Characterizing Behavioral Decisions with Choice Data[*]

Patricio S. Dalton

Tilburg University, CentER

Sayantan Ghosal

Adam Smith Business School, University of Glasgow

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Abstract

A number of different models with behavioral economics have a reduced form representation where potentially boundedly rational decision-makers do not necessarily internalize all the consequences of their actions on payoff relevant features (which we label as psychological states) of the choice environment. This paper studies the restrictions that such behavioral models impose on choice data and the implications they have for welfare analysis. First, we propose a welfare benchmark that is justified using standard axioms of rational choice and can be applied to a number of existing seminal behavioral economics models. Second, we show that Sen's axioms α and γ fully characterize choice data consistent with behavioral decision-makers. Third, we show how choice data to infer information about the normative significance of psychological states and establish the possibility of identifying welfare dominated choices.

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1 Introduction

There is considerable evidence from behavioral economics that individual behavior is systematically affected by intrinsic features of the decision-making environment that are assumed to be normatively irrelevant in a conventional account of rationality. Typical examples of such features are deadlines, default options, frames, reference points, expectations, aspirations, states of mind, emotions, moods, temptations, etc.

The presence of these features has challenged the way welfare analysis has been carried out since Samuelson's (1938) theory of revealed preferences. For example, preferences may reverse and hence, provide contradicting information about welfare: the decision-maker (hereafter DM) may choose option x over option y under feature A but y over x under feature B. To deal with this problem, Bernheim and Rangel (2009) (hereafter BR) proposed a welfare criterion that can be applied even when observed choices are inconsistent. Briefly, they state that x is (strictly) unambiguously chosen over y if y is never chosen when x is available.[1] Hence, regardless of how poorly behaved choice correspondences may be, their criterion implies that every action chosen (within a welfare-relevant domain) is a (weak) welfare optimum (BR, observation 1, pg. 62).

While BR's approach is able to deal with the inconsistency of choices, it is silent about the point that choices may be suboptimal. There is ample evidence of situations in which DMs choose against their own best interest systematically. That is, x may be chosen over y, but still be against the DM's best interest.[2] This is particularly relevant, for example, in models of addiction, projection bias, dynamic inconsistency or aspirations failure.

This paper studies the potential implications for welfare analysis of models of boundedly rational decision-making studied in behavioral economics. Unlike BR, we allow for the intrinsic features of the decision-making environment to be endogenous. We label such features as psychological states and we broadly interpret them to include reference points, beliefs, emotions, temptations, mood, aspirations, etc. Suboptimal behavior comes from the fact that DMs may mistakenly not internalize the endogeneity of psychological states.

In our framework, the DM chooses among mutually exclusive actions. Each action has an effect on payoffs both directly and indirectly through its effect on a psychological state, through a feedback function. The DM's preferences rank both actions and endogenous

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[1] Salant and Rubinstein (2008) make a similar point in their analysis of choice with exogenous frames.

[2] Köszegi and Rabin (2008) and Beshears et al. (2008) review empirical evidence of systematic mistakes people make. For example, in the "heat of the moment," people often take actions that they would not have intended to take (Loewenstein, 1996). Bernheim and Rangel (2007) also record situations in which it is clear that people act against themselves: an anorexic refusal to eat; people save less than what they would like; fail to take advantage of low interest loans available through life insurance policies; unsuccessfully attempt to quit smoking; maintain substantial balances on high-interest credit cards; etc.

psychological states.

We consider two types of decision procedures: a Standard Decision Procedure (SDP) and Behavioral Decision Procedure (BDP). In an SDP, the DM fully internalizes the feedback from actions to psychological states, and chooses an action that maximizes his welfare. This is equivalent to rational decision-making in a context with psychological states.[3] In an BDP, in contrast, a (behavioral) DM fails to internalize the effect of his action on his psychological state, and chooses an action taking as given his psychological state (although psychological states and actions are required to be mutually consistent at a BDP outcome). This is a form of boundedly rational decision-making. Note that, in this framework, choices can be systematically coherent (in BR's sense) but yet suboptimal.[4]

Despite its simplicity, our framework unifies seemingly disconnected models in the literature, from more recent positive behavioral economics models to older ones. In Section 3.3 we illustrate this feature by linking our framework with models of status-quo bias, reference-dependent consumption, dynamic-inconsistent preferences, adaptive preferences, anticipatory feelings and psychological games.

To study the link between welfare and the choices consistent with the models encompassed by our framework, we axiomatically characterize choice data compatible with BDPs and SDPs. We show that observed choices are compatible with an BDP if and only if choice data satisfy Sen's (1971) axioms α[5] and γ[6]. These two axioms are weaker than Sen's (1971) axioms α and β[7] that we show fully characterize an admissible SDP.[8]

The axiomatic characterization of an SDP and an BDP is important on its own, as it pins down the underlying choice structure of seemingly disconnected behavioral economic models in the literature. It shows that regardless how disconnected the behavioral models may seem to be, they are characterized by the same consistent choice-structure. Also, it

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[3]Example of standard DMs are people who self-impose deadlines to overcome procrastination (Ariely and Wertenbroch, 2002), who limit the number of alternatives (limited focus) as a self-control device to avoid regret (Carrillo and Mariotti, 2000) or who choose an optimal aspiration level as motivator of effort (Heath et al., 1999).

[4]In the Appendix, we extend our framework and allow for DMs who partially internalize the consequences of their actions. The main results of this paper still hold for this general case.

[5]Sen's axiom α states that the choice correspondence is (weakly) increasing as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set. This axiom was also introduced by Chernoff (1954).

[6]Sen's axiom γ states that if an action is chosen in each set in a class of sets, it must be also chosen in their union.

[7]Sen's axiom β states that when two actions are both chosen in a given set, and one of them is chosen in a larger set that includes the first set, then both are chosen in the larger set.

[8]In an *admissible* SDP, the ranking over consistent decision states is transitive. A *consistent* decision state is a pair of an action and psychological state so that the psychological state is an element of the feedback function.

tells us that boundedly rational behavior can be characterized by standard axioms of choice. Moreover, these results can also be used to examine the potential normative implications of behavioral economics for welfare analysis. Our results imply that choice data are compatible with SDP if and only if they are compatible with rational choice theory. This provides an axiomatic justification for an SDP to be the welfare benchmark that should be used in the models that are encompassed in our framework. Moreover, we show that choice data satisfying axioms α and β imply that we only need to know one psychological state to rationalize such data as the outcome of an SDP. Hence, when choice data satisfy α and β, psychological states are normatively irrelevant.

Next, we consider choice data that satisfy Sen's axioms α and γ (but violate axiom β). In such cases, we show that at least two psychological states are required for such data to be rationalized as the outcome of an admissible BDP.[9] Moreover, we are able to show that this key point can be inferred directly from choice data.

Clearly, the fact that a decision problem must require at least two psychological states to be rationalized as the outcome of an admissible BDP is a necessary (though not sufficient) condition to ensure the normative significance of psychological states. We establish the possibility of inferring welfare dominated BDP outcomes using choice data under a domain restriction.[10]

Finally, in the appendix, we show that our framework can be generalized to partially internalization of the impact of actions on psychological states. Moreover, we show that over a fixed domain preferences, the welfare implications of partial internalization may be perverse.

The remainder of the paper is organized as follows. Section 2 illustrate our framework with a simple example. Section 3 introduces the model, shows existence of a solution and describes some of the models encompassed in our general framework. Section 4 axiomatically characterizes both decision procedures. Section 5 discusses the insights for welfare analysis derived from our framework and Section 6 concludes. Additional generalizations and interpretations of our model, as well as the proof of existence of a solution are shown in the Appendix.

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[9]In an *admissible* BDP, the ranking over actions for a given psychological state is transitive.

[10]Specifically, we require that (i) both the ranking over consistent decision states and the ranking over actions for each psychological state to be transitive, and (ii) preferences over actions are neutral with respect to psychological states.

2 Example: Addiction

In this section, we motivate and illustrate the distinction between an SDP and an BDP with one simple example on addiction.

Consider a DM who is considering whether to drink alcohol. The psychological state will either be sober (if he does not drink) or inebriated (if he does). The payoff table below provides a quick summary of the decision problem:

	inebriated	sober
alcohol	$1 - 2$	$1 + 0$
no alcohol	$0 - 2$	$0 + 0$

In this example, the payoffs are an additive function of the action-based payoff and the psychological state-based payoff. Alcohol generates utility of 1; no alcohol generates utility of 0. Sobriety generates utility of 0; inebriation generates utility of -2.

A DM who uses an SDP to solve this problem recognizes that he has to choose between the on-diagonal elements. Alcohol goes together with the psychological state of inebriation. No alcohol goes together with the psychological state of sobriety. Hence, the off-diagonal paths are not options.

However, the behavioral DM mistakenly believes that he can change his alcohol consumption without changing his psychological state. Consequently, the behavioral DM decides to consume alcohol (since alcohol is always better, conditional on a fixed psychological state) and ends up inebriated (with net payoff -1). This is a mistake in the sense that the DM would be better off if he chose to drink no alcohol and ended up sober (with net payoff 0). In this sense, by using an BDP the DM imposes an externality on himself. Thus, the outcomes of an BDP can (although not necessarily) be welfare dominated.

3 The General Framework

3.1 The Model

The primitives of the model consist of a set A of actions, a set P of psychological states and a function $\pi : A \to P$ modeling the feedback effect from actions to psychological states. It is assumed that $\pi(a)$ is non-empty and single-valued for each $a \in A$. A decision state is a pair of an action and psychological state (a, p) where $a \in A$ and $p \in P$. A *consistent* decision state is a decision state (a, p) such that $p = \pi(a)$.

Following Harsanyi (1954) we assume intrapersonal comparability of utility. That is, the DM is not only able to rank different elements in A for a given p, but he is also able

to assess the subjective satisfaction he derives from an action when the psychological state is p with the subjective satisfaction he derives from another action when the psychological state is p'. In other words, we assume that the DM is able to rank elements in $A \times P$. Given intrapersonal comparability of utility, the preferences of the DM are denoted by $\succeq$, a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$.

A decision scenario is, thus, a collection $D = (A, P, \pi, \succeq)$.

We study two decision procedures:

1. Given a non-empty *feasible* set of actions $A' \subseteq A$, a *standard decision procedure* (SDP) is one where the DM chooses a consistent decision state (a, p), $a \in A'$ and $p = \pi(a)$. The outcomes of an SDP, denoted by $S \subseteq A \times P$, are

$$S = \left\{ (a, p) : (a, \pi(a)) \succeq (a', \pi(a')) \text{ for all } a' \in A' \right\}.$$

2. Given a non-empty *feasible* set of actions $A' \subseteq A$, a *behavioral decision procedure* (BDP) is one where the DM takes as given the psychological state p when choosing $a \in A'$. Define a preference relation $\succeq_p$ over A as follows:

$$a \succeq_p a' \Leftrightarrow (a, p) \succeq (a', p) \text{ for } p \in P.$$

The outcomes of an BDP, denoted by $B \subseteq A \times P$, are

$$B = \left\{ (a, p) : a \succeq_p a' \text{ for all } a' \in A', \, p = \pi(a) \right\}.$$

In both, SDPs and BDPs, a decision outcome must be a consistent decision state where the action is chosen from some feasible set of actions. In an SDP, the DM internalizes that his psychological state is determined by his action via the feedback effect when choosing an action from the set of feasible actions. In an BDP, the DM takes the psychological state as given when he chooses an action from the set of feasible actions although the psychological state is required to be consistent with the action actually chosen by the DM.[11]

3.2 Existence

Motivated by the literature of behavioral economics, we prove existence of solutions to an SDP and an BDP allowing for preferences to be incomplete, non-convex and acyclic (i.e. not

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[11]In Appendix 2, we extend our framework to one in which the psychological state is a vector of psychological states, and the DM correctly predicts the effect of his action on a subset of such vector and believes that he doesn't affect the complement. It turns out that both the existence result and the axiomatic characterization in this paper are the same in this generalized version of the model. Considering the possibility of partial prediction of psychological states has an interesting normative implication though, which we discuss also in the Appendix.

necessarily transitive).[12] We show (i) existence of a solution to an SDP applying Bergstrom (1975) and (ii) existence of a solution to an BDP extending Ghosal's (2011) result for normal-form games.[13]

Recall that the preferences of the DM is denoted by $\succeq$, a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$. Let $\succ_p$ denote the strict (asymmetric) preference relation corresponding to $\succeq_p$ i.e. $a \succ_p a'$ if and only if $a \succeq_p a'$ but $a' \not\succeq_p a$. Define the sets $\succ_p (a) = \{a' \in A : a' \succ_p a\}$ (the upper section of $\succ_p$), $\succ_p^{-1} (a) = \{a' \in A : a \succ_p a'\}$ (the lower section of $\succ_p$). Note that in this formulation, $\succ_p$ could be incomplete. Define a map $\Psi : P \to A$, where $\Psi(p) = \{a' \in A : \succ_p (a') = \emptyset\}$: for each $p \in P$, $\Psi(p)$ is the set of maximal elements of the preference relation $\succ_p$.

Consider the following assumptions:

(A1) It is assumed that for each $p \in P$,

(i) $\succ_p$ is acyclic i.e. there is no finite set $\{a^1, ..., a^T\}$ such that $a^{t-1} \succ_p a^t$, $t = 2, ..., T$, and $a^T \succ_p a^1$, or equivalently $\succeq_p$ is complete and P-acyclic.[14]

(ii) $\succ_p^{-1} (a)$ is open relative to A i.e. $\succ_p$ has an open lower section.[15]

(A2) A, P are both compact lattices with the vector ordering and π is an increasing continuous function.[16]

(A3) For each p, and a, a', (i) if $a \succeq_p \inf(a, a')$, then $\sup(a, a') \succeq_p a'$ (ii) if $a \succeq_p \sup (a, a')$ then $\inf (a, a') \succeq_p a'$ (quasi-supermodularity).

(A4) For each $a \geq a'$ and $p \geq p'$, (i) if $a \succeq_{p'} a'$, then $a \succeq_p a'$ and (ii) if $a' \succeq_p a$ then $a' \succeq_{p'} a$ (single-crossing property);

(A5) For each p and $a \geq a'$, (i) if $\succ_p (a') = \emptyset$ and $a \succeq_p a'$, then $\succ_p (a) = \emptyset$, and (ii) $\succ_p (a) = \emptyset$ and $a' \succeq_p a$, then $\succ_p (a') = \emptyset$,(monotone closure).

Assumptions (A3)-(A4) are quasi-supermodularity and single-crossing property defined

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[12] Mandler (2005) shows that incomplete preferences and intransitivity is required for "status quo maintenance" to be outcome rational. Tversky and Kahneman (1991) argue that reference-dependent preferences may not be convex.

[13] The seminal proof for existence of equilibria with incomplete preferences in Shafer and Sonnenschein (1975) requires convexity both to show existence of an optimal choice and to apply Kakutani's fix-point theorem.

[14] As for each p, $\succ_p$ is acyclic and therefore irreflexive, it follows that $\succeq_p$ is complete.

[15] The continuity assumption, that $\succ_p$ has an open lower section, is weaker than the continuity assumption made by Debreu (1959) (who requires that preferences have both open upper and lower sections), which in turn is weaker than the assumption by Shafer and Sonnenschein (1975) (who assume that preferences have open graphs). Note that assuming $\succ_p$ has an open lower section is consistent with $\succ_p$ being a lexicographic preference ordering over A.

[16] A lattice is a partially ordered subset of $\Re^k$ with the vector ordering (the usual component wise ordering: $x \geq y$ if and only if $x_i \geq y_i$ for each $i = 1, .., K$, and $x > y$ if and only if both $x \geq y$ and $x \neq y$, and $x \gg y$ if and only if $x_i > y_i$ for each $i = 1, .., K$) which contains the supremum and infimum of any two of its elements. A lattice that is compact (in the usual topology) is a compact lattice.

by Milgrom and Shannon (1994). Assumption (A5) was introduced in Ghosal (2011). Consider a pair of actions such that the first action is greater (in the usual vector ordering) than the second action. For a fixed p, suppose the two actions are unranked by $\succ_p$. Then, assumption (A5) requires that either both actions are maximal elements for $\succ_p$ or neither is maximal. The role played by this assumption in obtaining the monotone comparative statics is clarified in Ghosal (2011).

We are now in a position to state the following existence result:

Proposition 1. *(i) Suppose A is compact and π is a continuous function. Under assumption (A1), a solution to a SDP exists. (ii) Under assumptions (A1)-(A5), a solution to a BDP exists.*[17]

Proof. See appendix. ∎

3.3   Reduced Form Representation

We define psychological states as endogenous features of the decision-making environment that the DM may (mistakenly) not internalize. This broad definition of p allows us to unify, in a reduced form representation, seemingly disconnected models in the economic literature.

For example, take Tversky and Kahneman (1991)'s reference-dependent theory of riskless choice. In their framework, preferences do not only depend on consumption bundles but also on a reference consumption bundle which "usually corresponds to the decision-maker's current position" (Tversky and Kahneman, 1991, pp. 1046). They assume that the DM takes the reference point as exogenous at the moment to make a decision, so, in the lens of our framework, they consider exclusively an BDP. As an illustration, think of a DM who is considering whether to switch his service provider (e.g. electricity) from his current one. The psychological state (in this case the reference point) will either be the current supplier (if he keeps it) or the alternative supplier (if he changes). Assuming loss aversion, it is possible to write payoffs so that (a) the outcome of an SDP will be to switch to the alternative supplier, and (b) there exist two welfare ranked outcomes for the behavioral DM, one where the DM sticks with his current supplier and the reference point is the status-quo and the other where he switches suppliers and the reference point is the alternative.[18]

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[17]Note that the conditions for the existence of a solution to an SDP are weaker than the conditions for the existence of a solution to an BDP.

[18]Here is an example of such situation. There are two payoff-relevant dimensions of choice with outcome denoted by x_1 and x_2 and preferences $u(x) = x_1 + v(x_1 - r_1) + x_2 + v(x_2 - r_2)$ where $v(\cdot)$ is a Kahneman and Tversky (1979) value function with $v(z) = z$ if $z \geq 0$, $v(z) = \alpha z$, $\alpha > 2.5$ if $z < 0$ and $v(0) = 0$. The cost of switching is equal to 0.5. The status-quo (or current) position is defined by $(x_1 = 0, x_2 = 1)$ and the alternative option is $(x_1 = 2, x_2 = 0)$. When the psychological state is the status quo, then the reference point is $r_q = (r_1 = 0, r_2 = 1)$; when the psychological state is the alternative supplier, the reference point is $r_a = (r_1 = 2, r_2 = 0)$: therefore, the reference point corresponds to current choice of the DM (π is the

This is just one example of a (seminal) model that has a reduced form representation in our framework.[19] A comprehensive list of all the models that can be reduced to our framework is beyond the scope of this paper. Nonetheless, in what follows, we take five well-known behavioral economic models and make explicit the mapping from this literature to our framework, by associating π, P and A to each of them and indicating the decision-making procedure assumed maps to a SDP and a BDP.

3.3.1 Dynamic Inconsistency (Strotz, 1956; Peleg and Yaari, 1973; Laibson 1997).

The standard models of dynamically inconsistent preferences (e.g. Strotz, 1956, Peleg and Yaari, 1973 and Laibson, 1997) can be also reduced to our framework. Consider a three period problem $t = 1, 2, 3$ where at each t, the DM must choose action a_t, where $a_1 \in A_1$, $a_2 \in A_2(a_1)$, $a_3 \in A_3(a_1, a_2)$ and A_1, $A_2(a_1)$ for each a_1 and $A_3(a_1, a_2)$ for each a_1 and a_2 are non-empty sets of actions. Let $A_2 = \cup_{a_1 \in A_1} A_2(a_1)$ and $A_3 = \cup_{(a_1, a_2) \in A_1 \times A_2} A_3(a_1, a_2)$. The preferences of the DM over the action triple $(a_1, a_2, a_3) \in A_1 \times A_2 \times A_3$ are represented by $U_t = u(a_t) + \beta \left[\sum_{t'=t+1}^{3-t} \delta^{t'-t} u(a_{t'}) \right]$. Let

$$\widetilde{A}_3(a_1, a_2) = \arg \max_{a_3 \in A_3(a_1, a_2)} u(a_3), \qquad \widetilde{A}_2(a_1) = \arg \max_{a_2 \in A_1(a_1)} u(a_2) + \delta \beta u(\widetilde{A}_3(a_1, a_2)),$$

where it is assumed that both $\widetilde{A}_3(a_1, a_2)$ and $\widetilde{A}_2(a_1)$ are non-empty and single-valued. Let

$$p \in P = A_2 \times A_3 \text{ and } p = (p_2, p_3) = \left(\widetilde{A}_2(a_1), \widetilde{A}_3\left(a_1, \widetilde{A}_2(a_1)\right) \right) = \pi(a_1),$$

i.e. P is the set of psychological states. From the perspective of the current self at $t = 1$, the psychological states are precisely the actions a_2 and a_3 chosen by the future selves at $t = 2$ and $t = 3$ respectively and the feedback to psychological states from the perspective of the current self at $t = 1$ are the best responses of the future selves to his choice of action.

An SDP is equivalent to a Strotz equilibrium where the DM at $t = 1$ solves

$$Max_{a_1 \in A_1} u(a_1) + \beta \left[\delta u\left(\widetilde{A}_2(a_1) \right) + \delta^2 u\left(\widetilde{A}_3\left(a_1, \widetilde{A}_2(a_1)\right) \right) \right].$$

identity map). The payoff table below provides a quick summary of the decision problem:

	status quo	alternative
current supplier	1	$2 - 2\alpha$
alternative supplier	$3.5 - \alpha$	1.5

[19] Other models that have a reduced form representation in our framework include models of melioration (Herrnstein and Prelec, 1991), cognitive dissonance (Akerlof and Dickens, 1982), emotions (Bracha and Brown, 2007) and shrouded attributes (Gabaix and Laibson, 2006). In this latter case, for example, the psychological states can be interpreted as the (endogenous) costs of the add-ons (e.g. ink of a printer) that (behavioral) DMs fail to take into account at the moment of buying a base good (e.g. printer).

An BDP is equivalent to the Nash equilibrium of the intra-self game proposed by Peleg and Yaari (1973) defined as (a_1^*, p^*) such that (i) $a_1^* \in \arg\max_{a_1 \in A_1} u(a_1) + \beta \left[\delta u(p_2^*) + \delta^2 u(p_3^*) \right]$, and (ii) $p^* = (p_2^*, p_3^*) = (\pi(a_1^*), \pi(a_1^*))$

3.3.2 Adaptive Preferences (von Weizsacker, 1971; Hammond, 1976; Pollak, 1978)

In a number papers, von Weizsacker (1971), Hammond (1976) and Pollak (1978) study the steady states of adaptive preferences defined over consumption. We show the steady states of their models have a reduced form representation in our framework. As a by product, we also provide a dynamic interpretation of our framework.

Consider an adaptive preference adjustment mechanism where the preferences over actions at any t, denoted by $\succeq_{p_{t-1}}$, depends on the past psychological state. The statement $a \succeq_{p_{t-1}} a'$ means that the DM finds a at least as good as a' given the psychological state p_{t-1}. The DM takes as given the psychological state from the preceding period. Note that an outcome of a BDP corresponds to the steady state of an adjustment dynamics where the DM is myopic (i.e. does not anticipate that the psychological state at $t+1$ is affected by the action chosen at t). Let $h(p) = \{a \in A : a \succeq_p a', a' \in A\}$. For ease of exposition, assume that $h(p)$ is unique. Fix a $p_0 \in P$. A sequence of *short-run* outcomes is determined by the relations $a_t \in h(p_{t-1})$ and $p_t = \pi(a_t)$, $t = 1, 2, \dots$. At each step, the DM chooses a myopic best-response.[20] *Long-run* outcomes are denoted by a pair (a, p) with $p = \pi(a)$ where a is defined to be the steady-state solution to the short-run outcome functions, i.e. $a = h(\pi(a))$. In other words, long-run behavior corresponds to a subset of the set of consistent decision states, namely those that are the outcome of a BDP.

In contrast, in an SDP, the DM is farsighted (i.e. anticipates that the psychological state at $t + 1$ is affected by the action chosen at t). In this case, in each period t, the DM anticipates that p adjusts to a according to $\pi(\cdot)$ and taking this into account, chooses a. In an SDP therefore, at each t, the DM simply chooses between different consistent decision states: the outcome of an SDP at each t, is a pair (a_t, p_t) where $a_t \in \{a \in A : (a, \pi(a)) \succeq (a', \pi(a')), \text{ for all } a' \in A\}$ and $p_t = \pi(a_t)$. Note that in this simple framework, at each period t, the DM anticipates that there is instantaneous adjustment of the psychological state to the chosen action. Hence, the initial psychological state in period t, p_{t-1}, has no impact on the DM's choice. Moreover, with farsightedness, the dynamics of

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[20]Under the assumptions required to prove Proposition 1 (existence), $h(p)$ is increasing map of p so that the sequence of short-run outcomes is an (component-wise) increasing sequence (as by assumption contained in a compact set). Therefore it converges to its supremum, which is necessarily a BDP. So the existence result covers not only cases where a solution to a BDP (equivalently, a steady-state solution to the myopic preference adjustment mechanism) exists but also ensures that short-run outcomes converge to a BDP.

the preference adjustment mechanism is trivial as there is instantaneous adjustment to the the steady-state outcome.[21]

Finally, note that a farsighted DM does not regret his choice. Suppose that $(a, p) \succ (a', p')$ and $(a, p) \prec (a', p)$ with $p = \pi(a)$ and $p' = \pi(a')$. Then the DM solving an SDP would choose action a, but in the subsequent period, when state p is realized he will not regret his choice although $(a, p) \prec (a', p)$, the DM will anticipate that if he chooses a' the psychological state will adjust to p' and, by assumption, $(a, p) \succ (a', p')$.

3.3.3 Psychological Games (Geanakoplos, Pearce and Stachetti, 1989)

In Geanakoplos, Pearce and Stachetti (1989) (hereafter, GPS) psychological games, the payoff to each player depends not only on what every player does but also on high order beliefs (i.e. what the player believes every player believes, and on what he believes others believe he believes, and so on).[22] Each player takes beliefs and actions of other players as given when choosing his own action. In equilibrium, beliefs are assumed to correspond to actions actually chosen. In the special case where there is a single active player, the payoffs of this single active player depend on his own actions and the beliefs of other players over his own actions. These (endogenous) beliefs are psychological states in the lens of our framework.

For clarity, we illustrate their framework using a two player psychological game with a single active player (and one passive player). Player 1 is the active player with a set of pure actions S and mixed actions $\Sigma = \Delta(S)$. Player 2 has a singleton pure action set. Again, for simplicity of exposition, we restrict our attention to first and second order beliefs only. Let $\bar{b}_2^1 \in \Sigma$ be the first-order belief that player 2 has about the mixed strategy. Let $\bar{B}_i$ be the set of which each element is a sequence of coherent beliefs $\bar{b}_i = (\bar{b}_1^2, \bar{b}_i^2)$, where $\bar{b}_i^1$ are $i's$ are first order beliefs (i.e. beliefs about other player strategies), $\bar{b}_i^2 \in \Delta(\bar{B}_{-i}^1 \times \Sigma_{-i})$ with $\bar{B}_{-i}^1 = \times_{j \neq i} \bar{B}_j^1$ are $i's$ second order beliefs (i.e. joint beliefs about other players strategies and first-order beliefs).[23][24] Let $\bar{B} = \times_{i \in N} \bar{B}_i$ be the set of collectively coherent beliefs. Payoffs over pure strategies are given by $\bar{u}_i : \bar{B}_i \times A_1 \to \Re$ and the extension to payoffs over mixed strategies is given by $u_i(b_i, \sigma) = \sum_{a \in A} P_\sigma(a_1) \bar{u}_i(b_i, a_1)$.[25] In equilibrium all beliefs

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[21] Non-trivial dynamics would be associated with farsighted behavior if underlying preferences or action sets were time variant.

[22] Psychological games have been applied to model reciprocity concerns (Rabin, 1993; Charness and Rabin, 2002). However, this would require at least two active players and it falls outside the scope of this paper.

[23] Coherence requires that the marginal distribution of $i's$ belief with respect to Σ_{-i} coincides with $i's$ first-order beliefs.

[24] $\bar{b}_2^1 \in \bar{B}_2^1 = \Sigma$, $\bar{b}_1^1 \in \bar{B}_2^1 = \{1\}$, a singleton set (reflecting the fact that player 2 has only one action).

[25] Even in a two-player psychological game with one active player, the active player's payoff over actions may depend on his beliefs over the beliefs (over his actions) of the inactive player. For example, if player 1

must conform to a commonly held view of reality (i.e. if σ_1 is an equilibrium profile, then player 2 must believe with probability 1 that player 1 follows σ_1). Denote such a profile of beliefs by $\beta(\sigma_1) = (\beta_1(\sigma_1), \beta_2(\sigma_1)) \in \bar{B}$ i.e. each $\beta_i : \Sigma_1 \to \bar{B}_i$. A psychological Nash equilibrium is a pair $(\hat{b}, \hat{\sigma}_1) \in \bar{B} \times \Sigma_1$ s.t. (i) $\hat{b}_i = \beta_i(\hat{\sigma}_1)$ for each $i \in N$ and (ii) for player 1, $\sigma_1 \in \Sigma_1$, $u_1(\hat{b}_1, \hat{\sigma}_1) \geq u_1(\hat{b}_1, \sigma_1)$. Clearly, setting $A = \Sigma_1$, $P = \bar{B}_1 \times \bar{B}_1$ and $\pi(\sigma) = (\beta_1(\sigma), \beta_2(\sigma))$, ensures that a psychological equilibrium with one active player is an outcome of a BDP.[26] An SDP in a psychological game corresponds to a situation where the one active player acts as a Stackelberg leader and internalizes the impact of own (mixed) actions on the belief hierarchy of player 2.

3.3.4   Anticipatory Feelings (Caplin and Leahy, 2001)

In Caplin and Leahy's (2001) model of anticipatory feelings, preferences do not only depend on consumption today, but also on the feeling of anticipation of future consumption. These (endogenous) feelings correspond to psychological states in our framework. We illustrate the link of Caplin and Leahy's (2001) and our framework by using a simple two-period deterministic version of their model.[27] Consider a DM who, at each $t = 1, 2$, chooses an action $a_1 \in A_1$ and $a_2 \in A_2(a_1)$. Let $A_2 = \cup_{a_1 \in A_1} A_2(a_1)$. An anticipatory feeling (e.g. anxiety) is a psychological state that depends on the anticipated action. Formally, they define a function (equivalent to π in our framework) $\mu : A_2 \to P$ that associates an action in period 2 to a psychological state. The instantaneous utility at $t = 1$ is $u_1(a_1, p)$ and the instantaneous utility at $t = 2$ is $u_2(a_2)$. The preferences of the DM at $t = 1$ are $u_1(a_1, p) + u_2(a_2)$ and the preferences of the DM at $t = 2$ are $u_2(a_2)$. Caplin and Leahy assume that the DM solves this problem by backward induction. First, given a_1 and p, the DM solves $Max\ u_2(a_2)$ s.t. $a_2 \in A_2(a_1)$, with $\tilde{A}_2(a_1)$ being the set of solutions to this problem.[28] Then, he solves $Max\ u_1\left(a_1, \mu\left(\tilde{A}_2(a_1)\right)\right) + u_2\left(\tilde{A}_2(a_1)\right)$ s.t. $a_1 \in A_1$, with $\tilde{A}_1$ being the corresponding set of solutions. An optimal solution (equivalent to a Strotz equilibrium) is then defined as a pair $(\tilde{a}_1, \tilde{a}_2)$ such that $\tilde{a}_1 \in \tilde{A}_1$ and $\tilde{a}_2 = \tilde{A}_2(\tilde{a}_1)$. Note

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believes that player 2 believes he is going to behave in a foolhardy way, player 1 may well choose to do so even if, with a different configuration of beliefs, player 1 might have chosen to act cautiously.

[26]Clearly in this case, P is not a subset of a finite dimensional Euclidian space but of a complete, separable metric space. The existence results and the axiomatic characterization of our paper are stated and proved for the case when both A and P are subsets of a finite dimensional Euclidian space. We conjecture that our results can be extended to the general setting of a complete, separable metric space although we leave this for future research.

[27]We are aware that Caplin and Leahy (2001) is essentially a model of uncertainty. However, we chose a deterministic version only to avoid introducing new notation to the paper. By redefining actions and psychological states appropriately, it is possible to show that our framework is a reduced form representation of their model with uncertainty.

[28]For simplicity assume that $\tilde{A}_2(a_1)$ is non-empty and single valued for each a_1.

that Caplin and Leahy assume that DMs solve an SDP: they internalize the effect of their actions on their level of anxiety. Alternatively, if the DM was behavioral, he would solve the following maximization problem: $Max \ u_1(a_1, p) + u_2\left(\tilde{A}_2(a_1)\right)$ s.t. $a_1 \in A_1$. Defining $\hat{A}_1(p)$ as the set of solutions of the preceding maximization problem, the set of BDP outcomes (equivalent to the Nash equilibrium studied by Peleg and Yaari, 1973) would consist of a triple (a_1^*, a_2^*, p^*) such that $a_1^* \in \hat{A}_1(p^*)$, $a_2^* = \tilde{A}_2(a_1^*)$ and $p^* = \mu(a_2^*)$.[29]

3.3.5 Reference-dependent Consumption (Shalev, 2000; Kőszegi, 2005; Kőszegi and Rabin, 2006)

In Kőszegi and Rabin's (2006) model (see also Shalev, 2000), preferences not only depend on the consumption bundle chosen, but also on what the DM expects to consume in equilibrium. These (endogenous) expectations correspond to psychological states in our framework. Preferences are modeled as $u(c|r) = m(c) + n(c|r)$, where $m(c)$ is the intrinsic "consumption utility" that depends on a K-dimensional consumption bundle c, and $n(c|r)$, is the gain-loss utility relative to endogenous reference bundles, r. Both consumption utility and gain-loss utility are separable across dimensions, so that $m(c) = \sum_k m_k(c_k)$ and $n(c|r) = \sum_k n_k(c_k|r_k)$. They assume that $n_k(c_k|r_k) = \mu(m_k(c_k) - m_k(r_k))$, where $\mu(.)$ satisfies the properties of Kahneman and Tversky's (1979) value function. The reference bundles are determined in a Personal Equilibrium (Kőszegi, 2005) by the requirement that they must be consistent with the optimal c computed conditionally on rational forecasts of r. Thus, Kőszegi and Rabin's (2006) DM solves an BDP in our definition, and setting A and P to be the set of feasible consumption bundles and π to be the identity map, a Personal Equilibrium is equivalent to a BDP equilibrium.

3.4 Nash vs. Stackelberg in an Intra-self Game

In a formal sense, we can interpret the distinction between an SDP and an BDP as corresponding to the *Stackelberg* and, respectively, the *Nash* equilibrium of dual-self intra-personal game where one self chooses actions a and the other self chooses the psychological state p and $\pi(a)$ describes the best-response of the latter self for each $a \in A$. In a Stackelberg equilibrium, the self choosing actions anticipates that the other self chooses a psychological state according to the function $\pi(.)$. In a Nash equilibrium, both selves take the choices of the other self as given when making its own choices. Consistent with the dynamic interpretation of the general framework introduced above, in the definition of an SDP, internalization

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[29]Caplin and Leahy (2001) also provide a set of axioms so that the representation of underlying preferences with anticipatory feeling is possible in an expected utility setting. In this sense, the axiomatic characterization we provide in this paper complements their work.

(i.e. rationally anticipating the actual effects of one's actions) is equivalent to the DM anticipating the equilibrium (e.g. one's own actions is what one expects it to be, or what others expect it to be) and behaving accordingly.[30]

4 Characterization of BDPs and SDPs

4.1 Axiomatic Restrictions on Choice Data

Having demonstrated that our framework is a reduced form representation of a number of different models studied in behavioral economics, we now proceed to provide a choice theoretic axiomatic characterization of SDPs and BDPs in order to examine their normative implications. We ask under what conditions choice data can be rationalized as the outcome of an SDP or an BDP. In what follows, we show that both decision procedures are fully characterized by three observable properties of choice.

Fix $\succeq$, $\pi : A \rightarrow P$ and a family $\mathcal{A}$ of non-empty subsets of A. Define two choice correspondences, $\mathfrak{S}$ and $\mathfrak{B}$, from $\mathcal{A}$ to A as

$$\mathfrak{S}(A') = \big\{ a : (a, p) \succeq (a', p') \text{ for all } a' \in A', \ p' = \pi(a') \text{ and } p = \pi(a) \big\}$$

and

$$\mathfrak{B}(A') = \{ a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A' \},$$

as the choices corresponding to a standard and behavioral decision procedure, respectively. Note that $\mathfrak{S}(A') \subseteq A'$ and $\mathfrak{B}(A') \subseteq A'$ for each $A' \in \mathcal{A}$.

We say that $\mathfrak{S}(.)$ is *admissible* if the preference relation $\succeq$ is transitive over the set of consistent decision states. We say that $\mathfrak{B}(.)$ is *admissible* if for each $a \in A$, the preference relation $\succeq_{\pi(a)}$ is transitive over the set of actions.

Suppose we observe a non-empty correspondence C from $\mathcal{A}$ to A such that $C(A') \subseteq A'$ for each $A' \in \mathcal{A}$. We say that SDP (respectively, BDP) rationalizes C if there exist P, π and $\succeq$ such that $C(A') = \mathfrak{S}(A')$ (respectively, $C(A') = \mathfrak{B}(A')$).

Next, consider the following axioms introduced by Sen (1971).

Sen's axiom α. For all $A', A'' \subseteq A$, if $A'' \subseteq A'$ and $C(A') \cap A''$ is non-empty, then $C(A') \cap A'' \subseteq C(A'')$. In words, the choice correspondence is (weakly) increasing as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set.

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[30]For example, consider the model of cognitive dissonance (e.g. Akerlof and Dickens, 1982) where the psychological states are (endogenous) beliefs about the state of the world. In Akerlof and Dickens (1982), DMs manipulate their own beliefs to conform to their desired beliefs under a rational expectations assumption.

Sen's axiom β. For all $A', A'' \subseteq A$, if $A'' \subseteq A'$ and $a, a' \in C(A'')$, then $a \in C(A')$ if and only if $a' \in C(A')$. In words, when two actions are both chosen in a given set, and one of them is chosen in a larger set that includes the first set, then both are chosen in the larger set.

Sen's axiom γ. Let M be any class of sets $\{A'_k \subseteq A : k \geq 1\}$ and let V be the union of all sets in M. Then any a that belongs to $C(A')$ for all A' in M must belong to $C(V)$. In words, if an action is chosen in each set in a class of sets, it it must be also be chosen in their union.

We are now in a position to fully characterize choice data compatible with an SDP and an BDP. We begin with SDPs.

Proposition 2. *Choice data are rationalizable as the outcome of an admissible SDP if and only if both Sen's axioms α and β are satisfied.*

Proof. (i) We show that if choice data are rationalizable as the outcome of an admissible SDP, then, both Sen's axiom α and β hold. Fix $\succeq$, $\pi : A \to P$. For $A'' \subseteq A' \subseteq A$, if

$$a \in \mathfrak{S}(A') = \left\{ \begin{array}{c} a : (a, p) \succeq (a', p') \text{ for all } a' \in A', \, p' = \pi(a') \\ \text{and } p = \pi(a) \end{array} \right\}$$

then

$$a \in \mathfrak{S}(A'') = \left\{ \begin{array}{c} a : (a, p) \succeq (a', p') \text{ for all } a' \in A'', \, p' = \pi(a') \\ \text{and } p = \pi(a) \end{array} \right\}.$$

Therefore, $\mathfrak{S}(A') = C(A') \cap A'' \subseteq C(A'') = \mathfrak{S}(A'')$ so that Sen's axiom α is satisfied. Next, given $A'' \subseteq A'$, suppose $a', a'' \in C(A'') = \mathfrak{S}(A'')$ but $a' \in \mathfrak{S}(A')$ and $a'' \notin \mathfrak{S}(A')$. By construction, both $(a', p') \succeq (a'', p'')$ and $(a', p') \preceq (a'', p'')$ for $p' = \pi(a')$ and $p'' = \pi(a'')$. Therefore, by transitivity of $\succeq$ over consistent decision states, $a'' \in \mathfrak{S}(A')$, a contradiction so that Sen's axiom β is satisfied.

(ii) We show that if choice data satisfy Sen's axioms α and β, they are rationalizable as the outcome of an admissible SDP. To this end, we specify $\pi : A \to P$, $\#P \geq 1$ so that π is onto. Next we specify preferences $\succeq$: for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq$ satisfies the condition that $(a, p) \succeq (a', p')$ for all $a' \in A'$, $p = \pi(a)$ and $p' = \pi(a')$, $p, p' \in P$. Consider $C(A')$ for some non-empty $A' \subseteq A$. By construction if $a \in C(A') \Rightarrow$ $\mathfrak{S}(A')$ and therefore, $C(A') \subseteq \mathfrak{S}(A')$. We need to check that for the above specification of $\succeq$, $\pi : A \to P$, $\mathfrak{S}(A') \subseteq C(A')$. Suppose to the contrary, there exists $a' \in \mathfrak{S}(A')$ but $a' \notin C(A')$. It follows that $(a', \pi(a')) \succeq (b, \pi(b))$ for all $b \in A'$. Since $a' \notin C(A')$, by construction this is only possible if for each $b \in A'$, $a' \in C(A''_b)$ with $\{a, b\} \subseteq A''_b$. By Sen's axiom α, as $a' \in C(\{a, b\})$ and as $\{a, b\} \subseteq A'$, again by Sen's axiom α, $b \in C(\{a, b\})$ for $b \in C(A')$. Now, by construction, $A' = \cup_{b \in A'} \{a, b\}$. By Sen's axiom β, $a' \in C(A')$.

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Therefore, $\mathfrak{S}(A') = C(A')$. Finally, note that when choice data satisfy axioms α and β, $\succeq$ is transitive (Sen, 1971: Theorem 1) and therefore, $\mathfrak{S}(A')$ is admissible. $\blacksquare$

Proposition 2 has two implications. First, choice data are compatible with an admissible SDP if and only if they are compatible with rational choice theory. This is because rational choice theory is falsifiable if Arrow's (1959) axiom[31] holds (and hence, WARP[32] and menu independence[33]) which is in turn satisfied if and only if both Sen's axioms α and β are satisfied (Sen, 1971: Theorems 3 and 7). This provides an axiomatic justification for an SDP to be the welfare benchmark that should be used in the models that are encompassed in our framework. In Section 5.1 we expand this point with further details.

The second implication of Proposition 2 has to do with the identification of psychological states. Suppose that we are interested in identifying P. Inasmuch the data are rationalized as the outcome of an admissible SDP, all we need is to identify one psychological state (note that we can prove part (ii) of Proposition 2 by setting $\#P = 1$). For example, if the decision problem is one of addiction, we just need to know that no alcohol is consistent with sobriety, because the SDP outcome will be (no alcohol, sober). This is an important result if we are interested in identifying π or P, which are unobservable from choice data. Knowing that data satisfy axioms α and β implies that we don´t need to fully know π or all the set P, but only the p associated with the chosen action.

Now we move on and characterize choice data compatible with an BDP.

Proposition 3. *Choice data are rationalizable as the outcome of an BDP if and only if both Sen's axioms α and γ are satisfied.*

Proof. (i) We show that if choice data are rationalizable as the outcome of a BDP, then both *Sen's* α and γ hold. Fix $\succeq$, $\pi : A \to P$. For $A'' \subseteq A' \subseteq A$, if

$$a \in \mathfrak{B}(A') = \left\{ a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A' \right\}$$

then

$$a \in \mathfrak{B}(A'') = \left\{ a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'' \right\}.$$

Therefore, $C(A') \cap A'' \subseteq C(A'')$ as required so that Sen's axiom α is satisfied. Next, let M

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[31] Arrow (1959)'s axiom: If $A' \subseteq A$ and $C(A) \cap A'$ is non-empty, then $C(A') = C(A) \cap A'$. In words, when the set of feasible alternatives shrinks, the choice from the smaller set *consists precisely* of those alternatives chosen in the larger set and remain feasible, if there is any.

[32] WARP requires that for all non-empty $A', A'' \subseteq A$ and for all $a', a'' \in A' \cap A''$, if $a' \in C(A')$ and $a'' \in C(A'')$, then $a' \in C(A'')$. Richter (1966) carries out a revealed preferences analysis over the domain of linear budget sets. Thus, his analysis cannot be directly applied to the choice scenario studied here as we want to allow for finite actions sets.

[33] A menu is a non-empty subset A' of A. A menu-specific revealed preference for any $a, a' \in A'$, $aR_{A'}a' \Leftrightarrow a \in C(A')$. Menu independent choice requires the existence of a binary relation R_o over A such that for all non-empty $A' \subseteq A$ and for all $a, a' \in A'$, $aR_{A'}a' \Leftrightarrow aR_o a'$.

denote a class of sets $\{A'_k \subseteq A : k \geq 1\}$. If

$$a \in \mathfrak{B}(A'_k) = \{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'_k\}$$

and $V = \cup_{k \geq 1} A'_k$, it follows that

$$a \in \mathfrak{B}(V) = \{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in V\}$$

so that Sen's axiom γ is satisfied.

(ii) We show that if choice data satisfy both Sen's α and γ, they are rationalizable as the outcome of a BDP. To this end, we specify $\pi : A \rightarrow P$ so that $\#P \geq 1$ and π is onto. Next we specify preferences $\succeq$: for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq$ satisfies the condition that $a \succeq_p a'$ for all $a' \in A'$ and $p = \pi(a)$. Consider $C(A')$ for some non-empty $A' \subseteq A$. By construction if $a \in C(A')$, then $a \in \mathfrak{B}(A')$ and therefore, $C(A') \subseteq \mathfrak{B}(A')$. We need to check that for the above specification of $\succeq$, $\pi : A \rightarrow P$, $\mathfrak{B}(A') \subseteq C(A')$. Suppose to the contrary, there exists $a' \in \mathfrak{B}(A')$ but $a' \notin C(A')$. It follows that $a' \succeq_{p'} b$ for each $b \in A'$ and $p' = \pi(a')$. Since $a' \notin C(A')$, by construction this is only possible only if $a' \in C(A''_b)$ for some A''_b with $\{a', b\} \subseteq A''_b$. Let $A'' = \cup_{b \in A'} A''_b$. It follows that $a' \in A''$ and by Sen's axiom γ, $a' \in C(A'')$. As $A' \subseteq A''$ and $a' \in C(A'')$, by Sen's axiom α, $a' \in C(A')$ a contradiction. Therefore, $\mathfrak{B}(A') = C(A')$. $\blacksquare$

One implication of Proposition 3 is that the outcomes of an BDP violates the independence of irrelevant alternatives (Arrow's axiom).[34] Heuristically, its violation comes from the fact that alternatives that should be irrelevant from a rational point of view because they are never chosen by the DM, may not be irrelevant if the DM does not fully internalize the endogeneity of the psychological states.

4.2 Maximal and Minimal Number of Psychological States

In contrast to Proposition 2, Proposition 3 provides an axiomatic characterization of choice data compatible with any BDP whether admissible or not. Evidently, choice data generated by an admissible BDP will satisfy axioms α and γ. The following result characterizes the minimum number of psychological states required to rationalize choice data compatible with axioms α and γ (but not β) as the outcome of an *admissible* BDP.

Proposition 4. *Suppose $\#A \geq 3$. Then, choice data satisfying Sen's axioms α and γ (but not axiom β) can be rationalized as the outcome of an admissible BDP only if $\#P \geq 2$.*

Proof. If choice data satisfies Sen's axioms α and γ (but not axiom β) and $\#A \geq 3$, then there exists two non-empty sets A' and A'' with $A' \subseteq A$, $A'' \subseteq A$ and $A'' \subseteq A'$

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[34]Masatlioglu and Ok (2005)'s axiomatic characterization of rational choice with status quo bias (exogenous to the actions chosen by the DM) satisfies *Arrow's axiom* among other axioms.

such that $(C(A') \cap A'') \subset C(A'')$. Assume that $\#P = 1$ with $P = \{p\}$. Consider the preference relation defined over actions $\succeq_p$ where $P = \{p\}$ and for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq_p$ satisfies the condition that $a \succeq_p a'$ for all $a' \in A'$ and $p = \pi(a)$. We require that this choice data be rationalized as the outcome of a BDP (i.e. $\mathfrak{B}(A') = C(A')$, $A' \subseteq A$) with $\#P = 1$ and $P = \{p\}$, $p = \pi(a)$ for all $a \in A$ and $\succeq_p$ is transitive. Then, there exists $b, c \in A''$ s.t. $b \in (C(A') \cap A'')$, $c \in C(A'')$ but $c \notin C(A')$. Therefore, it follows that both $b \succeq_p c$ and $c \succeq_p b$ so that as $\succeq_p$ is transitive, $c \succeq_p a$ whenever $b \succeq_p a$ for any $a \in A$; therefore, $c \in C(A')$, a contradiction. It follows that $\#P > 1$ and so in the part (ii) of the proof of Proposition 3, we must have that $\#P \geq 2$. ∎

In proof of Proposition 2 - part (ii) - we show that when choice data can be rationalized as the outcome of an admissible SDP, the minimal number of psychological states that can be identified from choice data is $\#P = 1$. If $\#P = 1$, BDPs and SDPs would be necessarily indistinguishable and psychological states would be normatively irrelevant.[35] Proposition 4 puts a lower bound on the number of psychological states required to rationalize choice data satisfying Sen's axioms α and γ (but not axiom β) (as long as $\#A \geq 3$) as the outcome of an admissible BDP. Clearly, the requirement that $\#P > 1$ is a necessary (though not sufficient) condition to ensure the normative significance of psychological states. Also, note that Proposition 4 tells us that this key point can be inferred directly from choice data.

Below we provide two examples of choice data that satisfy Sen's axioms α and γ (but not β) which we rationalize as the outcome of an admissible BDP.

Example 1. Consider $A = \{a, b, c\}$, $C(\{a, b, c\}) = \{a, b\}$, $C(\{a, b\}) = \{a, b\}$, $C(\{a, c\}) = \{a, c\}$, $C(\{b, c\}) = \{b\}$ which satisfy axioms α and γ (but not β). By Proposition 4, we must have that $\#P \geq 2$. Suppose $\#P = 2$, with $P = \{p, q\}$ and $\pi(a) = \pi(b) = p$ and $\pi(c) = q$. Then, we have that $a \succeq_p b$, $b \succeq_p a$, $a \succeq_p c$, $b \succeq_p c$, $c \succeq_q a$, $b \succeq_q c$ so that (by transitivity of $\succeq_q$), $b \succeq_q a$: in this case with $\#P = 2$ it is possible to rationalize the choice data as the outcome of an admissible BDP. ∎

Example 2. Consider $A = \{a, b, c\}$, $C(\{a, b, c\}) = \{a\}$, $C(\{a, b\}) = \{a, b\}$, $C(\{a, c\}) = \{a\}$, $C(\{b, c\}) = \{c\}$ which satisfy axioms α and γ (but not β). By Proposition 4, we must have that $\#P \geq 2$. In fact, it is possible to go one step further and show that $\#P \geq 3$. Suppose $\#P = 2$, with $P = \{p, r\}$ and $\pi(a) = p$ and $\pi(c) = r$. Suppose $\pi(b) = p$: as both $a \succeq_p b$ and $b \succeq_p a$, as $\succeq_p$ is required to be transitive and $a \succeq_p c$, we must have that $b \succeq_p c$ so that $C(\{a, b, c\}) = \{a, b\}$, a contradiction. Suppose $\pi(b) = r$: as $b \succeq_r a$ and $c \succeq_r b$ by transitivity, $c \succeq_r a$ and therefore, $C(\{a, b, c\}) = \{a, c\}$, a contradiction. It follows that

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[35] Note that without the additional requirement that choice data satisfying axioms α and γ be rationalized as the outcome of an admissible BDP, it is without loss of generality to set $\#P = 1$ in part (ii) of the proof of Proposition 3. Sen (1971) has shown that choice data that satisfies axioms α and γ (but violates axiom β) can be represented by a preference relation that violates transitivity.

$\#P \geq 3.\blacksquare$

The two examples above show that the minimal number of psychological states required to rationalize choice data that satisfy Sen's axioms α and γ (but not β) depends on specific characteristics of the data and in some cases, could be at least as great the cardinality of the number of actions. This raises the question of whether it is possible to determine in general the maximal number of psychological states required to rationalize choice data.

We summarize the above discussion as the following proposition:

Proposition 5. *Suppose $\#A \geq 3$. Then, for choice data satisfying Sen's axioms α and γ (but not axiom β) to be rationalized as the outcome of an admissible BDP, $2 \leq \#P \leq \#A$.*

Proof. Consider two decision scenarios $D = (A, P, \pi, \succeq)$ and $\tilde{D} = (A, \tilde{P}, \tilde{\pi}, \tilde{\succeq})$. We say D is equivalent to $\tilde{D}$ if and only if the following two conditions are satisfied:

(i) $(a, \pi(a)) \succeq (a', \pi(a')) \Leftrightarrow (a, \tilde{\pi}(a)) \tilde{\succeq} (a', \tilde{\pi}(a'))$ for all $a, a' \in A$;

(ii) $(a, \pi(a)) \succeq (a', \pi(a)) \Leftrightarrow (a, \tilde{\pi}(a)) \tilde{\succeq} (a', \tilde{\pi}(a))$ for all $a, a' \in A$.

In words, two decision scenarios are equivalent if and only if (i) the unique ranking over actions induced by the ranking over consistent decision states in the two different decision scenarios is identical (so that these two rankings are normatively equivalent over actions), and (ii) the ranking over actions, relevant for the computation of BDP outcomes, is the same in the two decision scenarios (so that the two rankings are equivalent from a behavioral perspective over actions).

Consider a fixed decision scenario $D = (A, P, \pi, \succeq)$. Consider the decision scenario $D_{Id.} = (A, P = A, Id., \tilde{\succeq})$ (where $Id.$ denotes the identity function from A to itself) constructed as follows: (i) $(a, a) \tilde{\succeq} (a', a') \Leftrightarrow (a, \pi(a)) \succeq (a', \pi(a'))$ for all $a, a' \in A$, (ii) $(a, a) \tilde{\succeq} (a', a) \Leftrightarrow (a, \pi(a)) \succeq (a', \pi(a))$ for all $a, a' \in A$, with $\tilde{\succeq}$ arbitrarily defined otherwise. Then, $D_{Id.} = (A, P = A, Id., \tilde{\succeq})$ is, by construction, equivalent to $D = (A, P, \pi, \succeq)$. It follows that given any decision scenario, there is an equivalent (both from a normative and behavioral perspective) decision scenario where the set of psychological states is the set of actions and the function π is the identity function.

By Proposition 4, we already know that when $\#A \geq 3$ choice data satisfying Sen's axioms α and γ (but not axiom β) can be rationalized as the outcome of an admissible BDP only if $\#P \geq 2$. Therefore, the number of psychological states required to rationalize choice data is, in general, between 2 and $\#A.\blacksquare$

4.3 Axiomatic Characterization: Related Literature

There is an emerging literature that provides axiomatic characterizations of decision-making models with some specific behavioral flavor. Relevant contributions to this literature are Manzini and Mariotti (2007, 2012), Cherepanov et al. (2008) and Masatlioglu et al. (2012).

An BDP is observationally distinguishable from each of these models on the basis of choice data alone. To start with, choice data consistent with the different procedures of choice proposed by each of these papers can account for pairwise cycles, while choice data consistent with BDP cannot: pairwise cycles of choice are simply inconsistent with Sen's axiom α and γ. For example, suppose $A = \{a, b, c\}$ and $C(A) = \{a\}, C(\{a, b\}) = \{a\}, C(\{b, c\}) = \{b\}$ but $C(\{c, a\}) = \{c\}$. This choice function can be rationalized, for example, by Manzini and Mariotti's (2012) Categorize then Choose (CTC) procedure of choice, but is not consistent with a BDP. The choice data would be consistent with BDP if, for example, $C(\{c, a\}) = \{c, a\}$. Moreover, the Rationalized Shortlist Method (RSM) proposed by Manzini and Mariotti (2007) cannot accommodate menu dependence, whereas a BDP can.

Like us, Masatlioglu et al. (2012) model of Limited Attention allows for violations of menu independence, but in a form very different from (and incompatible with) our characterization of BDP. They define a consideration set (a subset of the set of feasible alternatives) and assume that the DM only pays attention to elements in the consideration set. In their paper revealed preferences are defined as follows: an alternative x is revealed preferred to y if x is chosen whenever y is present and x is not chosen when y is deleted. That is, the choice of an alternative from a set should be unaffected if an element which is not in the consideration set is deleted. If choice changes when an alternative is deleted, then the latter alternative was in the consideration set and clearly the chosen alternative was revealed preferred to it. This is a violation of independence of irrelevant alternatives, but in a form that is incompatible with Sen's axiom α. Such data cannot be rationalized as an outcome of a BDP, precisely because in a BDP (and also in a SDP), if x is chosen whenever y is present, x must be chosen when y is deleted.

5 Welfare Implications

5.1 Identification of Welfare Dominated Choices

The recent work on welfare analysis of non-rational choice relies on ordinal (i.e. choice data) information alone to derive a partial preference ordering based on pairwise coherence (BR, Salant and Rubinstein, 2008 (SR) and earlier Sen, 1971). BR (and also SR) generalize the standard revealed preference approach to allow for inconsistencies on choice correspondences such as preferences reversals. They adopt the normative position that what matters for welfare is a binary relation constructed solely on actions using data from behavior: psychological states (ancillary conditions in BR or frames in RS) are assumed to be normatively irrelevant. The question that still remains is whether it is possible to infer welfare dominated choices using choice data alone.

To establish the normative relevance of psychological states we proceed as follows. By Proposition 4, choice data that satisfies Sen's axioms α and γ (but not axiom β) can be rationalized as the outcome of an admissible BDP only if $\#P \geq 2$: this is clearly a necessary but not sufficient condition for establishing the normative significance of psychological states. In order to establish the possibility of inferring welfare dominated BDP outcomes using choice data, we will require a domain restriction defined as follows. A decision scenario $D = (A, P, \pi, \succeq)$ satisfies domain restriction R if:

(i) the preference relation $\succeq$ is transitive over the set of consistent decision states,

(ii) for each $a \in A$, the preference relation $\succeq_{\pi(a)}$ is transitive over the set of actions,

(iii) for each $a, a' \in A$, $(a, \pi(a)) \succeq (a', \pi(a'))$ if and only if $(a, \pi(a)) \succeq (a', \pi(a))$ and $(a, \pi(a')) \succeq (a', \pi(a'))$.[36]

Conditions (i) and (ii) are the two admissibility restrictions already imposed on an SDP and an BDP in Section 4.1. Condition (iii) states that the ranking of actions should be neutral with respect to psychological states.

As we show in the following proposition, under domain restriction R it is possible to infer the existence of welfare dominated choices:

Proposition 6. *Under the domain restriction R, there exists choice data satisfying Sen's axioms α and γ (but not axiom β) which can be only be rationalized by a BDP with welfare dominated outcomes.*

Proof: We prove the result by example. Let $A = \{a, b, c\}$. Suppose, $C(\{a, b, c\}) = \{a\}$, $C(\{a, b\}) = \{a\}$, $C(\{a, c\}) = \{a, c\}$, $C(\{b, c\}) = \{b\}$. Suppose we require that this choice data to be rationalized as the outcome of an admissible BDP satisfying the domain restriction R. First, note that the choice data satisfy Sen's axioms α and γ (but not β). Therefore, by Proposition 4, $\#P \geq 2$. Suppose $\#P = 2$ with $P = \{p_1, p_2\}$, $p_1 \neq p_2$ and $\pi(a) = p_1$ and $\pi(b) = \pi(c) = p_2$. Consider the preference relation defined over actions $\succeq_p$ where for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq_p$ satisfies the condition that $a \succeq_p a'$ for all $a' \in A'$ and $p = \pi(a)$. Then, $a \succeq_{p_1} b$, $a \succeq_{p_1} c$, $b \succeq_{p_2} c$, $c \succeq_{p_2} a$ and by transitivity of $\succeq_{p_2}$, $b \succeq_{p_2} a$ which implies that $C(\{a, b\}) = \{a, b\}$ a contradiction. Next, suppose that $\#P = 2$ with $P = \{p_1, p_2\}$, and $\pi(a) = \pi(c) = p_1$ and $\pi(b) = p_2$. Then, $a \succeq_{p_1} b$, $a \succeq_{p_1} c$, $b \succeq_{p_2} c$, $b \succeq_{p_2} c$, $c \succeq_{p_2} a$ and $a \succeq_{p_2} b$ and by transitivity of $\succeq_{p_1}$, $c \succeq_{p_1} b$ which implies that $C(\{a, b, c\}) = \{a, c\}$ a contradiction. So suppose $\#P = 2$ with

[36]This a strenghening of a similar condition used by Dalton and Ghosal (2012) in a model where a distinction is made between a pre-decision and a post-decision frame. They use this condition to relate their analysis of decision problems with endogeneous frames to choice with frames and ancillary conditions studied by Bernheim and Rangel and Rubinstein and Salant. The focus of this paper is completely different. Here we focus on adopting a choice-theoretic characterization of BDP and SDP outcomes and looking at the welfare implications of that (albeit with appropriate domain restrictions).

$P = \{p_1, p_2\}$, and $\pi(a) = \pi(b) = p_1$ and $\pi(c) = p_2$. Then, $a \succeq_{p_1} b$, $a \succeq_{p_1} c$, $b \succeq_{p_1} c$, $b \succeq_{p_2} c$, $c \succeq_{p_2} a$ and $b \succeq_{p_2} a$. It follows by domain restriction R that $(b, p_1) \succeq (c, p_2)$ and as $(a, p_1) \succeq (b, p_1)$, $(a, p_1) \succeq (c, p_2)$. Therefore, $C(\{a, c\}) = \{a, c\}$ contains the dominated action c. By Proposition 5, it remains to check the case when $p_1, p_2, p_3 \in P$ $p_1 \neq p_2 \neq p_3$ with $\pi(a) = p_1$, $\pi(b) = p_2$ and $\pi(c) = p_3$. Then, $a \succeq_{p_1} b$, $a \succeq_{p_1} c$, $a \succeq_{p_2} b$, $b \succeq_{p_2} c$, $b \succeq_{p_3} c$, $c \succeq_{p_3} a$ and $b \succeq_{p_3} a$. It follows that $(a, p_1) \succeq (b, p_2)$ and $(b, p_2) \succeq (c, p_3)$ so that $(a, p_1) \succeq (c, p_2)$ so that $C(\{a, c\}) = \{a, c\}$ contains the dominated action c. ∎

5.2   Welfare Benchmark for Existing Behavioral Economics Models

The literature of behavioral economics has not yet come to an agreement on which should be the appropriate welfare benchmark for behavioral economic models. Taken together with the assumption of intra-personal comparability of utility (Harasanyi, 1954), Proposition 2 gives an axiomatic justification for the preferences induced by an SDP to be used as the welfare benchmark of the models encompassed in our framework.[37] After all, the axiomatic characterization of an SDP is equivalent to the characterization of rational choice theory, which has been used since Samuelson's (1938) as the standard welfare benchmark in economics.

As an illustration, let's apply our welfare benchmark to the models discussed throughout the paper. In the example of addiction studied in the introduction, the action "no alcohol" and the psychological state "sober" welfare dominates all other consistent decision states. In models of dynamic inconsistent preferences, the (induced) preferences of the initial self (once the best-response of the future selves is taken onto account at a Strotz equilibrium) provides the welfare benchmark.[38] In models with endogenous reference points, the induced preferences over actions (internalizing the impact of actions on reference points) are the relevant welfare benchmark. In a decision problem with anticipatory feelings, the optimal solution of Caplin and Leahy (2001) provides the relevant benchmark. In a psychological game with one active player, the induced preferences of the active player over actions and beliefs when the active player acts as a Stackelberg leader provides the relevant normative benchmark. In a dual-self game, given the interpretation of an SDP and an BDP as corresponding, respectively, to a Stackelberg and Nash equilibrium of a dual-self game, the induced preferences of the self, acting as the Stackelberg leader, provides the relevant

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[37]Notice also that, the normative preferences $\succeq$ *over the set of consistent decision states* implied by an SDP directly induce a unique ranking of actions $(a, \pi(a)) \succeq (a', \pi(a'))$.

[38]In general, the best response the DMs self at $t = 1$ can be multi-valued. In this case, in our model, the feedback from actions to psychological states will be a correspondence. A consistent decision state will be a pair of an action and psychological state so that the psychological state is an element of the feedback correspondences. In such a scenario, consistent with the definition of a Strotz equilibrium, at an SDP, we will require that the DM is able to choose both a maximal action and psychological state pair.

normative benchmark.

Notice that the welfare benchmark we propose here contrasts other alternative welfare approaches adopted elsewhere in the literature. Some scholars have proposed to solve the model with one set of preference assumptions (e.g. hyperbolic discounting) and then to evaluate welfare using another set of assumptions (e.g. geometric discounting) (see, for example, O'Donoghue and Rabin, 2006). In contrast to this approach, the difference between an SDP outcome and an BDP outcome reflects a difference in decision-making procedures and not a shift in the preferences used to evaluate welfare. Another approach applied in the literature of dynamic inconsistent preferences is the multiself Pareto criterion (see Bernheim and Rangel, 2009), where the preferences of all the different selves in a dynamically inconsistent decision problem or the preferences of both selves in a dual-self game are explicitly taken into account. In our framework, in contrast, all that matters for welfare are the induced preferences of the initial self at a Strotz equilibrium.

6 Concluding Remarks

All of the welfare economics we know is based on the assumption that people choose what is best for them, and that we can accordingly use these choices as a guide to welfare policy. Once we build realistic behavioral features into our models, this foundation is lost. Can we still extract some normatively relevant information from choices in a context in which DMs may not be utility maximizers?

Arguably, this is an ongoing puzzle of utmost importance and we don't claim to give a complete answer to this question. However, we believe that this paper contributes with some ammunition towards a better understanding of the normative implications of behavioral economics.

The first contribution of this paper is to offer a simple, yet unifying platform that encompasses different existing work in the literature on behavioral economics. This platform is not meant to explain a new behavioral procedure of choice, but it constitutes a necessary initial step to address the general question of how to do welfare economics with agents who do not maximize.

Second, we offer a full choice characterization of behavioral decisions. If observed behavior is consistent with Sen's axioms α and γ (but not β), it is consistent with a decision-maker who doesn't fully internalize all the consequences of his actions.

Third, we propose a unified welfare benchmark for behavioral economics that is justified in standard axioms of choice (Sen's axioms α and β) and can be applied in existing seminal behavioral economics models. The benchmark proposed here has the same characterization of rational choice theory, which has been used since Samuelson's (1938) as the standard

welfare benchmark in economics.

Fourth, we show that it is possible to use only choice data to identify information about unobservable but normative relevant features of the choice environment that the decision-maker may fail to internalize. Moreover, under some restrictions, it is also possible to identify welfare dominated choices only with choice data.

All in all, this paper demonstrates that it is still possible to extract normatively relevant information from observed choices, even when we relax the full rationality assumption.

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Appendix 1: Proof of Proposition 1

(i) By assumption A is compact and π is a continuous function so that the set

$$\{p \in P : p = \pi(a) \text{ for some } a \in A\}$$

is compact and therefore, the set of consistent decision states is compact. Then, under the assumption that $\succ$ is acyclic and has open lower section, it follows that S is non-empty from Bergstrom (1975).

(ii) Propositions 1 and 2 in Ghosal (2011) show that assumptions (1)-(4), taken together, are sufficient to ensure that $\Psi(p)$ is non-empty and compact and for each $p \in P$, $\Psi(p)$ is a sublattice of A where both the maximal and minimal elements, denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively, are increasing functions on P. To complete the proof of Proposition 1, define a map $\Psi : A \times P \to A \times P$, $\Psi(a,p) = (\Psi_1(p), \Psi_2(a))$ as follows: for each (a,p), $\Psi_1(p) = \{a' \in A : \succ_p(a') = \phi\}$ and $\Psi_2(a) = \pi(a)$. It follows that $\Psi_1(p)$ is a compact (and consequently, complete) sublattice of A and has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively. By assumption 1, it also follows that for each a, $\pi(a)$ has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{\pi}(a)$ and $\underline{\pi}(a)$ respectively. Therefore, the map $(\bar{a}(p), \bar{\pi}(a))$ is an increasing function from $A \times P$ to itself and as $A \times P$ is a compact (and hence, complete) lattice, by applying Tarski's fix-point theorem, it follows that $(\bar{a}, \bar{p}) = (\bar{a}(\bar{p}), \bar{\pi}(\bar{a}))$ is a fix-point of Ψ and by a symmetric argument, $(\underline{a}(p), \underline{\pi}(a))$ is an increasing function from $A \times P$ to itself and $(\underline{a}, \underline{p}) = (\underline{a}(\underline{p}), \underline{\pi}(\underline{a}))$ is also a fix-point of Ψ; moreover, $(\bar{a}, \bar{p})$ and $(\underline{a}, \underline{p})$ are respectively the largest and smallest fix-points of Ψ.∎

Appendix 2: Extensions

Partial Prediction of Psychological States and Projection Bias

In Loewenstein et al. (2003) model of projection bias, future (endogenous) tastes are affected by current consumption but (behavioral) DMs partially fail to internalize this. In this subsection, we introduce partial prediction of psychological states to our framework and show that Loewenstein et al. (2003)'s model can be also reduced to our framework.

For clarity of exposition, assume that the binary relation $\succeq$ has an (expected) utility representation $u : A \times P \to \Re$. Assume also that the DM predicts that the psychological state will respond to their chosen actions with probability q, $0 \le q \le 1$. Let $v(a) = u(a, \pi(a))$ and define:

$$h(p; q) = \left\{ a \in A : a \in \arg \max_{a \in A} qv(a) + (1-q)u(a,p) \right\}.$$

Assume that $h(p; q)$ is unique. Fix a $p_0 \in P$. A sequence of *short-run* outcomes is determined by the relations $a_t \in h(p_{t-1}; q)$ and $p_t = \pi(a_t)$, $t = 1, 2, ...$: at each step, the

DM chooses a myopic best-response. *Long-run* outcomes are denoted by a pair a, p with $p = \pi(a)$ and a is defined to be the steady-state solution to the short-run outcome functions, i.e. $a = h(\pi(a); q)$. It follows that long-run behavior corresponds to the outcome of an BDP where the preferences are represented by a utility function $w(a, p; q) = qv(a) + (1-q)u(a, p)$. This formulation is formally equivalent to the modeling of projection bias in Loewenstein et al. (2003).

In this analysis, the value of q was kept constant throughout the adjustment dynamics. Our framework is consistent with an adaptive dynamics where the value of q can be adjusted over time so that, in principle, the DM could learn to internalize the consequences of his actions on the future evolution of psychological states. As long as at the limit point of the learning process the value of q is bounded away from one, the steady-state preferences corresponding to an adaptive preference mechanism can be represented as the outcomes of a BDP because the DM doesn't fully learn to internalize the feedback effect from actions to psychological states.

Partial Prediction of Long-term Psychological States

Up to now, our framework does not distinguish short from long-term effects of choices on psychological states. However, there may be cases in which the DM can anticipate changes in short-run psychological states but not in the long-run. In what follows, we extend our framework to account for this possibility.

Let $h^2(p) = h\left(\pi(h(p))\right)$ and define $h^t(p) = h\left(\pi(h^{t-1}(p))\right)$ iteratively $t = 1, 2, ...$ Fix a $p_0 \in P$. A sequence of *short-run* outcomes compatible with T-period (for some fixed, finite $T \geq 1$) forecasting is determined by the relations $a_t \in h^T(p_{t-1})$ and $p_t = \pi(a_t)$, $t = 1, 2, ...$: at each step, the DM chooses a best-response that anticipates the short-run psychological states within a T-period horizon.

Long-run outcomes compatible with T-period forecasting are denoted by a pair a', p' with $p' = \pi(a')$ and a' is defined to be the steady-state solution to the short-run outcome function i.e. $a' = h^T(\pi(a'))$. Long-run behavior corresponds to the outcome of a BDP where the feedback effect is defined to be $\pi'(a) = \pi(h^{T-1}(a))$.

Partial Prediction of Multi-Dimensional Psychological States

Our general framework can be extended to one in which the psychological state is multi-dimensional and the decision maker internalizes the effect of his action on a subset of such vector and believes that he doesn't affect the complement. Let $A \times P \subseteq \Re^K \times \Re^N$ and $\pi(a)$ be a non-empty and single-valued function for each $a \in A$, with $\pi(a) = (\pi_1(a), ..., \pi_N(a))$, and for clarity of exposition, assume that the binary relation $\succeq$ has a (expected) utility representation $u : A \times P \to \Re$. We will assume that the DM is able to

internalize the impact of choices on a subset of psychological states. As before, we write $\pi(a) = (\pi_1(a), ..., \pi_N(a))$. Suppose the DM is able to internalize the first M psychological states, $1 \leq M \leq N$. Let $\tilde{P}$ denote the projection of P onto $P \cap \Re^{N-M}$ with $\tilde{p}$ denoting a representative element of $\tilde{P}$. Let $\tilde{v}(a, \tilde{p}) = u(a, (\pi_1(a), ..., \pi_M(a), p_{M+1}, ..., p_N))$. Let $\tilde{h}(p) = \{a \in A : a \in \arg\max_{a \in A} \tilde{v}(a, \tilde{p})\}$. In what follows, we will assume that that $\tilde{h}(p)$ is unique. Fix a $p_0 \in P$. A sequence of *short-run* outcomes is determined by the relations $a_t \in \tilde{h}(p_{t-1})$ and $p_t = \pi(a_t)$, $t = 1, 2, ...$: at each step, the DM chooses a myopic best-response. *Long-run* outcomes are denoted by a pair a, p with $p = \pi(a)$ and a is defined to be the steady-state solution to the short-run outcome functions i.e. $a = \tilde{h}(\pi(a))$. It follows that long-run behavior corresponds to the outcome of a BDP where the preferences are represented by a utility function $\tilde{v}(a, \tilde{p}) = u(a, (\pi_1(a), ..., \pi_M(a), p_{M+1}, ..., p_N))$.

The Normative Implications of Partial Prediction

On a specific domain of preferences, a DM who is able to partially predict how psychological states evolve with actions may be worse-off than a DM who never predicts how psychological states evolve with actions as the following example shows:

Example. Consider the following example of a DM where there are two payoff relevant dimensions of choice with outcome denoted x_1 and x_2 and preferences $u(x) = x_1 + v_1(x_1 - r_1) + x_2 + v_2(x_2 - r_2)$ where $v(\cdot)$ is a Kahneman-Tversky value function with $v_i(z) = z$ if $z \geq 0$, $v(z) = \alpha_i z$, $\alpha_i > 1$ if $z < 0$ and $v(0) = 0$. There are two options. Option 1 is defined by $(x_1 = 3, x_2 = 2)$ and option 2 is $(x_1 = 6, x_2 = 0)$. We assume that π is the identity map so that in a consistent decision state the reference point corresponds to current choice of the DM.

Suppose the DM does not predict that reference point shifts in both dimensions 1 and 2. The payoff table below provides a quick summary of the decision problem in this case:

	reference point 1	reference point 2
option 1	5	$7 - 3\alpha_1$
option 2	$9 - 2\alpha_2$	6

A straightforward computation establishes that (option 2, reference point 2) is the unique BDP outcome whenever $\alpha_1 > \frac{7}{3}$ and $\alpha_2 < 2$.

Now suppose the DM is able to predict that the reference point will shift in the first dimension but not in the second dimension. The payoff table below provides a quick summary of the decision problem in this case:

	reference point 1	reference point 2
option 1	5	7
option 2	$9 - 2\alpha_2$	6

A straightforward computation shows that whenever $\alpha_2 \geq 1$, (option 1, reference point 1) is the unique BDP outcome.

As (option 2, reference point 2) always payoff dominates (option 1, reference point 1), partial prediction makes the DM worse-off.