Noncooperative Oligopoly in Markets with a Continuum of Traders: A Limit Theorem *à la Cournot*[*]

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Abstract

In this paper, we consider an exchange economy à la Shitovitz (1973), with atoms and an atomless set. We associate with it a strategic market game of the kind first proposed by Lloyd S. Shapley and known as the *Shapley window model*. We analyze the relationship between the set of the Cournot-Nash equilibrium allocations of the strategic market game and the Walras equilibrium allocations of the exchange economy with which it is associated. We show, with an example, that even when atoms are countably infinite, any Cournot-Nash equilibrium allocation of the game is not a Walras equilibrium of the underlying exchange economy. Accordingly, in the original spirit of Cournot (1838), we partially replicate the mixed exchange economy by increasing the number of atoms, without affecting the atomless part, and ensuring that the measure space of agents remains finite. We show that any sequence of Cournot-Nash equilibrium allocations of the strategic market games associated with the partially replicated exchange economies approximates a Walras equilibrium allocation of the original exchange economy.

Journal of Economic Literature Classification Numbers: C72, D51.

[*]Francesca Busetto and Giulio Codognato gratefully acknowledge financial support from MIUR (PRIN 20103S5RN3).

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1 Introduction

A large literature has been developed in the past decades, aiming at extending to a general equilibrium framework the classical Cournot (1838)'s theory of oligopoly, where oligopolistic agents that interact noncooperatively among them face a sector of consumers taking prices as given. The relevance of this issue nowadays has been affirmed by Jean Gabszewicz in a recent contribution (see Gabszewicz (2013), p. 6): "Direct observation of economic activity reveals that markets are the fields of "giants," operating simultaneously with a fringe of small competitors. [...] Behind the demand function there is a myriad of "small" price-taking agents, while the supply side is occupied by few agents appearing as giants, contrasting with the dwarfs on the demand side."

Most of the contributions on this issue belongs to two main lines of research: the Cournot-Walras equilibrium approach, initiated by Gabszewicz and Vial (1972), and the strategic market game approach, initiated by Shapley and Shubik (1977).

In their 1972's paper, Gabszewicz and Vial transposed to a general equilibrium setting Cournot's original idea of an asymmetric economy with production, in which firms with oligopolistic power that interact strategically on quantities face a sector of consumers behaving à la Walras. Nevertheless, in the same paper, they pointed out two major difficulties inherent in the standard Cournotian approach with strategic firms: first, profit maximization may not be a rational objective for firms that have influence on prices; second, the equilibrium is not independent from the rule chosen to normalize prices. These difficulties have been overcome within the Cournot-Walras approach by moving to the analysis of pure exchange economies (see Codognato and Gabszewicz (1993), d'Aspremont, Dos Santos Ferreira, and Gérard-Varet (1997), Gabszewicz and Michel (1997), Shitovitz (1997), among others). However, this latter class of models do not avoid a well-known problem inherent in the Cournot-Walras approach: there, an equilibrium may not exists, even in mixed strategies, since the Walras price correspondence may fail to admit a continuous selection (see Dierker and Grodal (1986)). A further fundamental problem common to the whole Cournot-Walras approach is that it leaves unexplained why some agents behave strategically while other agents behave competitively.

A different approach has been developed (still in pure exchange), that uses strategic market games à la Shapley and Shubik with the aim at providing a formal explanation of perfectly and imperfectly competitive behavior.

A fundamental contribution in this line is the paper of Okuno, Postlewaite and Roberts (1980). In particular, since the work of these authors, results in this direction have been obtained by incorporating within the framework of strategic market games a mixed measure space of traders à la Shitovitz (1973), composed by large traders, represented as atoms, and small traders, represented by an atomless part. In this setting, while all agents have a priori the same strategic position, some of them turn out to have influence on prices and some other turn out to be Walrasian, depending on their characteristics and their weight in the economy. Then, the asymmetric structure typical of the Cournotian theory is endogenously generated.

Busetto, Codognato, and Ghosal ((2008), (2011)) obtained a generalization of Okuno et al.'s work by using a mixed version of the Cournot-Nash equilibrium model first proposed by Lloyd S. Shapley, and known as the *Shapley window model* (this model was analyzed, in the case of finite economies, by Sahi and Yao (1989)). Busetto et al. (2011) provided an endogenous explanation of oligopolistic and competitive behavior (see also Busetto, Codognato, and Ghosal (2013)). Moreover, working within this strategic market game framework permitted them to show the existence of an equilibrium in pure strategies, and then to overcome the non-continuity problem which characterizes the Cournot-Walras approach.

This paper studies the link between Cournot and Walras equilibrium, with the aim at providing a noncooperative foundation to the theory of perfect competition. A mixed exchange economy à la Shitovitz is associated with the same strategic market game à la Shapley proposed by Busetto et al. (2011). Within this framework, the relationship between the set of the Cournot-Nash equilibrium allocations of the strategic market game and the Walras equilibrium allocations of the underlying exchange economy is examined. We show, with an example, that even when the set of atoms is countably infinite, there is the robust possibility that no Cournot-Nash equilibrium allocation of the strategic market game is a Walras equilibrium of the underlying exchange economy, because some atoms remain non-negligible in size. This non equivalence result then motivates us to analyze the asymptotic relationship between appropriately defined sequences of Cournot-Nash equilibrium allocations of the strategic market game and the Walras equilibrium allocations of the exchange economy. We do this by introducing a concept of replication which we call *à la Cournot*, since it extends to a general equilibrium context the original Cournotian idea of replication: that is, we partially replicate the economy by increasing only the number of atoms, this way making them asymptotically negligible, without affecting the atomless part. At the same time, the mechanism of replication of atoms is constructed in such a way that the measure space of traders remains finite. If this requirement was not satisfied, the general equilibrium model of oligopoly studied in the paper would not be well-defined. Our main theorem establishes that any sequence of Cournot-Nash equilibrium allocations of the strategic market game associated with the partially replicated exchange economies approaches a Walras equilibrium allocation of the original exchange economy.

To prove the limit theorem, we use analytical tools introduced by Sahi and Yao (1989) to show their convergence result for finite economies, and by Codognato and Ghosal (2000) to show their equivalence theorem à la Aumann. However, we have to tackle new technical issues due to the fact that the space of traders has a mixed nature. To this end, we exploit some tools previously applied to the Shapley's window model for mixed economies by Busetto et al. (2011). In particular, in order to determine the limit points of the sequences of Cournot-Nash equilibrium allocations, we use a version of the Fatou's lemma in several dimensions proved by Artstein (1979). Moreover, a key point in our paper is that since the Walrasian price taking behavior of small traders is not assumed a priori, as in the Cournot-Walras equilibrium models, but endogenously generated, we do not need to impose any continuity assumption on the selections from the Walras price correspondence.

The general equilibrium approach adopted here distinguishes our limit result from the well-known results, obtained within the Cournotian tradition in partial equilibrium establishing that the Cournot equilibrium approaches the competitive equilibrium as the number of oligopolists goes to infinity (see Frank (1965), Ruffin (1971), Novshek (1980), among others).

Our limit result is also different from existing results in the strategic market game literature: within their Shapley's window model for finite economies, Sahi and Yao (1989) showed the convergence of sequences of Cournot-Nash equilibrium allocations to a Walras equilibrium allocation by using a replication concept *à la Edgeworth*, that is in which all types of agents are replicated (on this kind of replication see Debreu and Scarf (1963)).

On the other hand, Codognato and Ghosal (2000) showed that the set of the Cournot-Nash equilibrium allocations of the Shapley's model and the set of the Walras equilibrium allocations coincide in economies à la Aumann (1964), where the space of agents is an atomless continuum.

Analogous results for limit exchange economies are obtained within the Cournot-Walras approach by Codognato and Gabszewicz (1993), for their prototypical model in pure exchange, and by Busetto et al. (2013), for the new version of this model proposed by the same authors in the 2008's paper.

Also limit results were obtained within the line opened by Gabszewicz and Vial: see Roberts (1980), Mas-Colell (1983), and Novshek and Sonnenschein ((1983), (1987)), among others. All of them are obtained by using replication concepts in which all types of agents are replicated. Moreover, as stressed by Mas-Colell (1982) (see, in particular, pp. 203-204)), these results depend in an essential way on the assumption that there exists a continuous selection from the Walras price correspondence.

The paper is organized as follows. In Section 2, we build the mathematical model. In Section 3, we introduce the notion of a δ-positive Cournot-Nash equilibrium used in the construction of the limit theorem, and we state a theorem on its existence (the proof is in the Appendix). In Section 4, we show the example on the non-equivalence between the sets of Cournot-Nash and Walras equilibrium allocations when the atoms are countably infinite. In Section 5, we introduce the replication à la Cournot. In Section 6, we state the existence of a δ-positive atom-type-symmetric Cournot-Nash equilibrium (the proof is in the Appendix). In Section 7, we state and prove the limit theorem.

2 The mathematical model

We consider a pure exchange economy, $\mathcal{E}$, with large traders, represented as atoms, and small traders, represented by an atomless part. The space of traders is denoted by the measure space $(T, \mathcal{T}, \mu)$, where T is the set of traders, $\mathcal{T}$ is the σ-algebra of all μ-measurable subsets of T, and μ is a real valued, non-negative, countably additive measure defined on $\mathcal{T}$. We assume that $(T, \mathcal{T}, \mu)$ is finite, i.e., $\mu(T) < \infty$. This implies that the measure space $(T, \mathcal{T}, \mu)$ contains at most countably many atoms. Let T_1 denote the set of atoms and $T_0 = T \setminus T_1$ the atomless part of T. A null set of traders is a set of measure 0. Null sets of traders are systematically ignored throughout the paper. Thus, a statement asserted for "all" traders, or "each" trader, or "each" trader in a certain set is to be understood to hold for all such traders except possibly for a null set of traders. The word "integrable" is to be understood in the sense of Lebesgue.

In the exchange economy, there are l different commodities. A commodity bundle is a point in R^l_+. An assignment (of commodity bundles to traders) is an integrable function $\mathbf{x}$: $T \to R^l_+$. There is a fixed initial assignment $\mathbf{w}$, satisfying the following assumption.

Assumption 1. $\mathbf{w}(t) > 0$, *for each* $t \in T$.

An allocation is an assignment $\mathbf{x}$ for which $\int_T \mathbf{x}(t)\, d\mu = \int_T \mathbf{w}(t)\, d\mu$. The preferences of each trader $t \in T$ are described by a utility function $u_t : R^l_+ \to R$, satisfying the following assumptions.

Assumption 2. $u_t : R^l_+ \to R$ *is continuous, strongly monotone, and quasiconcave, for each* $t \in T$.

Let $\mathcal{B}(R^l_+)$ denote the Borel σ-algebra of R^l_+. Moreover, let $\mathcal{T} \bigotimes \mathcal{B}$ denote the σ-algebra generated by all the sets $E \times F$ such that $E \in \mathcal{T}$ and $F \in \mathcal{B}$.

Assumption 3. $u : T \times R^l_+ \to R$, *given by* $u(t, x) = u_t(x)$, *for each* $t \in T$ *and for each* $x \in R^l_+$, *is* $\mathcal{T} \bigotimes \mathcal{B}$*-measurable.*

We also need the following assumption (see Sahi and Yao (1989)).

Assumption 4. *There are at least two traders in* T_1 *for whom* $\mathbf{w}(t) \gg 0$, u_t *is continuously differentiable in* R^l_{++}, *and* $\{x \in R^l_+ : u_t(x) = u_t(\mathbf{w}(t))\} \subset R^l_{++}$.

A price vector is a nonnull vector $p \in R^l_+$.

A Walras equilibrium of $\mathcal{E}$ is a pair $(p^*, \mathbf{x}^*)$, consisting of a price vector p^* and an allocation $\mathbf{x}^*$, such that, for each $t \in T$, $u_t(\mathbf{x}^*(t)) \geq u_t(y)$, for all $y \in \{x \in R^l_+ : p^* x = p^* \mathbf{w}(t)\}$.

We define now the strategic market game, Γ, associated with $\mathcal{E}$.

A strategy correspondence is a correspondence $\mathbf{B} : T \to \mathcal{P}(R^{l^2})$ such that, for each $t \in T$, $\mathbf{B}(t) = \{b \in R^{l^2}_+ : \sum_{j=1}^{l} b_{ij} \leq \mathbf{w}^i(t),\ i = 1, \ldots, l\}$, where b_{ij}, $i, j = 1, \ldots, l$, represents the amount of commodity i that trader t offers in exchange for commodity j. A strategy selection is an integrable function $\mathbf{b} : T \to R^{l^2}_+$, such that, for each $t \in T$, $\mathbf{b}(t) \in \mathbf{B}(t)$. Given a strategy selection $\mathbf{b}$, we define the aggregate matrix $\bar{\mathbf{B}} = (\int_T \mathbf{b}_{ij}(t)\, d\mu)$. Moreover, we denote by $\mathbf{b} \setminus b(t)$ a strategy selection obtained by replacing $\mathbf{b}(t)$ in $\mathbf{b}$ with $b \in \mathbf{B}(t)$. With a slight abuse of notation, $\mathbf{b} \setminus b(t)$ will also represent the value of the strategy selection $\mathbf{b} \setminus b(t)$ at t.

Then, we introduce two further definitions (see Sahi and Yao (1989)).

Definition 1. *A nonnegative square matrix* A *is said to be irreducible if, for every pair* (i, j), *with* $i \neq j$, *there is a positive integer* $k = k(i, j)$ *such that* $a^{(k)}_{ij} > 0$, *where* $a^{(k)}_{ij}$ *denotes the* ij*-th entry of the* k*-th power* A^k *of* A.

Definition 2. *Given a strategy selection* $\mathbf{b}$, *a price vector* p *is said to be market clearing if*

$$p \in R^l_{++}, \sum_{i=1}^{l} p^i \bar{\mathbf{b}}_{ij} = p^j (\sum_{i=1}^{l} \bar{\mathbf{b}}_{ji}), \; j = 1, \ldots, l. \qquad (1)$$

By Lemma 1 in Sahi and Yao (1989), there is a unique, up to a scalar multiple, price vector p satisfying (1) if and only if $\bar{\mathbf{B}}$ is irreducible. Then, we denote by $p(\mathbf{b})$ a function which associates with each strategy selection $\mathbf{b}$ the unique, up to a scalar multiple, price vector p satisfying (1), if $\bar{\mathbf{B}}$ is irreducible, and is equal to 0, otherwise.

Given a strategy selection $\mathbf{b}$ and a price vector p, consider the assignment determined as follows:

$$\begin{aligned}
\mathbf{x}^j(t, \mathbf{b}(t), p) &= \mathbf{w}^j(t) - \sum_{i=1}^{l} \mathbf{b}_{ji}(t) + \sum_{i=1}^{l} \mathbf{b}_{ij}(t) \frac{p^i}{p^j}, \text{ if } p \in R^l_{++}, \\
\mathbf{x}^j(t, \mathbf{b}(t), p) &= \mathbf{w}^j(t), \text{ otherwise,}
\end{aligned}$$

$j = 1, \ldots, l$, for each $t \in T$.

Given a strategy selection $\mathbf{b}$ and the function $p(\mathbf{b})$, the traders' final holdings are determined according with this rule and consequently expressed by the assignment

$$\mathbf{x}(t) = \mathbf{x}(t, \mathbf{b}(t), p(\mathbf{b})),$$

for each $t \in T$. It is straightforward to show that this assignment is an allocation.

We are now able to define a Cournot-Nash equilibrium of Γ (see Codognato and Ghosal (2000) and Busetto et al. (2011)).

Definition 3. *A strategy selection* $\hat{\mathbf{b}}$ *such that* $\hat{\bar{\mathbf{B}}}$ *is irreducible is a Cournot-Nash equilibrium of* Γ *if*

$$u_t(\mathbf{x}(t, \hat{\mathbf{b}}(t), p(\hat{\mathbf{b}}))) \geq u_t(\mathbf{x}(t, \hat{\mathbf{b}} \setminus b(t), p(\hat{\mathbf{b}} \setminus b(t)))),$$

for each $b \in \mathbf{B}(t)$ *and for each* $t \in T$.[1]

[1]Let us notice that, as this definition of a Cournot-Nash equilibrium explicitly refers to irreducible matrices, it applies only to active equilibria of Γ, (on this point, see Sahi and Yao (1989)).

3 The existence of a δ-positive Cournot-Nash equilibrium of Γ

In this section, we define the notion of a δ-positive Cournot-Nash equilibrium, which was first used by Sahi and Yao (1989) to prove their existence theorem for the finite version of the Shapley's model and their limit result. We will use it, in this paper, for analogous purposes.

Let $\bar{T}_1 \subset T_1$ be a set consisting of two traders in T_1 for whom Assumption 4 holds. Moreover, let $\delta = \min_{t\in\bar{T}_1}\{\frac{1}{l}\min\{\mathbf{w}^1(t),\ldots,\mathbf{w}^l(t)\}\}$. We say that the correspondence $\mathbf{B}^\delta : T \to \mathcal{P}(R_+^{l^2})$ is a δ-positive strategy correspondence if $\mathbf{B}^\delta(t) = \mathbf{B}(t) \cap \{b \in R_+^{l^2} : \sum_{i\notin J}\sum_{j\in J}(b_{ij} + b_{ji}) \geq \delta,$ for each $J \subseteq \{1,\ldots,l\}\}$, for each $t \in \bar{T}_1$ and if $\mathbf{B}^\delta(t) = \mathbf{B}(t)$, for the remaining traders $t \in T$. Moreover, we say that a strategy selection $\mathbf{b}$ is δ-positive if $\mathbf{b}(t) \in \mathbf{B}^\delta(t)$, for each $t \in T$. Finally, we say that a Cournot-Nash equilibrium $\hat{\mathbf{b}}$ of Γ is δ-positive if $\hat{\mathbf{b}}$ is a δ-positive strategy selection.

The following theorem shows the existence of a δ-positive Cournot-Nash equilibrium of Γ. It is a straightforward consequence of the existence theorem in Busetto et al. (2011).

Theorem 1. *Under Assumptions 1, 2, 3, and 4, there exists a δ-positive Cournot-Nash equilibrium of* Γ, $\hat{\mathbf{b}}$

Proof. See the Appendix. ∎

4 An example

Sahi and Yao (1989) formalized the Shapley's window model in the context of an exchange economy with a finite set of traders. Codognato and Ghosal (2000) reconsidered the Sahi and Yao's model within an exchange economy with an atomless continuum of traders. In this framework, they showed an equivalence result à la Aumann (1964) between the set of the Cournot-Nash equilibrium allocations of the window model and the set of the Walras equilibrium allocations of the underlying exchange economy. The mixed measure space we are considering here may contain countably infinite atoms. This raises the question whether an equivalence result can be obtained also in this case. The following example analyzes an exchange economy $\mathcal{E}$ with countably infinite atoms and it shows that any Cournot-Nash equilibrium allocation of the strategic market game Γ is not a Walras equilibrium allocation of $\mathcal{E}$.

Example. *Consider an exchange economy* $\mathcal{E}$ *where* $l = 2$, $T_1 = T_1' \cup T_1''$, $T_1' = \{2,3\}$, $T_1'' = \{4,5,\ldots\}$, $T_0 = [0,1]$; $\mathbf{w}(2) = \mathbf{w}(3) \gg 0$, $\mathbf{w}(t) = (0,1)$, *for each* $t \in T_1'' \cup T_0$; $u_t(\cdot)$ *satisfies Assumptions 2 and 3, for each* $t \in T$, $u_2(\cdot)$ *and* $u_3(\cdot)$ *satisfy Assumption 4,* $u_2(x) = u_3(x)$, $u_t(x) > u_t(y)$, *whenever* $x \in R^l_{++}$ *and* $y \in (R^l_+ \setminus R^l_{++})$, *for each* $t \in T_1'' \cup T_0$; μ *is the Lebesgue measure, when restricted to* T_0, *and* $\mu(t) = (\frac{1}{2})^t$, *for each* $t \in T_1$. *Then, if* $\hat{\mathbf{b}}$ *is a Cournot-Nash equilibrium of* Γ, *the pair* $(\hat{p}, \hat{\mathbf{x}})$ *such that* $\hat{p} = p(\hat{\mathbf{b}})$ *and* $\hat{\mathbf{x}}(t) = \mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$, *for each* $t \in T$, *is not a Walras equilibrium of* $\mathcal{E}$.

Proof. Suppose that $\hat{\mathbf{b}}$ is a Cournot-Nash equilibrium of Γ and that the pair $(\hat{p}, \hat{\mathbf{x}})$ such that $\hat{p} = p(\hat{\mathbf{b}})$ and $\hat{\mathbf{x}}(t) = \mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$, for each $t \in T$, is a Walras equilibrium of $\mathcal{E}$. Clearly, $\hat{\mathbf{b}}_{21}(t) > 0$, for each $t \in T_1'' \cup T_0$. Let $v = \int_{T_1'' \cup T_0} \hat{\mathbf{b}}_{21}(t)\, d\mu$. At a Cournot-Nash equilibrium, for each $t \in T_1'$, the marginal rate of substitution must be equal to the rate at which he can trade off commodity 1 for commodity 2 (see Okuno et al. (1980)). Moreover, at a Walras equilibrium, this marginal price must be in turn equal to the relative price of commodity 1 in terms of commodity 2. These two conditions are expressed by the following equations:

$$\frac{dx_2}{dx_1} = -(\hat{p}^1)^2 \frac{\hat{\mathbf{b}}_{12}(t)}{\hat{\mathbf{b}}_{21}(t) + v} = -\hat{p}^1,$$

for each $t \in T_1'$. But then, we must also have

$$\frac{\hat{\mathbf{b}}_{21}(2) + v}{\hat{\mathbf{b}}_{12}(2)} = \frac{\hat{\mathbf{b}}_{21}(2) + \hat{\mathbf{b}}_{21}(3) + v}{\hat{\mathbf{b}}_{12}(2) + \hat{\mathbf{b}}_{12}(3)} = \frac{\hat{\mathbf{b}}_{21}(3) + v}{\hat{\mathbf{b}}_{12}(3)}. \tag{2}$$

The last equality in (2) holds if and only if $\hat{\mathbf{b}}_{21}(2) = z(\hat{\mathbf{b}}_{21}(3) + v)$ and $\hat{\mathbf{b}}_{12}(2) = z\hat{\mathbf{b}}_{12}(3)$, with $z > 0$. But then, the first and the last members of (2) cannot be equal because

$$\frac{z(\hat{\mathbf{b}}_{21}(3) + v) + v}{z\hat{\mathbf{b}}_{12}(3)} \neq \frac{\hat{\mathbf{b}}_{21}(3) + v}{\hat{\mathbf{b}}_{12}(3)}.$$

This implies that the pair $(\hat{p}, \hat{\mathbf{x}})$ such that $\hat{p} = p(\hat{\mathbf{b}})$ and $\hat{\mathbf{x}}(t) = \mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$, for each $t \in T$, cannot be a Walras equilibrium of $\mathcal{E}$. ■

The example shows that the condition that $\mathcal{E}$ contains a countably infinite number of atoms is not sufficient to guarantee that the set of the Cournot-Nash equilibrium allocations of Γ coincides with the set of the Walras equilibrium allocations of $\mathcal{E}$.

5 The replication à la Cournot of $\mathcal{E}$

The non equivalence result obtained in the previous section leads us to deal with the question whether a limit result can be instead obtained by replicating the exchange economy $\mathcal{E}$ and by generating sequences of Cournot-Nash equilibrium allocations which converge to a Walras equilibrium allocation.

We address this question by introducing a concept of replication in the original spirit of Cournot (1838). By analogy with the replication proposed by Cournot in a partial equilibrium framework, the concept we propose is obtained in fact by replicating only the atoms of $\mathcal{E}$, while making them asymptotically negligible, and without affecting the atomless part.

This partial replication à la Cournot of $\mathcal{E}$ can be formalized as follows. Let $\mathcal{E}^n$ be an exchange economy characterized as in Section 2, where each atom is replicated n times. For each $t \in T_1$, let tr denote the r-th element of the n-fold replication of t. We assume that, for each $t \in T_1$, $\mathbf{w}(tr) = \mathbf{w}(ts) = \mathbf{w}(t)$, $u_{tr}(\cdot) = u_{ts}(\cdot) = u_t(\cdot)$, $r, s = 1, \ldots, n$, and $\mu(tr) = \frac{\mu(t)}{n}$, $r = 1, \ldots, n$. Clearly, $\mathcal{E}^1$coincides with $\mathcal{E}$.

Then, the strategic market game Γ^n associated with $\mathcal{E}^n$ can be characterized, *mutatis mutandis*, as in Section 2. Clearly, Γ^1 coincides with Γ. A strategy selection $\mathbf{b}$ of Γ^n is said to be atom-type-symmetric if $\mathbf{b}^n(tr) = \mathbf{b}^n(ts)$, $r, s = 1, \ldots, n$, for each $t \in T_1$.

We provide now the definition of an atom-type-symmetric Cournot-Nash equilibrium of Γ^n.

Definition 4. *A strategy selection $\hat{\mathbf{b}}$ such that $\bar{\hat{\mathbf{B}}}$ is irreducible is an atom-type-symmetric Cournot-Nash equilibrium of Γ^n if $\hat{\mathbf{b}}$ is atom-type-symmetric and if*

$$u_{tr}(\mathbf{x}(tr, \hat{\mathbf{b}}(tr), p(\hat{\mathbf{b}}))) \geq u_{tr}(\mathbf{x}(tr, \hat{\mathbf{b}} \setminus b(tr), p(\hat{\mathbf{b}} \setminus b(tr)))),$$

for each $b \in \mathbf{B}(tr)$, $r = 1, \ldots, n$, and for each $t \in T_1$;

$$u_t(\mathbf{x}(t, \hat{\mathbf{b}}(t), p(\hat{\mathbf{b}}))) \geq u_t(\mathbf{x}(t, \hat{\mathbf{b}} \setminus b(t), p(\hat{\mathbf{b}} \setminus b(t)))),$$

for each $b \in \mathbf{B}(t)$ and for each $t \in T_0$.

6 The existence of a δ-positive atom-type-symmetric Cournot-Nash equilibrium of Γ^n

In this section, we introduce the notion of a δ-positive atom-type-symmetric Cournot-Nash equilibrium of Γ^n. Moreover, we state and prove that an equilibrium of this kind exists under Assumptions 1, 2, 3, and 4. This result is needed to show our limit theorem by using the replication à la Cournot introduced in Section 5.

Let δ be determined as in Section 3. Also a δ-positive strategy correspondence, $\mathbf{B}^{\delta}$, is defined, *mutatis mutandis*, as in Section 3. Notice that $\mathbf{B}^{\delta}(tr) = \mathbf{B}^{\delta}(ts)$, $r, s = 1, \ldots, n$, for each $t \in T_1$. We say that a strategy selection $\mathbf{b}$ is δ-positive if $\mathbf{b}(tr) \in \mathbf{B}^{\delta}(tr)$, $r = 1, \ldots, n$, for each $t \in T_1$, and $\mathbf{b}(t) \in \mathbf{B}^{\delta}(t)$, for each $t \in T_0$. Then, we say that an atom-type-symmetric Cournot-Nash equilibrium $\hat{\mathbf{b}}$ of Γ^n is δ-positive if $\hat{\mathbf{b}}$ is a δ-positive strategy selection.

The following theorem establishes the existence of a δ-positive atom-type-symmetric Cournot-Nash equilibrium of Γ^n. The proof of the theorem adapts to our context tools and arguments developed by Say and Yao (1989) and Busetto et al (2011) to show the existence of a Cournot-Nash equilibrium of the Shapley's window model, respectively in the case of a finite set of traders and in the case of a mixed measure space of traders à la Shitovitz.

Theorem 2. *Under Assumptions 1, 2, 3, and 4, there exists a δ-positive atom-type-symmetric Cournot-Nash equilibrium of Γ^n, $\hat{\mathbf{b}}$.*

Proof. See the Appendix. ■

7 The limit theorem

In this section, we state and prove the limit theorem. It establishes that the sequences of Cournot-Nash equilibrium allocations generated by replicating $\mathcal{E}$ à la Cournot approximate a Walras equilibrium allocation of the economy. In particular, the theorem shows that, given a sequence of atom-type-symmetric Cournot-Nash equilibrium allocations of Γ^n, for $n = 1, 2, \ldots$, there exists a Walras equilibrium allocation of $\mathcal{E}$ with this property: for each trader $t \in T$, the assigment corresponding to this Walras equilibrium allocation is a limit point of the sequence of final holdings of t associated with the sequence of atom-type-symmetric Cournot-Nash equilibria of Γ^n, for $n = 1, 2, \ldots$.

Theorem 3. *Under Assumptions 1, 2, 3, and 4, let $\{\hat{\mathbf{b}}^n\}$ be a sequence of strategy selections of Γ and let $\{\hat{p}^n\}$ be a sequence of prices such that $\hat{\mathbf{b}}^n(t) = \hat{\mathbf{b}}^{\Gamma^n}(tr)$, $r = 1, \ldots, n$, for each $t \in T_1$, $\hat{\mathbf{b}}^n(t) = \hat{\mathbf{b}}^{\Gamma^n}(t)$, for each $t \in T_0$, and $\hat{p}^n = p(\hat{\mathbf{b}}^{\Gamma^n})$, where $\hat{\mathbf{b}}^{\Gamma^n}$ is a δ-positive atom-type-symmetric Cournot-Nash equilibrium of Γ^n, for $n = 1, 2, \ldots$. Then, (i) there exists a subsequence*

$\{\hat{\mathbf{b}}^{k_n}\}$ *of the sequence* $\{\hat{\mathbf{b}}^n\}$*, a subsequence* $\{\hat{p}^{k_n}\}$ *of the sequence* $\{\hat{p}^n\}$*, a strategy selection* $\hat{\mathbf{b}}$ *of* Γ*, and a price vector* $\hat{p}$*, with* $\hat{p} \gg 0$*, such that* $\hat{\mathbf{b}}(t)$ *is the limit of the sequence* $\{\hat{\mathbf{b}}^{k_n}(t)\}$*, for each* $t \in T_1$*,* $\hat{\mathbf{b}}(t)$ *is a limit point of the sequence* $\{\hat{\mathbf{b}}^{k_n}(t)\}$*, for each* $t \in T_0$*, the sequence* $\{\overline{\hat{\mathbf{B}}^{k_n}}\}$ *converges to* $\bar{\hat{\mathbf{B}}}$*, and the sequence* $\{\hat{p}^{k_n}\}$ *converges to* $\hat{p}$*; (ii)* $\hat{\mathbf{x}}(t)$ *is the limit of the sequence* $\{\hat{\mathbf{x}}^{k_n}(t)\}$*, for each* $t \in T_1$*, and* $\hat{\mathbf{x}}(t)$ *is a limit point of the sequence* $\{\hat{\mathbf{x}}^{k_n}(t)\}$*, for each* $t \in T_0$*, where* $\hat{\mathbf{x}}(t) = \mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$ *for each* $t \in T$*,* $\hat{\mathbf{x}}^{k_n}(t) = \mathbf{x}(t, \hat{\mathbf{b}}^{k_n}(t), \hat{p}^{k_n})$*, for each* $t \in T$*, and for* $n = 1, 2, \ldots$*; (iii) the pair* $(\hat{p}, \hat{\mathbf{x}})$ *is a Walras equilibrium of* $\mathcal{E}$*.*

Proof. (i) Let $\{\hat{\mathbf{b}}^n\}$ be a sequence of strategy selections of Γ and let $\{\hat{p}^n\}$ be a sequence of prices such that $\hat{\mathbf{b}}^n(t) = \hat{\mathbf{b}}^{\Gamma^n}(tr)$, $r = 1, \ldots, n$, for each $t \in T_1$, $\hat{\mathbf{b}}^n(t) = \hat{\mathbf{b}}^{\Gamma^n}(t)$, for each $t \in T_0$, and $\hat{p}^n = p(\hat{\mathbf{b}}^{\Gamma^n})$, where $\hat{\mathbf{b}}^{\Gamma^n}$ is a δ-positive atom-type-symmetric Cournot-Nash equilibrium of Γ^n, for $n = 1, 2, \ldots$. The fact that the sequence $\{\overline{\hat{\mathbf{B}}^n}\}$ belongs to the compact set $\{b_{ij} \in R^{l^2} : b_{ij} \leq \int_T \mathbf{w}^i(t)\, d\mu,\, i, j = 1, \ldots, l,\, \sum_{i \notin J} \sum_{j \in J}(b_{ij} + b_{ji}) \geq \int_{\bar{T}_1} \delta\, d\mu,$ for each $J \subseteq \{1, \ldots, l\}\}$, the sequence $\{\hat{\mathbf{b}}^n(t)\}$ belongs to the compact set $\mathbf{B}^\delta(t)$, for each $t \in T_1$, and the sequence $\{\hat{p}^n\}$, belongs, by Lemma 9 in Sahi and Yao, to a compact set P, implies that there is a subsequence $\{\overline{\hat{\mathbf{B}}^{k_n}}\}$ of the sequence $\{\overline{\hat{\mathbf{B}}^n}\}$ which converges to an element of the set $\{b_{ij} \in R^{l^2} : b_{ij} \leq \int_T \mathbf{w}^i(t)\, d\mu,\, i, j = 1, \ldots, l,\, \sum_{i \notin J} \sum_{j \in J}(b_{ij} + b_{ji}) \geq \int_{\bar{T}_1} \delta\, d\mu,$ for each $J \subseteq \{1, \ldots, l\}\}$, a subsequence $\{\hat{\mathbf{b}}^{k_n}(t)\}$ of the sequence $\{\hat{\mathbf{b}}^n(t)\}$ which converges to an element of the set $\mathbf{B}^\delta(t)$, for each $t \in T_1$, and a subsequence $\{\hat{p}^{k_n}\}$ of the sequence $\{\hat{p}^n\}$ which converges to an element $\hat{p}$ of the set P. Moreover, by Lemma 9 in Sahi and Yao, $\hat{p} \gg 0$. Since the sequence $\{\hat{\mathbf{b}}^{k_n}\}$ satisfies the assumptions of Theorem A in Artstein (1979), there is a function $\hat{\mathbf{b}}$ such that $\hat{\mathbf{b}}(t)$ is the limit of the sequence $\{\hat{\mathbf{b}}^{k_n}(t)\}$, for each $t \in T_1$, $\hat{\mathbf{b}}(t)$ is a limit point of the sequence $\{\hat{\mathbf{b}}^{k_n}(t)\}$, for each $t \in T_0$, and such that the sequence $\{\overline{\hat{\mathbf{B}}^{k_n}}\}$ converges to $\bar{\hat{\mathbf{B}}}$. (ii) Let $\hat{\mathbf{x}}(t) = \mathbf{x}(t, \hat{\mathbf{b}}(t), \hat{p})$ for each $t \in T$, $\hat{\mathbf{x}}^{k_n}(t) = \mathbf{x}(t, \hat{\mathbf{b}}^{k_n}(t), \hat{p}^{k_n})$, for each $t \in T$, and for $n = 1, 2, \ldots$. Then, $\hat{\mathbf{x}}(t)$ is the limit of the sequence $\{\hat{\mathbf{x}}^{k_n}(t)\}$, for each $t \in T_1$, as $\hat{\mathbf{b}}(t)$ is the limit of the sequence $\{\hat{\mathbf{b}}^{k_n}(t)\}$, for each $t \in T_1$, and the sequence $\{\hat{p}^{k_n}\}$ converges to $\hat{p}$, $\hat{\mathbf{x}}(t)$ is a limit point of the sequence $\{\hat{\mathbf{x}}^{k_n}(t)\}$, for each $t \in T_0$, as $\hat{\mathbf{b}}(t)$ is a limit point of the sequence $\{\hat{\mathbf{b}}^{k_n}(t)\}$, for each $t \in T_0$, and the sequence $\{\hat{p}^{k_n}\}$ converges to $\hat{p}$. (iii) $\overline{\hat{\mathbf{B}}^{\Gamma^n}} = \overline{\hat{\mathbf{B}}^n}$ as $\overline{\hat{\mathbf{b}}_{ij}^{\Gamma^n}} = \sum_{t \in T_1} \sum_{r=1}^n \hat{\mathbf{b}}_{ij}^{\Gamma^n}(tr)\mu(tr) + \int_{t \in T_0} \hat{\mathbf{b}}_{ij}^{\Gamma^n}(t)\, d\mu = \sum_{t \in T_1} n \hat{\mathbf{b}}_{ij}^n(t) \frac{\mu(t)}{n} + \int_{t \in T_0} \hat{\mathbf{b}}_{ij}^n(t)\, d\mu = \sum_{t \in T_1} \hat{\mathbf{b}}_{ij}^n(t)\mu(t) + \int_{t \in T_0} \hat{\mathbf{b}}_{ij}^n(t)\, d\mu = \overline{\hat{\mathbf{b}}_{ij}^n}$,

$i, j = 1, \ldots, l$, for $n = 1, 2, \ldots$. Then, $\hat{p}^n = p(\hat{\mathbf{b}}^n)$ as $\hat{p}^n$ and $\hat{\mathbf{b}}^n$ satisfy (1), for $n = 1, 2, \ldots$. But then, by continuity, $\hat{p}$ and $\hat{\mathbf{b}}$ must satisfy (1). Therefore, Lemma 1 in Sahi and Yao implies that $\bar{\hat{\mathbf{B}}}$ is completely reducible. Moreover, $\hat{\mathbf{b}}(t) \in \mathbf{B}^\delta(t)$ since $\hat{\mathbf{b}}(t)$ is a limit point of the sequence $\{\hat{\mathbf{b}}^{k_n}(t)\}$, for all $t \in T$. Then, $\hat{\mathbf{b}}$ is δ-positive. But then, by Remark 3 in Sahi and Yao, $\bar{\hat{\mathbf{B}}}$ must be irreducible. Consider the pair $(\hat{p}, \hat{\mathbf{x}})$. It is straightforward to show that the assignment $\hat{\mathbf{x}}$ is an allocation as $\hat{p}$ and $\hat{\mathbf{b}}$ satisfy (1) and that $\hat{\mathbf{x}}(t) \in \{x \in R^l_+ : \hat{p}x = \hat{p}\mathbf{w}(t)\}$, for all $t \in T$. Suppose that $(\hat{p}, \hat{\mathbf{x}})$ is not a Walras equilibrium of $\mathcal{E}$. Then, there exists a trader $\tau \in T$ and a commodity bundle $\tilde{x} \in \{x \in R^l_+ : \hat{p}x = \hat{p}\mathbf{w}(\tau)\}$ such that $u_\tau(\tilde{x}) > u_\tau(\hat{\mathbf{x}}(\tau))$. By Lemma 5 in Codognato and Ghosal (2000), there exist $\tilde{\lambda}^j \geq 0$, $\sum_{j=1}^l \tilde{\lambda}^j = 1$, such that

$$\tilde{x}^j = \tilde{\lambda}^j \frac{\sum_{i=1}^l \hat{p}^i \mathbf{w}^i(\tau)}{\hat{p}^j}, \; j = 1, \ldots, l.$$

Let $\tilde{b}_{ij} = \mathbf{w}^i(\tau)\tilde{\lambda}^j$, $i, j = 1, \ldots, l$. Then, it is straightforward to verify that

$$\tilde{x}^j = \mathbf{w}^j(\tau) - \sum_{i=1}^l \tilde{b}_{ji} + \sum_{i=1}^l \tilde{b}_{ij} \frac{\hat{p}^i}{\hat{p}^j},$$

for each $j = 1, \ldots, l$. Consider the following cases.

Case 1. $\tau \in T_1$. Let $\{h_n\}$ denote a sequence such that $h_n = k_n$, if $k_1 > 1$, $h_n = k_{n+1}$, otherwise, for $n = 1, 2, \ldots$. Let ρ denote the k_1-th element of the n-fold replication of $\mathcal{E}$ and let $\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} \setminus \tilde{b}(\tau\rho)$ denote the aggregate matrix corresponding to the strategy selection $\hat{\mathbf{b}}^{\Gamma^{h_n}} \setminus \tilde{b}(\tau\rho)$, where $\tilde{b}(\tau\rho) = \tilde{b}$, for $n = 1, 2, \ldots$. Let $\Delta\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}}$, $\Delta\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}} \setminus \tilde{b}(\tau\rho)}$, and $\Delta\overline{\hat{\mathbf{B}}^{h_n}}$ denote the diagonal matrices of row sums of, respectively, $\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}}$, $\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} \setminus \tilde{b}(\tau\rho)$, and $\overline{\hat{\mathbf{B}}^{h_n}}$, for $n = 1, 2, \ldots$. Moreover, let $q^{\Gamma^{h_n}}$, $q^{\Gamma^{h_n}}_{\tau\rho}$, and q^{h_n} denote the vectors of the cofactors of the first column of, respectively, $\Delta\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} - \overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}}$, $\Delta\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}} \setminus \tilde{b}(\tau\rho)} - \overline{\hat{\mathbf{B}}^{\Gamma^{h_n}} \setminus \tilde{b}(\tau\rho)}$, and $\Delta\overline{\hat{\mathbf{B}}^{h_n}} - \overline{\hat{\mathbf{B}}^{h_n}}$, for $n = 1, 2, \ldots$. Clearly, $q^{\Gamma^{h_n}} = q^{h_n}$ as $\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} = \overline{\hat{\mathbf{B}}^{h_n}}$, for $n = 1, 2, \ldots$. Let $\Delta\bar{\hat{\mathbf{B}}}$ be the diagonal matrix of row sums of $\bar{\hat{\mathbf{B}}}$ and q be the cofactors of the first column of $\Delta\bar{\hat{\mathbf{B}}} - \bar{\hat{\mathbf{B}}}$. The sequences $\{q^{\Gamma^{h_n}}\}$ and $\{q^{h_n}\}$ converge to q as the sequence $\overline{\hat{\mathbf{B}}^{h_n}}$ converges to $\bar{\hat{\mathbf{B}}}$ and $q^{\Gamma^{h_n}} = q^{h_n}$, for $n = 1, 2, \ldots$. Let $\bar{w} = \max\{\mathbf{w}^1(\tau), \ldots, \mathbf{w}^l(\tau)\}$. Consider the matrix $\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} - \overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} \setminus \tilde{b}(\tau\rho)$, for $n = 1, 2, \ldots$. Then, $\overline{\hat{\mathbf{b}}^{\Gamma^{h_n}}_{ij}} - \overline{\hat{\mathbf{b}}^{\Gamma^{h_n}}_{ij}} \setminus \tilde{b}_{ij}(\tau\rho) = (\frac{1}{n}\hat{\mathbf{b}}^{\Gamma^{h_n}}_{ij}(\tau\rho) - \frac{1}{n}\tilde{b}_{ij}(\tau\rho))$, $i, j = 1, \ldots, l$, for $n = 1, 2, \ldots$. But then, the sequence of Euclidean distances $\{\|\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} - \overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} \setminus \tilde{b}(\tau\rho)\|\}$

converges to 0 as $|\frac{1}{n}\hat{\mathbf{b}}_{ij}^{\Gamma^{h_n}}(\tau\rho) - \frac{1}{n}\tilde{b}_{ij}(\tau\rho)| = \frac{1}{n}|\hat{\mathbf{b}}_{ij}^{\Gamma^{h_n}}(\tau\rho) - \tilde{b}_{ij}(\tau\rho)| \leq \frac{1}{n}\bar{w}$, $i, j = 1, \ldots, l$, $n = 1, 2, \ldots$. The sequence $\{\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} \setminus \tilde{b}(\tau\rho)\}$ converges to $\bar{\hat{\mathbf{B}}}$ as, by the triangle inequality, $\|\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} \setminus \tilde{b}(\tau\rho)\} - \bar{\hat{\mathbf{B}}}\| \leq \|\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} - \overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} \setminus \tilde{b}(\tau\rho)\| + \|\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} - \bar{\hat{\mathbf{B}}}\| = \|\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} - \overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} \setminus \tilde{b}(\tau\rho)\| + \|\overline{\hat{\mathbf{B}}^{h_n}} - \bar{\hat{\mathbf{B}}}\|$, for $n = 1, 2, \ldots$, and the sequences $\{\|\overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} - \overline{\hat{\mathbf{B}}^{\Gamma^{h_n}}} \setminus \tilde{b}(\tau\rho)\|\}$ and $\{\|\overline{\hat{\mathbf{B}}^{h_n}} - \bar{\hat{\mathbf{B}}}\|\}$ converge to 0. Then, the sequence $\{q_{\tau\rho}^{\Gamma^{h_n}}\}$ converges to q. $u_{\tau\rho}(\mathbf{x}(\tau\rho, \hat{\mathbf{b}}^{\Gamma^{h_n}}(\tau\rho), p(\hat{\mathbf{b}}^{\Gamma^{h_n}}))) \geq u_{\tau\rho}(\mathbf{x}(\tau\rho, \hat{\mathbf{b}}^{\Gamma^{h_n}} \setminus \tilde{b}(\tau\rho), p(\hat{\mathbf{b}}^{\Gamma^{h_n}} \setminus \tilde{b}(\tau\rho))))$ as $\hat{\mathbf{b}}^{\Gamma^{h_n}}$ is a δ-positive atom-type-symmetric Cournot-Nash equilibrium of Γ^{h_n}, for $n = 1, 2, \ldots$. Let $\hat{\mathbf{b}}^{h_n} \setminus \tilde{b}(\tau)$ be a strategy selection obtained by replacing $\hat{\mathbf{b}}^{h_n}(\tau)$ in $\hat{\mathbf{b}}^{h_n}$ with $\tilde{b}$, for $n = 1, 2, \ldots$. Then, $u_\tau(\mathbf{x}(\tau, \hat{\mathbf{b}}^{h_n}(\tau), q^{\Gamma^{h_n}})) \geq u_\tau(\mathbf{x}(\tau, \hat{\mathbf{b}}^{h_n} \setminus \tilde{b}(\tau), q_{\tau\rho}^{\Gamma^{h_n}}))$ as $\hat{\mathbf{b}}^{h_n}(\tau) = \hat{\mathbf{b}}^{\Gamma^{h_n}}(\tau\rho)$, $p(\hat{\mathbf{b}}^{\Gamma^{h_n}}) = \alpha_{h_n} q^{\Gamma^{h_n}}$, with $\alpha_{h_n} > 0$, by Lemma 2 in Sahi and Yao, $\hat{\mathbf{b}}^{\Gamma^{h_n}} \setminus \tilde{b}(\tau\rho) = \hat{\mathbf{b}}^{h_n} \setminus \tilde{b}(\tau)$, and $p(\hat{\mathbf{b}}^{\Gamma^{h_n}} \setminus \tilde{b}(\tau\rho)) = \beta_{h_n} q_{\tau\rho}^{\Gamma^{h_n}}$, with $\beta_{h_n} > 0$, by Lemma 2 in Sahi and Yao, for $n = 1, 2, \ldots$. But then, $u_\tau(\hat{\mathbf{x}}(\tau)) \geq u_\tau(\tilde{x})$, by Assumption 2, as the sequence $\{\hat{\mathbf{b}}^{h_n}(\tau)\}$ converges to $\hat{\mathbf{b}}(\tau)$, the sequence $\{q^{\Gamma^{h_n}}\}$ converges to q, the sequence $\{q_{\tau\rho}^{\Gamma^{h_n}}\}$ converges to q, and $\hat{p} = \theta q$, with $\theta > 0$, by Lemma 2 in Sahi and Yao, a contradiction.
Case 2. $\tau \in T_0$. Let $\{\hat{\mathbf{b}}^{h_{k_n}}(\tau)\}$ be a subsequence of the sequence $\{\hat{\mathbf{b}}^{k_n}(\tau)\}$ which converges to $\hat{\mathbf{b}}(\tau)$. Moreover, let $\hat{\mathbf{b}}^{\Gamma^{h_{k_n}}} \setminus \tilde{b}(\tau)$ be a strategy selection obtained by replacing $\hat{\mathbf{b}}^{h_{k_n}}(\tau)$ in $\hat{\mathbf{b}}^{\Gamma^{h_{k_n}}}$ with $\tilde{b}$, for $n = 1, 2, \ldots$. $u_\tau(\mathbf{x}(\tau, \hat{\mathbf{b}}^{\Gamma^{h_{k_n}}}(\tau), p(\hat{\mathbf{b}}^{\Gamma^{h_{k_n}}}))) \geq u_\tau(\mathbf{x}(\tau, \hat{\mathbf{b}}^{\Gamma^{h_{k_n}}} \setminus \tilde{b}(\tau), p(\hat{\mathbf{b}}^{\Gamma^{h_{k_n}}} \setminus \tilde{b}(\tau))))$ as $\hat{\mathbf{b}}^{\Gamma^{h_{k_n}}}$ is a δ-positive atom-type-symmetric Cournot-Nash equilibrium of $\Gamma^{h_{k_n}}$, for $n = 1, 2, \ldots$. Let $\hat{\mathbf{b}}^{h_{k_n}} \setminus \tilde{b}(\tau)$ be a strategy selection obtained by replacing $\hat{\mathbf{b}}^{h_{k_n}}(\tau)$ in $\hat{\mathbf{b}}^{h_{k_n}}$ with $\tilde{b}$, for $n = 1, 2, \ldots$.. Then, $u_\tau(\mathbf{x}(\tau, \hat{\mathbf{b}}^{h_{k_n}}(\tau), \hat{p}^{h_{k_n}})) \geq u_\tau(\mathbf{x}(\tau, \hat{\mathbf{b}}^{h_{k_n}} \setminus \tilde{b}(\tau), \hat{p}^{h_{k_n}}))$ as $\hat{\mathbf{b}}^{h_{k_n}}(\tau) = \hat{\mathbf{b}}^{\Gamma^{h_{k_n}}}(\tau)$, $\hat{p}^{h_{k_n}} = p(\hat{\mathbf{b}}^{\Gamma^{h_{k_n}}})$, $\hat{\mathbf{b}}^{\Gamma^{h_{k_n}}} \setminus \tilde{b}(\tau) = \hat{\mathbf{b}}^{h_{k_n}} \setminus \tilde{b}(\tau)$, and $\hat{p}^{h_{k_n}} = p(\hat{\mathbf{b}}^{\Gamma^{h_{k_n}}} \setminus \tilde{b}(\tau))$. But then, $u_\tau(\hat{\mathbf{x}}(\tau)) \geq u_\tau(\tilde{x})$, by Assumption 2, as the sequence $\{\hat{\mathbf{b}}^{h_{k_n}}(\tau)\}$ converges to $\hat{\mathbf{b}}(\tau)$ and the sequence $\{p^{h_{k_n}}\}$ converges to $\hat{p}$, a contradiction.

Hence, the pair $(\hat{p}, \hat{\mathbf{x}})$ is a Walras equilibrium of $\mathcal{E}$. ■

8 Appendix

Proof of Theorem 1. Busetto et al. (2011) showed that, under Assumptions 1, 2, 3, and 4, there exists a Cournot-Nash equilibrium of Γ, $\hat{\mathbf{b}}$, such that, for each $t \in T$, $\hat{\mathbf{b}}(t) \in \mathbf{B}^\delta(t)$. This implies that $\hat{\mathbf{b}}$ is a δ-positive Cournot-Nash equilibrium of Γ. ■

Proof of Theorem 2. Let us introduce a slightly perturbed version of the game Γ^n, denoted by $\Gamma^n(\epsilon)$. Given $\epsilon > 0$ and a strategy selection $\mathbf{b}$, we define the aggregate bid matrix $\bar{\mathbf{B}}^\epsilon = (\bar{\mathbf{b}}_{ij} + \epsilon)$. Clearly, the matrix $\bar{\mathbf{B}}^\epsilon$ is irreducible. The interpretation is that an outside agency places fixed bids of ϵ for each pair of commodities (i, j). Given $\epsilon > 0$, we denote by $p^\epsilon(\mathbf{b})$ the function which associates, with each strategy selection $\mathbf{b}$, the unique, up to a scalar multiple, price vector which satisfies

$$\sum_{i=1}^{l} p^i(\bar{\mathbf{b}}_{ij} + \epsilon) = p^j(\sum_{i=1}^{l}(\bar{\mathbf{b}}_{ji} + \epsilon)),\ j = 1, \ldots, l. \tag{3}$$

Then, let us introduce the following notion of equilibrium for $\Gamma^n(\epsilon)$.

Definition 5. *Given $\epsilon > 0$, a strategy selection $\hat{\mathbf{b}}^\epsilon$ is an atom-type-symmetric ϵ-Cournot-Nash equilibrium of $\Gamma^n(\epsilon)$ if $\hat{\mathbf{b}}^\epsilon$ is atom-type-symmetric and*

$$u_{tr}(\mathbf{x}(tr, \hat{\mathbf{b}}^\epsilon(tr), p^\epsilon(\hat{\mathbf{b}}^\epsilon))) \geq u_{tr}(tr, \hat{\mathbf{b}}^\epsilon \setminus b(tr), p^\epsilon(\hat{\mathbf{b}}^\epsilon \setminus b(tr)))),$$

for all $b \in \mathbf{B}(tr)$, $r = 1, \ldots, n$, and for each $t \in T_1$;

$$u_t(\mathbf{x}(t, \hat{\mathbf{b}}^\epsilon(t), p^\epsilon(\hat{\mathbf{b}}^\epsilon))) \geq u_t(t, \hat{\mathbf{b}}^\epsilon \setminus b(t), p^\epsilon(\hat{\mathbf{b}}^\epsilon \setminus b(t)))),$$

for all $b \in \mathbf{B}(t)$ and for each $t \in T_0$.

Moreover, we say that an atom-type-symmetric ϵ-Cournot-Nash equilibrium $\hat{\mathbf{b}}^\epsilon$ of $\Gamma^n(\epsilon)$ is δ-positive if $\hat{\mathbf{b}}^\epsilon$ is a δ-positive strategy selection.

To show Theorem 2, we first need to prove the existence of a δ-positive atom-type-symmetric ϵ-Cournot-Nash equilibrium of $\Gamma^n(\epsilon)$. To do so, we apply, as in Busetto et al. (2011), the Kakutani-Fan-Glicksberg Theorem (see Theorem 17.55 in Aliprantis and Border (2006), p. 583).

We neglect, as usual, the distinction between integrable functions and equivalence classes of such functions and denote by $L_1(\mu, R^{l^2})$ the set of integrable functions taking values in R^{l^2}, by $L_1(\mu, \mathbf{B}(\cdot))$ the set of strategy selections, and by $L_1(\mu, \mathbf{B}^*(\cdot))$ the set of atom-type-symmetric strategy selections. Note that the locally convex Hausdorff space we shall be working in is $L_1(\mu, R^{l^2})$, endowed with its weak topology.

The proof of existence of a δ-positive atom-type-symmetric ϵ-Cournot-Nash equilibrium of $\Gamma^n(\epsilon)$ is articulated in three lemmas.

The first lemma establishes the properties of $L_1(\mu, \mathbf{B}^*(\cdot))$ required to apply the Kakutani-Fan-Glicksberg Theorem.

Lemma 1. *The set $L_1(\mu, \mathbf{B}^*(\cdot))$ is nonempty, convex and weakly compact.*

Proof. $L_1(\mu, \mathbf{B}^*(\cdot))$ is nonempty, convex and it has a weakly compact closure by the same argument used by Busetto et al. (2011) to prove their Lemma 1. Now, let $\{\mathbf{b}^m\}$ be a convergent sequence of $L_1(\mu, \mathbf{B}^*(\cdot))$. Since $L_1(\mu, R^{l^2})$ is complete, $\{\mathbf{b}^m\}$ converges in the mean to an integrable function $\mathbf{b}$. But then, there exists a subsequence $\{\mathbf{b}^{k_m}\}$ of $\{\mathbf{b}^m\}$ such that $\mathbf{b}^{k_m}(tr)$ converges to $\mathbf{b}(tr)$, $r = 1, \ldots, n$, for each $t \in T_1$, and $\mathbf{b}^{k_m}(t)$ converges to $\mathbf{b}(t)$, for each $t \in T_0$ (see Theorem 25.5 in Aliprantis and Burkinshaw (1998), p. 203). The compactness of $\mathbf{B}(t)$, for each $t \in T$, and the fact that $\mathbf{b}^{k_m}(tr) = \mathbf{b}^{k_m}(ts)$, $r, s = 1, \ldots, n$, for each $t \in T_1$, implies that $\mathbf{b} \in L_1(\mu, \mathbf{B}^*(\cdot))$. Hence $L_1(\mu, \mathbf{B}^*(\cdot))$ is norm closed and, since it is also convex, it is weakly closed (see Corollary 4 in Diestel (1984), p. 12). ∎

Now, given $\epsilon > 0$, let $\alpha^\epsilon_{tr} : L_1(\mu, \mathbf{B}^*(\cdot)) \to \mathbf{B}(tr)$ be a correspondence such that $\alpha^\epsilon_{tr}(\mathbf{b}) = \text{argmax}\{u_{tr}(\mathbf{x}(t, \mathbf{b} \setminus b(tr), p^\epsilon(\mathbf{b} \setminus b(tr)))) : b \in \mathbf{B}(tr)\}$, $r = 1, \ldots, n$, for each $t \in T_1$, and let $\alpha^\epsilon_t : L_1(\mu, \mathbf{B}(\cdot)) \to \mathbf{B}(t)$ be a correspondence such that $\alpha^\epsilon_t(\mathbf{b}) = \text{argmax}\{u_t(\mathbf{x}(t, \mathbf{b} \setminus b(t), p^\epsilon(\mathbf{b} \setminus b(t)))) : b \in \mathbf{B}(t)\}$, for each $t \in T_0$. Let $\alpha^\epsilon : L_1(\mu, \mathbf{B}^*(\cdot)) \to L_1(\mu, \mathbf{B}(\cdot))$ be a correspondence such that $\alpha^\epsilon(\mathbf{b}) = \{\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot)) : \mathbf{b}(tr) \in \alpha^\epsilon_{tr}(\mathbf{b}),\ r = 1, \ldots, n,$ for each $t \in T_1$, and $\mathbf{b}(t) \in \alpha^\epsilon_t(\mathbf{b})$, for each $t \in T_0\}$.

The second lemma provides us with the properties of the correspondence α^ϵ. The proof is obtained by readapting to our context the arguments used to show Lemma 2 in Busetto et al. (2011), and we omit it here.

Lemma 2. *Given $\epsilon > 0$, the correspondence $\alpha^\epsilon : L_1(\mu, \mathbf{B}^*(\cdot)) \to L_1(\mu, \mathbf{B}(\cdot))$ is such that the set $\alpha^\epsilon(\mathbf{b})$ is nonempty and convex, for all $\mathbf{b} \in L_1(\mu, \mathbf{B}^*(\cdot))$, and it has a weakly closed graph.*

Now, given $\epsilon > 0$, let $\alpha^{\epsilon\delta}_{tr} : L_1(\mu, \mathbf{B}^*(\cdot)) \to \mathbf{B}(tr)$ be a correspondence such that $\alpha^{\epsilon\delta}_{tr}(\mathbf{b}) = \alpha^\epsilon_{tr}(\mathbf{b}) \cap \mathbf{B}^\delta(tr)$, $r = 1, \ldots, n$, for each $t \in T_1$, and let $\alpha^\epsilon_t : L_1(\mu, \mathbf{B}(\cdot)) \to \mathbf{B}(t)$ be a correspondence such that $\alpha^{\epsilon\delta}_t(\mathbf{b}) = \alpha^\epsilon_t(\mathbf{b}) \cap \mathbf{B}^\delta(t)$, for each $t \in T_0$. Let $\alpha^{\epsilon\delta} : L_1(\mu, \mathbf{B}^*(\cdot)) \to L_1(\mu, \mathbf{B}(\cdot))$ be a correspondence such that $\alpha^{\epsilon\delta}(\mathbf{b}) = \{\mathbf{b} \in L_1(\mu, \mathbf{B}(\cdot)) : \mathbf{b}(tr) \in \alpha^{\epsilon\delta}_{tr}(\mathbf{b}),\ r = 1, \ldots, n,$ for each $t \in T_1$, and $\mathbf{b}(t) \in \alpha^{\epsilon\delta}_t(\mathbf{b})$, for each $t \in T_0\}$. Moreover, let $\alpha^{\epsilon\delta*} : L_1(\mu, \mathbf{B}^*(\cdot)) \to L_1(\mu, \mathbf{B}^*(\cdot))$ be a correspondence such that $\alpha^{\epsilon\delta*}(\mathbf{b}) = \alpha^{\epsilon\delta}(\mathbf{b}) \cap L_1(\mu, \mathbf{B}^*(\cdot))$.

We are ready to prove the existence of a δ-positive atom-type-symmetric ϵ-Cournot-Nash equilibrium of $\Gamma^n(\epsilon)$.

Lemma 3. *Given $\epsilon > 0$, there exists a δ-positive atom-type-symmetric ϵ-Cournot-Nash equilibrium of $\Gamma^n(\epsilon)$, $\hat{\mathbf{b}}^\epsilon$.*

Proof. Let $\epsilon > 0$ be given. By Lemma 6 in Sahi and Yao (1989), we know that, for each $\mathbf{b} \in L_1(\mu, \mathbf{B}^*(\cdot))$, $\alpha_{tr}^{\epsilon\delta}(\mathbf{b})$ is nonempty, $r = 1, \ldots, n$, for each $t \in \bar{T}_1$. Moreover, for each $\mathbf{b} \in L_1(\mu, \mathbf{B}^*(\cdot))$ and for each $t \in T_1$, there exists $\bar{b} \in \mathbf{B}(t)$ such that $\bar{b} \in \alpha_{tr}^{\epsilon\delta}(\mathbf{b})$, $r = 1, \ldots, n$ as $\mathbf{b}$ is an atom-type-symmetric strategy profile. But then, by the same argument of Lemma 2 in Busetto et al. (2011), $\alpha^{\epsilon\delta *}(\mathbf{b})$ is nonempty. The convexity of $\alpha^{\epsilon\delta}(\mathbf{b})$, for each $\mathbf{b} \in L_1(\mu, \mathbf{B}^*(\cdot))$, is a straightforward consequence of the convexity of $\alpha_{tr}^{\epsilon}(\mathbf{b})$ and $\mathbf{B}^{\delta}(t)$, $r = 1, \ldots, n$, for each $t \in T_1$, and of $\alpha_t^{\epsilon}(\mathbf{b})$ and $\mathbf{B}^{\delta}(t)$, for each $t \in T_0$. But then, $\alpha^{\epsilon\delta *}$ is convex valued as $L_1(\mu, \mathbf{B}^*(\cdot))$ is convex. $\alpha_{tr}^{\epsilon\delta}$ is upper hemicontinuous and compact valued, $r = 1, \ldots, n$, for each $t \in T_1$, as it is the intersection of the correspondence α_{tr}^{ϵ}, which is upper hemicontinuous and compact valued by Lemma 2 in Busetto et al. (2011), and the continuous and compact valued correspondence which assigns to each strategy selection $\mathbf{b} \in L_1(\mu, \mathbf{B}^*(\cdot))$ the strategy set $\mathbf{B}^{\delta}(tr)$ (see Theorem 17.25 in Aliprantis and Border (2006), p. 567). Moreover, $\alpha_t^{\epsilon\delta}$ is upper hemicontinuous and compact valued, for each $t \in T_0$, using the same argument. Therefore, $\alpha^{\epsilon\delta}$ has a weakly closed graph, by the same argument used in the proof of Lemma 2. Finally, $\alpha^{\epsilon\delta *}$ has a weakly closed graph as it is the intersection of the correspondence $\alpha^{\epsilon\delta}$ and the continuous correspondence which assigns to each strategy selection $\mathbf{b} \in L_1(\mu, \mathbf{B}^*(\cdot))$ the weakly closed set $L_1(\mu, \mathbf{B}^*(\cdot))$ which, by the Closed Graph Theorem (see Theorem 17.11 in Aliprantis and Border (2006), p. 561), has a weakly closed graph (see Theorem 17.25 in Aliprantis and Border (2006), p. 567). But then, by the Kakutani-Fan-Glicksberg Theorem (see Theorem 17.55 in Aliprantis and Border (2006), p. 583), there exists a fixed point $\hat{\mathbf{b}}^{\epsilon}$ of the correspondence $\alpha^{\epsilon\delta *}$ and hence a δ-positive atom-type-symmetric ϵ-Cournot-Nash equilibrium of $\Gamma^n(\epsilon)$. ■

To complete the proof of Theorem 2, we have to show that there exists the limit of a sequence of δ-positive atom-type-symmetric ϵ-Cournot-Nash equilibria and that this limit is a δ-positive atom-type-symmetric ϵ-Cournot-Nash equilibrium of Γ^n. Following Busetto et al. (2011), in this part of the proof we essentially refer to Lemma 9 in Sahi and Yao (1989) and a generalization of the Fatou's lemma in several dimensions provided by Artstein (1979).

Then, let $\epsilon_m = \frac{1}{m}$, $m = 1, 2, \ldots$. By Lemma 3, for each $m = 1, 2, \ldots$, there is a δ-positive atom-type-symmetric ϵ-Cournot-Nash equilibrium $\hat{\mathbf{b}}^{\epsilon_m}$. The fact that the sequence $\{\overline{\hat{\mathbf{B}}^{\epsilon_m}}\}$ belongs to the compact set $\{b_{ij} \in R^{l^2} : b_{ij} \leq n \int_{T_1} \mathbf{w}^i(t)\, d\mu + \int_{T_0} \mathbf{w}^i(t)\, d\mu,\ i, j = 1, \ldots, l,\ \sum_{i \notin J} \sum_{j \in J} (b_{ij} + b_{ji}) \geq n \int_{\bar{T}_1} \delta\, d\mu,$ for each $J \subseteq \{1, \ldots, l\}\}$, the sequence $\{\hat{\mathbf{b}}^{\epsilon_m}(tr)\}$ belongs to the compact set $\mathbf{B}^{\delta}(tr)$, $r = 1, \ldots, n$, for each $t \in T_1$, and the sequence $\{\hat{p}^{\epsilon_m}\}$, where $\hat{p}^{\epsilon_m} = p(\hat{\mathbf{b}}^{\epsilon_m})$, for each $m = 1, 2, \ldots$, belongs, by Lemma 9 in Sahi and Yao (1989), to a compact set P, implies that there is a subsequence $\{\overline{\hat{\mathbf{B}}^{\epsilon_{k_m}}}\}$ of the sequence $\{\overline{\hat{\mathbf{B}}^{\epsilon_m}}\}$ which converges to an element of the set $\{b_{ij} \in R^{l^2} : b_{ij} \leq n \int_{T_1} \mathbf{w}^i(t)\, d\mu + \int_{T_0} \mathbf{w}^i(t)\, d\mu,\ i, j = 1, \ldots, l,\ \sum_{i \notin J} \sum_{j \in J}(b_{ij} + b_{ji}) \geq n \int_{\bar{T}_1} \delta\, d\mu,$ for each $J \subseteq \{1, \ldots, l\}\}$, a subsequence $\{\hat{\mathbf{b}}^{\epsilon_{k_m}}(tr)\}$ of the sequence $\{\hat{\mathbf{b}}^{\epsilon_m}(tr)\}$ which converges to an element of the set $\mathbf{B}^{\delta}(tr)$, $r = 1, \ldots, n$, for each $t \in T_1$, and a subsequence $\{\hat{p}^{\epsilon_{k_m}}\}$ of the sequence $\{\hat{p}^{\epsilon_m}\}$ which converges to an element of the set P. Since the sequence $\{\hat{\mathbf{b}}^{\epsilon_{k_m}}\}$ satisfies the assumptions of Theorem A in Artstein (1979), there is a function $\hat{\mathbf{b}}$ such that $\hat{\mathbf{b}}(tr)$ is the limit of the sequence $\{\hat{\mathbf{b}}^{\epsilon_{k_m}}(tr)\}$, $r = 1, \ldots, n$, for each $t \in T_1$, $\hat{\mathbf{b}}(t)$ is a limit point of the sequence $\{\hat{\mathbf{b}}^{\epsilon_{k_m}}(t)\}$, for each $t \in T_0$, and such that the sequence $\{\overline{\hat{\mathbf{B}}^{\epsilon_{k_m}}}\}$ converges to $\bar{\hat{\mathbf{B}}}$. Then, $\hat{\mathbf{b}}(tr) = \hat{\mathbf{b}}(ts)$ as $\{\hat{\mathbf{b}}^{\epsilon_{k_m}}(tr)\} = \{\hat{\mathbf{b}}^{\epsilon_{k_m}}(ts)\}$, $r, s = 1, \ldots, n$, for each $t \in T_1$, and $\hat{\mathbf{b}}(tr)$ is the limit of the sequence $\{\hat{\mathbf{b}}^{\epsilon_{k_m}}(tr)\}$, $r = 1, \ldots, n$, for each $t \in T_1$. Hence, it can be proved, by the same argument used by Busetto et al. (2011) to show their existence theorem, that $\hat{\mathbf{b}}$ is an atom-type-symmetric δ-positive Cournot-Nash equilibrium of $\Gamma^n(\epsilon)$. ■

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