels of the real interest rate differential. We illustrate this in the context of a bi-variate system of the form:[2]

$$\mathbf{A}(\mathbf{L})\mathbf{x}_t = \mathbf{x}_t - \sum_{l=1}^{p} \mathbf{A}^l \mathbf{x}_{t-l} = \varepsilon_t, \tag{6}$$

where $\mathbf{A}(\mathbf{L})$ is a matrix polynomial in the lag operator or order p, and ε_t is an $i.i.d.$ error vector, with covariance matrix $\mathbf{\Omega}$ and

$$\mathbf{x}_t = \left[\begin{array}{c} r_t - r_t^* \\ \Delta q_t \end{array} \right].$$

We write the VAR in companion form so that

$$\mathbf{z}_t = \mathbf{\Gamma}\mathbf{z}_{t-1} + \mathbf{u}_t,$$

where

$$\mathbf{\Gamma} = \left[\begin{array}{cccc} \mathbf{A}_1 & \mathbf{A}_2 & ... & \mathbf{A}_p \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ : & \mathbf{I} & : & : \\ \mathbf{0} & ... & \mathbf{I} & \mathbf{0} \end{array} \right], \ \mathbf{z}_t = \left[\begin{array}{c} \mathbf{x}_t \\ \mathbf{x}_{t-1} \\ ... \\ \mathbf{x}_{t-p} \end{array} \right], \ \text{and } \mathbf{u}_t = \left[\begin{array}{c} \varepsilon_t \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{array} \right].$$

Now we can use the VAR to back out $\mathbf{E}\left(\Delta q_{t+k}\right)$ as

$$\mathbf{E}\left(\Delta q_{t+k}\right) = \mathbf{e}_2'\mathbf{\Gamma}^k\mathbf{z}_t. \tag{7}$$

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[2]We now drop the index for the maturity horizon and use the shorthand notation $r_t - r_t^*$ to denote long-term real interest rate differential at horizon k. We will henceforth adopt this simplified notation whenever the exact maturity horizon does not matter in our derivations.

where $\mathbf{e}'_2 = \begin{bmatrix} 0 & 1 & 0 & .. & 0 \end{bmatrix}$ is the second $p \times 2$ unit vector. We can then write the RERI as

$$\mathbf{E}\left(q_{t+k} - q_t\right) = \mathbf{e}'_2 \sum_{l=1}^{k} \mathbf{\Gamma}^l \mathbf{z}_t = r_t - r_t^*. \tag{8}$$

This equation suggests that a natural way of examining the empirical validity of the RERI is to inspect how closely $\mathbf{E}\left(q_{t+k} - q_t\right)$ is related to $(r_t - r_t^*)$ by simply looking at correlations between the two series. This is the approach taken in the literature inspired by the seminal work of Campbell and Shiller (1987) and it offers interesting perspectives on some of the earlier literature on the RERI. For example, Baxter (1994) was among the first to argue, in the context of the RERI derivation discussed above, that the real interest rate differential should be a stationary variable and therefore correlating it with a nonstationary variable does not make sense. Instead, she proposes correlating the real interest differential with the transitory, or stationary, component of the real interest differential extracted from the real exchange rate using a multivariate Beveridge-Nelson (1981) decomposition. This approach is shown to be successful in the sense that such correlations are significant throughout, though not always correctly signed. Although our approach also uses a permanent-transitory decomposition, it differs from Baxter's in the important respect that our multivariate decomposition involves the real interest rate differential itself.[3]

One central idea underlying the Campbell and Shiller approach is that - under the null of the present value model – the spread variable is a sufficient statistic for agents' expectations of changes in the fundamentals. Therefore, the econometrician, who will generally only have limited information, should include this variable in her forecasting equation. Once we interpret the RERI as a present value relationship in which the discount factor is restricted to unity, it becomes clear that this idea should carry over to the RERI relationship. As we will see, the inclusion of the real interest rate differential, and its interpretation as the spread, greatly improves the proxy of expected exchange rate changes vis-a-vis earlier studies and allows us to identify the RERI relationship quite robustly. Emphasizing the structural similarity of the RERI with a present value model is also useful in evaluating to what extent we should expect to identify this relationship in the data and what possible sources of a failure to detect this relationship may be. After a brief description of our data, the next section will present the results of our empirical implementation of (8) along with a detailed discussion of these issues.

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[3]Baxter's multivariate decomposition was derived from a bivariate VAR in monthly changes of the real exchange rate and inflation differential.

3   Re-evaluating the RERI

3.1   Data

Our data set consists of quarterly data for the G7 countries, the United States, Japan, Germany, France, Italy, the United Kingdom and Canada, over the period 1978:Q1 to 1997:Q4. All data are sourced from the IMF's International Financial Statistics (IFS).

The nominal interest rates are long bond yields (line 61) and the price indices are consumer prices (line 64). We constructed bilateral CPI-based real exchange rates vis-a-vis the United States using average quarterly dollar exchange rates. The output data measure real GDP denominated in domestic currency (code 99B). These were converted into US dollars using the mean nominal exchange rate over the sample period. We then expressed GDP data in per capita terms using annual population data, also from the IFS, before constructing relative output levels, again vis-a-vis the U.S.

In order to obtain long-term real interest rates, we first constructed an estimate of average inflation expectations over the maturity horizon of the underlying government bonds (typically 10 years). This was achieved by running a univariate autoregression of CPI-inflation with 5 lags.[4] We then generated forecasts of quarterly inflation 40 periods ahead. To generate the average expected annual inflation rate we finally divided the cumulative sum of inflation rates by the bond's maturity horizon.

[Figure 1 about here]

3.2   Results of Present Value tests

Figure 1 provides a first impression of the RERI link by plotting the data. An ocular inspection seems to reveal a clear link between real interest rate differentials and exchange rates. Periods of low interest rate differentials coincide with high levels of q, i.e. with periods of very depreciated real exchange rates.

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[4]To check our results for robustness, we varied the lag length in the construction of expected inflation between 1 and 9 lags. All the results in the paper were found to be robust to this change in the construction of real interest rates.

We specified our VARs with seven lags for most countries, although somewhat shorter lag lengths eventually proved sufficient for Germany (5), France (3) and Italy (3). In keeping with the maturity horizons of the government bonds we consider here, we project the expectations 10 years, or 40 quarters, into the future. In Figures 2-7 we plot the expected annualized real rate of depreciation, generated from the VARs, of the US dollar vis-a-vis the currencies of the other G7 countries. The results are quite striking and would seem to suggest that there is considerable support for the RERI in the data. For virtually all countries, the predicted rate of depreciation is highly correlated with the real interest rate differential, though the interest rate differential is generally more volatile. We also obtain a measure of the uncertainty surrounding our forecast of exchange rate changes based on 100 bootstrap replications. In figures 2-7, the dotted lines represent the 90 percent quantile of the small sample distribution of FC, our estimate of $\mathbf{E}_t(q_{t+k} - q_t)$, thus obtained.[5] It is noteworthy that the conditional forecast distribution covers the interest rate differential for most of the sample period or is at least very close to it.

The impression obtained from the graphical analysis is confirmed by the results in Table 1, where we report correlation coefficients. The correlations range from a minimum of 0.56 for Japan to 0.94 for Germany, the average of the correlation coefficients across countries is 0.8. An examination of the relative standard deviations, reported in the second column, reveals that the predicted exchange rate change is generally just half as volatile as the real interest rate differential, with Canada being the exception. The RERI seems to do much better in terms of the correlation between real exchange rates and real interest rates than in terms of their relative volatility. To assess the robustness of this conclusion, we obtain 100 bootstrap replications of the model and tabulate the probability that the correlation coefficient is bigger than 0, 0.5 and 0.8 respectively. The results are given in rectangular brackets in the first column of table 1. The probability mass of the empirical distribution of the correlation coefficients is concentrated in the positive unit interval and in four out of the six countries (Canada, France, Germany and Italy) at least 70 percent of the bootstrapped correlation coefficients are bigger than 0.5 and at least about a quarter even exceed 0.8. Hence, the empirical distributions

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[5]It may not be surprising that this quantile almost always covers zero – exchange rate changes are hard to predict, and particularly so based on a VAR deliberately set up as parsimoniously as ours. Thus, while the hypothesis that $FC = 0$ is hard to reject at conventional significance levels, the confidence intervals also suggest that the bulk of the probability mass is actually changing the side of zero quite frequently.

tabulated here suggest that the correlation coefficients are also statistically close to unity. Conversely, the 90% confidence intervals of the relative standard deviations of the forecasted exchange rate change and the real interest rate differential – reported in parentheses in the second column – do not cover unity in 5 out of six cases, with Canada being the sole exception; if the RERI is statistically rejected, it is so because expected real exchange rate changes are much less volatile than real interest rate differentials.[6]

[Table 1 about here]

Graphs such as those presented in figures 2-7 have played an important role in convincing macroeconomists that simple present value models – be it of the New Keynesian Phillips curve (Sbordone (2002)), the term structure of interest rates (Campbell and Shiller (1987)), or of consumption (Campbell (1987)) – should not be dismissed prematurely, even though the exact cross-equation restrictions imposed by these models have often been statistically rejected. In many of these applications, the statistical rejection can be traced back to the fact that the present value model fails to replicate the exact variability of the forecasting or 'spread' variable, while the model typically does well in terms of the correlation of the predicted value with the 'spread' variable. As our graphs and the results in table 1 suggest, the RERI is no exception in this regard.

The cross-equation equation restrictions imposed by the RERI are easily obtained by noting that equation (8) must hold for all realizations of $\mathbf{z}_t$:

$$\mathbf{e}_2' \mathbf{\Gamma} \left[\mathbf{I} - \mathbf{\Gamma}^k \right] \left[\mathbf{I} - \mathbf{\Gamma} \right]^{-1} = \mathbf{e}_1'.$$

Here, in addition, we have used the formula for the $k-$th partial sum of the geometric series and the fact that $r_t - r_t^* = \mathbf{e}_1' \mathbf{z}_t$. We report the p-values of Wald tests of this set of restrictions in the third column of table 1. Statistically, the cross equation restrictions are rejected in all six bilateral exchange rates. What interpretation should be placed on such rejections, given that the correlations in the first column of Table 1, in addition to the graphs, convey a much more positive message? In order to understand the meaning of this statistical rejection, we turn to the economic interpretation of the cross-equation restrictions in (8). To this end, we use the definition of the real excess return on holding a currency over k periods as

$$\sigma_t(k) = q_{t+k} - q_t - (r_t(k) - r_t^*(k)). \tag{9}$$

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[6]We obtained very similar results from a trivarate VAR that also included relative output growth as an additional endogenous variable.

Taking expecations, we see that the cross-equation restrictions imply that $\mathbf{E}_t(\sigma_t(k)) = 0$ – excess returns should not be predictable. Taking conditional expectations, re-arranging terms and taking variances of both sides, we can decompose the variance of the real interest rate differential as follows:

$$\frac{\mathbf{cov}(\mathbf{E}_t(q_{t+k} - q_t), r_t(k) - r_t^*(k))}{\mathbf{var}(r_t(k) - r_t^*(k))} - \frac{\mathbf{cov}(\mathbf{E}_t(\sigma_t(k)), r_t(k) - r_t^*(k))}{\mathbf{var}(r_t(k) - r_t^*(k))} = \beta_q - \beta_\sigma = 1.$$
(10)

This decomposition provides us with an alternative, economically more interpretable measure of fit of the RERI. It adapts the 'good beta, bad beta' methodology of Campbell and Voultenaaho (2004) to the RERI: in the language of Campbell and Voultenaaho (2004) and Froot and Ramadorai (2001), β_q can be thought of as measuring the contribution of cash flow news, whereas β_σ measures the impact of expected variation in the discount factor on the interest rate differential. A strict interpretation of the RERI and the cross-equation restrictions implies that $\begin{bmatrix} \beta_q & \beta_\sigma \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Clearly this assumption could be violated if there is a risk premium on the currency that varies in a predictable way. But if there is such a premium, then, according to (10), the real interest rate differential must be correlated with expected excess returns. Hence, either the RERI holds or excess returns on the currency are predictable. It is not logically possible to reject both return predictability and the RERI.[7] The joint hypothesis $\begin{bmatrix} \beta_q & \beta_\sigma \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ may well be rejected in the data, and this is what tests of the cross-equation restrictions indeed suggest. Nonetheless, there may be an economically important link between the real interest rate differential and expected rates of change of the real exchange rate; even though β_q may not be identically one, it can still be statistically significantly different from zero in the data and it then measures the extent to which the RERI relationship can explain the variation in the real interest rate differential. The above decomposition is easily obtained by regressing our expected depreciation measure on the real interest rate differential.[8] Table 2 reports our results. The RERI explains a significant portion of the observed variation in real interest rates in all six currency pairs; we consistently reject $\beta_q = 0$ at very high levels of significance and our point estimates of β_q range from 0.36 to 0.88. For some countries, the estimated value of β_q gets remarkably close to one. But – with the exception

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[7]This result is quite analogous to Cochrane's (2001) observation that it is not possible to reject both dividend predictability and the predictability of excess returns in a stock price model

[8]Clearly, our estimate of the coefficient β_q will correspond to the product of the correlation and relative standard deviation reported in table 1. But performing the regression independently allows us to obtain a direct measure of the significance of β_q.

of Canada – we also consistently reject $\beta_\sigma = 0$ which, because of $\beta_\sigma = \beta_q - 1$, is equivalent to rejecting $\beta_q = 1$. These findings provide us with an economic interpretation of the rejection of the exact cross-equation restrictions implied by the RERI: the interest rate differential has some predictive power for long-term real excess returns on holding the US dollar – a finding which is consistent with the literature on the predictive properties of the forward foreign exchange premium (see Engel (1995) and MacDonald (2006)).

[Table 2 about here]

It is instructive to compare the results of our variance decomposition to the findings reported by Baxter (1994). Baxter constructs Beveridge-Nelson (1981) measures of the transitory component of the real exchange rate, q_t^T. She then regresses this transitory component on the real interest rate differential. Our coefficient β_q is analogous to the coefficient recovered from Baxter's regressions and, as in Baxter (1994), we find this coefficient to be significant in all six bilateral exchange rates. But in her paper, Baxter also encounters a puzzle: while always significant, the regression of q^T on the interest rate differential sometimes gives a positive, sometimes a negative coefficient. Our coefficient β_q is unambiguously, and significantly, positive in all six cases. Our interpretation of this finding is the following: we can only hope to identfy the RERI link with the correct theoretical sign if we have a sufficiently good measure of expected exchange rate changes. Including the real interest in the construction of such a transitory component of real exchange rates – as we advocate here – may therefore be important in consistently identifying the RERI link.

In appraising the economic significance of our results, it may also be useful to compare them to other extant findings in the wider empirical literature on present value models. For example, in the context of a test of the permanent income model, Campbell (1986) writes that '*the permament income hypothesis is worth taking seriously. [...] More generally, models which are strongly rejected statistically may be good approximations of the behavior of economic variables*' (p.29). Campbell and Shiller (1987) reach similarly positive conclusions about the term structure model of interest rates, but are more skeptical about the fixed-discount factor model of stock prices. Indeed subsequent to their work, it has been demonstrated that the present-value model of stock prices fails because the dividend price ratio does not reveal variation in dividends; rather, it uncovers variation in both stock prices and excess returns (see the discussion in Cochrane (1994) and Cochrane (2001)).

As our results here show, the RERI is also rejected statistically. This rejection can be traced back to the fact there is some predictability in the real excess returns on holding a currency. But as we have also shown, in spite of this, there is still a significant link between the expected depreciation and the real interest rate differential: the RERI explains a significant fraction of the variation in real interest rates and provides a reasonable first-order approximation of expected exchange rate changes. In this respect, the RERI certainly performs no worse than most applications of the present value model, and, in fact, seems to perform as well as the relatively more sucessful implementations of the model. We illustrate this point further in the next section, where we conduct a systematic examination of how the RERI relationship is affected by various types of structural shocks.

4 Structural shocks to the RERI relation

The RERI predicts that fluctuations in the real interest rate differential should be associated with temporary fluctuations in real exchange rates. More specifically, a widening interest rate differential in favour of the home country should be indicative of a future depreciation of the real exchange rate. Given a fixed long-run value of the exchange rate, this implies that the real exchange rate should appreciate after a shock to the real interest rate differential. In our notation, this implies that q will have to drop when $r - r^*$ rises: the impact responses of the two variables after a temporary shock should have opposite sign. In this section, we explore this prediction in a structural VAR framework and examine the robustness of our conclusions with respect to different identifying assumptions.

4.0.1 Choleski and Blanchard-Quah identification schemes

Consider again our baseline VAR-specificaton, discussed in (6) above:

$$\mathbf{A}(\mathbf{L})\mathbf{x}_t = \boldsymbol{\varepsilon}_t. \tag{11}$$

Following the structural VAR literature, we postulate that the reduced-form residual, $\boldsymbol{\varepsilon}_t$, is a linear function of the vector, $\mathbf{v}_t$, of structural shocks, so that $\boldsymbol{\varepsilon}_t = \mathbf{S}\mathbf{v}_t$, where $\mathbf{S}$ is a non-singular square matrix of dimension 2. Let $\boldsymbol{\Omega}$ be the variance covariance matrix of $\boldsymbol{\varepsilon}_t$. Furthermore, we assume that the structural shocks are mutually uncorrelated, so that $\mathbf{E}(\mathbf{v}_t\mathbf{v}_t') = \mathbf{I}$ and:

$$\boldsymbol{\Omega} = \mathbf{S}\mathbf{S}'. \tag{12}$$

In our two-dimensional VAR, equation (12) imposes three non-redundant restrictions on $\mathbf{S}$. To just-identify the vector of shocks, $\mathbf{v}_t$, and the associated impulse response functions, we therefore need an additonal restriction. It is customary, to impose $S_{12} = 0$, which amounts to a Choleski-decomposition of $\mathbf{\Omega}$. However, there is an entire manifold of possible choices for $\mathbf{S}$. Let $\mathbf{S}_0$ and $\mathbf{S}_1$ be two such alternative choices. Then

$$\mathbf{v}_{0t} = \mathbf{S}_0^{-1}\boldsymbol{\varepsilon}_t = \mathbf{S}_0^{-1}\mathbf{S}_1\mathbf{v}_{1t} = \mathbf{P}\mathbf{v}_{2t}, \tag{13}$$

and since both $\mathbf{v}_{0t}$ and $\mathbf{v}_{1t}$ have unit variance

$$\mathbf{I} = \mathbf{var}(\mathbf{v}_{0t}\mathbf{v}_{0t}') = \mathbf{P}\mathbf{var}(\mathbf{v}_{1t}\mathbf{v}_{1t}')\mathbf{P} = \mathbf{PP}'.$$

Hence, the mapping between two orthogonalized shock vectors $\mathbf{v}_{0t}$ and $\mathbf{v}_{1t}$, given by $\mathbf{v}_{0t} = \mathbf{P}\mathbf{v}_{1t}$, is orthogonal. The set of two-dimensional orthogonal matrices can be parametrized as

$$\mathbf{P} = \left[\begin{array}{cc} \rho & \sqrt{1-\rho^2} \\ \sqrt{1-\rho^2} & \rho \end{array} \right] = \left[\begin{array}{cc} \cos\lambda & -\sin\lambda \\ \sin\lambda & \cos\lambda \end{array} \right], \tag{14}$$

where $-1 < \rho < 1$ and $0 < \lambda < 2\pi$ and therefore we can write $\mathbf{P}$ as a function of λ, so that $\mathbf{P}(\boldsymbol{\lambda})$ defines a rotation. Two possible time series of orthogonal shocks - $\mathbf{v}_{0t}$ and $\mathbf{v}_{1t}$ − can then simply be rotated onto each other by an appropriate choice of λ. For an initial choice of $\mathbf{S}_0$ with $\boldsymbol{\varepsilon}_t = \mathbf{S}_0\mathbf{v}_{0t}$, we therefore consider the space of alternative rotations $\mathbf{S}_\lambda = \mathbf{P}(\boldsymbol{\lambda})'\mathbf{S}_0$, where we let λ vary between 0 and 2π. Each choice of λ identifies a vector of mutually orthogonal shocks, $\mathbf{v}_\lambda = \mathbf{S}_\lambda^{-1}\boldsymbol{\varepsilon}_t$, to which we can obtain the impulse responses. In this way, we can determine the robustness of our conclusions with respect to the key question addressed here: are shocks to the real interest rate differential associated with temporary fluctuations in the real interest rate differential?

Our approach is similar in spirit to that used in King and Watson (1997), who examine the robustness of long-run monetary neutrality under different identification schemes in a bivariate VAR setting. The main difference between our approach and theirs is that King and Watson estimate the contemporaneous interaction between the variables using simultaneous equation methods, whereas in our setting the contemporaneous interaction is given by $\mathbf{S}$ which is just identified from the set of orthogonality restrictions (12) and the additional identifying restriction as defined by $\mathbf{P}(\boldsymbol{\lambda})$.

Clearly, not all choices for $\mathbf{S}_0$ and $\mathbf{P}(\boldsymbol{\lambda})$ are equally plausible. We therefore start by considering two particularly important, and possibly plausible, identifying restrictions on $\mathbf{S}$. The first is the Choleski identification, in which

we choose $S_{12} = 0$. We will argue that this identification can yield important insights into the economic relevance of the RERI relation. To make this point, we write $\mathbf{x}_t$ in moving average form as:

$$\mathbf{x}_t = \mathbf{A}^{-1}(\mathbf{L})\boldsymbol{\varepsilon}_t = \mathbf{C}(\mathbf{L})\boldsymbol{\varepsilon}_t = \mathbf{C}(\mathbf{L})\mathbf{S}\mathbf{v}_{0t},$$

where

$$\mathbf{C}(\mathbf{L}) = \mathbf{I} + \sum_{k=1}^{\infty} \mathbf{C}_k \mathbf{L}^k.$$

Recall that we defined $\mathbf{x}_t = \begin{bmatrix} r_t - r_t^* & \Delta q_t \end{bmatrix}'$, so that the real interest rate differential is ordered first. Hence, with $S_{12} = 0$, we identify one shock that affects both the real interest rate differential and the real exchange rate and one shock that only affects the real exchange rate. According to the RERI, the former should: a) have only a transitory effect on the real exchange rate; and b) trigger an impact response of the real exchange rate that has the opposite sign of the response in the real interest rate differential, i.e. $S_{21}/S_{11} < 0$. Conversely, permanent variations in the real exchange rate should mainly be driven by those shocks that leave the real interest rate differential unaffected. Therefore, one test of the economic relevance of the RERI is to impose a Choleski identification and to test whether the impulse responses comply with the overidentifying restrictions just discussed.

An alternative test is to use a long-run identification in the spirit of Blanchard and Quah (1989). Such a restriction can be applied to the RERI by requiring that shocks to the interest rate differential should not have a long-run impact on the real exchange rate. We obtain this restriction by acknowledging that the long-run response of $\mathbf{x}_t$ is given by $\mathbf{D}(\mathbf{1}) = \mathbf{C}(\mathbf{1})\mathbf{S}$. Given the ordering of our variables, requiring that the shock to the interest rate differential does not have an impact on the long-run level of the exchange rate, this amounts to $D_{21} = 0$. Here, the set of overidentifying restrictions implied by the RERI would be that the transitory shock should account for the bulk of the dynamics in the real interest rate differential and that the response of the interest rate differential to such a shock should have the opposite sign of the real exchange rate response.

4.0.2 Impulse response results

>From our discussion, it is apparent that the RERI actually implies that both the Choleski and the Blanchard-Quah decompositions should give us the same pattern of responses: the response to the transitory shock in the Blanchard-Quah decomposition should just correspond to the response to an interest rate shock in the Choleski-decomposition.

For all six country pairs, Figure 8 presents the impulse responses of the VAR in (11) obtained under the Choleski and the Blanchard decompostions respectively. The first key point to note from these graphs is that the choice between the two identification schemes does not strongly affect the results: the responses obtained under the Choleski and the Blanchard-Quah schemes move very closely together in all six countries. In many cases, the Blanchard-Quah response even falls into the 90 percent bootstrapped confidence interval of the Choleski decomposition, so that – at least in a macroeconomic sample of the size we have here – it is statistically not possible to tell the two identification schemes apart. Even more encouragingly, the relative sign of the responses matches the predictions of the RERI – a transitory appreciation of the real exchange rate is typically associated with an increase in the real interest rate differential. Furthermore, the two responses are often of roughly the same absolute size, i.e. the on-impact percentage increase in $r - r^*$ almost matches the percentage decrease in q.

The graphs also give some indication as to why the cross-equation restrictions imposed by the RERI, discussed in the previous section, may be statistically rejected. Specifically, while we find that the shape of the real exchange rate response to an interest rate shock is very similar to its response to a transitory shock, the two responses often diverge in the long run – there seems to be a small permanent component in the real exchange rate response to the first Choleski-shock; that is, the shock in the real interest rate differential. Based on our bootstrapped confidence intervals, the difference between the two responses does not, for most countries, appear to be statistically significant. But to the extent that the difference *is* significant, the shocks to the real interest rate (as identified through the Choleski scheme) must be correlated with permanent shocks to the real exchange rate (as identified from the BQ-scheme). To see this, recall from our discussion above that we can always map the two schemes onto each other by writing

$$\mathbf{S}_{BQ} = \mathbf{P}(\boldsymbol{\lambda})' \mathbf{S}_{Chol},$$

where $\mathbf{P}(\boldsymbol{\lambda})$ is the appropriate rotation-matrix. Clearly, if the two schemes yield identical responses, $\lambda = 0$ so that $\mathbf{P}(\boldsymbol{\lambda}) = \mathbf{I}$. For $\lambda \neq 0$, however, we can immediately infer from the definition of $\mathbf{P}(\boldsymbol{\lambda})$ in (14) above that the off-diagonal entries of $\mathbf{P}(\boldsymbol{\lambda})$ must generally be non-zero. Note further that $\mathbf{P}(\boldsymbol{\lambda})$ is also the correlation between the shocks identified under the Choleski- and the BQ-schemes. Therefore, unless $\mathbf{P}(\boldsymbol{\lambda}) = \mathbf{I}$, the interest rate shock will be correlated with the permanent shock. We contend that the economic interpretation of this finding is the same as that of the rejection of

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the overidentifying restrictions in the previous sub-section. If the Blanchard-Quah and the Choleski schemes do not identify the same responses, this will be a reflection of the predictability of excess returns. To see this, note that we can use equation (9) to write the interest rate shock, identified from the Choleski identification as

$$
\begin{aligned}
r_t(k) - r_t^*(k) - \mathbf{E}_{t-1}(r_t(k) - r_t^*(k)) &= \sigma_t(k) - [q_{t+k} - q_t] - \mathbf{E}_{t-1}\left[\sigma_t(k) - [q_{t+k} - q_t]\right], \\
&= \sigma_t(k) - \mathbf{E}_{t-1}\sigma_t(k) - [q_{t+k} - q_t - E_{t-1}\left[q_{t+k} - q_t\right]].
\end{aligned}
$$

Taking expectations as of time t, we obtain

$$
r_t(k) - r_t^*(k) - \mathbf{E}_{t-1}(r_t(k) - r_t^*(k)) = \mathbf{E}_t - \mathbf{E}_{t-1}\left(\sigma_t(k)\right) - \mathbf{E}_t - \mathbf{E}_{t-1}\left[q_{t+k} - q_t\right].
$$

The first term on the right hand side is the shock to expected excess returns. The second term on the right hand side is the change in the expected rate of depreciation – the innovation in the transitory component of q_t. As we have argued above, under a strict reading of the RERI, expected excess returns should be unpredictable, so that in particular $\mathbf{E}_t - \mathbf{E}_{t-1}\left(\sigma_t(k)\right) = 0$. If excess returns are unpredictable, they cannot be expected to be offset in the future; hence, under the RERI, excess returns must correspond to the permanent shock in the real exchange rate and the real interest rate differential will not be correlated with these permanent shocks. Only in this case, will the Choleski and the Blanchard–Quah schemes yield identical responses. However, if excess returns are predictable then the real interest rate will also be correlated with the permanent component of the real exchange rate.

In this context, it is useful to inspect once again the bootstrapped confidence intervals in Figure 8: based on these, we cannot actually differentiate the responses of the exchange rate based on either identification scheme. Hence, the evidence for the predictability of excess returns, and therefore for a statistical rejection of the RERI, is a lot weaker once we base our inference on approximations of the underlying small sample distributions.

Again, we conclude from these findings that predictable variation in the discount factor may lead to a statistical rejection of the RERI, but that this does not invalidate the RERI as an economically significant relationship. Taking account of the additional sampling uncertainty in small samples reinforces this point, because it suggests that researchers should be even more wary not to prematurely reject the RERI based on asymptotic distributions.

4.0.3 Robustness of identification schemes

Theoretical considerations suggest that the Blanchard-Quah and Choleski decompositions deserve special consideration in the context of the RERI.

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But as we discussed initially, there is an infinity of potential identification schemes, and, clearly, not all of these schemes are equally plausible on economic grounds. But it is nonetheless informative to examine the robustness of our conclusions with respect to different identification schemes for the structural shocks.

According to equations (13) and (14), any possible identification of structural shocks to the RERI can be recovered through an appropriate rotation of the shocks recovered from the Choleski-identification. Let $\mathbf{S}_\lambda$ be any matrix fulfilling the orthogonality restrictions (12), then

$$\mathbf{S}_\lambda = \mathbf{P}(\boldsymbol{\lambda})' \mathbf{S}_{Chol},$$

for some rotation matrix $\mathbf{P}(\boldsymbol{\lambda})$. Hence, in order to explore how the RERI relationship is affected by different identifying assumptions, we simply have to vary λ . Specifically, we choose values of λ in the interval $\begin{bmatrix} 0, & 2\pi \end{bmatrix}$ with a step width of one degree, i.e. $2\pi/360$. For each $\mathbf{S}_\lambda$ thus obtained, we obtain the impulse responses to the two structural shocks. We normalize the interest rate response to the first shock and the exchange rate response to the second shock to be positive. For convenience we therefore continue to call the first shock the interest rate shock and the second the exchange rate, or excess-return, shock. We then average the impulse response functions over the 360 different realizations and we also calculate the median response in order to obtain an impression of the distribution of the underlying responses. The results of this exercise are plotted in Figure 9, panels a-f.

[Figure 9 about here]

The characteristic pattern of the response to an interest rate shock that we established from the Choleski- and Blanchard-Quah decompositions, turns out to be very robust to changes in the identifying assumptions: for most countries, the first shock leads to a fall - an appreciation - in the real exchange rate. This appreciation is then generally offset as the interest rate differential starts to narrow. In the long-run, this shock does not have a pronounced impact on the real exchange rate. The second (excess return) shock generally has a permanent effect on the exchange rate and it is also generally associated with temporary fluctuations in the real interest rate differential, but only to the extent that the real exchange rate intitially underadjusts to the permanent shock. The message from the various panels in Figure 9 is the same, irrespective of whether we consider the median or the average response. We note, however, that the stylized pattern is generally even more

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pronounced once we consider the median response, and particularly for the real exchange rate. This suggests that for the majority of all possible identification restrictions, the response of the two variables to structural shocks complies well with the predictions of the RERI.

5   Conclusion

In this paper we have re-examined the real exchange rate - real interest rate (RERI) relationship using data for six US dollar bilateral exchange rates, over the period 1978 to 1997. Many previous tests of this relationship have involved attempting to cointegrate measures of a real exchange rate with a measure of a country's real interest differential. However, following Baxter (1994), the derivation of the RERI relationship suggests that such a method is likely to be flawed since if the real exchange rate is integrated of order one, the real interest differential must be stationary.

Building on the work of Baxter (1994), we proposed interpreting the RERI as a present-value relation and to test it using the VAR-based approach of Campbell and Shiller (1987). This involves taking the projection for the change in the real exchange rate from a bivariate VAR, consisting of the change in the real exchange rate and the real interest differential, and correlating this with the real interest differential. We argued that this kind of test is much closer in spirit to the RERI relationship than many extant tests and it produces measures of long-run expected changes in the exchange rate which are highly correlated with real interest rate differentials. While the entire set of cross-equation restrictions that arise from our model is statistically rejected, this rejection can be traced back to the presence of predictable excess returns and does not invalidate the RERI link as an economically significant relation. The upshot of our results is that the RERI is no more elusive than other important relationships in macroeconomics and finance that have been tested in a present value context, such as: the stock price / dividend relationship, the consumption - income relationship, the term structure of interest rates and the new Keynesian Phillips-Curve. Such models are often statistically rejected in a present value setting, but the statistical rejection is usually associated with a fixed discount-factor assumption.

Further evidence in support of the RERI is provided by our attempts to identify structural shocks to the RERI relationship. We find that shocks to the real interest rate differential, in general, only produce temporary responses in the real exchange rate and these responses have the right sign: on impact, a widening interest rate differential leads to a temporary appreciation that is then offset through a subsequent depreciation as relative price

levels start to converge and as the interest rate differential starts to narrow again. This result turns out to be independent of the particular identification scheme imposed on our VAR model. The evidence we have reported in this paper therefore strongly suports the important conclusion of Baxter (1994) that the RERI as an economic relationship should be taken seriously: real interest rate differentials constitute a good proxy for the temporary component in real exchange rates!

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Table 1: Comovement of $FC = E(\widehat{q_{t+k} - q_t})$ with $r - r^*$.

	Correlation	Relative Std. Dev.	Wald test		
		$(\sigma_{FC}/\sigma_{r-r^*})$	χ^2	p-value	DOF
Canada	0.80	1.13	30.40	0.01	15
	[0.91,0.86,0.23]	[0.15-1.45]			
France	0.87	0.50	1.68×10^7	0.00	7
	[0.87,0.76,0.45]	[0.11-0.74]			
Germany	0.94	0.56	141.31	0.00	11
	[0.79,0.7,0.56]	[0.20-1.10]			
Italy	0.90	0.61	16131.00	0.00	7
	[0.99,0.98,0.87]	[0.09-0.80]			
Japan	0.56	0.52	430.78	0.00	15
	[0.75,0.30,0.01]	[0.22-0.75]			
United Kingdom	0.74	0.41	112.53	0.00	15
	[0.82,0.46,0.08]	[0.19-0.73]			
G7 average	0.80	0.62			

NOTES: In the column 'Correlation', the numbers in parentheses give the probability that the correlation is bigger than 0,0.5 and 0.8 respectively.In the column 'relative Std. Dev.' the numbers in parentheses give the 90% confidence intervals. The confidence measures in both columns are obtained from 100 bootstrap replications of the model. Column 'DOF' gives degrees of freedom

Table 2: Variance decomposition of $r(k) - r^*(k)$

		coefficient	t-stat	R^2
Canada	β_q	0.88	11.04	0.64
	β_σ	-0.11	-1.45	0.03
France	β_q	0.43	15.00	0.75
	β_σ	-0.57	-20.03	0.84
Germany	β_q	0.51	23.45	0.88
	β_σ	-0.49	-22.89	0.88
Italy	β_q	0.55	17.65	0.81
	β_σ	-0.45	-14.53	0.74
Japan	β_q	0.31	5.71	0.32
	β_σ	-0.69	-12.87	0.70
United Kingdom	β_q	0.36	9.15	0.54
	β_σ	-0.64	-16.53	0.80

Notes: coefficients β_q and β_σ obtained from OLS-regressions of the form
$$\mathbf{E}(\widehat{q_{t+k} - q_t}) = \beta_q \left(r_t(k) - r_t^*(k) \right) + v_t \text{ and } \sigma_t(k) = \beta_\sigma \left(r_t(k) - r_t^*(k) \right) + v_t$$
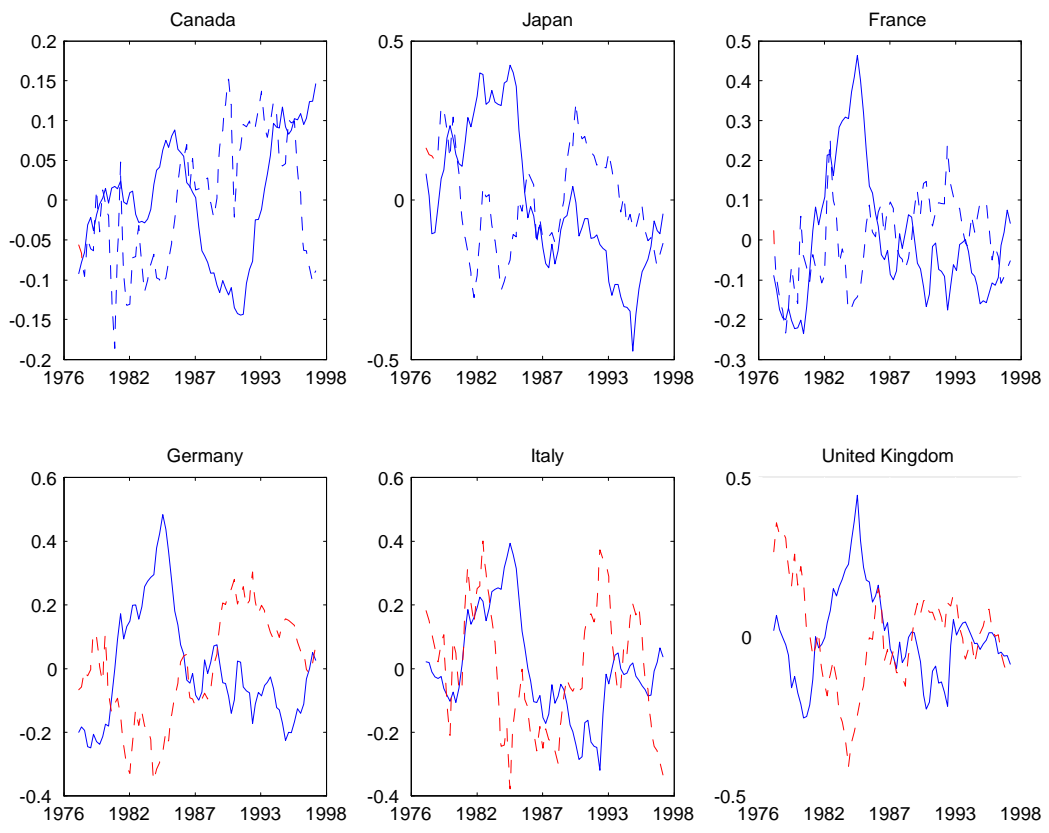
Figure 1: U.S. bilateral CPI real exchange rates (solid line) and real interest differential (in %*10^{-1})
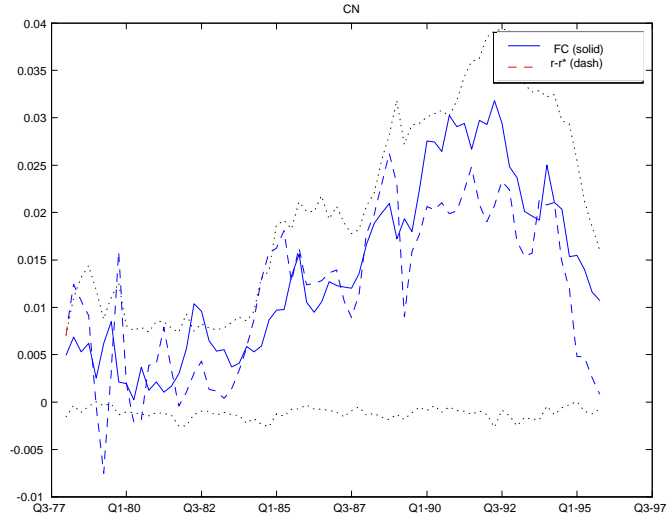
Figure 2: **Canada** – Expected Rate of Depreciation, $FC = \mathbf{E}_t(\widehat{q_{t+k} - q_t})$, (solid/blue) and real interest rate differential (dashed/red). Dotted/black line gives 90% confidence intervals of FC.
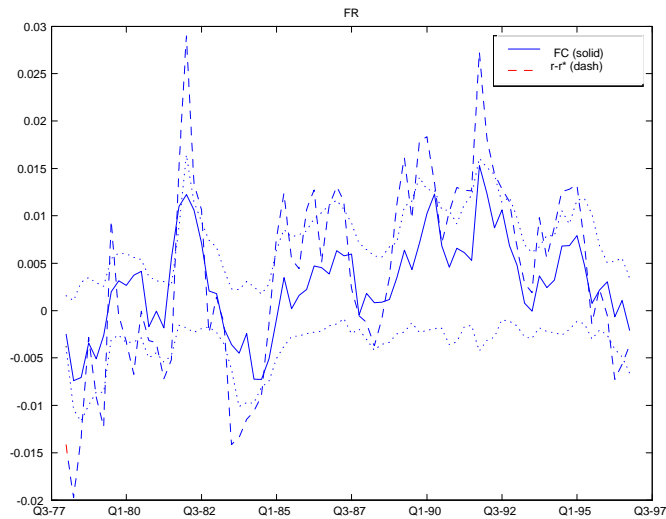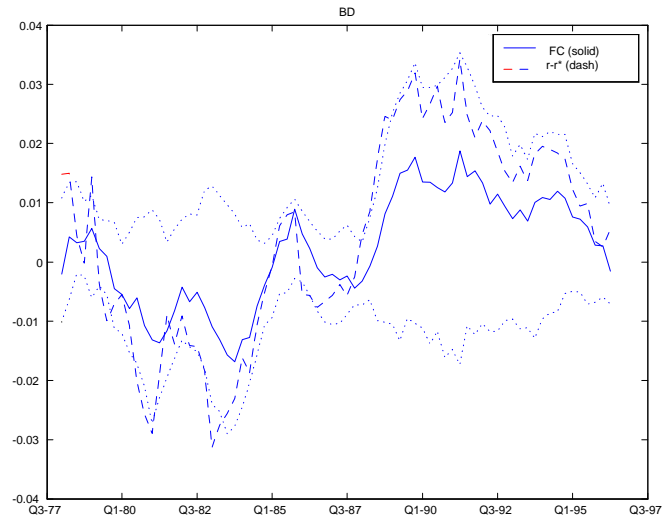

Figure 3: **France** – for notes see figure 2.

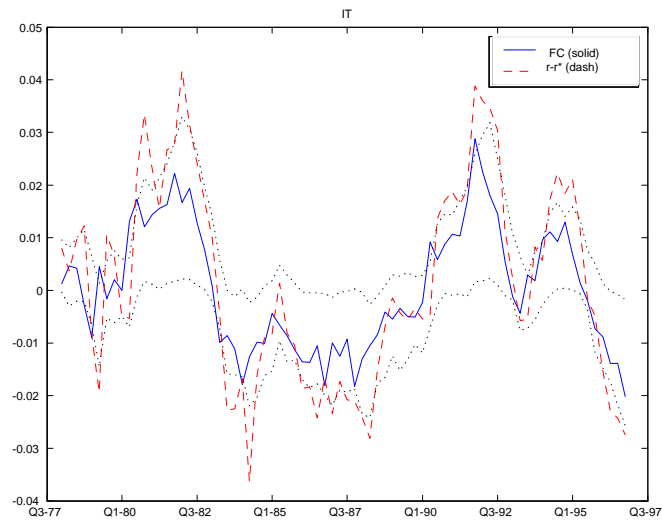24

Figure 4: **Germany** – for notes see figure 2
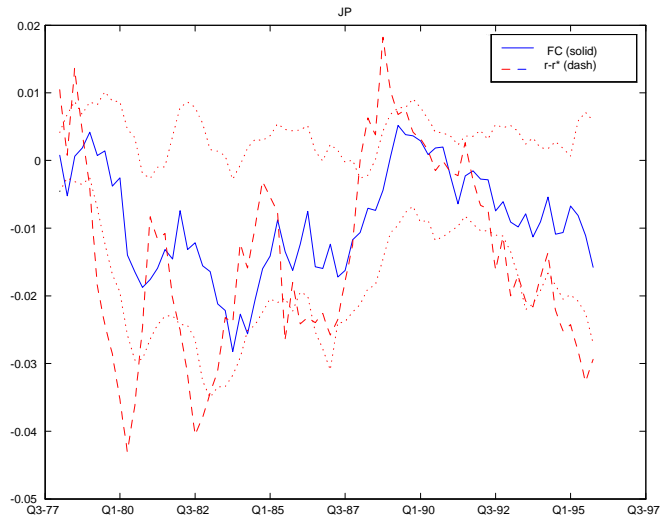

Figure 5: **Italy** – for notes see figure 2
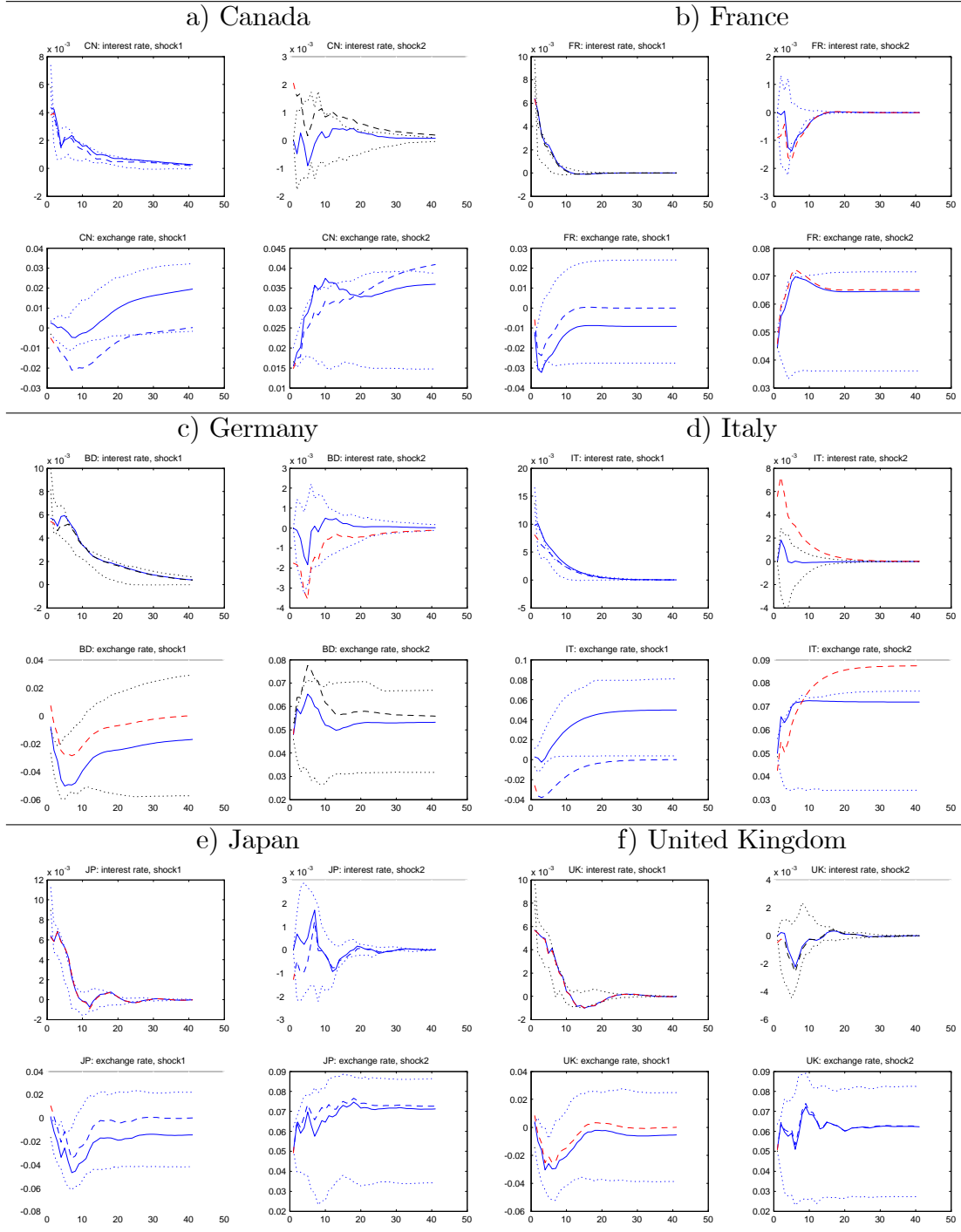
25

Figure 6: **Japan** – for notes see figure 2
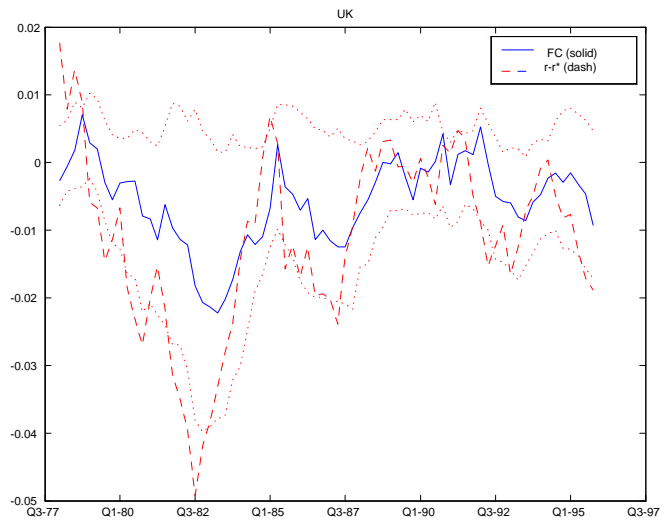
Figure 7: **United Kingdom** – for notes see figure 2

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Figure 8: Impulse responses obtained from Choleski and BQ-decompositions

a) Canada

b) France

c) Germany

d) Italy

e) Japan

f) United Kingdom

Figure 9: Average and median Impulse responses obtained through rotation

a) Canada

b) France

c) Germany

d) Italy

e) Japan

f) United Kingdom
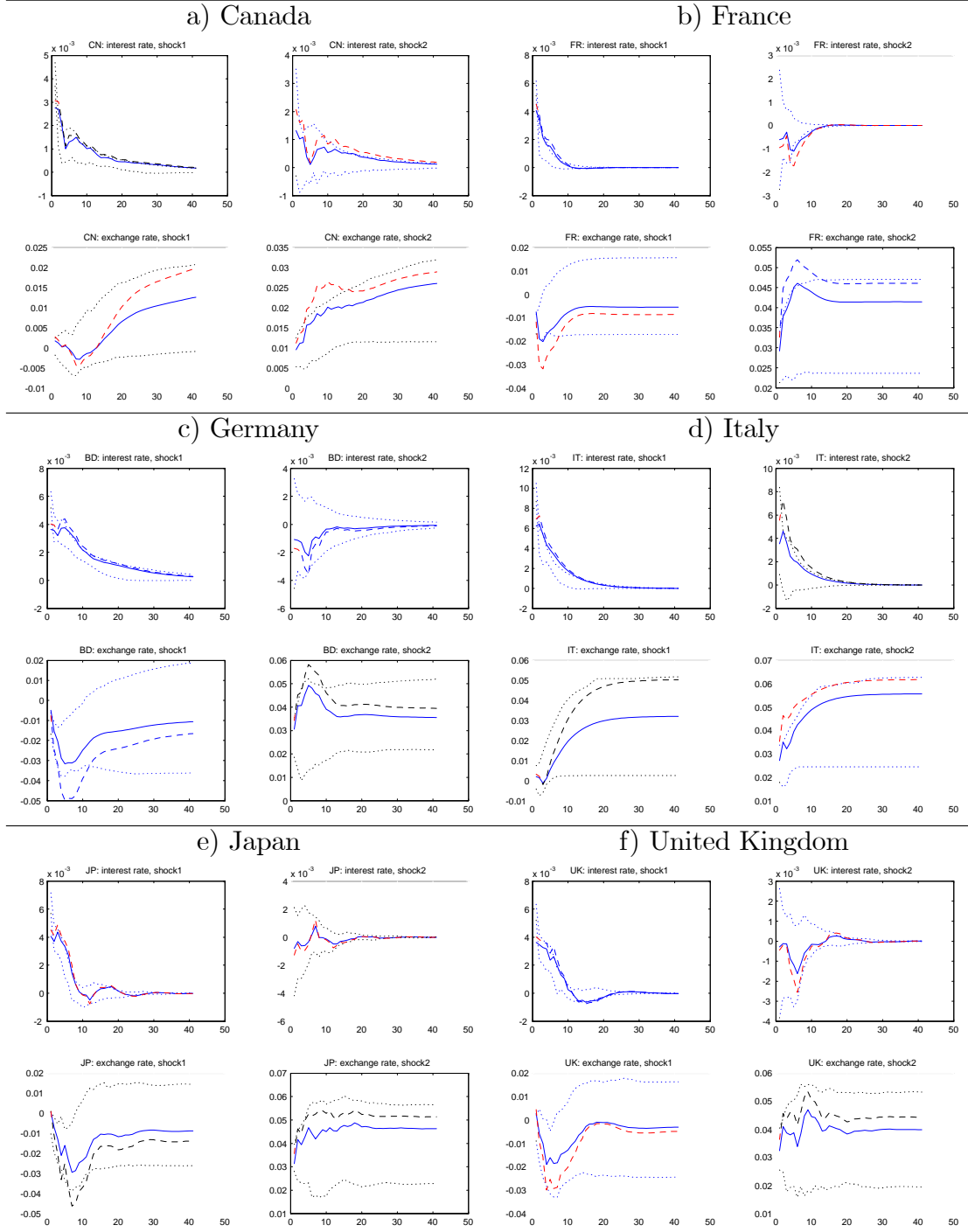
NOTES: Mean (blue, solid) and median (red, dashed) across all responses obtained from $P(\lambda)'S_{Chol}$ for $0 < \lambda < 2\pi$. Black, dotted lines are 10% confidence intervals of the mean response obtained by 100 bootstrap replications.

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