On the cost of rent-seeking by government bureaucrats in a Real-Business-Cycle framework

Aleksandar Vasilev*

September 22, 2013

Abstract

This paper studies the wasteful effect of bureaucracy on the economy by addressing the link between rent-seeking behavior of government bureaucrats and the public sector wage bill, which is taken to represent the rent component. In particular, public officials are modeled as individuals competing for a larger share of those public funds. The rent-seeking extraction technology in the government administration is modeled as in Murphy *et al.* (1991) and incorporated in an otherwise standard Real-Business-Cycle (RBC) framework with public sector. The model is calibrated to German data for the period 1970-2007. The main findings are: (i) Due to the existence of a significant public sector wage premium and the high public sector employment, a substantial amount of working time is spent rent-seeking, which in turn leads to significant losses in terms of output; (ii) The measures for the rent-seeking cost obtained from the model for the major EU countries are highly-correlated to indices of bureaucratic inefficiency; (iii) Under the optimal fiscal policy regime,steady-state rent-seeking is smaller relative to the exogenous policy case, as the government chooses a higher public wage premium, but sets a much lower public employment, thus achieving a decrease in rent-seeking.

JEL Classification: E69, E62, E32, J45

Keywords: Rent-seeking, bureaucracy, public employment, government wages

---

*Asst. Professor, American University in Bulgaria, Balkanski Academic Center, Blagoevgrad 2700, Bulgaria. This paper is based on Ch.3 from the author's PhD thesis. The author thanks James Malley, Konstantinos Angelopoulos, Miltos Makris, participants at SAEe 2013, and the RES PhD Meeting 2013, for the excellent comments. E-mail address for correspondence: avasilev@aubg.bg.

Contents

Contents

List of Figures

List of Tables

1 Introduction

The social cost of rent-seeking and administrative corruption in Europe can cause a significant loss for the economy, as argued in Rose-Akerman (1999). Rent-seeking behavior, however, can take many forms and in many instances corruption schemes are not obvious. In particular, this paper focuses on the non-productive activities that occur inside public administration and models them in a dynamic general-equilibrium setting. This is achieved by adding public employment and rent-seeking by government officials to an otherwise standard RBC model. As in Wallenius and Prescott (2011), public sector labor choice is shown to be important not only for fiscal policy, but also for political economy issues. The framework in this paper is then used to generate a theory-based measure for the cost of the waste imposed on the economy, and proceeds to compare and contrast it with the value obtained from the optimal fiscal policy case.

In line with Eliott (1997), Goel and Nelson (1998), and Persson and Tabellini (2000, p.8.), the focus in this paper will be on particular types of government expenditure, namely spending on wages, and its potential to produce rent-seeking behavior. In particular, the sharp increase in public sector employment observed in EU member states in the post-WWII era, together with the existence of a significant public wage premium, could be driven by the tendency for bureaucracy to self-breed and expand independently. Borcherding (1977) was the first to provide some evidence for such a hypothesis by demonstrating that only half of the increase in real government spending can be rationalized with changes in relative prices, or demand factors such as increase in real income, and population growth. In addition, LaPalombara (1994) finds that the size of government budget relative to output is positively correlated with levels of corruption (p.338).

Importantly, this bloating process in the administration and the subsequent expansion of the public sector wage bill should raise concerns in policy makers, since larger governments tend to lose efficiency progressively with size, an issue first expressed in Parkinson (1962). Furthermore, a classic study by von Mises (1944) provides a possible justification for this claim by elaborating on some of the important differences between a bureaucrat and a private sector worker. First, labor productivity in the public sector is difficult to measure. Moreover, a quantity corresponding to government production is also hard to define. This is driven by the fact that public services are to a great extent composed of non-market output. Lastly, a government employee often comes into office through the political system - by election or by appointment of other bureaucrats, usually from the same party. Therefore, a bureaucrat is both an employer and an employee. Once organized, bureaucrats concentrate on increasing in numbers, keeping the *status quo*, and resisting change.[1]

In the literature on bureaucracy, the studies by von Mises (1944), Parkinson (1957), Niskanen (1971), Warwick (1975), Tullock (1976) and Tinbergen (1985) all focus on the strong competition for advancement within bureaus and the inter-unit conflicts. In particular, as noted in Box (2004), "public sector bureaucrats want their agencies to grow so that their status and freedom to act are increased," and thus each government official has a vested interest in promotion. Furthermore, the contemporary bureaucrat is not the ideal type envisioned and described in Weber (1963), the impartial official and the competent professional who shows a strong *ethos* for working in the public administration,[2] but rather a self-interested individual. In particular, Peter (1969) describes the process employed by a government worker to obtain promotions as "acquiring a pull" (p.48). This can occur when the employee finds a patron, a person superior in the hierarchy, who can help with the employee's promotion.[3] In some instances, multiple patrons may be chosen, producing a network effect, as individual patrons talk to one another about the employee's career prospects and advancement. In his monograph, Klitgaard (1991) pays particular attention to bureaucratic corruption, and investigates different cases that result from bureaucratic employment. In particular, his study argues that administrative corruption, and often the use of patronage allow bureaucrats to supplement salaries with public funds.

In an important study, Rose-Akerman (1999) also argues that corruption, or rent-seeking behavior, is embedded in the hierarchical structure of public administration. For example, subordinates in the administration are treated as "family," and some of the gains obtained by their superiors through rent-seeking are shared with staff members, who are lower in the hierarchy. Thus, Rose-Akerman (1978) not only distinguishes between high-level and low-

[1] Often, the expansion in bureaucracy is justified by bureaucrats themselves, who claim there is an increase in the perceived demand for public services and thus there is need for more regulation, hence larger administration.

[2] After all, Fry (1989) reminds that "Weber's ideal type is not a description of reality." (p. 21)

[3] Another author who discusses the patronage system is Gortner (1977).

level bureaucrats, but also emphasizes inter-official competition.[4] In addition, as observed in Parkinson (1957), officials can increase and multiply by making work for each other through the redundancy of repetitive tasks and overlapping authorities and responsibilities. In this sense, as in McKenzie and Tullock (1978) and Reisman (1990), public professionals can be regarded as self-interested maximizers of their position in a bureaucratic world, who pursue career advancement, financial security, and try to use the organization where they work to serve their personal interests. In a more recent study, Lambsdorff (2007) claims that if corruption involves a rent-seeking government whose members attempt to enrich themselves, then the size of the government itself should be significantly decreased (p.4). Similar views are presented in Rose-Akerman (1978), who associates bureaucratic corruption with bloated agency budgets.[5]

Very few economists, with the notable exception of Buchanan and Tullock (1962), Tullock (1965), and Niskanen (1971), have focused on the presence of a large bureaucracy and provided evidence of its importance in the macroeconomic context. In addition, a very small economic literature exists on the internal organization of the state and the incentives of government bureaucracy, *e.g.*, Acemoglu and Verdier (1998), Acemoglu (2005), and Becker and Mulligan (2003). Guriev (2004), Dixit (2006, 2010), and Dodlova (2013) also point out to the multi-tier structure of the government bureaucracy, the principal-bureaucrat-agent hierarchy, as the main culprit for state inefficiency, due to the agency problem generated within such an organizational arrangement. More specifically, elected politicians (agents) need to elect experts (bureaucrats) to implement policies in the interest of the citizens (principal). Thus, as argued in Tirole (1994) and Aghion and Tirole (1997), bureaucrats possess the real authority in the government, as they have an "effective control over decisions" (even though they don't have the "formal authority"). In addition, Niskanen (1971), Bendor *et al.* (1985), and Horn (1995) argue that bureaucrats use their superior information when the budget is decided to inflate the costs of labor input.[6]

The focus in those studies also fall on the emergence of inefficient states, where the inefficiency is measured in terms of extracted rents and the excessive spending on wages of

[4] Promotions and prizes in bureaucracy will also be treated as rent-seeking activities in the model.

[5] Bhagwati (1982) names the process "internal corruption," and views rent-seeking activities as "directly unproductive."

[6] For literature on the incentives of bureaucrats, see Laffont (2000), and Laffont and Martimort (2002).

bureaucrats. Gregory and Lazarev (2003) and Acemoglu *et al.* (2007) also documents cases of overemployment of bureaucrats to boost encumbant's party's votes.[7] Furthermore, government workers could be compensated with higher wages, as a favor by policymakers in exchange of bureaucrats' loyalty, or an an efficiency wage aiming to solve monitoring problems (Acemoglu and Verdier 2000). Makris (2006) also emphasizes the importance of the high cost of production of government services for the budget, while Migue and Belanger (1974) argue that bureaucrats oversupply public services, while at the same time enjoying an excessive budget, the size of which is unrelated to their labor productivity.[8] The latter could be rationalized, at least partially, with the moral hazard problem in the government employment sector, as bureaucrats have a better information about the level of the effort exerted, relative to the superiors.[9]

Despite being a possibly better description of the internal organization of the state and the incentives of the bureaucracy, the modeling choice in those partial-equilibrium setups cannot be easily translated and adopted in general equilibrium. Possible extensions of the work in this chapter along those lines is left for future research, as it does not serve our current purpose of quantifying the cost of rent seeking for the aggregate economic activity. This chapter would therefore present an alternative channel used by bureaucrats to rent-seek and lobby for government funds.[10] Bureaucracy in the setup will be implicitly modeled as a collection of competing bureaus, as in Niskanen (1971): every bureaucrat, in each bureau, would want to control more and more subordinates. In addition, the bureaucrat's preference for power and prestige will bring utility through higher labor income, and thus higher consumption.

[7]Egorov and Sonin (2009) point out that often policy makers would prefer a loyal bureaucrat to a capable one.

[8]Roett (1999) points out that there is a public perception that public sector workers are overpaid and underworked.

[9]See Dewatripont *et al.* (1999), Francois (2000), Dixit (2002), Prendergast (2002) and Besley and Ghatak (2005) for the implications of moral hazard in government non-profit organizations. In addition, Tarschys (1975) and Wildavsky (1974) argue that government bureaucracies develop internal pressures for self-aggrandizement and expansion. Lastly, interaction among bureaucrats for budget appropriation are discussed in Miller and Moe (1983), Moe (1997), Bendor et al. (1985), Bos (2001), and Casas-Pardo and Puchades-Navarro (2001).

[10]Indeed, most of government consumption spending is on the wage bill for staff compensation (OECD 1982).

To illustrate the processes taking place within public administration, the model setup in this paper incorporates a symmetric non-cooperative game that is played among government bureaucrats themselves to increase individual income at the expense of the other public officials earnings.[11] The symmetric rent-seeking process is modeled as in Murphy *et al.* (1991)[12]: each individual allocates time optimally on both productive activities and rent-seeking. Additionally, the interaction between agents in the public sector generates strategic complementarities, as individual rent-seeking is positively related to opponents' choice of rent-seeking. As in Burnside and Eichenbaum (1993, 1996), rent-seeking increases one's own capacity and at the same time decreases others' capacity level. In reality, as pointed out in Tinbergen (1985), this correspond to public employees redistributing residual bureau funds, expropriating vacation money (Mieczkowski 1984, p. 164), or applying bonus schemes methodology according to rank and experience. The benefit from engaging in rent-seeking comes at the expense of a cost incurred, which is measured in terms of time, similar to the approach used in Angelopoulos *et al.* (2009).

The value-added of this paper is the focus on the link between the rent-seeking behavior in bureaucracy and the government wage bill, and the resulting cost imposed on the economy as a result of the non-productive activities taking place in the public sector. Another novelty here is that rent-seeking occurs in a non-competitive labor market, the public sector one, where wage rate is set above private sector pay. This stimulates entry of labor in the sector, and as a result, public employment eventually becomes too high. In particular, both the high public wage and employment stimulate rent-seeking by generating a positive benefit of engaging in wasteful activities. In turn, a higher wage bill requires higher tax rates to finance government spending. In the private sector, high taxes reduce incentives to supply

[11] Note that when a large number of bureaucrats attempt to maximize identical objective functions, which are subject to the same set of constraints, this results in a theory covering bureaucracy as a whole, as was first attempted in Niskanen (1971).

[12] An incomplete list of the extraction literature that uses rent-seeking mechanism is: Krueger (1974), Posner (1974, 1975), Tullock (1975, 1980), Congleton (1980), Bhagwati and Srinivasan (1980), Corcoran and Karels (1985), Appelbaum and Katz (1987), Hillman and Riley (1989), Murphy et al. (1991, 1993), Perez-Castrillo and Verdier (1992), Nitzan (1994), Tirole (1986), Khan and Sundaran (2000), Mohtadi and Roe (1998, 2003) and Mauro (2004). Park et al. (2005) for similar setups in which rent-seekers compete with each other for fiscal favors. For surveys of the literature of rent-seeking see Drazen (2000, Ch.11), Persson and Tabellini (2000, Ch. 14.4) and Mueller (2003).

labor and accumulate capital, and decrease consumption and output. Thus rent-seeking has a negative impact on the economy, and this paper attempts to quantify the loss for the economy in a general-equilibrium framework.

The study in this paper is also complementary to Park *et. al* (2005), and Angelopoulos *et al.* (2009, 2011), who all address rent-seeking issues using DSGE models.[13] Their focus, however, falls on problems of tax collection and/or protection of property rights, this study concentrates on the inefficiencies on the government spending side, and the wage bill in particular. In addition, earlier studies consider exogenous policy only, and focus on the non-cooperative Nash (1953) solution of the rent-seeking game, while only briefly discussing the existence (and attainability) of other subgame-perfect equilibria.[14]

Lastly, the rent-seeking process in the public sector can be also viewed as a "coordination failure" problem, as first described in Cooper and John (1988) and Cooper (1999). There is a bad equilibrium, from the non-cooperative game, but also a good equilibrium, where all agents coordinate on the zero rent-seeking. A positive value of rent-seeking time chosen by bureaucrats is socially costly, as the return comes not from productive effort, but rather from the distribution of public funds. At the same time, the positive amount of time dedicated to opportunistic activities is an efficient outcome from an individual worker's point of view, as all agents are fully rational and maximize their utility levels. Thus, in equilibrium, individual bureaucratic rent-seeking efforts will adjust to the point where the value of additional resources spent per bureaucrat equals the benefit that accrues to that individual.[15] Moreover, the higher the level of output, the higher the tax revenue, and thus the larger the pie available for redistribution.This paper finds non-trivial welfare gains of cooperation in the exogenous policy case, which is not considered in previous studies. In addition, a comparative statics exercise is performed to show that significant gains can be realized when the economy moves to a steady-state in which wages are equalized across sectors, and thus rent-seeking is brought down to zero.

[13]Park *et al.* (2004), Economides *et al.* (2007) and Angelopoulos and Economides (2008) address rent-seeking in models with electoral uncertainty.

[14]This infinitely-repeated game can generate a very rich class of subgame-perfect equilibria, which are all sustainable (Mas-Colell *et al.* 1995).

[15]Note that the first-best solution (the jointly optimal level) in the model results in zero rent-seeking.

Next, using the model-generated measure, rent-seeking across the EU-12 is compared to indices of institutional quality in the government sector. Angelopoulos *et al.* (2009, 2011) use the ICRG index as a proxy for rent-seeking, while this paper considers additional indicators that specifically focus on public administration quality. In general, rent-seeking from the government budget is expected to be associated with low quality of government, high corruption, and heavy bureaucratization.[16]

The study then proceeds to discuss the optimal fiscal policy, where not only tax rates, but also different categories of government spending, as well as rent-seeking (from the non-cooperative Nash equilibrium) are optimally chosen by a benevolent Ramsey planner. Due to rent-seeking, public investment is lower, and government consumption (wage bill) is too high. Furthermore, the rent-seeking estimates can be evaluated against findings from studies using static models with rent-seeking of tariff revenues, monopoly profits, and regulations, as well as the costs computed in Angelopoulos *et al.* (2009) in a general-equilibrium framework. Thus, using tools of modern dynamic economics, the study in this paper contributes to the understanding of the wasteful effect of bureaucracy for the economy. It also provides an integrated framework to address public economics and political economy issues, such as public sector labor supply in a non-competitive market, as well as the optimal production and provision of congestible government services.

The main findings of the study are that: (i) Due to the existence of a significant public sector wage premium and the high public sector employment, a substantial amount of working time is spent rent-seeking, which in turn leads to significant losses in terms of output; (ii) The measures for the rent-seeking cost obtained from the model for the major EU countries are highly-correlated to indices of bureaucratic inefficiency; (iii) Under the optimal fiscal policy regime, and with congestible public goods, steady-state rent-seeking is significantly smaller relative to the exogenous policy case; (iv) In addition to the zero capital tax rate, and the higher labor tax rate, the benevolent government planner invests more in public capital, chooses a higher public wage premium, but sets a much lower public employment, and thus achieves a decrease in the level of rent-seeking relative to the value obtained in the

[16]The ICRG index from the IRIS dataset has been used in, among others, Knack and Kiefer (1995), Barro (1997), Hall and Jones (1999), Rodrick (1999), Acemoglu et al. (2001, 2002), Fisman and Gatti (2002) and Persson et al. (2003).

exogenous case.

The paper is organized as follows: Section 2 presents the model setup in the context of the relevant literature. Section 3 explains the data used and model calibration. Section 4 solves for the steady-state, and section 5 calculates the cost of rent-seeking on the economy; Section 6 presents the model solution procedure, discusses the effect of technology shocks and the impulse responses of variables. Section 7 discusses the optimal policy (Ramsey) framework and solves for the steady-state. Section 8 presents the optimal reaction of fiscal policy instruments to technology shocks, and compares and contrasts it to the exogenous policy case. Section 9 acknowledges the limitations of the study, and Section 10 concludes.

2 Model Setup

2.1 Description of the model

There are N_t households, as well as a representative firm. Each household owns physical capital and labor, which it supplies to the firm. Time can be spent working in the private and/or public sector, rent-seeking, or dedicated to leisure. Working in the government sector imposes an additional convex transaction cost, which decreases leisure time. The perfectly-competitive firm produces output using labor and capital. The government produces utility-enhancing labor-intensive public services and uses tax revenues from consumption expenditure, labor and capital income to finance: (1) government transfers, (2) government investment, and (3) public wage bill.

The public sector wage in this paper is modeled as featuring a time-variant mark-up over the private sector wage. Next, individual hours supplied in the public sector can be augmented by using rent-seeking time, and the coefficient of capacity utilization positively depends on one's own rent-seeking time and negatively related to other households' rent-seeking time. This is similar to the study by Angelopoulos *et al.* (2009, 2011), in which households were able to extract part of the tax revenues, or output directly, respectively. In this paper the resource extraction is a little more sophisticated, as public wages are financed from government tax revenues, and therefore government resources are only indirectly expropriated.

2.2 Households

There are N_t representative households in the model economy, who are infinitely-lived.[17] There is no population growth. As in Baxter and King (1993), household h maximizes the following expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^{ph}, S_t^g, L_t^h), \tag{2.2.1}$$

where E_0 is the expectation operator as of period 0; C_t^{ph}, S_t^g and L_t^h are household's private consumption, per household consumption of government services, and leisure enjoyed by household h at time t, respectively. The parameter β is the discount factor, $0 < \beta < 1$. The instantaneous utility function $U(.,.,.)$ is increasing in each argument and satisfies the Inada conditions. The particular functional form for instantaneous utility used is as follows:

$$U(C_t^{ph}, S_t^g, L_t^h) = \psi_1 \frac{(C_t^{ph})^{(1-\sigma_c)}}{1-\sigma_c} + \psi_2 \frac{(L_t^h)^{(1-\sigma_l)}}{1-\sigma_l} + \psi_3 \frac{(S_t^g)^{1-\sigma_s}}{1-\sigma_s} \tag{2.2.2}$$

where $\sigma_c, \sigma_l, \sigma_s > 0$ are the curvature parameters of private consumption, leisure, and government services utility component. Parameters ψ_1, ψ_2, ψ_3 are the weight attached to private consumption, leisure and public services components in utility, respectively, where $0 < \psi_1, \psi_2, \psi_3 < 1$, and $\psi_1 + \psi_2 + \psi_3 = 1$.

Total time available to each household is split between work, N_t^h, rent-seeking in the public sector, RS_t^h, and leisure, L_t^h, Households can supply hours of work in the public sector, N_t^{gh}, in the private one, N_t^{ph}, with $N_t^h = N_t^{ph} + N_t^{gh}$. Given a positive public sector wage, every household will optimally choose to supply a positive amount of hours in the public sector. Thus, the model allows everyone to engage in public sector rent-seeking.

In addition, it will be assumed that the household incurs a quadratic transaction cost from government work, $\varphi(N_t^{gh})^2$, where $\varphi > 0$. The modeling choice tries to capture some of the market imperfections existing in the public sector labor markets, such as the high unionization and the monopsony situation, as well as to help the model accommodate different wage rates in the two sectors, as well as public hours labor choice in the framework. This assumption is also in line with evidence from case studies in Box (2004), which shows that

[17] This number is countably infinite, and the households could be thought of being uniformly distributed on the $[0, 1]$ interval.

working in the public sector is different from working in the private sector, as the two sectors operate under different institutional settings.

In the private sector, efficiency level is constant and for convenience will be normalized to unity, while utilization rate in the public sector can vary because of the rent-seeking. Similar to the approach adopted in Cho and Cooley (1994), and Hayashi and Prescott (2007), in this framework contracted and effective public hours enter the household's utility function through different functional forms. The wage rates per efficiency unit of labor in the private and the public sector are denoted by w_t^p and w_t^g, respectively. In addition, public wage rate will carry a premium over the private wage rate, which is allowed to vary over time. As pointed out in Bellante and Jackson (1979), the overpayment of public employees could be regarded as rent as it is "a pay level higher necessary to attract the requisite quality and quantity of labor in the public sector (p. 248), and thus can be viewed as an efficiency wage.[18]

Next, after joining the public sector, rent-seeking occurs, as it brings a positive benefit from engaging in opportunistic behavior. The form of the corruption is a non-transaction type, and can be interpreted as an abuse of power for personal advantage, or putting one's own interests first in the performance of a public duty. In particular, by using his or her own rent-seeking time, an individual's public sector labor income can be augmented by increasing the effective hours worked in the government: By supplying N_t^{gh} contract hours in the public sector, and spending RS_t^h hours on rent-seeking, each households generates $\frac{RS_t^h}{RS_t}N_t^{gh}$ of "efficiency units of labor," as in Burnside and Eichenbaum (1993, 1996), hence total public sector labor income becomes $w_t^g\frac{RS_t^h}{RS_t}N_t^{gh}$.[19] At the same time, predatory behavior decreases the capacity utilization of labor of the other workers in the public sector. Note that each household is atomistic, so it takes the aggregate quantity of rent-seeking RS_t as given. (In

[18]This is in line with the evidence that public sector employees are more skilled than those in the private sector. On the other hand, Beaumont (1980) argues that public unions usually maintain that "a public employer should be the best employer, that its wage policy should be based on the highest rates being paid for comparable work in the private sector" (p. 32). In addition, Bender (1998, 2003) provides empirical support that part of the public wage premium is due to political economy factors, *e.g.*, public employees vote more often than the average private sector worker (Jensen *et al.* 2009).

[19]Thus $\frac{RS^h}{RS}$ can be interpreted as a shift parameter of the rent-seeking extraction technology. As in Grossman (2002), this technology representation is a useful short-cut to model rent-seeking. In addition, note that at the aggregate level, the efficiency issues disappears, as $\sum_h \frac{RS^h}{RS}N^{gh} = \sum_h \frac{RS}{RS}N^{gh} = N^g$ by first applying the symmetry, and then aggregating over households.

equilibrium, $RS_t = \sum_h RS_t^h$.) Thus, even though total public employment is exogenous,[20] the individual public hours are endogenous.

As in Burnside and Eichenbaum (1993, 1996), it is assumed that each household cares about effective hours of work only. Thus, the time constraint that each household faces in each period (in efficiency terms) is as follows:

$$N_t^{ph} + \frac{RS_t^h}{RS_t} N_t^{gh} + \varphi(N_t^{gh})^2 + RS_t^h + L_t^h = 1. \tag{2.2.3}$$

The rent-seeking technology described is a special case of a standard symmetric contestable prize function used in the literature.[21] This approach models rent-seeking as an optimal choice made by each government bureaucrat. In addition, the size of the total pie available to government workers will be endogenously determined, as each bureaucrat chooses individual public hours optimally. Thus each official has an incentive to choose the optimal size of their effective "slice." The only modeling difference in this paper from the earlier general-equilibrium studies on rent-seeking, *e.g.* Angelopoulos *et al.* (2009, 2011), is that the cost of resources spent on influencing the probability of winning, $\frac{RS_t^h}{RS_t}$, is measured in terms of time and thus in utility of leisure terms instead of output/income directly. For practical purposes, the specification used in the current model setup can also be interpreted as an auction in which competing bureaus lobby for a larger share of the contestable transfer, and the endogenous sharing rule defines the rent-seeking technology. Moreover, a larger share of the pie means higher effective public hours, which can be associated with promotion in the hierarchical structure, higher prestige, more subordinates, more power by entrenchment in an organization and thus the achievement of security and convenience.

In addition to the labor income received, each household saves by investing in private capital I_t^h. As an owner of capital, the household receives interest income $r_t K_t^{ph}$ from renting the capital to the firms; r_t is the return to physical capital and K_t^{ph} denotes physical capital

[20] The mechanism has also empirical foundations: as established in Staaf (1977), in the US public education sector, the supervisory salaries are correlated with the number of subordinates (teachers in the district).

[21] An inexhaustive list of studies includes Krueger (1974), Posner (1974, 1975), Tullock (1975, 1980), Congleton (1980), Bhagwati and Srinivasan (1980), Corcoran and Karels (1985), Appelbaum and Katz (1987), Hillman and Riley (1989), Castrillo and Verdier (1992), Nitzan (1994) and Khan and Sundaran (2000). For an extensive review of "extraction technologies" the interested reader should consult Mueller (2003) Ch. 15.

stock in the beginning of period t.

The household's physical capital evolves according to the following law of motion

$$K_{t+1}^{ph} = I_t^h + (1-\delta^p)K_t^{ph}, \tag{2.2.4}$$

where $0 < \delta^p < 1$ is the depreciation rate of private physical capital.

Finally, consumers are owners of the firms in the economy, and receive equal share of the profit (Π_t^h) in the form of dividends. The budget constraint for each household is

$$C_t^{ph} + I_t^h \leq (1-\tau_t^l)\left[w_t^p N_t^{ph} + w_t^g \frac{RS_t^h}{RS_t} N_t^{gh}\right] + (1-\tau_t^k) r_t K_t^{ph} + \Pi_t^h + G_t^{th}, \tag{2.2.5}$$

where τ_t^l, τ_t^k are the proportional tax rates on labor and capital income, respectively, and G_t^{th} denotes the level of per household lump-sum government transfer.

Each household h acts competitively by taking prices $\{w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$, tax rates $\{\tau_t^l, \tau_t^k\}$ and policy variables $\{G_t^i, S_t^g, G_t^t, K_{t+1}^g\}_{t=0}^{\infty}$ as given, it chooses allocations $\{C_t^{ph}, N_t^{ph}, N_t^{gh}, RS_t^h, I_t^h,$ $K_{t+1}^h\}_{t=0}^{\infty}$ to maximize Equation (2.2.1) subject to Equations (2.2.2)-(2.2.5), and the initial condition for physical and public capital stocks $\{K_0^{ph}, K_0^g\}$.

The optimality conditions from the household's problem, together with the transversality condition (TVC) for the private physical capital stock, are as follows:[22]

$$C_t^{ph} : \frac{\psi_1}{[C_t^{ph}]^{\sigma_c}} \quad = \quad \Lambda_t \tag{2.2.6}$$

$$K_{t+1}^{ph} : \Lambda_t \quad = \quad \beta E_t \Lambda_{t+1}\Big[(1-\tau_{t+1}^k) r_{t+1} + (1-\delta^p)\Big] \tag{2.2.7}$$

$$N_t^{ph} : \frac{\psi_2}{(L_t)^{\sigma_l}} = \Lambda_t (1-\tau_t^l) w_t^p \tag{2.2.8}$$

$$N_t^{gh} : \frac{\psi_2}{(L_t)^{\sigma_l}}\left[\frac{RS_t^h}{RS_t} + 2\varphi N_t^{gh}\right] = \Lambda_t (1-\tau_t^l) w_t^g \frac{RS_t^h}{RS_t} \tag{2.2.9}$$

$$RS_t^h : \frac{\psi_2}{(L_t)^{\sigma_l}}\left[1 + \frac{N_t^{gh}}{RS_t}\right] = \Lambda_t (1-\tau_t^l) w_t^g \frac{N_t^{gh}}{RS_t} \tag{2.2.10}$$

[22]Detailed derivations in Appendix 11.1.2.

$$\lim_{t\to\infty} \beta^t \Lambda_t K_{t+1}^{ph} \quad = \quad 0, \tag{2.2.11}$$

where Λ_t is the Lagrange multiplier on the household's budget constraint. The household equates marginal utility from consumption with the marginal cost imposed on its budget. Next, the Euler equation describes the optimal capital accumulation rule, and implicitly characterizes the optimal consumption allocations chosen in any two neighboring periods. Private hours are chosen so that the disutility of an hour work in the private sector at the margin equals the after-tax return to labor. The disutility of an hour of rent-seeking time equals the marginal increase in after-tax public sector labor income. At the margin, the benefit of engaging in rent-seeking equals the utility cost of doing so. The last expressions, (2.1.11), is the so-called "transversality condition" (TVC), imposed to ensure that the value of the private physical capital that remains after the optimization horizon is zero. This boundary conditions guarantees that the model equilibrium is well-defined by ruling out explosive solution paths.

Divide (2.2.9) by (2.2.8), and impose symmetry (hence $RS_t^h = RS_t$, $N_t^{gh} = N_t^g$) to obtain

$$1 + 2\varphi N_t^g = \frac{w_t^g}{w_t^p}. \tag{2.2.12}$$

Eq. (2.2.12) is a typical labor supply relationship, and characterized in this framework by a positive relationship between total public hours and the public/private wage ratio.[23] Next, divide (2.2.10) by (2.2.8) to obtain

$$\frac{w_t^g N_t^{gh}}{w_t^p RS_t} = 1 + \frac{N_t^{gh}}{RS_t}. \tag{2.2.13}$$

After some rearrangement, and by imposing symmetry once again, it can be shown that

$$RS_t = \left[\frac{w_t^g}{w_t^p} - 1\right] N_t^g. \tag{2.2.14}$$

Optimality condition (2.2.15) is new in the literature on rent-seeking. As seen from above, rent-seeking time is a product of public employment, N_t^g, and the net wage premium, $\frac{w_t^g}{w_t^p} - 1$. In other words, the corruption problem in this framework could be split into two parts: the high public employment ("extensive margin"), and the high public wage premium ("intensive

[23]Alternatively, the equation can be interpreted as a "wage curve" equation, similar to the one described in Blanchflower and Oswald (1996).

margin").[24] Therefore, Eq. (2.2.14) suggests that cuts in the public wage bill are important for curbing the size of the contestable prize and thus effectively restraining the rent-seeking behavior of government bureaucrats.[25]

2.3 Firms

There is also a representative private firm. It produces a homogeneous final product using a production function that requires physical capital, K_t and labor hours H_t^p. The production function is as follows:

$$Y_t = A_t (N_t^p)^{\alpha} (K_t^p)^{1-\alpha}, \tag{2.3.1}$$

where A_t measures the total factor productivity in period t; $0 < \alpha, (1-\alpha) < 1$ are the productivity of labor and private physical capital, respectively.

The representative firm acts competitively by taking prices $\{w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$ and policy variables $\{\tau_t^k, \tau_t^l, G_t^i, G_t^t, K_t^g\}_{t=0}^{\infty}$ as given. Accordingly, K_t^p, and N_t^p are chosen every period to maximize static aggregate profit,

$$\Pi_t = A_t (N_t^p)^{\alpha} (K_t^p)^{1-\alpha} - r_t K_t^p - w_t^p N_t^p. \tag{2.3.2}$$

In equilibrium, capital and labor receive their marginal products, *i.e.*[26]

$$r_t = (1-\alpha)\frac{Y_t}{K_t^p}, \tag{2.3.3}$$

$$w_t^p = \alpha \frac{Y_t}{N_t^p}. \tag{2.3.4}$$

Hence, equilibrium per-period profits are zero.

2.4 Government

Government invests in capital, G_t^i, which is used in the provision of the utility-enhancing government services. In addition, government hires labor N_t^g at a wage level w_t^g to produce

[24] Hence $RS_t = 0$ when $w_t^p = w_t^g$, and/or $N_t^g = 0$.

[25] Another instance when equilibrium rent-seeking is zero, is when all households decide to play the co-operative solution. The government can use a "coordination device," *e.g.* by providing a forum for free exchange of information that would ultimately induce workers to cooperate on the "good equilibrium."

[26] For a detailed derivation, see Appendix 11.1.1.

public consumption goods and distributes transfers $G^t_t(\equiv \sum_h G^{th})$. The production function for public consumption is as in Cavallo (2005), Linnemann (2009) and Economides *et al.* (2011):

$$S^g_t = (N^g_t)^{\gamma}(K^g_t)^{(1-\gamma)}, \tag{2.4.1}$$

where $0 < \gamma < 1$ is the share of public employment. Since the household takes the level of government services as given, in competitive equilibrium there will be externalities arising from the presence of public employment and investment in the government services production function: More hours in the public sector generate more government services (a higher level of the public good available for public consumption), which directly increase utility. In addition, holding all else equal, an increase in public employment raises welfare indirectly by increasing the after-tax public sector labor income, and hence consumption. Lastly, more hours spent in the public sector decrease the amount of leisure the household can enjoy in a certain period (and also increase rent-seeking), and thus lower welfare. The quantitative effect will be determined by the value of the curvature parameter σ_s in the household's utility function. Therefore, in the general case it is unclear *ex ante* whether public employment creates a positive or a negative externality in the economy.[27]

Total government expenditure, $G^t_t + G^i_t + w^g_t H^g_t$, is financed by levying proportional taxes on capital and labor income. Thus, the government budget constraint is as follows:

$$G^t_t + G^i_t + w^g_t N^g_t = \tau^k_t r_t K^p_t + \tau^l_t \Big[w^p_t N^p_t + w^g_t N^g_t \Big]. \tag{2.4.2}$$

Next, the law for government capital accumulation is as follows:

$$K^g_{t+1} = G^i_t + (1-\delta^g)K^g_t, \tag{2.4.3}$$

where $0 < \delta^g < 1$ is the depreciation rate of public capital.

Government takes market prices $\{w^p_t, r_t\}^{\infty}_{t=0}$ and allocations $\{N^p_t, N^g_t, K^p_t\}^{\infty}_{t=0}$ as given. Finally, only four of the five policy instruments, $\{\tau^l_t, \tau^k_t, w^g_t, G^i_t, G^t_t\}^{\infty}_{t=0}$, can be exogenously set. Government investment share in output, $G^{iy}_t = \frac{G^i_t}{Y_t}$, as well as the two tax rates $\{\tau^l_t, \tau^k_t\}$ will be fixed to their corresponding data average in all time periods; Thus, the level of government investment will react to private output. Note that public capital stock series will

[27] *Ex post*, in the optimal policy framework, it will be shown that public hours create a negative externality.

be residually determined from a given initial stock, and public investment sequence. Next, government transfers $\{G_t^t\}_{t=0}^{\infty}$ will be set to match the employment ratio in data. Lastly, the public wage rate will be determined residually to ensure that the government budget constraint is satisfied in every period.

In other words, the government controls the labor demand in the public sector, and facing a supply schedule for labor services in the government sector, sets the price of labor to clear the market. Despite the market-clearing property in this market, however, the situation is one of imperfect competition, as the price of labor is decoupled from the marginal productivity in the public sector and, rather, determined by budgetary considerations. In a sense, public sector labor markets will operate inside the production possibilities frontier.

2.5 Stochastic processes for the policy variables

Total factor productivity, A_t, will be assumed to follow AR(1) processes in logs, in particular

$$\ln A_{t+1} = (1-\rho^a)\ln A_0 + \rho^a \ln A_t + \epsilon_{t+1}^a, \tag{2.5.1}$$

where $A_0 = A > 0$ is steady-state level value of the total factor productivity process, $0 < \rho^a < 1$ is the first-order autoregressive persistence parameter and $\epsilon_t^a \sim iidN(0, \sigma_a^2)$ are random shocks to the total factor productivity progress. Hence, the innovations ϵ_t^a represent unexpected changes in total factor productivity process.

2.6 Symmetric Decentralized Competitive Equilibrium

Given the paths of the policy instruments $\{G_t^i, G_t^t\}_{t=0}^{\infty}$, the exogenous process followed by total factor productivity, $\{A_t\}_{t=0}^{\infty}$, and initial conditions for the state variables $\{A_0, K_0^{ph}, K_0^g\}$, a symmetric decentralized competitive equilibrium (DCE) is defined to be a sequence of allocations $\{C_t^{ph}, N_t^{ph}, N_t^{gh}, RS_t^h, I_t^h, K_{t+1}^{ph}, K_{t+1}^g\}_{t=0}^{\infty}$, $\forall h$, prices $\{r_t, w_t^p, w_t^g\}_{t=0}^{\infty}$ and the tax rates $\{\tau_t^k, \tau_t^l\}$ so that (i) all households maximize utility; (ii) firms maximize profits; (iii) the government budget constraint is satisfied in each time period, and (iv) all markets clear.[28]

[28] The symmetric DCE system of equations for the general case, and the steady-state system is presented in Appendices 11.2 and 11.3.

3 Data and model calibration

The model in this paper is calibrated to German data at annual frequency. The choice of the particular economy was made based on the large public employment share, as well as the significant public wage premium observed in this country. Since there is no EU-wide fiscal authority, an individual country was chosen, instead of calibrating the model for the EU Area as a whole. In addition, payment in the public sector in the model is determined not by marginal productivity of labor, but rather by factors outside the model.

The paper follows the methodology used in Kydland and Prescott (1982), as it is the standard approach in the literature. Both the data set and steady-state DCE relationships of the models will be used to set the parameter values, in order to replicate relevant long-run moments of the reference economy for the question investigated in this paper.

3.1 Model-consistent German data

Due to data limitations, the model calibrated for Germany will be for the period 1970-2007 only, while the sub-period 1970-91 covers West Germany only.[29] For Germany, data on real output per capita, household consumption per capita, government transfers and population was taken from the World Development Indicators (WDI) database. The Organization for Economic Co-operation and Development (OECD) statistical database was used to extract the long-term interest rate on 10-year generic bonds, CPI inflation, average annual earnings in the private and public sector, average hours, private, public and total employment in Germany. Investment and capital stock series were obtained from the EU Klems database (2009). The German average annual real public compensation per employee was estimated by dividing the real government wage bill (OECD 2011) by the number of public employees.

3.2 Calibrating model parameters to the German data

In the German data, the average public/private employment ratio over the period 1970-2007 is $n^g/n^p = 0.17$, and the average public/private wage ratio is $w^g/w^p = 1.20$. Next, the

[29]The time period is particularly suitable for the study of public employment, and government wage bill spending; Hughes (1994), for example, argues that "[i]n the 1970s, intellectual arguments were mounted by conservative economists that government was the economic problem restricting economic growth and freedom."(p. 11)

average effective tax rates on labor and physical capital, obtained from McDaniel's (2009) dataset are $\tau^l = 0.409$ and $\tau^k = 0.16$, respectively. McDaniel's approach was preferred to the one used by Mendoza et al. (1984) and the subsequent updates, *e.g.* Martinez-Mongay (2000), Carey and Tchilinguirian (2000) and Carey and Rabesona (2002), due to its more careful treatment of property and import taxes in the former. The labor share, $\alpha = 0.71$, was computed as the average ratio of compensation of employees in total output. Alternatively, average capital share, $1 - \alpha = 0.29$, can be obtained as the mean ratio of gross capital compensation in output from the EU Klems Database (2009). The private capital depreciation rate was found to be $\delta^p = 0.082$, while public capital depreciation rate was $\delta^g = 0.037$ over the period.

The discount rate $\beta = 0.979$ was calibrated from the steady-state consumption Euler equation to match the average private capital-to-output ratio in data. Next, parameter $\gamma = 0.62$, which measures the weight on public sector hours in the public good production was obtained as the average ratio of the public sector wage bill to total government expenditure less transfers and subsidies, as in Cavallo (2005) and Linnemann (2009). The value is consistent with OECD (1982) estimates for the period 1960-78 for Germany, which was obtained from a log-linear regression estimation. Additionally, the calibrated value of public capital elasticity, $1-\gamma = 0.38$, is consistent with the government capital effect estimated in Aschauer (1989) and Hjerppe *et al.* (2006).

As in Cavallo (2005) and Linnemann (2009), a logarithmic specification was chosen for the utility of private consumption, namely $\sigma_c = 1$. This follows Merz (1995) and Gomes (2012), who argue that workers from the private and public sector are able to pool their resources together, and thus achieve complete insurance.[30] Similarly, as in Gali (2008), the inverse of the Frisch elasticity of labor supply is assumed to be approximately unity, *i.e.*, $\sigma_l = 1$. In addition, the logarithmic form for the utility of leisure has empirical support, *e.g.* Asch and Heaton (2008) and Falsch (2008), who find that public labor supply elasticity in two representative sectors, secondary education and defense, does not differ significantly from unity. The logarithmic specifications for private consumption and leisure may seem restrictive at first sight, but it assists greatly in matching hours across sectors, which is the

[30] A similar claim is made in Blundell *et al.* (2012) in the context of family labor supply and consumption smoothing.

dimension of interest in this paper, rather a focus on issues such as risk aversion, and/or the Frisch elasticity of labor supply.[31]

Next, as in Chatterjee and Ghosh (2009), the curvature of the government services utility component was set to $\sigma_s = 0.95$ to reflect the "degree of relative congestibility associated with the utility benefits derived from the public goods." Alternatively, $1/\sigma_s$ measures the intertemporal elasticity of substitution of government services, or how responsive the median household (voter) is to growth in public services with respect to the changes in the median household's income.[32] Given that government services are modeled as a non-market output, and the normalization of private consumption good to unity, total income is a good proxy for the willingness of pay, as it represents the tax base on which the government levies taxes to finance the provision of public services. The value for σ_s used in the calibration is in line with the findings in Falvey and Gemmel (1996), who estimate the elasticity $\frac{1}{\sigma_s}$ to fall in the $[1.04, 1.07]$ interval for general government services (*i.e.* $\sigma_s \in [0.93, 0.95]$), and Gibson (1980), who estimates that the income elasticities for public services, such as social care, education, pollution control, parks and recreational areas, as well as highway construction and maintenance, are slightly higher than unity. In other words, agents are elastic, but not greatly so: still, they do not want to accept large variations in the level of public services provided over time. Furthermore, in the exogenous policy setup, parameter σ_s does not affect allocations (but it affects the level of utility), since the household ignores the externality. Thus, the level of government services will be residually determined given the steady-state public employment and government capital stock.[33]

The average steady-state total hours of work in data as a share of total hours available is $n = 0.296$, hence total employment in the model is consistent with the estimates in Ghez and Becker (1975) of the fraction of time spent working. Together with the public/private employment ratio, this yields the model-consistent steady-state values for private and public

[31] Allowing for a general CRRA representation of the utility of consumption and leisure leads to convergence problems in the optimal policy framework.

[32] Since all households are the same, the median household is the same as the average, or the representative one in the model.

[33] In the optimal policy case, the congestibility condition for public services, $\sigma_s < 1$, turns out to be a necessary and sufficient to produce a decrease in steady-state decrease in the rent-seeking time relative to the exogenous policy case.

hours, $n^p = 0.253$ and $n^g = 0.043$, respectively. Steady-state public hours in the model are set to the value in data. The weight on utility from government services was set to $\psi_3 = 0.15$, which is consistent with the value used in Finn (1994) and Klein *et al.* (2008). Next, the weights attached to private consumption $\psi_1 = 0.35$ and $\psi_2 = 0.50$ are set to match exactly both types of hours in data.[34] Note that ψ_1 is set larger than the average time spent working, as suggested in Kydland (1995), due to the presence of government work transaction cost, and the existence of rent-seeking in the public sector. On the other hand, the model is roughly consistent with Bouakez and Rebel (2007), Leeper *et al.* (2009), and Conesa *et al.* (2009), who argue that private consumption good is on average twice as valuable as government services, as $\psi_1/\psi_3 = 2.33$, and leisure is twice as valuable as the private consumption good, as $\psi_1/\psi_1 = 1.43$. The scale parameter of the transaction cost associated with government work, $\varphi = 2.318$, is calibrated to match the average public/private wage ratio in the data.

Total factor productivity moments, $\rho^a = 0.943$ and $\sigma^a = 0.013$, were obtained in several steps: First, using the model's aggregate production function specification and data series for physical capital and labor, Solow residuals (SR) were computed in the following way:

$$\ln SR_t = \ln y_t - (1-\alpha)\ln k_t^p - \alpha \ln n_t^p. \tag{3.2.1}$$

The logged series were then regressed on a linear trend ($b > 0$) to obtain

$$\ln SR_t = bt + \epsilon_t^{SR}. \tag{3.2.2}$$

Observe that the residuals from the regression above,

$$\epsilon_t^{SR} = \ln SR_t - bt \equiv \ln a_t, \tag{3.2.3}$$

represent the stationary, or detrended, component of the logged TFP series.

Next, the AR(1) regression

$$\ln a_t = \beta_0 + \beta_1 \ln a_{t-1} + \epsilon_t^a \tag{3.2.4}$$

was run using ordinary least squares (OLS) to produce the estimates (denoted by the "hat" symbol) for the persistence and standard deviation parameters of the total factor productivity

[34] Observe that given the pre-set value for ψ_3 and the fact that $\psi_1 + \psi_2 + \psi_3 = 1$, by setting ψ_1, ψ_2 will be residually determined.

process to be used in the calibration of the model. In particular,

$$\hat{\beta}_1 = \rho^a \tag{3.2.5}$$

$$\hat{\epsilon}_t^a \sim N(0, \sigma_a^2). \tag{3.2.6}$$

Table 1 on the next page summarizes all model parameters used in the calibration.

Table 1: Model Parameters

Param.	Value	Definition	Source
β	0.979	Discount factor	Calibrated
α	0.710	Labor income share	Data average
δ^p	0.082	Depreciation rate on private capital	Data average
δ^g	0.037	Depreciation rate on public capital	Data average
ψ_1	0.350	Weight on consumption in utility	Calibrated
ψ_2	0.500	Weight on leisure in utility	Calibrated
ψ_3	0.150	Weight on government services in utility	Set
σ_c	1.000	Curvature parameter of the private consumption utility component	Set
σ_l	1.000	Curvature parameter of the leisure utility component	Set
σ_s	0.950	Curvature parameter of the government services utility component	Set
φ	2.318	Scale parameter of government work transaction cost	Calibrated
γ	0.620	Labor share in public services production	Data average
$1-\gamma$	0.380	Capital share in public services production	Calibrated
τ^k	0.160	Effective tax rate on capital income	Data average
τ^l	0.409	Effective tax rate on labor income	Data average
A	1.000	Steady-state level of total factor productivity	Set
ρ^a	0.943	AR(1) parameter total factor productivity	Estimated
σ_a	0.013	SD of total factor productivity innovation	Estimated

4 Steady state results

Once model parameters were obtained, the unique steady-state of the system was computed numerically for the Germany-calibrated model. Results are reported in Table 2 below, where $\bar{r} = (1-\tau^k)(r-\delta^p)$ denotes the after-tax net of depreciation real return to private capital.

Table 2: Data averages and long-run solution

	Description	GE Data	Model
c/y	Consumption-to-output ratio	0.590	0.784
i/y	Private investment-to-output ratio	0.210	0.192
g^i/y	Public investment-to-output ratio	0.023	0.023
k^p/y	Private capital-to-output ratio	2.350	2.346
k^g/y	Public capital-to-output ratio	0.630	0.630
s^g/y	Public services-to-output ratio	0.193	0.225
g^t/y	Public transfers-to-output	0.170	0.228
$w^p n^p/y$	Private labor share in output	0.710	0.710
$w^g n^g/y$	Public wage bill share in output	0.130	0.145
rk/y	Private capital share in output	0.290	0.290
w^g/w^p	Public/Private wage ratio	1.200	1.200
w^p	Private sector wage rate	-	1.006
w^g	Public sector wage rate	-	1.207
$\tilde{w}^p$	After-tax private wage rate	-	0.595
$\tilde{w}^g$	After-tax public wage rate	-	0.714
n	Total employment	0.296	0.296
n^p	Private employment level	0.253	0.253
n^g	Public employment level	0.043	0.043
rs	Rent-seeking level	-	0.009
n^g/n^p	Public/private employment ratio	0.170	0.170
$\bar{r}$	After-tax net return to capital	0.036	0.035

The model performs relatively well *vis-a-vis* data. It slightly overestimates average consumption and underestimates the investment shares in output. This mismatch is due to the fact that the model treats government wage bill consumption as a transfer payment, and not as final public consumption, as is the case in the national accounts. This is not an issue here as the main objective of the model is to replicate the stylized facts in the labor markets. However, the model accurately captures the long-run after-tax capital return, where the latter is proxied by the average return on 10-year generic bonds net of CPI inflation. Moreover, the imputed government services is also predicted to make a significant share of output.

Along the labor market dimension, the model was calibrated to match the average time spent working, and the wage and employment ratios in data. Given the focus on the effects of rent-seeking in the public sector, the framework was calibrated to reproduce those stylized facts in the steady state, as this framework will provide an important benchmark for the measures use to quantify the loss from rent-seeking activities on the economy. Next, the ratio of time spent rent-seeking to public employment is a non-trivial figure in steady-state: Using $p \equiv \frac{w^g}{w^p}$ to denote the steady-state wage ratio, one can obtain $\frac{rs}{n^g} = \frac{(p-1)n^g}{n^g} = p - 1 = 0.20$. This value is consistent with the results obtained in Angelopoulos *et al.* (2009), who found that 18 % of time in Germany is spent rent-seeking. Thus, the non-productive rent-seeking in the public sector is also likely to generate a significant waste on aggregate level.[35]

5 Long-Run Cost of Rent-seeking

The model in this paper naturally suggests estimates of rent-seeking time. It also provides estimates that aim to quantify the loss from rent-seeking in terms of output. In turn, given the calibration objective in this paper to match hours in each sector, the values for other EU countries can be easily obtained from data averages after some transformations. Given the calibrated values for different countries in EU, a ranking can be constructed for different countries. Finally, the model-based estimates are compared to empirical measures of institutional quality. One such index is the compound International Country Risk Guide (ICRG), where the values for a selected set of European countries were obtained from Angelopoulos *et al.* (2009).[36] Additionally, a second set of indicators, the Worldwide Governance Indicators (WGI) were extracted from the WDI database. Detailed description of the indices used in this paper is provided in Appendix 11.4. The chosen indices reflected government size, control of corruption, expenditure effectiveness of public funds, government effectiveness, and

[35] Note that given the structure of the problem, and the symmetry imposed, a first-best solution is to set rent-seeking to zero, as this results in a higher welfare. This is investigated Appendix 11.3.1. The other special case, when the government sets equal wages in the two sectors, is presented in Appendix 11.3.2.

[36] For more detailed discussion of this index, interested readers should consult Knack and Keefer (1995)

the efficiency of public administration.[37]

The first measures to be used in the comparison with indices is the steady-state rent-seeking time itself, which was computed as:

$$rs = (p-1)n^g. \tag{5.0.7}$$

Second, rent-seeking time is also expressed in relative terms as a share of public hours[38] to obtain:

$$\frac{rs}{n^g} = \frac{(p-1)n^g}{n^g} = p-1. \tag{5.0.8}$$

Next, several estimates of the loss imposed on the economy, in terms of output, were also calculated. The first such expression is named "wasteful lobbying cost," as it represents the opportunity cost of using time to engage in rent-seeking activities, which is not directly productive (but only indirectly increases the probability of winning the contestable prize, and/or increases the labor income from government work), instead of using the time to produce public services, which have utility-generating effect. The analytical representation of this cost is as follows:

$$\frac{w^g rs}{y} = \frac{pw^p(p-1)n^g}{y} = \frac{(p-1)pn^g}{n^p}\frac{w^p n^p}{y} = \alpha(p-1)p\frac{n^g}{n^p}. \tag{5.0.9}$$

Furthermore, the value of the contestable transfer, the government wage bill, could also be regarded as a wasteful expenditure. This is because public sector wage and employment are determined in a non-competitive market, and public consumption is valued much less than the private consumption good. In other words, the wage paid to government employees is unrelated to productivity of labor in the government services production function. Moreover, the share of government wage bill in output can be also represented as a product of "primitives" as follows:

$$\frac{w^g n^g}{y} = \frac{pw^p n^p}{y}\frac{n^g}{n^p} = p\alpha\frac{n^g}{n^p}. \tag{5.0.10}$$

[37]World Values Studies compute a "general trust" measure, which is also highly correlated with indices of corruption and institutional quality (La Porta *et al.* 1997) and where the measure aims to capture the level of social trust and confidence in the government.

[38]Since rent-seeking occurs only in the public sector, it does not make much sense to express it relative to the total labor supply.

Therefore, the total waste in the economy is the sum of the lobbying cost and the wage bill share. Given that government employees are not entirely wasteful, the combined measure presented below could be regarded as the upward bound of the total loss in the economy from rent-seeking, expressed relative to output. The analytical representation obtained is as follows:

$$\frac{w^g rs}{y} + \frac{w^g n^g}{y} = \alpha(p-1)p\frac{n^g}{n^p} + p\alpha\frac{n^g}{n^p} = \alpha p^2 \frac{n^g}{n^p}. \tag{5.0.11}$$

As seen from above, in the long run, the cost of rent seeking as a share in output depends only on the private labor share in output, the gross public wage mark-up p (*i.e.* the average public/private wage ratio), as well as the average public/private employment ratio. Thus, the model predicts that countries with a high labor share in aggregate production function, a high public employment share in total, and high wages in the public relative to the private sector, will feature the highest losses.

Since the model was constructed to match those dimensions in data, estimates of the measures above were easily computed from OECD (2011) data for a cross-section of EU countries, without explicitly calibrating the model for all the countries, but rather by simply computing the required averages for the corresponding country from OECD data directly. Following Angelopoulos *et al.* (2009, 2011), all measures are presented and ranked in Table 3 on the next page, together with the ICRG index first. A lower rent-seeking cost corresponds to a higher ranking. A higher value of the ICRG index reflects better institutions, and a higher ranking for the country.[39]

Results in Table 3 above show that the cost of lobbying is 2.9 % of GDP for Germany, but can reach 9 % of GDP in Greece, 11.8 % in the Netherlands, and 11.32 % in Spain. The magnitude of these is in line with Magee *et al.* (1989), who show in a static model that 5-15 % of an economy's capital and labor is lost in predatory lobbying. Next, when the share of the public wage bill is added, the costs rise significantly. Germany is still the leader with the lowest loss (17 %), while Greece features the highest figure (31 %), followed immediately by Belgium (30 %), Italy (30 %), and Spain (30 %). These values are also comparable with earlier studies, *e.g.*, using a static framework, Mohammad and Whalley (1994) compute redistributive activity costs to be 25-40 % of Indian GNP, while Ross (1984) calculates it

[39] Overall, countries with larger shares of the government wage bill in output also feature higher tax rates. However, since this paper focuses on the relationship between rent-seeking and the government wage bill, and not on the effect of rent seeking on tax revenues, this stylized fact in data is not discussed.

Table 3: Rent-seeking results in EU member countries

Country	rs	p	n^g/n^p	rs/n^g	$\frac{w^g rs}{y}$	$\frac{w^g n^g}{y}$	$\frac{w^g(n^g+rs)}{y}$	ICRG
Austria	0.016(5)	1.28(5)	0.207(4)	0.28(5)	0.050(5)	0.180(5)	0.23(5)	47.22(5)
Belgium	0.021(7)	1.28(6)	0.285(9)	0.28(6)	0.066(7)	0.231(10)	0.30(7)	47.46(4)
Finland	0.003(2)	1.03(2)	0.353(11)	0.03(2)	0.008(2)	0.226(9)	0.23(6)	48.76(3)
France	0.001(1)	1.01(1)	0.320(10)	0.01(1)	0.002(1)	0.204(7)	0.21(3)	46.62(6)
Germany	0.010(3)	1.20(3)	0.170(2)	0.20(3)	0.029(3)	0.145(1)	0.17(1)	48.92(2)
Greece	0.038(11)	1.41(9)	0.260(7)	0.41(9)	0.090(9)	0.220(8)	0.31(11)	34.36(11)
Ireland	0.015(4)	1.22(4)	0.236(6)	0.22(4)	0.036(4)	0.169(2)	0.21(2)	44.37(7)
Italy	0.025(8)	1.30(7)	0.266(8)	0.30(7)	0.070(8)	0.232(11)	0.30(10)	40.90(8)
Netherlands	0.028(9)	1.69(11)	0.166(1)	0.69(11)	0.118(11)	0.171(3)	0.29(8)	49.40(1)
Portugal	0.020(6)	1.30(8)	0.217(5)	0.30(8)	0.052(6)	0.175(4)	0.23(4)	40.13(10)
Spain	0.034(10)	1.60(10)	0.195(3)	0.60(10)	0.112(10)	0.187(6)	0.30(9)	40.40(9)

as 38 % of Kenyan GNP.[40] Lastly, the size of the rent-seeking cost in terms of output is comparable to the estimates in Angelopoulos *et al.* (2009), who use a DSGE framework with a rent-seeking extraction of the government tax revenue to calculate the cost to be in the range of 0-16 % of GDP across the EU-12 countries.

Next, as documented in Table 4 below, rent-seeking estimates and loss measures are found to be moderately- to highly-correlated to other indices of institutional quality. As expected, rent-seeking time in steady-state is very strongly negatively related to the indices of bureaucratic efficiency, where the values range between -0.50 and -0.73. The public wage premium is also moderately negatively related to institutional quality. This could be an indicator that public sector wages are indeed determined within a political economy environment. Lastly, the public/private employment ratio is essentially uncorrelated with the index values. The two loss measures, the lobbying cost and government wage bill as shares in output, are moderately to strongly negatively correlated with different indicators of bureaucratic efficiency.[41]

[40] These studies, however, focus on bureaucrats whose rent-seeking activity is tariff revenue extraction.

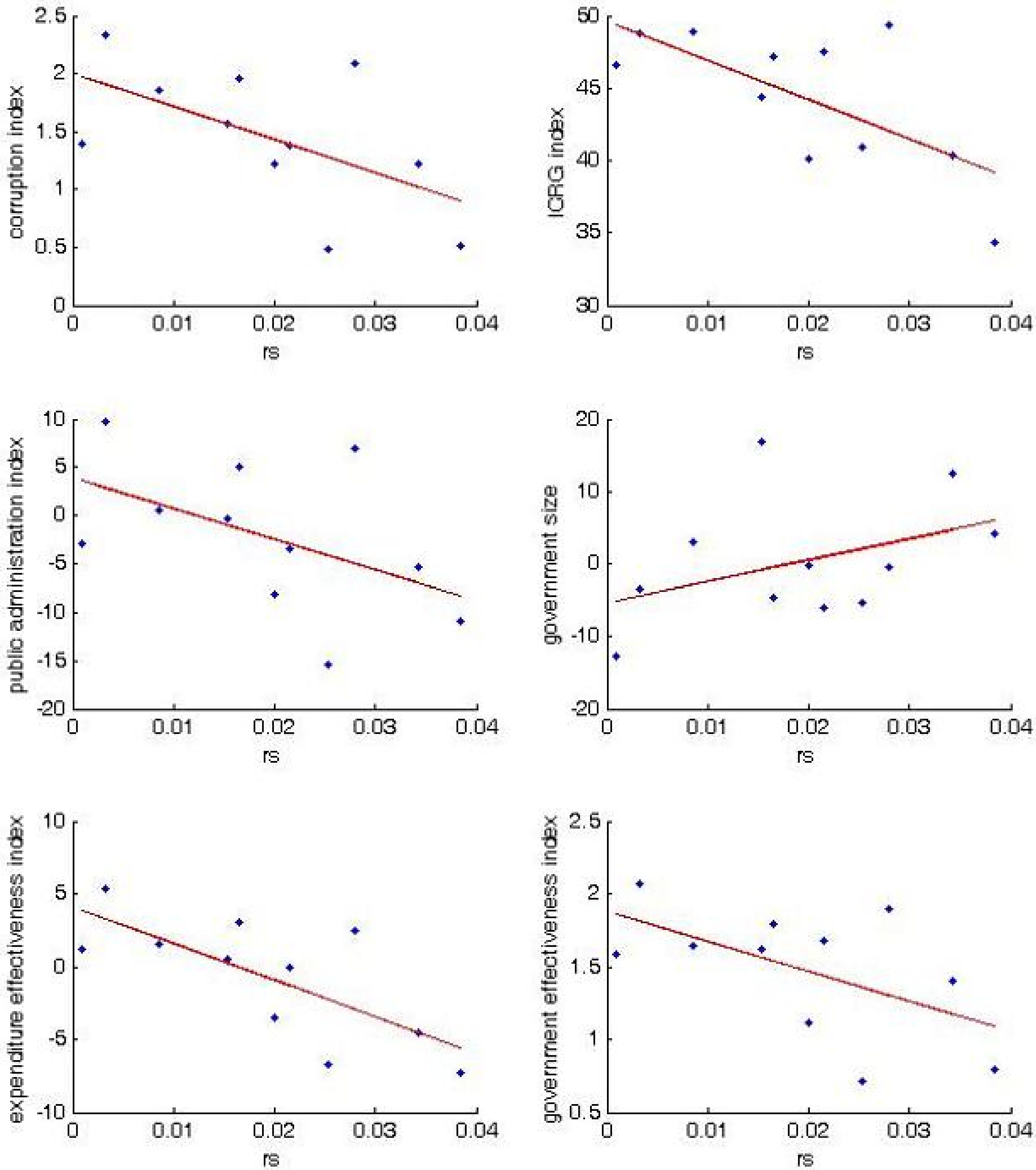
[41] Interestingly, the two loss measures produce the same correlations with the indices. This effect, however, is a almost direct consequence of how the two measures were constructed,as the lobbying cost is proportional to the government wage bill, with the coefficient of proportionality equal to $(p-1)$.

Table 4: Correlation matrix

Index	rs	p	n^g/n^p	rs/n^g	$\frac{w^g rs}{y}$	$\frac{w^g n^g}{y}$	$\frac{w^g(n^g+rs)}{y}$
ICRG index	-0.68	-0.27	0.01	-0.27	-0.39	-0.39	-0.39
control of corruption index	-0.58	-0.16	-0.09	-0.16	-0.32	-0.32	-0.32
public administration index	-0.50	-0.12	-0.02	-0.12	-0.26	-0.26	-0.26
expenditure effectiveness index	-0.73	-0.37	0.12	-0.37	-0.50	-0.50	-0.50
government effectiveness index	-0.57	-0.19	0.04	-0.19	-0.31	-0.31	-0.31

In the next step in the analysis, rent-seeking time was plotted against the indices. As seen from Fig. 1 on the next page, this generated a good fit in a cross-section of EU countries. There is a clear negative relationship with the indices of institutional quality, and a positive one with the size of government. In the next section, the behavior of the model outside of the steady-state is investigated. In particular, the transitional dynamics of the model economy and the responses of the variables in the face of a surprise technological innovation are presented and discussed.

Figure 1: Rent-seeking time *vs.* indices of institutional quality



6 Model solution and impulse responses

Since there is no closed-form general solution for the model in this paper, a typical approach followed in the RBC literature is to log-linearize the stationary DCE equations around the steady state, where $\hat{x}_t = \ln x_t - \ln x$, and then solve the linearized version of the model. The log-linearized system of model equations is derived and summarized in Appendix 11.5-11.6. The linearized DCE system can be represented in the form of first-order linear stochastic difference equations as in King, Plosser and Rebello (1988, 1999):

$$\mathbf{A}E_{t+1}\mathbf{\hat{x}_t} = \mathbf{B}\mathbf{\hat{x}_t} + \mathbf{C}\varepsilon_\mathbf{t}, \tag{6.0.12}$$

where $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ are coefficient matrices, $\varepsilon_\mathbf{t}$ is a matrix of innovations, and $\mathbf{\hat{x}_t}$ is the stacked vector of state (also called 'predetermined') variables, $\mathbf{\hat{s}_t} = \begin{bmatrix} \hat{a}_t & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\mathbf{\hat{z}_t} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{rs}_t & \hat{g}_t^i & \hat{s}_t^g \end{bmatrix}'$. Klein's (2000) generalized eigenvalue decomposition algorithm was used to solve the model. Using the model solution, the impulse response functions (IRFs) were computed to analyze the transitional dynamics of model variables to a surprise innovation to productivity.

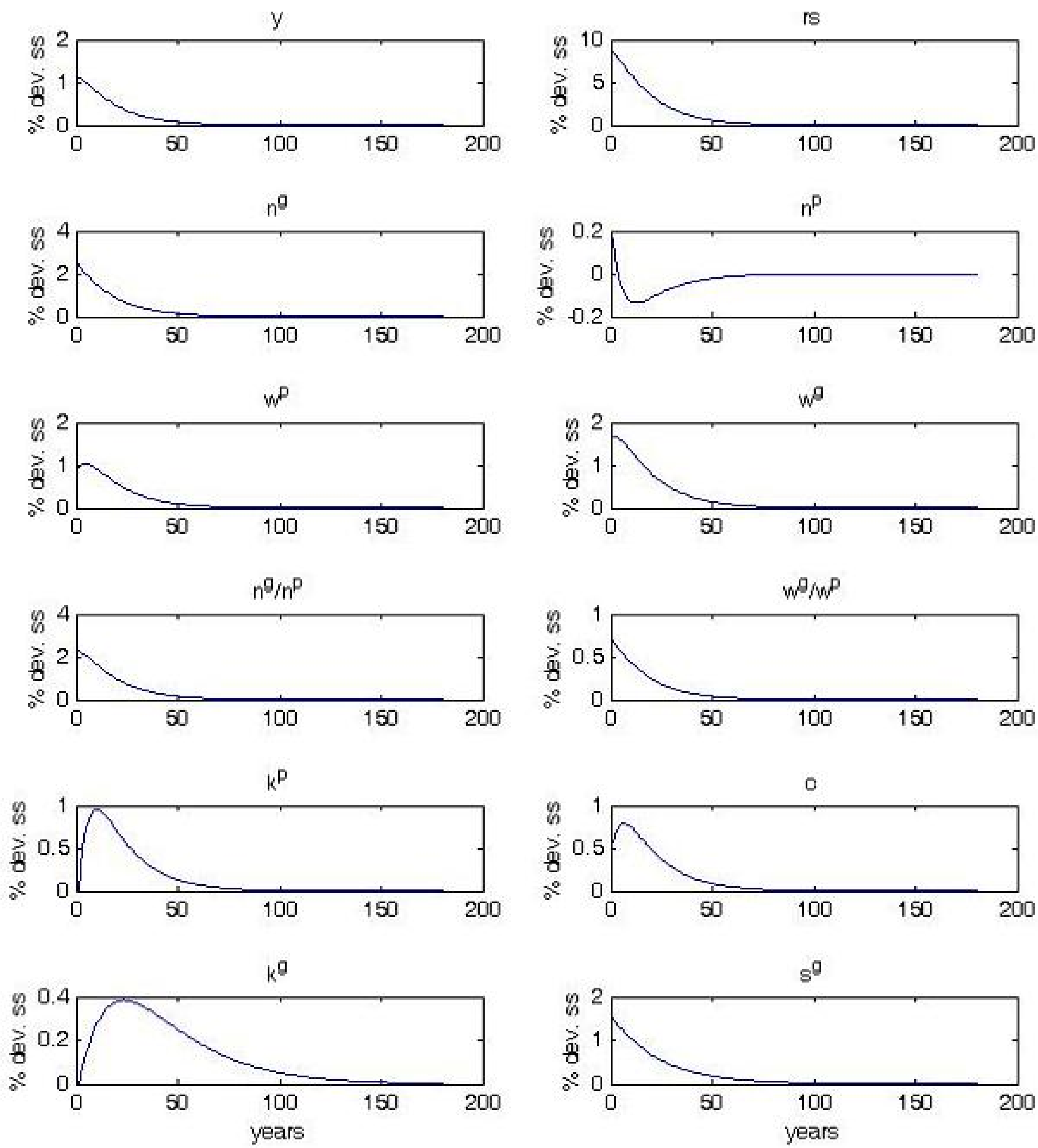
6.1 The Effect of a positive productivity shock

Figure 1 shows the impact of a 1% surprise TFP innovation on the model economy. There are two main channels through which the TFP shock affects the model economy. A higher TFP increases output directly upon impact. This constitutes a positive wealth effect, as there is a higher availability of final goods, which could be used for private and public consumption, as well as for investment. From the rule for the government investment in levels, a higher output translates into higher level of expenditure in that category (not pictured, identical to output response). Next, the positive TFP shock increases both the marginal product of capital and labor, hence the real interest rate (not illustrated) and the private wage rate increase. The household responds to the price signals and supplies more hours in the private sector, as well as increasing investment. This increase is also driven from both the intertemporal consumption smoothing and the intra-temporal substitution between private consumption and leisure. In terms of the labor-leisure trade-off, the income effect ("work more") produced by the increase in the private wage dominates the substitution effect ("work less"). Furthermore, the increase of private hours expands output further, thus both output and government spending categories increase slightly more than the amount of the shock upon impact. Over time, as private physical capital stock accumulates, marginal product of capital falls, which decreases the incentive to invest. In the long-run, all variables return to their old steady-state values. Due to the highly-persistent TFP process, the effect of the shock is still present after 50 periods.

With regard to public sector labor dynamics, however, there is the additional effect of an increase in productivity leading to an increase in income and consumption. Higher income and consumption lead to greater tax revenue. In particular, the growth in government revenue exceeds the increase in the fiscal spending instruments. As a result, the additional funds available are spent on government investment and the wage bill. In turn, the increase in the latter leads to an expansion in both public sector wages and hours. In addition, the model in this paper generates an interesting dynamics in the wage and hours ratio, which is not present in models with stochastic public employment, such as Finn (1998), Cavallo (2005) or Linnemann (2009). The two wage rates, as well as the two types of hours move together, but less than perfectly so, thus making the model consistent with the empirical evidence presented in Lamo, Perez and Schuknecht (2007, 2008). In addition, as in the data, public sector labor variables react much more strongly to positive technological innovations than do their private sector counterparts.

Given that both public wages and hours react strongly and positively to technological improvements, the new variable in the model, rent-seeking time also increases. The intuition behind this result is that during unexpectedly good times, tax revenues are larger than usual, increasing the amount of funds "up for grabs," which are then expropriated by government bureaucrats in the form of excessive salaries to government bureaucrats. Rent-seeking time in the model thus responds very strongly to the dynamics of output, as it is related to the tax base in the labor income generated in the economy.

Figure 2: Impulse Responses to a positive 1% productivity shock in Germany



Overall, a positive innovation to total factor productivity has a positive effect on the allocations and prices in the economy. The novelty is that the endogenous public sector hours model generates an important difference in the composition of household's labor income with the public sector share increasing at a much faster rate than the private sector labor income. Another important observation to make is that the TFP shocks, being the main driving force in the model, induce pro-cyclical behavior in public wages and hours. The shock effects are smaller and variables reach their peak very quickly. This means that the impulse effect dies out relatively fast. However, the transition period can still take up to 100 years. This illustrates the important long-run effects of TFP shocks on the wage- and hours ratios.

However, an important limitation of the exogenous policy analysis performed so far is that both tax rates were taken to be fixed. In addition, government investment share was exogenously set, and public wage rate was a residually-determined instrument that always adjusted accordingly to balance the budget. In effect, all interaction between the two tax rates was precluded by construction - by fixing each to the corresponding average effective rate in data over the chosen period of study. These restrictions will be lifted in the next section, and the optimal fiscal policy framework will be considered in an environment, where the two tax rates, government investment, public employment (and hence also government services), public sector wage rate, and thus effectively the optimal rent-seeking time, are chosen jointly by a benevolent government, whose preferences are perfectly aligned with the household's utility function.

7 The Ramsey problem (optimal fiscal policy under full commitment)

In this section, the government assumes the role of a benevolent planner, who takes into account that the representative household and the firm behave in their own best interest, taking fiscal policy variables as given. The instruments under government's control in this section are labor and capital tax rates, next-period public capital (and hence public investment), public employment and public sector wage rate (and thus the Ramsey planner effectively determines rent-seeking time[42]). Government transfers are fixed at the level from the exogenous policy case. It is assumed that only linear taxes are allowed, and that the government can credibly commit to those. Thus, given the restriction to a set of linear distortionary tax rates, only a second-best outcome is feasible. However, the emphasis on the second-best theory makes the setup more realistic, and thus it can be taken as a better approximation of the environment in which policymakers decide on a particular fiscal policy.

It is important to emphasize that each set of fiscal policy instruments implies a feasible allocation that fully reflects the optimal behavioral responses of the household and firm. Alternatively, each set of fiscal policy instruments can be thought of as generating a different competitive equilibrium allocation, *i.e.* allocations and prices are contingent on the particular values chosen for the fiscal instruments. The difference from the analysis performed so far in the paper, is that in the Ramsey framework, the government chooses all instruments, instead of taking them as being exogenous. At the same time, the government also optimally selects the allocations of agents, as dictated by the dual approach to the Ramsey problem as in Chamley (1986).[43] It it also assumed that the government discounts time at the same rate as the households, and treats each household the same. The constraints which the government takes into account when maximizing households' welfare include the government budget constraints, and the behavioral responses of both the household and the

[42]Note that the rent-seeking is the one from the non-cooperative Nash equilibrium, as the Ramsey planner takes the DCE system of equations as the constraint set for the maximization problem, and thus also takes the symmetry imposed as given.

[43]In contrast, the primal approach all the policy variables and prices are solved as functions of the allocations, thus the government decides only on the optimal allocation.

firm. These are summarized in the symmetric DCE of the exogenous fiscal policy case.[44,45] In other words, in the dual approach of Ramsey problem, which will be utilized in this section, the choice variables for the government are $\{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, w_t^p, w_t^g, r_t\}_{t=0}^{\infty}$ plus the two tax rates $\{\tau_t^l, \tau_t^k\}_{t=0}^{\infty}$.[46] The initial conditions for the state variable $\{A_0, K_0^p, K_0^g\}$, as well as the sequence of government transfers $\{G_t^t\}_{t=0}^{\infty}$ and the process followed by total factor productivity $\{A_t\}_{t=0}^{\infty}$ are taken as given.

Following the procedure in Chamley (1986) and Sargent and Ljungqvist (2004), the Ramsey problem will be transformed and simplified, so that the government chooses the after-tax interest rate $\tilde{r}_t$ and wage rates $\tilde{w}_t^p$ and $\tilde{w}_t^p$ directly, instead of setting tax rates and prices separately, where

$$\tilde{r}_t \equiv (1 - \tau_t^k) r_t, \tag{7.0.1}$$

$$\tilde{w}_t^p \equiv (1 - \tau_t^l) w_t^p, \tag{7.0.2}$$

$$\tilde{w}_t^g \equiv (1 - \tau_t^l) w_t^g. \tag{7.0.3}$$

Thus, the transformed government budget constraint becomes

$$A_t(N_t)^{\alpha} K_t^{1-\alpha} - \tilde{r}_t K_t - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1 - \delta^g) K_t^g + G_t^t. \tag{7.0.4}$$

Once the optimal after-tax returns are solved for, the expression for the before-tax real interest rate and private wage can be obtained from the DCE system. Solving for optimal capital and labor tax rates is then trivial.

The transformed symmetric Ramsey problem (note that rent-seeking is already substituted

[44] The DCE system is summarized in Appendix 11.5.

[45] Stockman (2001) shows that the absence of debt and thus the inability of the government to run surpluses and deficits has no dramatic effect on the optimal policies in the full commitment case.

[46] Note that by choosing next-period public capital, the planner is choosing public investment $\{G_t^i\}_{t=0}^{\infty}$ optimally. Similarly, by choosing public employment and the wage ratio optimally, the government chooses rent-seeking time $\{RS_t\}_{t=0}^{\infty}$ optimally as well.

out in the household's utility function) then becomes:

$$\max_{C_t,N_t^p,N_t^g,K_{t+1}^p,K_{t+1}^g,\tilde{w}_t^p,\tilde{w}_t^g,\tilde{r}_t} E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \psi_1 \frac{C_t^{\sigma_c}}{1-\sigma_c}$$

$$+\psi_2 \frac{\left[1 - N_t^p - N_t^g - N_t^g[(\tilde{w}_t^g/\tilde{w}_t^p) - 1] - \varphi(N_t^g)^2\right]^{1-\sigma_l}}{1-\sigma_l}$$

$$+\frac{\psi_3}{1-\sigma_s}\left[(N_t^g)^\gamma (K_t^g)^{1-\gamma}\right]^{(1-\sigma_s)} \Bigg\} \tag{7.0.5}$$

s.t

$$\frac{1}{C_t^{\sigma_c}} = \beta E_t \frac{1}{C_{t+1}^{\sigma_c}} \left[1 - \delta^p + (1-\tau_{t+1}^k)(1-\alpha)\frac{Y_{t+1}}{K_{t+1}^p}\right] \tag{7.0.6}$$

$$\psi_2 C_t^{\sigma_c} = \psi_1 \left[1 - N_t^p - N_t^g - N_t^g[(\tilde{w}_t^g/\tilde{w}_t^p) - 1] - \varphi(N_t^g)^2\right]^{\sigma_l} (1-\tau_t^l) w_t^p \tag{7.0.7}$$

$$\psi_2 C_t^{\sigma_c}[1 + 2\varphi N_t^g] = \psi_1 \left[1 - N_t^p - N_t^g - N_t^g[(\tilde{w}_t^g/\tilde{w}_t^p) - 1] - \varphi(N_t^g)^2\right]^{\sigma_l} (1-\tau_t^l) w_t^g \tag{7.0.8}$$

$$RS_t = N_t^g \left[(\tilde{w}_t^g/\tilde{w}_t^p) - 1\right] \tag{7.0.9}$$

$$A_t (N_t^p)^\alpha K_t^{(1-\alpha)} = C_t + K_{t+1}^g - (1-\delta^g)K_t^g + K_{t+1}^p - (1-\delta^p)K_t^p \tag{7.0.10}$$

$$A_t (N_t)^\alpha K_t^{1-\alpha} - \tilde{r}_t K_t - \tilde{w}_t^p N_t^p = \tilde{w}_t^g N_t^g + K_{t+1}^g - (1-\delta^g)K_t^g + G_t^t \tag{7.0.11}$$

$$K_{t+1}^p = I_t + (1-\delta^p)K_t^p \tag{7.0.12}$$

$$r_t = (1-\alpha)\frac{Y_t}{K_t^p} \tag{7.0.13}$$

$$w_t^p = \alpha \frac{Y_t}{N_t^p} \tag{7.0.14}$$

$$S_t^g = (N_t^g)^\gamma (K_t^g)^{(1-\gamma)} \tag{7.0.15}$$

$$K_{t+1}^g = G_t^i + (1-\delta^g)K_t^g. \tag{7.0.16}$$

After numerically solving for the unique steady-state, the full characterization of the long-run Ramsey equilibrium is summarized in Table 6 on the next page, where the same values for the parameters from the exogenous policy section (see Table 1) were used.[47]

As in Lucas (1990), Cooley and Hansen (1992) and Ohanian (1997), parameter ξ is introduced to measure the consumption-equivalent long-run welfare gain of moving from the steady-state allocations in the exogenous policy case to the equilibrium values obtained under Ramsey policy. In other words, the value of ξ measures the share of steady-state consumption under the exogenous policy that the household has to be compensated with, in order to achieve the same level of utility as the one under the Ramsey policy. A fraction $\xi > 0$, which is the case reported in Table 6 on the next page, demonstrates that the agent is better-off under Ramsey, while $\xi < 0$ would have implied that the agent is worse-off under Ramsey.

There are several additional important findings in the Ramsey equilibrium that can be seen in Table 6 on the next page. First, as expected, total discounted welfare is higher under the Ramsey regime.[48] Next, private consumption share is lower, while private capital- and investment shares are higher, and thus interest rate is lower. The model generates a zero steady-state optimal capital tax, and a higher labor tax rate. All these results are consistent with the findings in earlier studies, *e.g.* Judd (1985), Chamley (1986), Zhu (1992), Sargent and Ljungqvist (2004) and Kocherlakota (2010). In addition, earlier studies that use the representative-agent setup, *e.g.* Lucas (1990) and Cooley and Hansen (1992), have shown that tax reforms which abolish capital taxation, even at the expense of a higher tax burden on labor, still produce significant welfare gains for the society.

[47] The dynamic version of the model was also solved to check that the model possesses saddle-path stability, *i.e.* that for $t \geq 1$, given initial conditions for K^p, K^g the model has a unique set of sequences for $\{C_t, N_t^p, N_t^g, K_{t+1}^p, K_{t+1}^g, w_t^p, w_t^g, r_t, \tau_t^l, \tau_t^k\}_{t=0}^{\infty}$ that converges to the steady-state.

[48] The positive values of utility are due to the domination of the government services term, given that $\sigma_s < 1$ (but close to unity).

Table 5: Data averages and long-run solution: exogenous vs. optimal policy

	Description	GE Data	Exogenous	Ramsey
c/y	Consumption-to-output ratio	0.590	0.784	0.718
i/y	Private investment-to-output ratio	0.210	0.192	0.229
g^i/y	Public investment-to-output ratio	0.023	0.023	0.053
k^p/y	Private capital-to-output ratio	2.350	2.346	2.793
k^g/y	Public capital-to-output ratio	0.630	0.630	1.442
s^g/y	Public services-to-output ratio	0.193	0.225	0.229
$w^p n^p/y$	Private labor share in output	0.710	0.710	0.710
$w^g n^g/y$	Public wage bill share in output	0.130	0.145	0.102
rk/y	Private capital share in output	0.290	0.290	0.290
w^g/w^p	Public/Private wage ratio	1.200	1.200	1.277
w^p	Private sector wage rate	-	1.006	1.080
w^g	Public sector wage rate	-	1.207	1.379
$\tilde{w}^p$	After-tax private wage rate	-	0.595	0.603
$\tilde{w}^g$	After-tax public wage rate	-	0.714	0.770
n	Total employment	0.296	0.296	0.294
n^p	Private employment level	0.253	0.253	0.264
n^g	Public employment level	0.043	0.043	0.030
rs	Rent-seeking time	-	0.009	0.008
n^g/n^p	Public/private employment ratio	0.170	0.170	0.112
$\bar{r}$	After-tax net return to capital	0.036	0.035	0.022
τ^k	Capital income tax rate	0.160	0.160	0.000
τ^l	Labor income tax rate	0.409	0.409	0.442
U	Total discounted welfare	-	95.02	96.47
ξ	Welfare gain	-	0	0.095

Next, due to the presence of a second labor market, as well as the endogenous public sector hours, sophisticated labor market interactions are generated. In the framework presented in this paper, the labor market structure allows for labor flows between sectors. Furthermore, the government internalizes the public services externality in its choice. Thus, it picks the socially optimal levels of public hours and capital stock to provide the optimal level of the public consumption good. In addition, the planner chooses a different mix between the inputs used in the provision of government services: a higher level of government investment is undertaken, while fewer public hours are employed than the in the DCE solution. As a result, public investment (and thus public capital) share is more than double than that of the exogenous policy case. As a result, the amount of the public good produced relative to output is also slightly higher. In addition, public hours are substituted for private hours, keeping the total virtually unchanged.

In terms of the relative price of labor in the two labor markets, both the after-tax private and public wage rates increase slightly. The higher public/private wage ratio, and thus the higher public wage premium in the optimal policy case overcompensate for the increase in the labor tax. Furthermore, the public/private hours ratio is lower, due to the substitution away from labor in the government sector. In other words, the increase in the public wage premium is driven by budgetary considerations, as the public wage is the residually-determined fiscal instrument that balances the per-period government budget constraint. In addition, the result is consistent with economic logic and scarcity argument: relatively fewer hours are employed in the public sector, thus the steady-state public wage rate is higher. Furthermore, optimal government wage consumption is significantly lower. In turn, optimally chosen rent-seeking level is also lower, as the public employment effect dominated the public wage premium effect.[49,50]

[49]This effect is due to the fact that public goods are congestible, or that $0 < \sigma_s < 1$. For a case when $\sigma_s \geq 1$, a case not supported by empirical data, optimal rent-seeking is higher than in the exogenous case, which is counterintuitive, and thus not considered here. In particular, a value higher than unity for σ_s results in a higher public wage premium and government investment, but dampens the negative effect on public employment.

[50]Alternatively, the rent-seeking chosen in the exogenous policy case can be interpreted as being "third-best," as households ignore the utility effect of public hours working through the government services production function, and thus the DCE choice is inferior to the second-best choice made by the benevolent Ramsey government.

The value-added of the rent-seeking model with endogenous public hours and wages is that it generates predictions about the long-run effects of fiscal policy through labor markets, as well as the level of rent-seeking in the government sector, which is in line with earlier studies. In particular, the benevolent Ramsey planner corrects two inefficiencies in the government sector, the excessive employment and the scarcity of public capital.[51] Moreover, the wage- and employment ratios, the optimal composition of the government wage bill consumption, as well as the distribution of spending across government expenditure categories were all important elements of the analysis on the optimal amount of rent-seeking activity within the public administration. The novel results obtained in this paper were generated from the incorporation of a richer government spending side, are new and interesting for policy makers, as previous research had ignored these important dimensions.

The result that cuts in the wage bill have expansionary effect on the economy is not new to the empirical macroeconomic studies, *e.g.* Algan *et al.* (2002), Alesina (1997), Alesina *et al.* (2001), Alesina *et al.* (2002), and Giavazzi and Pagano (1990). However, the optimal public wage and employment aspects in the analysis are novel in the modern macroeconomic literature, given the predominance of setups with single wage rates, and exogenously-determined public employment. In addition, given the doubling in public investment share, the fixed level of government transfers, and the reduction of the public wage share in output, the loss of capital income tax revenue requires steady-state labor tax to increase by only 3.3% relative to the rate used in the exogenous policy case. The changes in the distribution of spending, as well as the optimal amount of rent-seeking, are new results in both the optimal policy and political economy literatures. As seen in Table 6 on the previous page, if these aspects are ignored, important public finance aspects are missed.

Finally, note that the restriction $\sigma_s < 1$ is a necessary and sufficient condition to generate lower rent-seeking under Ramsey. This value does not deal with rent-seeking theory *per se*, but rather captures an important characteristic of public goods, namely their congestible nature.[52] This plausible assumption can be viewed as a technical condition: In the general

[51] These features of the public sector are first noted in Baumol (1965).

[52] In addition, $1/\sigma_s$ is also the intertemporal elasticity of substitution of government services, which in data is slightly higher than unity, $1/\sigma_s \in [1.03, 1.07]$, hence $\sigma_s \in [0.93, 0.97]$.

case, with CRRA utility for government services, optimal public wages are higher, and optimal public employment is lower. However, only when $\sigma_s < 1$, the compositional effect on the wage bill is such that optimal rent-seeking is lower than the value in the exogenous policy case.

In the next section, the analysis is extended to the behavior of the Ramsey economy outside of the steady-state. The transitional dynamics of model variables, and rent-seeking in particular, under optimal policy setup is also analyzed. In particular, the optimal responses of the fiscal instruments and the other prices and allocations to positive shocks to TFP is presented and discussed.

8 Optimal reaction of fiscal policy instruments to productivity shocks

The optimal policy model is now solved using the first-order linearization procedure from Schmitt-Grohe and Uribe (2004) to study the dynamics of prices and allocations outside the steady-state.[53] The model solution is then used to study transitional behavior in response to a surprise innovation in total factor productivity. Under the optimal policy (Ramsey) regime, endogenous variables would generally react differently to the responses to a positive technology shock under the exogenous fiscal policy case. Fig. 3 summarizes all responses to a 1 % surprise innovation to total factor productivity. To highlight differences across regimes, Fig. 4 plots on the same graph both the IRFs from the exogenous policy case and the optimal ones. The new variables in the system are the five fiscal policy instruments - capital and labor taxes, as well as public investment (hence public capital), public wage rate and public employment. Note that by choosing the two wage rates, and employment in the public sector, the planner determines the optimal amount of rent-seeking time. Therefore, by intervening in the public sector labor market, the benevolent government can influence private sector labor market, and thus affect the course of the economy. In addition, the government can use the available fiscal instruments at its disposal to affect rent-seeking among bureaucrtats, and thus reduce the loss due to these counter-productive activities.

[53] Given the absence of curvature in the model, the second-order approximation to the equilibrium system of equations did not change results significantly.

In period 0, after the realization of the unexpected technology innovation, capital tax stays unchanged.[54] This result is in line with previous findings in the literature, *e.g.* Chari and Kehoe (1994, 1999) who show that in a standard RBC model capital tax rate does not respond to productivity shocks. In other words, the benevolent government would not deviate from the optimal zero steady-state capital tax rate even in the face of uncertain productivity shocks.[55] Next, labor income tax rate increases upon the impact of the positive surprise innovation in TFP and then slowly returns to its old steady-state; the substantial persistence observed is in line with previous studies (Chari and Kehoe, 1994, 1999). However, due to the richer structure of the and endogenously-determined government spending, the magnitude of the response in the labor tax is higher.

Furthermore, given that public spending categories are optimally chosen in this framework, the setup generates considerable more interaction among the variables than does standard RBC model. For example, public investment increases substantially, as the government under the Ramsey regime also chooses public capital and government services optimally as well. Next, as in the exogenous policy case, there will be greater increase in public sector wages than in the private sector wages. The higher volatility in public wages, as discussed in earlier sections, is an artifact of the presence of transaction costs from government work. The change in the public/private wage ratio in turn triggers a reallocation of labor resources from the private to the public sector. Next, the outflow of hours from the firm leads to an increase in the marginal product of capital; hence, the real interest rate increases. However, from the complementarity between private labor and capital in the Cobb-Douglas production function, private capital decreases. Therefore, due to the fall in the levels of the two private inputs, output increases by less than the size of the technology shock.

[54]At first glance the huge percentage deviation from the steady-state for capital tax can be misleading. However, noting that the steady-state capital tax rate under Ramsey is of order $10^{-10} \approx 0$, it follows that the log-deviation from the steady-state is a very very large number indeed, as the denominator is close to zero, although the absolute value of change is minute.

[55]This is also a result of the logarithmic specification of the household's utility of consumption.

Figure 3: Impulse Responses to a positive 1% productivity shock under Ramsey policy

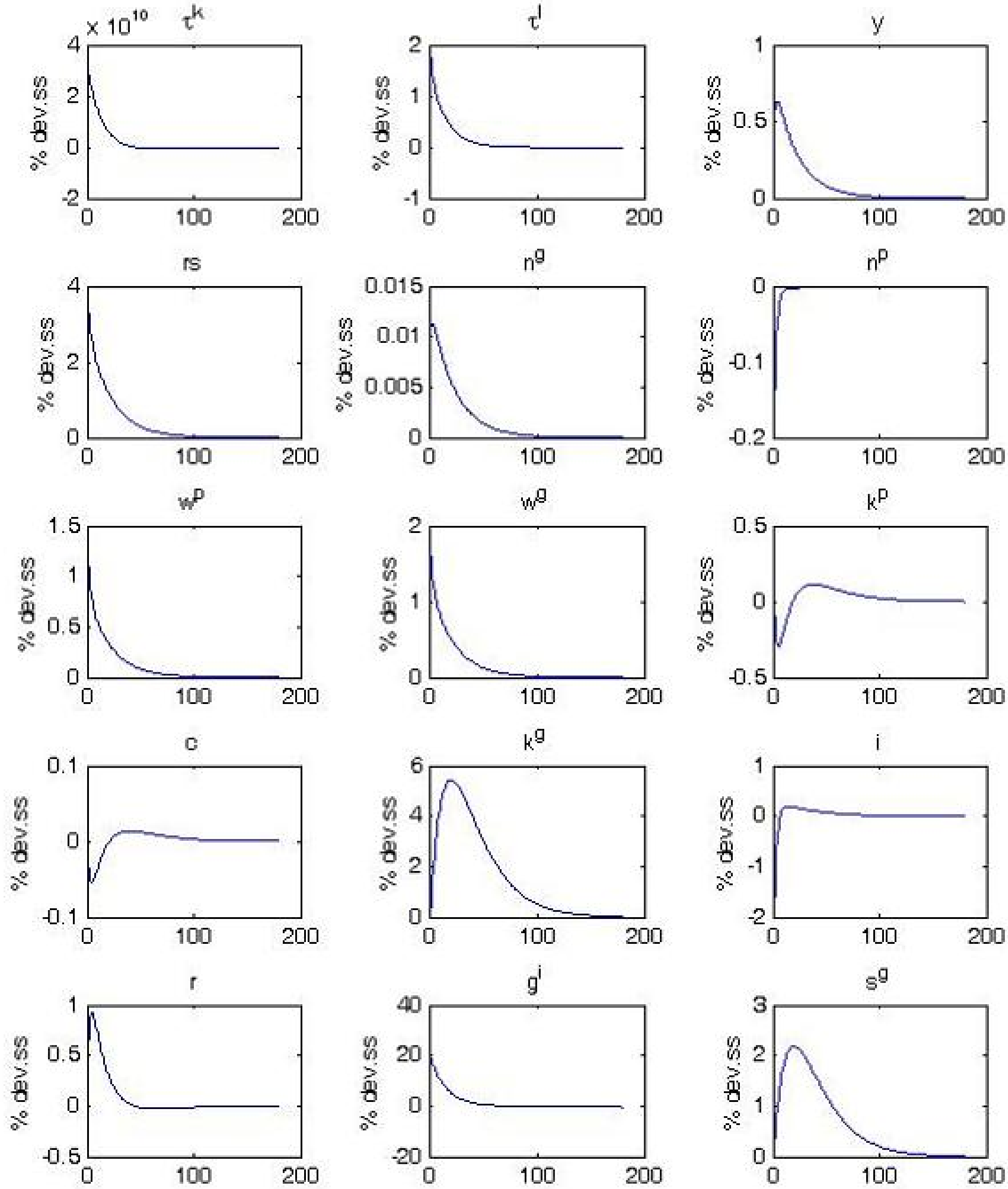
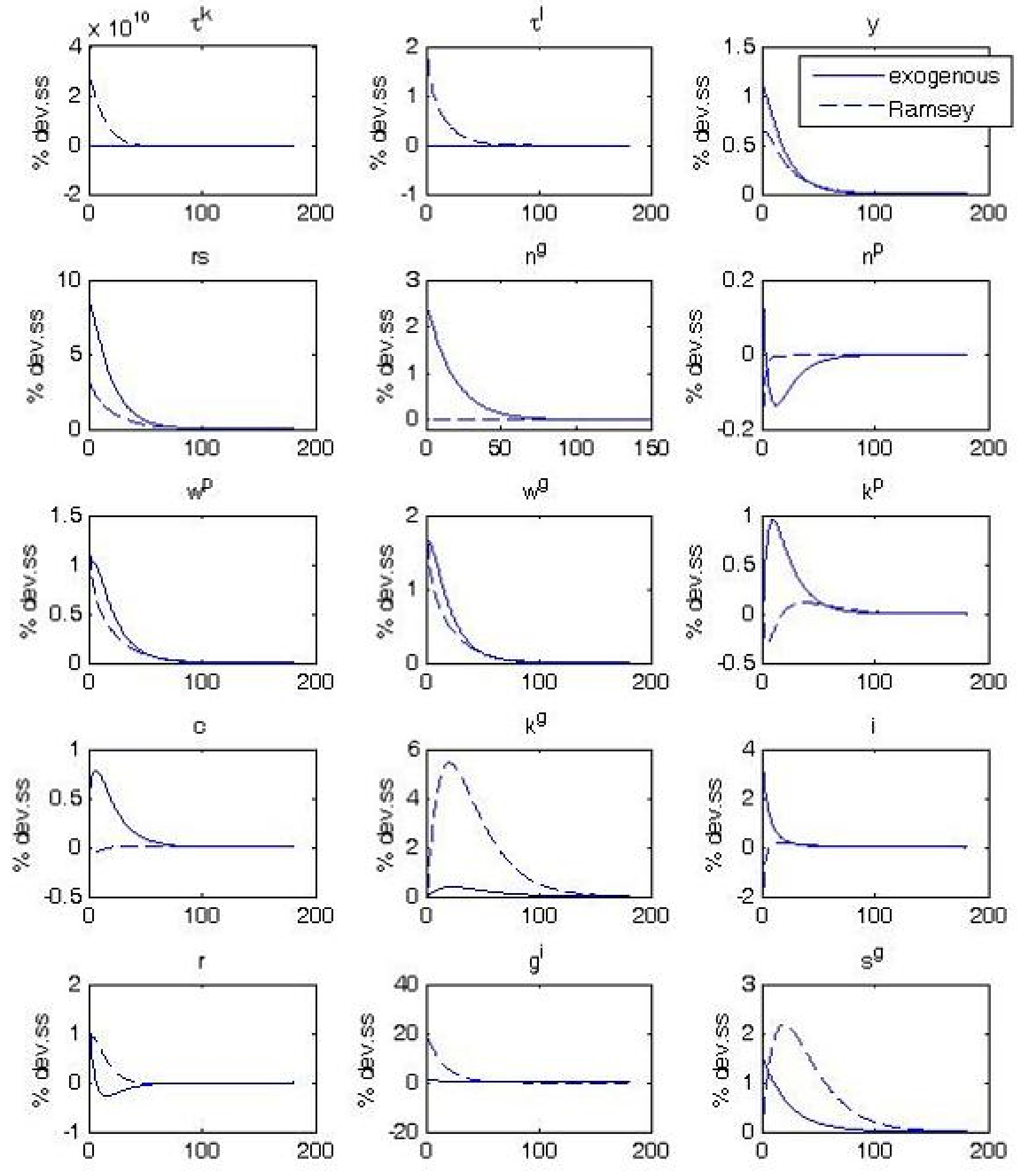


Figure 4: Impulse Responses to a positive 1% productivity shock under exogenous and Ramsey policy



In addition, given the jump in government investment, private consumption and investment fall upon the impact of the shock. Overall, the difference in the dynamics in the main model variables under the Ramsey regime is due to the fact that the government chooses the optimal levels of public hours and capital (and hence also public investment). Over time, attracted by the above-steady-state real interest rate, more private investment is undertaken by the government. In turn, private capital accumulation increases, and the usual hump-shape dynamics appears. Higher capital input increases the marginal productivity of labor, and private labor starts slowly to recover to its old steady-state. As time passes, private consumption response also turns positive, and the shape if its response follows the dynamic path of private capital. Lastly, the benevolent Ramsey planner chooses to suppress the positive response of rent-seeking to technological improvements. This follows directly from the fact that in the optimal policy case, the government chooses optimally both public employment and the public/private wage ratio.

Overall, the positive innovations to TFP have a positive effect on the economy. Additionally, there is a long-lasting internal propagation effect on the economy. This is due to the fact that there are two labor markets featuring different wage rates, and labor can flow between sectors in response to changes in the relative wage. Moreover, there is complementarity between public hours and the public capital, which reinforces the complementarity between private and public consumption in the household's utility. The quantitative effect of public sector labor market, however, completely dominates capital response in terms of the initial dynamics. Nevertheless, in the long-run, the private capital accumulation effects becomes the dominant one, as dictated by the standard neoclassical RBC model.

An interesting result is that a significant portion of the private gains are channeled to the public sector in the form of higher spending on public wages and public investment. Indeed, in this model both categories are productive expenditures, as labor and capital are combined in the provision of the public good. Even though the household suffers a little from the lower private consumption, this negative effect is overcompensated for by the increase in leisure (as there is a greater fall in private hours relative to the increase in private hours), the decrease in rent-seeking, and a higher level of public good consumption. Overall, it takes more than 100 years for all the model variables to return to their old steady states.

9 Limitations of the study and suggestions for future research

This section analyses the key assumptions of the model and proposes some possible extensions to the current framework. First, it was assumed that the government sector wage bill is the pool of public resources, which are up for grabs. In reality a much larger rent component is the tax revenues, an avenue pursued in Angelopoulos *et al.* (2009), and thus not discussed here. In the model in this paper, a positive rent exist because of the higher public wages, and the high public employment. In addition, given the individual decision making in a DCE, a positive amount of rent-seeking time is chosen by each government bureaucrat. Rent-seeking in the model disappears if wages are equalized across sectors, or in case all agents decide to play the cooperative solution to the rent-seeking game.

Second, the model assumed that each individual could work in both the private and the public sector, or equivalently, that workers from different sectors could safely pool together their resources and thus achieve complete insurance against variations in consumption. This was the rationale for the households being modeled as risk-neutral agents. Since this study is intended as the beginning of a long research agenda, the usual starting point is precisely the representative household paradigm. The simple macro model was then used to see how it matches data, and what the forces at work are. As a follow-up to the research in this paper, other specific issues, requiring a much more detailed data match, could be pursued. One such possible extension is to model government officials and private sector workers separately, as their preferences, and their attitude to risk might differ. This modeling choice, however, would complicate the algebra too much with a limited promise of providing analytically tractable and interesting results. Nonetheless, this line of research is left on the agenda for future work.

Third, it was assumed that only public bureaucrats were allowed to engage in rent-seeking, and the only rents available were the funds from the wage bill. In reality, a much larger flow of funds are tax revenues, which can be expropriated for private gains by either public bureaucrats, or business people. A model along these lines, but with private sector individuals, is presented in Angelopoulos *et al.* (2009, 2011). Furthermore, such schemes are usually organized and jointly implemented by public bureaucrats and firm-owners. There will be insiders and outsiders to the scheme, both in the private and in the government sector, or honest and corrupt individuals. However, in the model presented in this paper, all consumers own shares in the firm, and work as bureaucrats at the same time, so there are no outsiders. A very simple attempt in a partial-equilibrium setting is considered in Hillman and Ursprung (2000), but in the current setup, it greatly increases the complexity of the problem, and thus is left for further research.

Fourth, the model did not elaborate on the rent-seeking function. The simplest possible form of the contestable logit function was chosen to abstract away from possible non-linearities. The framework ignores rent-seeking in groups, and possible asymmetries in the distribution of the "prize." Such extensions are possible (see Congleton *et al.* 2008), but make the model cumbersome, and thus were not considered in this paper. The simpler, and much more elegant representation of the auctioning mechanism was preferred instead in order to preserve model elegance and ensure analytical tractability.

10 Conclusions

This paper studied the wasteful effect of bureaucracy on the economy by addressing the link between the rent-seeking behavior of government bureaucrats and the public sector wage bill, which was taken to represent the rent component. In particular, public officials were modeled as individuals competing for a larger share of those public funds. The rent-seeking extraction technology in the government administration was modeled as in Murphy *et al.* (1991) and incorporated in an otherwise standard Real-Business-Cycle (RBC) framework with public sector. The model was calibrated to German data for the period 1970-2007. The main findings are: (i) Due to the existence of a significant public sector wage premium and the high public sector employment, a substantial amount of working time is spent rent-seeking, which in turn leads to significant losses in terms of output; (ii) The measures for the rent-seeking cost obtained from the model for the major EU countries are highly-correlated to indices of bureaucratic inefficiency; (iii) Under the optimal fiscal policy regime, and with congestible public goods, steady-state rent-seeking is significantly smaller relative to the time spent in the exogenous policy case; (iv) In addition to the zero capital tax rate, and the higher labor tax rate, the benevolent government planner invests more in public capital and chooses a higher public wage premium, but sets a much lower public employment, thus achieving an overall decrease in the level of rent-seeking relative to the value obtained in the exogenous case.

References

[1] Acemoglu, D. (2005) "Politics and economics in weak and strong states," *Journal of Monetary Economics* 52, 1199-1226.

[2] Acemoglu, D., D. Ticchi, and A. Vindigni (2007) "Emergence and Persistence of Inefficient States," *Working Papers* 0707, University of Urbino Carlo Bo, Department of Economics.

[3] Acemoglu, D. and T. Verdier (1998) "Property rights, corruption, and the allocation of talent: a general equilibrium approach," *Economic Journal* 108, 1381-1403.

[4] Acemoglu, D., and Verdier, T. (2000). "The Choice Between Market Failures and Corruption," *American Economic Review*, vol. 90(1), pp. 194-211.

[5] Acemoglu, D., Johnson, S. and Robinson, J.A. (2001). "The colonial origins of comparative development: An empirical investigation," *American Economic Review*, 91, 1369-1401.

[6] Acemoglu, D., Johnson, S. and Robinson, J.A. (2002). "Reversal of fortune: Geography and institutions in the making of the modern world income distribution," *Quarterly Journal of Economics*, 118, 1231-1294.

[7] Aghion, P. and J. Tirole (1997) "Formal and Real Authority in Organizations," *Journal of Political Economy*, 105(1), 1-29.

[8] Alesina, A., Ardagna, S., Perotti, R. and Schiantarelli, F. (2002) "Fiscal Policy, Profits and Investment," *American Economic Review*, June 2002, vol. 92, No. 3, pp. 571-589.

[9] Alesina, A., Danninger, S. and Rostagno, M. (2001). "Redistribution Through Public Employment: The Case of Italy," *IMF Staff Papers*, Palgrave Macmillan Journals, vol. 48.

[10] Alesina, A. and Perotti, R. (1997). "Fiscal Adjustment in OECD Countries: Composition and Macroeconomic Effects," *IMF Staff Papers*(June): 210-248.

[11] Algan, Y., Cahuc, P. and Zylberberg, A. (2002). "Public Employment. Does it increase Unemployment?" *Economic Policy*, April 2002, pp. 9-65.

[12] Angelopoulos, K., Philippopoulos, A., and Vassilatos, V.(2009). "The social cost of rent seeking in Europe," *European Journal of Political Economy* 25, pp. 280-299.

[13] Angelopoulos, K., Economides, G., and Vassilatos, V. (2011). "Do institutions matter for economic fluctuations? Weak property rights in a business cycle model for Mexico," *Review of Economic Dynamics*, vol. 14(3), pp. 511-531.

[14] Appelbaum, E. and Katz, E. (1987). "Seeking Rents by Setting Rents: The Political Economy of Rent-Seeking," *The Economic Journal*, vol. 97, pp. 685-99.

[15] Asch, B. and Heaton, P. (2008). "Monopsony and Labor Supply in the Army and Navy," Industrial Relations Working Paper 537, Princeton University.

[16] Aschauer, D. (1989). "Is public Expenditure Productive?," *Journal of Monetary Economics* 23, 177-200.

[17] Baker, G., R. Gibbons, and K. J. Murphy (1999) "Informal Authority in Organizations," *Journal of Law, Economics and Organization*, 15(1), 56-73.

[18] Barro, R. (1997). *Determinants of Economic Growth: A Cross-Country Empirical Study*. The MIT Press: Cambridge, Mass.

[19] Baumol, W. J. (1965). *Welfare Economics and the theory of the state*. LSE: London, UK.

[20] Beaumont, P.B. (1980). "Government as Employer - Setting an Example?" *RIPA Studies*: London, UK.

[21] Becker, G. and C. Muligan (2003) "Deadweight costs and the size of government," *Journal of Law and Economics*, 46, 293-340.

[22] Bellante, D. and Jackson, M. (1979) *Labor Economics: Choice in Labor Markets.* McGraw-Hill Book Company: NY, US.

[23] Bender, K. (1998) "The central government-private sector wage differential," *Journal of Economic Surveys* 12, pp. 177-220.

[24] Bender, K. (1993) "Examining equality between public- and private-sector wage differential," *Economic Inquiry* 41, pp.62-79.

[25] Bendor, J., S. Taylor, and R. van Gaalen. (1985) "Bureaucratic expertise vs. legislative authority: A model of deception and monitoring in budgeting," *American Political Science Review* 79, 1041-1060.

[26] Bendor, J., S. Taylor, and R. van Gaalen. (1987) "Politicians, bureaucrats and asymmetric information," *American Journal of Political Science* 31, 796-828.

[27] Besley, T., and M. Ghatak (2003) "Incentives, Choice, and Accountability in the Provision of Public Services," *Oxford Review of Economic Policy*, 19(2), 235-249.

[28] Bhagwati, J.N. (1982). "Directly unproductive, Profit-seeking (DUP) Activities," *Journal of Political Economy*, University of Chicago Press, vol. 90, pp. 988-1002.

[29] Bhagwati, J.N. and Srinivasan, T.N. (1984). "DUP Activities in Economic Theory," in: *Neoclassical Political Economy. The Analysis of Rent-Seeking and DUP Activities*, ed. by D.C. Colander, Balinger Publishing Company: Cambridge, MA,pp.17-32.

[30] Blanchflower, D. G. and Oswald, A. J. (1996) *The Wage Curve*. The MIT Press: London, UK.

[31] Blundell, R., Pistaferi, L. and Saporta-Eksten, I. (2012) "Consumption smoothing and family labor-supply," Stanford University, USA. Mimeo.

[32] Borcherding, T.E. (ed.) (1977) *Budgets and Bureaucrats: The Sources of Government Growth*. Duke University Press: Durham, North Carolina.

[33] Bos, D. (2001) "Bureaucrats and Public Procurement", *CESifo Working Paper* 515.

[34] Bouakez, H. and Rebei, N. (2007). "Why does private consumption rise after a government spending shock?," *Canadian Journal of Economics*, Canadian Economic Assotiation, vol. 40(3), pp. 954-979.

[35] Box, R. (2004). *Public administration and society: Critical issues in American Governance*. M. E. Sharpe: London, England.

[36] Breton, A. (1974) *The Economic Theory of Representative Government*. The MacMillan Press Ltd: London, UK.

[37] Breton, A. and R. Wintrobe (1975) "The equilibrium size of a budget-maximizing bureau: A note on Niskanen's theory of bureaucracy". *Journal of Political Economy*, 82, 195-207.

[38] Buchanan, J.M. and Tullock, G. (1962). *The Calculus of Consent.* University of Michigan Press: Ann Arbor, Michigan.

[39] Burnside, C., Eichenbaum, M., and Rebello, S. (1993). "Labor Hoarding and the Business Cycle," *Journal of Political Economy*, vol. 101(2), pp. 245-73.

[40] Burnside, C. and Eichenbaum, M. (1996). "Factor Hoarding and the Propagation of Business-Cycle Shocks," *American Economic Review*, vol. 86(5), pp. 1154-74.

[41] Burgess, S., and M. Ratto (2003) "The Role of Incentives in the Public Sector: Issues and Evidence," *Oxford Review of Economic Policy*, 19(2), 285-300.

[42] Burnside, C., Eichenbaum, M., and Fisher, J. (2004). "Fiscal shocks and their consequences," *Journal of Economic Theory* 115, pp. 89-117.

[43] Cameron, D. R. (1978) "The Expansion of the Public Economy: A Comparative Analysis," *American Political Science Review*, 72(4), 1243-1261.

[44] Carey, D. and Rabesona, J. (2002). "Tax ratios on labour and capital income and consumption," *OECD Economic Studies*, No. 35.

[45] Carey, D. and Tchilinguirian, H. (2000). "Average Effective Tax Rates on Capital, Labour and Consumption," *OECD Economics Department Working Papers*, No. 258.

[46] Casas-Pardo, J. and M. Puchades-Navarro. (2001) "A critical comment on Niskanen's model," *Public Choice*, 107, 147-167.

[47] Cavallo, Michele. (2005). "Government employment expenditure and the effects of fiscal policy shocks," *Federal Reserve Bank of San Francisco Working Paper 2005-16.*

[48] Chamley, C. (1986). "Optimal taxation of capital income in general equilibrium," *Econometrica*, vol. 54, pp. 604-622.

[49] Chari, V.V. and Kehoe, P. (1994). "Optimal Fiscal Policy in a Real Business Cycle Model," *Journal of Political Economy*, vol.102, no.4 (Aug.), pp. 617-652.

[50] Chari, V.V. and Kehoe, P. (1999). "Optimal Fiscal and Monetary Policy," in Taylor, J. and Woodford, M. (eds.), *Handbook of Macroeconomics*, vol.1 (Amsterdam: North-Holland).

[51] Chatterjee, S. and Ghosh, S. (2009). "Public Goods, Congestion, and Fiscal Policy: Do Consumption-Based Instruments Matter?," *Economics and Finance Working Papers*, Brunel University, UK.

[52] Cho, J. and Cooley, T.F. (1994). "Employment and Hours over the Business Cycle," *Journal of Economic Dynamics and Control*, 18, pp. 411-432.

[53] Conesa, J. C., Kitao, S. and Krueger, D. (2009). "Taxing Capital? Not a bad idea after all!," *American Economic Review*, 99(1), pp. 25-48.

[54] Congleton, P. (1980). "Competitive Process, Competitive Waste, and Institutions," *Toward a Theory of Rent-Seeking Society*, ed. by J.M. Buchanan, R.D. Tollison, and G. Tullock,Texas A&M University Press: College Station, TX, pp. 153-79.

[55] Congleton, R.D., Hillman, A.L., Konrad, K.A. (2008) "Forty Years of Research on Rent-Seeking: an Overview." In: Congleton *et al.* (eds.) *Forty Years of Research on Rent-Seeking*, Vol. 1: Theory of Rent-Seeking. Springer, Berlin, pp. 1-42.

[56] Corcoran, W.J., and Karels, G.V. (1985). "Rent-seeking behavior in the long-run," *Public Choice* 46: 227-246.

[57] Cooley, T. and Hansen, G. (1992). "Tax distortions in a neoclassical monetary economy," *Journal of Economic Theory* 58, pp. 290-316.

[58] Cooper, R. (1999). *Coordination Games: Complementarities and Macroeconomics*, Cambridge University Press: Cambridge, UK.

[59] Cooper, R. and John, A. (1988). "Coordinating Corrdination Failures in Keynesian Models," *Quarterly Journal of Economics* 103, 441-463.

[60] Cowling, K. and Mueller, D. (1978). "The Social Cost of Monopoly," *European Journal* vol 88, pp. 727-48.

[61] De Fraja, G. (1993) "Productive efficiency in public and private firms," *Journal of Public Economics*, 50, 15-30.

[62] Dewatripont, M., I. Jewitt, and J. Tirole (1999) "The economics of career concerns, Part II: Application to missions and accountability of government agencies," *Review of Economic Studies*, 66, 199-217.

[63] Dixit, A. (2002). "Incentives and Organizations in the Public Sector: An Interpretative Review," *Journal of Human Resources*, 37(4), 696-727.

[64] Dixit, A. (2006) "Predatory States and Failing States: An Agency Perspective," *Working Papers* 71, Princeton University, Department of Economics, Center for Economic Policy Studies.

[65] Dixit, A. (2010) "Democracy, Autocracy and Bureaucracy," *Journal of Globalization and Development*, 1(1).

[66] Dodlova, M. (2013) "Political Accountability and Real Authority of Government Bureaucrats", Paris School of Economics, Mimeo.

[67] Drazen, Allan. (2000) *Political Economy in Macroeconomics*. Princeton University Press: Princeton, NJ.

[68] Economides, G., Park, H., and Philippopoulos, A. (2007). "Optimal Protection of Property Rights in a General Equilibrium Model of Growth," *Scandinavian Journal of Economics* 109(1), 153-175.

[69] Economides, G., Philippopoulos, A. and Malley, J. (2004). "Electoral Uncertainty, Fiscal Policies and Growth: Theory And Evidence From Germany, The UK And The US," *Royal Economic Society Annual Conference 39, Royal Economic Society.*

[70] Economides, G., Philippopoulos, A. and Vassilatos, V. (2011). "Public Providers, versus Private Providers, of Public Goods: A General Equilibrium Study of the Role of the State." *CES Ifo Working Paper*, No. 3487.

[71] Egorov, G., and K. Sonin (2009) "Dictators and Their Viziers: Endogenizing the Loyalty- Competence Trade-Off," *Journal of European Economic Association*, forthcoming.

[72] Elliott, K. A. (1997). "Corruption as an International Policy Problem: Overview and Recommendations," *Corruption and the Global Economy*, ed. by K.A. Elliott, Institute for International Economics, pp. 175-223.

[73] European Commission (2009). *Public Finances in the EMU*, pp. 71-82.

[74] EU Klems Database (2009). Available on-line at: www.euklems.net

[75] Falsch, T. (2008). "The elasticity of labor supply at establishment level," Industrial Relations Working Paper 536, Princeton University.

[76] Falvey, R. E. (1996). "Are services income-elastic? Some new evidence," *Review of Income and Wealth*, Series 42, No. 3.

[77] Finn, M. (1994). "Is all government capital productive?," Federal Reserve Bank of Richmond *Economics Quarterly*, Vol. 79/4.

[78] Finn, Mary G. (1998). "Cyclical Effects of Government's Employment and Goods Purchases." *International Economic Review*, Vol. 39, No. 3 (Aug.), pp. 635-657.

[79] Fisman, R. and Gatti, R. (2002). "Decentralization and corruption: evidence across countries," *Journal of Public Economics*, 83, 325-345.

[80] Francois, P. (2000) "Public service motivation as an argument for government provision," *Journal of Public Economics*, 78, 777-95.

[81] Fry, B. (1989) *Mastering Public Administration: From Max Weber to Dwight Waldo.* Chatham House Publishers: Chatham, NJ.

[82] Gali, J. (2008). *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press: Princeton, USA.

[83] Ghez, Gilbert, and Becker, Gary S. (1975). *The Allocation of Time and Goods over the Life Cycle*, National Bureau of Economic Research, Inc.

[84] Giavazzi, F. and Pagano, M. (1990). "Can Severe Fiscal Contractions be Expansionary? Tales of Two Small European Countries," in Blanchard, O. and Fischer, S. (eds.), *NBER Macroeconomics Annual*, MIT Press.

[85] Gibson, B. (1980). "Estimating Demand Elasticities from Survey Data," *The American Economic Review*, vol. 70, no. 5, pp. 1069-1076.

[86] Goel, R. K. and Nelson, M. A. (1998) "Corruption and Government size: A disaggregated analysis," *Public Choice*, 97, 107-120.

[87] Gomes, P. (2012). "Optimal Public Sector Wages." Carlos III University Working papers, Mimeo.

[88] Gortner, H. (1977). *Administration in the public sector.* John Wiley&Sons: London, UK.

[89] Gregory, P. and V. Lazarev (2003) "Commissars and Cars: A Case Study in the Political Economy of Dictatorship," *Journal of Comparative Economics*, 31(1), 1-19.

[90] Grossman, H. (2002). "Make Us a King: Anarchy, Predation and the State," *European Journal of Political Economy* 18, 31-46.

[91] Guriev, S. (2004) "Red tape and Corruption," *Journal of Development Economics*, 73(2), pp. 489-504.

[92] Hall, R. and Jones, C. (1999). "Why Do Some Countries Produce So Much More Output Per Worker than Hours," *Quarterly Journal of Economics*, 114, 83-116.

[93] Hayashi, F. and Prescott, E. C. (2007). "The 1990s in Japan: A Lost Decade," in: *Great Depressions of the Twentieth Century*, T. J. Kehoe and E. C. Prescott (eds.). Federal Reserve Bank of Minneapolis: Minnesota, USA.

[94] Hillman, A. L., and Riley, J.G. (1989). "Politically contestable rents and transfers," *Economics and Politics*, 1(1), 17-39.

[95] Hillman, A.L., and Ursprung, H.W. (2000). "Political culture and economic decline." *European Journal of Political Economy* 16, 189-213. In: Congleton *et al.* (eds.) *Forty Years of Research on Rent-Seeking*, Vol. 2: Applications: Rent-Seeking in Practice. Springer, Berlin, pp. 219-243.

[96] Hjerppe, R., Kiander, J., Viren, M. (2006). "Are Government Expenditure Productive? Measuring the effects on private sector production," *Working paper*, February 2006. Government Institute for Economic Research: Helsinki, Finland.

[97] Horn, M. J. (1995) *The Political Economy of Public Administration*, Cambridge University Press.

[98] Hughes, O.E. (1994) *Public Management and Administration: an Introduction*. St. Martin's Press: NY, USA.

[99] Judd, Kenneth. (1985) "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, vol. 28, pp. 59-83.

[100] Kaplan, G. (2012) "Inequality and the Lifecycle," Mimeo, University of Pennsylvania.

[101] Khan, M. and Sundaran, J.K. (2000) *Rents, rent-seeking and economic development: Theory and evidence in Asia*, Cambridge University Press: Cambridge, UK.

[102] King, Robert G., Plosser, Charles I. and Sergio T. Rebello. (1988) "Production, growth, and business cycles I: The basic neoclassical model," *Journal of Monetary Economics*, 21(2), pp. 195-232.

[103] King R. and S. Rebelo (1999). "Resuscitating real business cycles." In: *Handbook of Macroeconomics*, vol. 1B, eds J. Taylor and M. Woodford, North Holland.

[104] Klein, Paul. (2000) "Using the generalized Schur form to solve a multivariate linear rational expectations model," *Journal of Economic Dynamics and Control* 24, pp. 1405-1423.

[105] Klein, Paul. (2000) MATLAB Code for "Using the generalized Schur form to solve a multivariate linear rational expectations model," *Journal of Economic Dynamics and Control* 24, pp. 1405-1423.

[106] Klitgaard, R. (1991). *Controlling corruption.* University of California Press Ltd: London, UK.

[107] Knack, S. and Kiefer, P. (1995). "Institutions and economic performance: cross-country tests using alternative institutional measures," *Economics and Politics* 7, 207-227.

[108] Krueger, A. (1974). "The Political Economy of the Rent Seeking Society," *American Economic Review*, vol. 64, pp. 291-303.

[109] Kydland, F. (1995) "Business cycles and aggregate labor market fluctuations." In: T. Cooley (ed.), *Frontiers of Business Cycle Research*, Princeton University Press: Princeton, NJ.

[110] Kydland, Finn E. and Prescott, E.C. (1982). "Time to build and aggregate fluctuations," *Econometrica* 50(6), Nov 1982, pp. 1345-1369.

[111] Laband, D. and Sophocleus, J. (1988). "The Social Cost of rent-seeking: First Estimates," *Public Choice* 58, 269-276.

[112] Laffont, J. J. (2000) *Incentives and Political Economy*, Oxford University Press, New York.

[113] Laffont, J. J., and D. Martimort (2002) *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, Princeton.

[114] Lambsdorff, J. G. (2007). *Institutional Economics of Corruption and Reform*. Cambridge University Press: Cambridge, UK.

[115] Lamo, A., J. Perez and L. Schuknecht (2007). "The cyclicality of consumption, wages and employment in the public sector in the Euro Area," *ECB Working paper Series* 757, May.

[116] Lamo, A., J. Perez and L. Schuknecht (2008). "Public and Private Sector Wages: Co-movement and Causality," *ECB WDN Working Paper Series* 963, November.

[117] LaPalombara, J. (1994). "Structural and Institutional Aspects of Corruption," *Social Research*, vol. 61, pp. 325-50.

[118] La Porta, R., Lopez-de-Silane, F., Shleifer, A.,Vishny, R.W. (1997). "Trust in Large Organizations," *American Economic Review*, vol. 87(2), pp. 333-38.

[119] Leeper, E.M., Walker, T.B., and Yang, S. (2009). "Government Investment and Fiscal Stimulus in the Short and Long Run," NBER Working Papers 15153, National Bureau of Economic Research, Inc.

[120] Linnemann, Ludger (2009) "Macroeconomic Effects of Shocks to Public Employment," *Journal of Macroeconomics* 31(2), 252-267.

[121] Little, W., Posada-Carbo, E. (eds) (1996). *Political Corruption in Europe and Latin America*. MacMillan Press Ltd: London, UK.

[122] Lucas, R. (1990). "Supply-side economics: an analytical review," *Oxford Economic Papers*, 42(2), 293-316.

[123] Makris, M. (2006) "Political authority, expertise and government bureaucracies," *Public Choice* 127, pp. 275-292.

[124] Martinez-Mongay, C. (2000). "ECFIN?s effective tax rates: Properties and comparisons with other tax indicators"', Economic Paper no. 146, Brussels: European Commission, Directorate for Economic and Financial Affairs.

[125] Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic Theory.* Oxford University Press: Oxford, UK.

[126] Mauro, P. (2004). "The persistence of corruption and slow economic growth," *IMF Staff Papers*, 51, 1-18.

[127] McDaniel, Cara (2007) "Average tax rates on consumption, investment, labor and capital in the OECD 1950-2003." Available on-line at: http://www.caramcdaniel.com/researchpapers. Retrieved on Aug.7, 2010.

[128] McDaniel, Cara (2009) *Updated tax series OECD 1950-2009.* Available on-line at at: http://www.caramcdaniel.com/researchpapers. Retrieved on Aug.7, 2010.

[129] McKenzie, R. B., and Tullock, G. (1978) *Modern Political Economy: An Introduction to Economics.* McGraw Hill, US.

[130] Mendoza, E.G., Razin A., and Tesar, L. (1994) "Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption." *Journal of Monetary Economics* 34 (December): 297-323.

[131] Merz, M. (1995). "Search in the labor market and the real business cycle," *Journal of Monetary Economics*, vol. 36(2), pp. 269-300.

[132] Mieczkowski, B. and Zinam, O. (1984). *Bureaucracy, Ideology, Technology: Quality of Life; East and West.* Association for the Study of the nationalities (USSR and East Europe) Inc: Charleston, US.

[133] Migue, J. L., and G. Belanger (1974) "Toward a general theory of managerial discretion," *Public Choice*, 17, 27-43.

[134] Miller, G. J., and T. M. Moe (1983) "Bureaucrats, legislators, and the size of government," *American Political Science Review* 77, 297-322.

[135] Mohtadi, H. and Roe, T. (1998). "Growth, lobbying and public goods," *European Journal of Political Economy*, 14, 453-473.

[136] Mohtadi, H. and Roe, T. (2003). "Democracy, rent-seeking, public spending and growth," *Journal of Public Economics*, 87, 445-466.

[137] Moe, T. M. (1997) "The positive theory of public bureaucracy." In: D. C. Mueller (ed.), *Perspectives on Public Choice*, Cambridge University Press, pp. 455-480.

[138] Mueller, D. (2003). *Public Choice III.* Cambridge University Press: Cambridge, UK.

[139] Murphy, K., Shleifer, A. and Vishny, R. (1991). "The Allocation of Talent: Implications for Growth," *Quarterly Journal of Economics* 106, 503-530.

[140] Murphy, K., Shleifer, A. and Vishny, R. (1993). "Why is Rent Seeking So Costly to Growth?," *American Economic Review, Papers and Proceedings* 83, 409-414.

[141] Nash, J. (1953). "Two-Person Cooperative Games," *Econometrica*, vol 21(1), p. 128-140.

[142] Niskanen, W. N. (1971). *Bureaucracy and Representative Government.* Aldine-Alberton Press: Chicago, US.

[143] Nitzan, S. (1994). "Modeling rent-seeking contests," *European Journal of Political Economy* 10: 41-60.

[144] OECD (1982). *Working in the public sector*. Paris, France.

[145] OECD Statistical Database (2011). Available on-line at: stats.oecd.org. Accessed on Aug.7, 2010.

[146] Ohanian, L. (1997). "The macroeconomic effects of war finance in the United States: World War II and the Korean war," *American Economic Review* vol. 87(1), pp. 23-40.

[147] Park, H., Philippopoulos, A., Vassilatos, V. (2005) "Choosing the size of the public sector under rent seeking from state coffers." *European Journal of Political Economy* 21, 830-850.

[148] Parkinson, C. Northcote (1957) *Parkinson's Law, or the Pursuit of Progress.* John Murray: London, Great Britain.

[149] Perez-Castrillo, J. D. and Verdier, T. (1992). "A General Analysis of Rent-Seeking Games," *Public Choice* vol 73(3), pp. 335-50.

[150] Persson, T. and Tabellini, G. (2000). *Political Economics: Explaining Economic Policy.* The MIT Press: London, UK.

[151] Persson, T. and Tabellini, G. (2003). *The Economic Effect of Constitutions*. The MIT Press: Cambridge, Mass.

[152] Perz-Castrillo, J.D. and Verdier, T. (1992). "A general analysis of rent-seeking games," *Public Choice* 73: 335-350.

[153] Peter, L. J. and Hull, R. (1969). *The Peter Principle: "Every Employee Tends to Rise to His Level of Incompetence."* Pan Books Ltd: London, UK.

[154] Posner, R.A. (1974). "'Theories of Economic Regulation," *Bell Journal of Economics and Management*, Vol. 5(2), pp. 335-58.

[155] Posner, R.A. (1975). "The Social Cost of Monopoly and Regulation," *Journal of Political Economy*, vol. 83, pp. 807-27.

[156] Prendergast, C. (2002) "The limits of bureaucratic efficiency," *Journal of Political Economy*, forthcoming.

[157] Riordan, R. (1999) *Brazil: Politics in a patrimonial society*. Praeger.

[158] Rodrick, D. (1999). "Where did all the growth go? External shocks, social conflict and growth collapses, *Journal of Economic Growth*, 4, 385-412.

[159] Rose-Akerman, S. (1978) *Corruption: A Study in Political Economy*. Academic Press: London, UK.

[160] Rose-Akerman, S. (1999) *Corruption and Government: Causes, Consequences, and Reform*. Cambridge University Press: Cambridge, UK.

[161] Reisman, D. (1990). *The Political Economy of James Buchanan.* MacMillan Press: London, UK.

[162] Sargent, T. and Ljungqvist, L. (2004) *Recursive Macroeconomic Theory 2nd ed*. The MIT Press: Mass., USA.

[163] Schmitt-Grohe, S. and Uribe, M. (1997). "Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability," *Journal of Political Economy*, vol. 105, No. 5 (October), pp. 976-1000.

[164] Schmitt-Grohe, S. and Uribe, M. (2004). "Solving Dynamic General Equilibrium Models Using a Second Order Approximation to the policy function," *Journal of Economic Dynamics and Control* 28, 755-775.

[165] Staaf, R. (1977). "The Public School System in Transition: Do Teachers make a Difference?" In: *Budgets and Bureaucrats: The Sources of Government Growth*, Thomas E. Borcherding (ed.). Duke University Press: Durham, North Carolina.

[166] Stockman, D. (2001). "Balance-Budget Rules: Welfare Loss and Optimal Policies," *Review of Economic Dynamics* vol. 4, pp. 438-59.

[167] Tarschys, D. (1975) "The Growth of Public Expenditures: Nine Models of Explanation," *Scandinavian Political Studies* 10: 9-31.

[168] Tinbergen, J. (1985). *Production, Income and Welfare: The Search for an Optimal Social Order*. Harvester Press Group: Salisbury, UK.

[169] Tirole, J. (1996). "A Theory of Collective Reputations (With Applications to the Persistence of Corruption and the Firm Quality)," *Review of Economic Studies*, 63, 1-22.

[170] Tirole, J. (1994) "The Internal Organization of Government," *Oxford Economic Papers*, 46(1), pp. 1-29.

[171] Tullock, G. (1965). *The Politics of Bureaucracy*. Public Affairs Press: Washington, D.C.

[172] Tullock, G. (1974) "Dynamic Hypothesis on Bureaucracy," *Public Choice* 19(1), pp. 127-131.

[173] Tullock, G. (1977). "What is to be done?" In: *Budgets and Bureaucrats: The Sources of Government Growth*, Thomas E. Borcherding (ed.). Duke University Press: Durham, North Carolina.

[174] Tullock, G. (1980). "Efficient rent-seeking," in J.M. Buchanan, R.D. Tollison and G. Tullock, *Toward a theory of the rent-seeking society*. Texas A&M University Press.

[175] von Mises, L. (1944). *Bureaucracy.* Yale University Press: New Haven, USA.

[176] Wallenius, J. and Prescott, E. C. (2011) "Aggregate Labor Supply," *Federal Reserve Bank of Minneapolis Research Department Staff Report* 457.

[177] Warwick, D. (1975). *A Theory of Public Bureaucracy: Politics, Personality and Organization in the State Department.* Harvard University Press: Cambridge, Mass.

[178] Weber, M. (1983) *Capitalism, Bureaucracy and Religion* George Allen and Urwin: London, UK.

[179] Wildavsky, A. (1974) *The Politics of the Budgetary Process*, 2nd ed. Boston: Little, Brown.

[180] Wilson, J. Q. (1989) *Bureaucracy: What Government Agencies Do and Why They Do It*, Basic Books.

[181] Wintrobe, R. (1997) "Modern Bureaucratic theory." In: D. C. Mueller (ed.), *Perspectives on Public Choice*, Cambridge University Press, pp. 429-454.

[182] World Bank, Worldwide Governance Indicators (WGI). Available on-line at: http://info.worldbank.org/governance/wgi/index.asp. Accessed February 3, 2012.

[183] Zhu, X. (1992). "Optimal Fiscal Policy in a Stochastic Growth Model," *Journal of Monetary Economics*, vol. 58, pp. 250-289.

11 Technical Appendix

11.1 Optimality Conditions

11.1.1 Firm's Problem

The profit function is maximized when the derivatives of that function are set to zero. Therefore, the optimal amount of capital - holding the level of technology A_t and labor input N_t^p constant - is determined by setting the derivative of the profit function with respect to K_t^p equal to zero. This derivative is

$$(1-\alpha)A_t(N_t^p)^\alpha K_t^{-\alpha} - r_t \quad = \quad 0 \tag{11.1.1}$$

where $(1-\alpha)A_t(N_t^p)^\alpha K_t^{-\alpha} - r_t$ is the marginal product of capital because it expresses how much output will increase if capital increases by one unit. The economic interpretation of this First-Order Condition (FOC) is that in equilibrium, firms will rent capital up to the point where the benefit of renting an additional unit of capital, which is the marginal product of capital, equals the rental cost, i.e the interest rate.

$$r_t \quad = \quad (1-\alpha)A_t(N_t^p)^\alpha K_t^{-\alpha} \tag{11.1.2}$$

Now, multiply by K_t^p and rearrange terms. This gives the following relationship:

$$K_t(1-\alpha)A_t(N_t^p)^\alpha K_t^{-\alpha} = r_t K_t \quad \text{or} \quad (1-\alpha)Y_t = r_t K_t \tag{11.1.3}$$

because

$$K_t(1-\alpha)A_t(N_t^p)^\alpha K_t^{-\alpha} = (1-\alpha)A_t(N_t^p)^\alpha K_t^{1-\alpha} \quad = \quad (1-\alpha)Y_t \tag{11.1.4}$$

To derive firms' optimal labor demand, set the derivative of the profit function with respect to the labor input equal to zero, holding technology and capital constant:

$$\alpha A_t(N_t^p)^{\alpha-1}K_t^{1-\alpha} - w_t^p = 0 \text{ or } w_t^p = \alpha A_t(N_t^p)^{\alpha-1}K_t^{1-\alpha} \tag{11.1.5}$$

In equilibrium, firms will hire labor up to the point where the benefit of hiring an additional hour of labor services, which is the marginal product of labor, equals the cost, i.e the hourly wage rate.

Now multiply both sides of the equation by N_t^p and rearrange terms to yield

$$N_t^p \alpha A_t(N_t^p)^{\alpha-1}K_t^{1-\alpha} = w_t^p N_t^p \quad \text{or} \quad \alpha Y_t = w_t^p N_t^p \tag{11.1.6}$$

Next, it will be shown that in equilibrium, economic profits are zero. Using the results above one can obtain

$$\Pi_t \quad = \quad Y_t - r_t K_t - w_t^p N_t^p = Y_t - (1-\theta)Y_t - \theta Y_t = 0 \tag{11.1.7}$$

Indeed, in equilibrium, economic profits are zero.

11.1.2 Consumer problem

Set up the Lagrangian

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \Bigg\{ \psi_1 \frac{C_t^{1-\sigma_c}}{1-\sigma_c} + \psi_2 \frac{\left[1 - N_t^p - \frac{RS_t^h}{RS_t} N_t^{gh} - RS_t^h - \gamma (N_t^{gh})^2\right]^{1-\sigma_l}}{1-\sigma_l} + \psi_3 \frac{(S_t^g)^{1-\sigma_s}}{1-\sigma_s}$$
$$+\Lambda_t \left[(1-\tau_t^l)(w_t^p N_t^p + w_t^g \frac{RS_t^h}{RS_t} N_t^{gh}) + (1-\tau_t^k) r_t K_t^p + G_t^t - C_t - K_{t+1}^p + (1-\delta) K_t^p \right] \Bigg\} \tag{11.1.8}$$

This is a concave programming problem, so the FOCs, together with the additional, boundary (”transversality”) conditions for private physical capital and government bonds are both necessary and sufficient for an optimum.

To derive the FOCs, first take the derivative of the Lagrangian w.r.t C_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{C_t} = 0$. That will result in the following expression

$$\beta^t \left\{ \frac{\psi_1}{(C_t)^{\sigma_c}} - \Lambda_t \right\} = 0 \quad \text{or} \quad \frac{\psi_1}{(C_t)^{\sigma_c}} = \Lambda_t \tag{11.1.9}$$

This optimality condition equates marginal utility of consumption to the marginal utility of wealth.

Now take the derivative of the Lagrangian w.r.t K_{t+1}^p (holding all other variables unchanged) and set it to 0, *i.e.* $L_{K_{t+1}^p} = 0$. That will result in the following expression

$$\beta^t \left\{ -\Lambda_t + E_t \Lambda_{t+1} \left[(1-\tau_{t+1}^k) r_{t+1} + (1-\delta^p) \right] \right\} \quad = \quad 0 \tag{11.1.10}$$

Cancel the β^t term to obtain

$$-\Lambda_t + \beta E_t \Lambda_{t+1} \left[(1-\tau_{t+1}^k) r_{t+1} + \tau^k \delta^p + (1-\delta^p) \right] = 0 \tag{11.1.11}$$

Move Λ_t to the right so that

$$\beta E_t \Lambda_{t+1} \Bigg[(1-\tau^k_{t+1}) r_{t+1} + (1-\delta^p) \Bigg] = \Lambda_t \tag{11.1.12}$$

Using the expression for the real interest rate shifted one period forward one can obtain

$$r_{t+1} = (1-\alpha) \frac{Y_{t+1}}{K^p_{t+1}}$$

$$\beta E_t \Lambda_{t+1} \Bigg[(1-\tau^k_{t+1})(1-\alpha) \frac{Y_{t+1}}{K^p_{t+1}} + (1-\delta^p) \Bigg] = \Lambda_t \tag{11.1.13}$$

This is the Euler equation, which determines how consumption is allocated across periods.

Take now the derivative of the Lagrangian w.r.t N^p_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N^p_t} = 0$. That will result in the following expression

$$\beta^t \Bigg\{ - \frac{\psi_2}{[1 - N^p_t - \frac{RS^h_t}{RS_t} N^g_t - RS^h_t - \gamma (N^{gh}_t)^2]^{\sigma_l}} + \Lambda_t (1-\tau^l_t) w^p_t \Bigg\} \;\; = \;\; 0 \tag{11.1.14}$$

Cancel the β^t term to obtain

$$- \frac{\psi_2}{[1 - N^p_t - \frac{RS^h_t}{RS_t} N^{gh}_t - RS^h_t - \varphi (N^{gh}_t)^2]^{\sigma_l}} + \Lambda_t (1-\tau^l_t) w^p_t = 0 \tag{11.1.15}$$

Rearranging, one can obtain

$$\frac{\psi_2}{[1 - N^p_t - \frac{RS^h_t}{RS_t} N^{gh}_t - RS^h_t - \varphi (N^{gh}_t)^2]^{\sigma_l}} = \Lambda_t (1-\tau^l_t) w^p_t \tag{11.1.16}$$

Plug in the expression for w^h_t, that is,

$$w^p_t = \alpha \frac{Y_t}{N^p_t} \tag{11.1.17}$$

into the equation above. Rearranging, one can obtain

$$\frac{\psi_2}{[1 - N^p_t - \frac{RS^h_t}{RS_t} N^{gh}_t - RS^h_t - \varphi (N^{gh}_t)^2]^{\sigma_l}} + \Lambda_t (1-\tau^l_t) \theta \frac{Y_t}{N^p_t} \tag{11.1.18}$$

Take now the derivative of the Lagrangian w.r.t N^g_t (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N^g_t} = 0$. That will result in the following expression

$$\beta^t \Bigg\{ - \frac{\psi_2 (1 + 2\varphi N^{gh}_t)}{[1 - N^p_t - \frac{RS^h_t}{RS_t} N^{gh}_t - RS^h_t - \varphi (N^{gh}_t)^2]^{\sigma_l}} + \Lambda_t (1-\tau^l_t) w^g_t \frac{RS^h_t}{RS_t} \Bigg\} \;\; = \;\; 0 \tag{11.1.19}$$

Cancel the β^t term to obtain

$$-\frac{\psi_2(1+2\varphi N_t^{gh})}{[1-N_t^p-\frac{RS_t^h}{RS_t}N_t^{gh}-RS_t^h-\varphi(N_t^{gh})^2]^{\sigma_l}}+\Lambda_t(1-\tau_t^l)w_t^g\frac{RS_t^h}{RS_t}=0 \tag{11.1.20}$$

Rearranging, one can obtain

$$\frac{\psi_2(1+2\varphi N_t^{gh})}{[1-N_t^p-\frac{RS_t^h}{RS_t}N_t^{gh}-RS_t^h-\varphi(N_t^{gh})^2]^{\sigma_l}}=\Lambda_t(1-\tau_t^l)w_t^g\frac{RS_t^h}{RS_t} \tag{11.1.21}$$

Take now the derivative of the Lagrangian w.r.t RS_t^h (holding all other variables unchanged) and set it to 0, *i.e.* $\mathcal{L}_{N_t^{gh}}=0$. That will result in the following expression

$$\beta^t\left\{-\frac{\psi_2(1+\frac{N_t^{gh}}{RS_t})}{[1-N_t^p-\frac{RS_t^h}{RS_t}N_t^{gh}-RS_t^h-\varphi(N_t^{gh})^2]^{\sigma_l}}+\Lambda_t(1-\tau_t^l)w_t^g\frac{N_t^{gh}}{RS_t}\right\} = 0 \tag{11.1.22}$$

Cancel the β^t term to obtain

$$-\frac{\psi_2(1+\frac{N_t^{gh}}{RS_t})}{[1-N_t^p-\frac{RS_t^h}{RS_t}N_t^{gh}-RS_t^h-\varphi(N_t^{gh})^2]^{\sigma_l}}+\Lambda_t(1-\tau_t^l)w_t^g\frac{N_t^{gh}}{RS_t}=0 \tag{11.1.23}$$

Rearranging, one can obtain

$$\frac{\psi_2(1+\frac{N_t^{gh}}{RS_t})}{[1-N_t^p-\frac{RS_t^h}{RS_t}N_t^{gh}-RS_t^h-\varphi(N_t^{gh})^2]^{\sigma_l}}=\Lambda_t(1-\tau_t^l)w_t^g\frac{N_t^{gh}}{RS_t} \tag{11.1.24}$$

Lastly, a transversality condition need to be imposed to prevent Ponzi schemes, i.e borrowing bigger and bigger amounts every subsequent period and never paying it off.

$$\lim_{t\to\infty}\beta^t\Lambda_t K_{t+1}^p = 0 \tag{11.1.25}$$

11.2 Per capita stationary symmetric DCE

Since the model in stationary and per capita terms by definition, there is no need to transform the optimality conditions, but only impose symmetry *i.e.* $Z_t^h = Z_t = z_t$. Thus, the system of equations that describes the DCE is as follows:

$$y_t = a_t(k_t^p)^{1-\alpha}(n_t^p)^{\alpha} \tag{11.2.1}$$

$$y_t = c_t + k_{t+1}^p - (1-\delta^p)k_t^p + g_t^i \tag{11.2.2}$$

$$\frac{\psi_1}{(c_t)^{\sigma_c}} = \lambda_t \tag{11.2.3}$$

$$\lambda_t = \beta E_t \lambda_{t+1}\left[1-\delta^p + (1-\tau^k)(1-\alpha)\frac{y_{t+1}}{k_{t+1}^p}\right] \tag{11.2.4}$$

$$\frac{\psi_2}{[1-n_t^p - n_t^g - rs_t - \varphi(n_t^g)^2]^{\sigma_l}} = \frac{\psi_1}{(c_t)^{\sigma_c}}(1-\tau^l)\alpha\frac{y_t}{n_t^p} \tag{11.2.5}$$

$$\frac{\psi_2}{[1-n_t^p - n_t^g - rs_t - \varphi(n_t^g)^2]^{\sigma_l}}[1+2\gamma n_t^g] = \frac{\psi_1}{(c_t)^{\sigma_c}}(1-\tau^l)w_t^g \tag{11.2.6}$$

$$k_{t+1}^p = i_t + (1-\delta^p)k_t^p \tag{11.2.7}$$

$$r_t = (1-\alpha)\frac{y_t}{k_t^p} \tag{11.2.8}$$

$$w_t^p = \alpha\frac{y_t}{n_t^p} \tag{11.2.9}$$

$$g_t^t + g_t^i + w_t^g n_t^g = \tau^k r_t k_t^p + \tau^l\left[w_t^p n_t^p + w_t^g n_t^g\right]. \tag{11.2.10}$$

$$k_{t+1}^g = g_t^i + (1-\delta^g)k_t^g \tag{11.2.11}$$

$$g_t^i = g_t^{iy} y_t \tag{11.2.12}$$

$$rs_t = n_t^g\left[\frac{w_t^g}{w_t^p} - 1\right] \tag{11.2.13}$$

$$s_t^g = (n_t^g)^{\gamma}(k_t^g)^{1-\gamma} \tag{11.2.14}$$

Therefore, the DCE is summarized by Equations (13.1.1)-(13.1.14) in the paths of the following 14 variables $\{y_t, c_t, i_t, k_t^p, k_t^g, g_t^i, rs_t, n_t^p, n_t^g, s_t^g, w_t^p, w_t^g, r_t, \lambda_t\}_{t=0}^{\infty}$ given the process followed by total factor productivity $\{a_t\}_{t=0}^{\infty}$, the values of government investment shares g^{iy}, the fixed level of government transfers g^t and capital and labor tax rates $\{\tau^k, \tau^l\}$.

11.3 Steady-state

In steady-state, there is no uncertainty and variables do not change. Thus, eliminate all stochasticity and time subscripts to obtain

$$y = a(k^p)^{1-\alpha}(n^p)^\alpha \tag{11.3.1}$$

$$y = c + \delta^p k^p + g^i \tag{11.3.2}$$

$$\frac{\psi_1}{(c)^{\sigma_c}} = \lambda \tag{11.3.3}$$

$$1 = \beta\left[1 - \delta^p + (1-\tau^k)(1-\alpha)\frac{y}{k^p}\right] \tag{11.3.4}$$

$$\frac{\psi_2}{[1-n^p-n^g-rs-\varphi(n^g)^2]^{\sigma_l}} = \frac{\psi_1}{(c)^{\sigma_c}}(1-\tau^l)\theta\frac{y}{n^p} \tag{11.3.5}$$

$$\frac{\psi_2}{[1-n^p-n^g-rs-\varphi(n^g)^2]^{\sigma_l}}[1+2\varphi n^g] = \frac{\psi_1}{(c)^{\sigma_c}}(1-\tau^l)w^g \tag{11.3.6}$$

$$i^p = \delta^p k^p \tag{11.3.7}$$

$$r = (1-\alpha)\frac{y}{k^p} \tag{11.3.8}$$

$$w^p = \alpha\frac{y}{n^p} \tag{11.3.9}$$

$$g^t + g^i + w^g n^g = \tau^k r k^p + \tau^l\left[w^p n^p + w^g n^g\right]. \tag{11.3.10}$$

$$g^i = \delta^g k^g \tag{11.3.11}$$

$$g^i = g^{iy} y \tag{11.3.12}$$

$$rs = n^g\left[\frac{w^g}{w^p} - 1\right] \tag{11.3.13}$$

$$s^g = (n^g)^\gamma (k^g)^{1-\gamma} \tag{11.3.14}$$

11.3.1 Cooperative solution

This subsection compares the non-cooperative Nash equilibrium solution to a case where all households coordinate on the first-best solution for rent-seeking. In particular, in every period, before choosing allocations, all households meet and discuss the possibility of engaging in rent-seeking activities. Throughout the discussion, they realize that if rent-seeking, they bargain again themselves, and thus agree not to rent-seek, as that would be jointly socially optimal outcome. Indeed, such a pre-communication results in sizable welfare gains, as shown in Table 7 on the next page. Aside from some slight differences in hours, there are no significant differences between the allocations in the tho equilibria. Note also that the transaction cost parameter in the cooperative solution is slightly lower.

Table 6: Data averages and long-run solution: non-cooperative vs. cooperative equilibrium

	Description	GE Data	Non-cooperative	Cooperative
c/y	Consumption-to-output ratio	0.590	0.784	0.784
i/y	Private investment-to-output ratio	0.210	0.192	0.192
g^i/y	Public investment-to-output ratio	0.023	0.023	0.023
k^p/y	Private capital-to-output ratio	2.350	2.346	2.346
k^g/y	Public capital-to-output ratio	0.630	0.630	0.630
s^g/y	Public services-to-output ratio	0.193	0.225	0.225
g^t/y	0.170	0.228	0.228	
$w^p n^p/y$	Private labor share in output	0.710	0.710	0.710
$w^g n^g/y$	Public wage bill share in output	0.130	0.145	0.145
rk/y	Private capital share in output	0.290	0.290	0.290
w^g/w^p	Public/Private wage ratio	1.200	1.200	1.200
w^p	Private sector wage rate	-	1.006	1.006
w^g	Public sector wage rate	-	1.207	1.207
$\tilde{w}^p$	After-tax private wage rate	-	0.595	0.595
$\tilde{w}^g$	After-tax public wage rate	-	0.714	0.714
n	Total employment	0.296	0.296	0.300
n^p	Private employment level	0.253	0.253	0.256
n^g	Public employment level	0.043	0.043	0.044
rs	Rent-seeking time	-	0.009	0.000
n^g/n^p	Public/private employment ratio	0.170	0.170	0.170
$\bar{r}$	After-tax net return to capital	0.036	0.035	0.035
φ	Transaction cost parameter	-	2.318	2.298
U	Total discounted welfare	-	95.02	95.42
ξ	Welfare gain	-	0	0.025

11.3.2 Wage equalization across sectors

The second special case is when the government sets equal wages across sectors, and thus eliminates rent-seeking. This is still a non-cooperative Nash solution, in which transaction costs from working in the government are no longer present, as $\gamma = 0$ (there is no public wage premium). Again, as seen in Table 8, with the exception of hours, there are no other differences in steady-state allocations across equilibria. Still, the welfare gains of setting wages equal across sectors brings substantial gains in the exogenous policy case.

Table 7: Data averages and long-run solution: non-cooperative vs. cooperative equilibrium

	Description	GE Data	Non-cooperative	Cooperative
c/y	Consumption-to-output ratio	0.590	0.784	0.784
i/y	Private investment-to-output ratio	0.210	0.192	0.192
g^i/y	Public investment-to-output ratio	0.023	0.023	0.023
k^p/y	Private capital-to-output ratio	2.350	2.346	2.346
k^g/y	Public capital-to-output ratio	0.630	0.630	0.630
s^g/y	Public services-to-output ratio	0.193	0.225	0.225
g^t/y	0.170	0.228	0.242	
$w^p n^p/y$	Private labor share in output	0.710	0.710	0.710
$w^g n^g/y$	Public wage bill share in output	0.130	0.145	0.121
rk/y	Private capital share in output	0.290	0.290	0.290
w^g/w^p	Public/Private wage ratio	1.200	1.200	1.000
w^p	Private sector wage rate	-	1.006	1.006
w^g	Public sector wage rate	-	1.207	1.006
$\tilde{w}^p$	After-tax private wage rate	-	0.595	0.595
$\tilde{w}^g$	After-tax public wage rate	-	0.714	0.595
n	Total employment	0.296	0.296	0.301
n^p	Private employment level	0.253	0.253	0.257
n^g	Public employment level	0.043	0.043	0.044
rs	Rent-seeking time	-	0.009	0.000
n^g/n^p	Public/private employment ratio	0.170	0.170	0.170
$\bar{r}$	After-tax net return to capital	0.036	0.035	0.035
φ	Transaction cost parameter	-	2.318	0.000
U	Total discounted welfare	-	95.02	95.82
ξ	Welfare gain	-	0	0.052

11.4 Appendix: Data description

ICRG: The ICRG index is based on annual values for indicators of the quality of governance, corruption and violation of property rights over the period 1982-1997. It has been constructed by Stephen Knack and the IRIS Center, University of Maryland, from monthly ICRG data provided by Political Risk Services. This index takes values within the range 0-50, with higher values indicating better institutional quality. The reported numbers are the averages over 1982-1997, and are taken from Angelopoulos *et al.* (2009). Knack and Keefer (1995) explain in detail the how the index was constructed.

Control of corruption index: Control of Corruption index measures perceptions of the extent to which public power is exercised for private gain, including both petty and grand forms of corruption, as well as "capture" of the state by elites and private interests. The index is obtained from the World Bank Worldwide Governance Indicators (WGI). The units in which the control of corruption is measured follow a normal distribution with a mean of zero and a standard deviation of one in each period. This implies that virtually all scores lie between -2.5 and 2.5, with higher scores corresponding to better outcomes. The values are averaged over the 1996-2009 period.

Quality of Public Finances (Size of government, Public Administration, Government expenditure effectiveness): The Quality of Public Finances (QPF) composite index is composed of sub-indices which distinguish between five dimensions through which public finances can impact long-term economic growth drawing on the theoretical and empirical literature on the links between public finances and long-term economic growth. The dimensions considered in this paper are: (i) the size of government (dimension QPF 1),(ii) the composition, efficiency and effectiveness of expenditure (dimension QPF 3), and (iv) the structure and efficiency of the public administration (dimension QPF 4) QPF is defined as all fiscal policy arrangements and operations that support achieving macroeconomic goals of fiscal policy, in particular long-term economic growth. Scores range from -30 to +30 with an EU-15 average of 0. Assuming a normal distribution a value between -10 and -30 is deemed as very poor, between -4 and -10 as poor, between -4 and +4 as average, between +4 and +10 as good, and between +10 and +30 as very good. Scores were calculated using linear unweighted average. More information on the index is contained in the European Commission (2009) report.

Government effectiveness index: The scores lie between -2.5 and 2.5 (distributed according to a standard normal distribution), with higher scores corresponding to better outcomes. In "Government Effectiveness" category the quality of public service provision, the quality of the bureaucracy, the competence of civil servants, the independence of the civil service from political pressures, and the credibility of the governments commitment to policies is combined to one index. The main focus of this index is on "inputs" required for the government to be able to produce and implement good policies and deliver public goods. The values are averaged over the 1996-2009 period.

11.5 Log-linearization

11.5.1 Log-linearized production function

$$y_t = a_t (k_t^p)^{1-\alpha}(n_t^p)^\alpha \tag{11.5.1}$$

Take natural logs from both sides to obtain

$$\ln y_t \quad = \quad \ln a_t + (1-\alpha)\ln k_t^p + \alpha \ln n_t^p \tag{11.5.2}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln y_t}{dt} \quad = \quad \frac{d\ln a_t}{dt} + (1-\alpha)\frac{d\ln k_t^p}{dt} + \alpha\frac{d\ln n_t^p}{dt} \tag{11.5.3}$$

$$\frac{1}{y}\frac{dy_t}{dt} \quad = \quad \frac{1}{a}\frac{da_t}{dt} + \frac{1-\alpha}{k^p}\frac{dk_t}{dt} + \frac{\alpha}{n^p}\frac{dn_t^p}{dt} \tag{11.5.4}$$

Pass to log-deviations to obtain

$$0 \quad = \quad -\hat{y}_t + (1-\alpha)\hat{k}_t^p + \hat{a}_t + \alpha\hat{n}_t^p \tag{11.5.5}$$

11.5.2 Linearized market clearing

$$c_t + k_{t+1}^p - (1-\delta)k_t^p + g_t^i \quad = \quad y_t \tag{11.5.6}$$

Take logs from both sides to obtain

$$\ln[c_t + k_{t+1}^p - (1-\delta)k_t^p + g_t^i] \quad = \quad \ln(y_t) \tag{11.5.7}$$

Totally differentiate with respect to time

$$\frac{d\ln[c_t + k_{t+1}^p - (1-\delta)k_t^p + g_t^i]}{dt} \quad = \quad d\ln(y_t) \tag{11.5.8}$$

$$\frac{1}{c+\delta k^p + g^c}\left[\frac{dc_t}{dt}\frac{c}{c} + \frac{dk_{t+1}^p}{dt}\frac{k^p}{k^p} - (1-\delta^p)\frac{dk_t^p}{dt}\frac{k^p}{k^p} + \frac{dg_t^i}{dt}\frac{g^i}{g^i}\right] = \frac{dy_t}{dt}\frac{1}{y} \tag{11.5.9}$$

Define $\hat{z} = \frac{dz_t}{dt}\frac{1}{z}$. Thus passing to log-deviations

$$\frac{1}{y}\left[\hat{c}_t c + \hat{k}_{t+1}^p k^p - (1-\delta^p)\hat{k}_t^p k^p + g^i \hat{g}_t^i\right] \quad = \quad \hat{y}_t \tag{11.5.10}$$

$$\hat{c}_t c + \hat{k}_{t+1}^p k^p - (1-\delta^p)\hat{k}_t^p k^p + g^i\hat{g}_t^i \quad = \quad y\hat{y}_t \tag{11.5.11}$$

$$k^p\hat{k}_{t+1}^p \quad = \quad y\hat{y}_t - c\hat{c}_t + (1-\delta)k^p\hat{k}_t^p - g^i\hat{g}_t^i \tag{11.5.12}$$

11.5.3 Linearized FOC consumption

$$\frac{\psi_1}{c_t} = \lambda_t \tag{11.5.13}$$

Take natural logarithms from both sides to obtain

$$\ln \psi_1 - \ln(c_t) = \ln \lambda_t \tag{11.5.14}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln \psi_1}{dt} - \frac{d \ln c_t}{dt} = \frac{d \ln \lambda_t}{dt} \tag{11.5.15}$$

or

$$-\frac{d \ln c_t}{dt} = \frac{d \ln \lambda_t}{dt} \tag{11.5.16}$$

$$-\frac{dc_t}{dt}\frac{1}{c} = \frac{d\lambda_t}{dt}\frac{1}{\lambda} \tag{11.5.17}$$

Pass to log-deviations to obtain

$$-\hat{c}_t = \hat{\lambda}_t \tag{11.5.18}$$

11.5.4 Linearized no-arbitrage condition for capital

$$\lambda_t \quad = \quad \beta E_t \lambda_{t+1}[(1 - \tau^k_{t+1})r_{t+1} + (1 - \delta^p)] \tag{11.5.19}$$

Substitute out r_{t+1} on the right hand side of the equation to obtain

$$\lambda_t \quad = \quad \beta E_t[\lambda_{t+1}((1 - \tau^k_{t+1})(1-\alpha)\frac{y_{t+1}}{k^p_{t+1}} + 1 - \delta^p)] \tag{11.5.20}$$

Take natural logs from both sides of the equation to obtain

$$\ln \lambda_t \quad = \quad \ln E_t[\lambda_{t+1}((1 - \tau^k_{t+1})(1-\alpha)\frac{y_{t+1}}{k^p_{t+1}} + 1 - \delta^p)] \tag{11.5.21}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln \lambda_t}{dt} = \frac{d \ln E_t[\lambda_{t+1}((1 - \tau^k_{t+1})(1-\alpha)\frac{y_{t+1}}{k^p_{t+1}} + (1 - \delta^p)]}{dt} \tag{11.5.22}$$

$$\frac{1}{\lambda}\frac{d\lambda_t}{dt} = E_t\left\{\frac{1}{\lambda((1-\tau^k_{t+1})(1-\alpha)\frac{y}{k^p}+1-\delta^p}\left[((1-\tau^k)(1-\alpha)\frac{y}{k^p}+1-\delta^p)\frac{d\lambda_{t+1}}{dt}\frac{\lambda}{\lambda}\right.\right.$$
$$\left.\left.+\frac{\lambda(1-\tau^k)(1-\alpha)}{k^p}\frac{dy_{t+1}}{dt}\frac{y}{y}-\left[\frac{\lambda(1-\tau^k)(1-\alpha)y}{(k^p)^2}\right]\frac{dk^p_{t+1}}{dt}\frac{k^p}{k^p}\right]\right\} \quad (11.5.23)$$

Pass to log-deviations to obtain

$$\hat{\lambda}_t = E_t\left\{\hat{\lambda}_{t+1}+\left[\frac{(1-\tau^k)(1-\alpha)y}{((1-\tau^k)(1-\alpha)\frac{y_{t+1}}{k^p_{t+1}}+1-\delta^p)k^p}\hat{y}_{t+1}\right.\right.$$
$$\left.\left.-\frac{(1-\tau^k)(1-\alpha)y}{((1-\alpha)\frac{y_{t+1}}{k^p_{t+1}}+1-\delta^p)k^p}\hat{k}^p_{t+1}\right]\right\} \quad (11.5.24)$$

Observe that

$$(1-\tau^k)(1-\alpha)\frac{y}{k^p}+1-\delta^p \quad = \quad 1/\beta \quad (11.5.25)$$

Plug it into the equation to obtain

$$\hat{\lambda}_t \quad = \quad E_t\left[\hat{\lambda}_{t+1}+\frac{\beta(1-\tau^k)(1-\alpha)y}{k^p}\hat{y}_{t+1}-\frac{\beta(1-\tau^k)(1-\alpha)y}{k^p}\hat{k}^p_{t+1}\right] \quad (11.5.26)$$

$$\hat{\lambda}_t \quad = \quad E_t\hat{\lambda}_{t+1}+\frac{\beta(1-\tau^k)(1-\alpha)y}{k^p}E_t\hat{y}_{t+1}-\frac{\beta(1-\tau^k)(1-\alpha)y}{k^p}E_t\hat{k}^p_{t+1} \quad (11.5.27)$$

11.5.5 Linearized MRS(c_t, n^p_t)

$$\psi_2 c_t \quad = \quad \psi_1[1-n^p_t-rs_t-n^g_t-\varphi(n_t)^2](1-\tau^l)\alpha\frac{y_t}{n^p_t} \quad (11.5.28)$$

Take natural logs from both sides of the equation to obtain

$$\ln\psi_2 c_t \quad = \quad \ln\psi_1[1-n^p_t-n^g_t-rs_t-\varphi(n_t)^2](1-\tau^l)\alpha\frac{y_t}{n^p_t} \quad (11.5.29)$$

$$\ln\psi_2+\ln c_t \quad = \quad \ln\psi_1+\ln[1-n^p_t-n^g_t-rs_t-\varphi(n_t)^2]$$
$$+\ln(1-\tau^l_t)+\ln y_t-\ln n^p_t \quad (11.5.30)$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln\psi_2}{dt}+\frac{d\ln c_t}{dt}=\frac{d\ln\psi_1}{dt}+\frac{d\ln[1-n^p_t-n^g_t-rs_t-\varphi(n_t)^2]}{dt}$$
$$+\frac{d\ln(1-\tau^l_t)}{dt}+\frac{d\ln y_t}{dt}-\frac{d\ln n^p_t}{dt} \quad (11.5.31)$$

$$\frac{1}{c}\left[\frac{dc_t}{dt}\right] = -\frac{1}{1-n^p-n^g-rs_t-\varphi(n^g)^2}\frac{d\left[n_t^p+n_t^g+rs_t+\varphi(n_t)^2\right]}{dt}$$

$$-\frac{d\tau_t^l}{dt}\frac{1}{1-\tau^l}+\frac{1}{y}\frac{dy_t}{dt}-\frac{1}{n^p}\frac{dn_t^p}{dt} \qquad (11.5.32)$$

$$\frac{dc_t}{dt}\frac{1}{c} = -\frac{n^p}{1-n^p-n^g-rs-\varphi(n^g)^2}\frac{dn_t^p}{dt}\frac{1}{n^p}$$

$$-\frac{rs}{1-n^p-n^g-rs-\varphi(n^g)^2}\frac{drs_t}{dt}\frac{1}{rs}$$

$$-\frac{n^g(1+2\varphi n^g)}{1-n^p-n^g-rs-\varphi(n^g)^2}\frac{dn_t^g}{dt}\frac{1}{n^g}-\frac{\tau^l}{1-\tau^l}\frac{d\tau_t^l}{dt}\frac{1}{\tau^l}+\frac{1}{y}\frac{dy_t}{dt}-\frac{1}{n^p}\frac{dn_t^p}{dt} \qquad (11.5.33)$$

Pass to log-deviations to obtain

$$\hat{c}_t = -\frac{n^p}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{n}_t^p-\frac{rs}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{rs}_t$$

$$-\frac{n^g(1+2\varphi n^g)}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{n}_t^g-\frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l+\hat{y}_t-\hat{n}_t^p \qquad (11.5.34)$$

Group terms to obtain

$$\hat{c}_t = -\frac{1-n^g-\varphi(n^g)^2}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{n}_t^p$$

$$-\frac{rs}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{rs}_t-\frac{n^g(1+2\varphi n^g)}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{n}_t^g-\frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l+\hat{y}_t (11.5.35)$$

11.5.6 Linearized MRS(c_t, n_t^g)

$$\psi_2 c_t = \psi_1[1-n_t^p-n_t^g-rs_t-\varphi(n_t)^2](1-\tau^l)w_t^g \qquad (11.5.36)$$

Take natural logs from both sides of the equation to obtain

$$\ln\psi_2 c_t = \ln\psi_1[1-n_t^p-n_t^g-rs_t-\varphi(n_t)^2](1-\tau^l)w_t^g \qquad (11.5.37)$$

$$\ln\psi_2+\ln c_t = \ln\psi_1+\ln[1-n_t^p-n_t^g-rs_t-\varphi(n_t)^2]+\ln(1-\tau_t^l)+\ln w_t^g (11.5.38)$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln\psi_2}{dt}+\frac{d\ln c_t}{dt} = \frac{d\ln\psi_1}{dt}+\frac{d\ln[1-n_t^p-n_t^g-rs_t-\varphi(n_t)^2]}{dt}$$

$$+\frac{d\ln(1-\tau_t^l)}{dt}+\frac{d\ln w_t^g}{dt} \qquad (11.5.39)$$

$$\frac{1}{c}\left[\frac{dc_t}{dt}\right] = -\frac{1}{1-n^p-n^g-rs-\varphi(n^g)^2}\frac{d\left[n_t^p+n_t^g+rs_t+\varphi(n_t)^2\right]}{dt} - \frac{d\tau_t^l}{dt}\frac{1}{1-\tau^l} + \frac{dw_t^g}{dt}\frac{1}{w^g} \tag{11.5.40}$$

$$\frac{dc_t}{dt}\frac{1}{c} = -\frac{n^p}{1-n^p-n^g-rs-\varphi(n^g)^2}\frac{dn_t^p}{dt}\frac{1}{n^p} - \frac{rs}{1-n^p-n^g-rs-\varphi(n^g)^2}\frac{drs_t}{dt}\frac{1}{rs} - \frac{n^g(1+2\varphi n^g)}{1-n^p-n^g-rs-\varphi(n^g)^2}\frac{dn_t^g}{dt}\frac{1}{n^g} - \frac{\tau^l}{1-\tau^l}\frac{d\tau_t^l}{dt}\frac{1}{\tau^l} + \frac{1}{w^g}\frac{dw_t^g}{dt} \tag{11.5.41}$$

Pass to log-deviations to obtain

$$\hat{c}_t = -\frac{n^p}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{n}_t^p - \frac{rs}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{rs}_t - \frac{n^g(1+2\varphi n^g)}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{n}_t^g - \frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l + \hat{w}_t^g \tag{11.5.42}$$

11.5.7 Linearized private physical capital accumulation

$$k_{t+1}^p = i_t + (1-\delta^p)k_t^p \tag{11.5.43}$$

Take natural logs from both sides of the equation to obtain

$$\ln k_{t+1}^p = \ln(i_t + (1-\delta^p)k_t^p) \tag{11.5.44}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k_{t+1}^p}{dt} = \frac{1}{i+(1-\delta)k^p}\frac{d(i_t+(1-\delta^p)k_t^p)}{dt} \tag{11.5.45}$$

Observe that since

$$i = \delta^p k^p, \text{ it follows that } \quad i + (1-\delta^p)k^p = \delta^p k^p + (1-\delta^p)k^p = k^p. \text{ Then} \tag{11.5.46}$$

$$\frac{dk_{t+1}^p}{dt}\frac{1}{k^p} = \frac{1}{k^p}\frac{di_t}{dt}\frac{i}{i} + \frac{k^p}{i+(1-\delta^p)k_t^p}\frac{dk_t^p}{dt}\frac{k^p}{k^p} \tag{11.5.47}$$

Pass to log-deviations to obtain

$$\hat{k}_{t+1}^p = \frac{\delta^p k^p}{k^p}\hat{i}_t + \frac{(1-\delta^p)k^p}{k^p}\hat{k}_t^p \tag{11.5.48}$$

$$\hat{k}_{t+1}^p = \delta^p\hat{i}_t + (1-\delta^p)\hat{k}_t^p \tag{11.5.49}$$

11.5.8 Linearized government physical capital accumulation

$$k^g_{t+1} = g^i_t + (1-\delta^g)k^g_t \tag{11.5.50}$$

Take natural logs from both sides of the equation to obtain

$$\ln k^g_{t+1} = \ln(g^i_t + (1-\delta^g)k^g_t) \tag{11.5.51}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln k^g_{t+1}}{dt} = \frac{1}{g^i + (1-\delta^g)k^g}\frac{d(i_t + (1-\delta^g)k^g_t)}{dt} \tag{11.5.52}$$

Observe that since

$$g^i = \delta^g k^g, \text{ it follows that } \quad g^i + (1-\delta^g)k^g = \delta^g k^g + (1-\delta^g)k^g = k^g. \text{ Then} \tag{11.5.53}$$

$$\frac{dk^g_{t+1}}{dt}\frac{1}{k^g} = \frac{1}{k^g}\frac{dg^i_t}{dt}\frac{g^i}{g^i} + \frac{k^g}{{}^i + (1-\delta^g)k^g_t}\frac{dk^g_t}{dt}\frac{k^g}{k^g} \tag{11.5.54}$$

Pass to log-deviations to obtain

$$\hat{k}^g_{t+1} = \frac{\delta^g k^g}{k^g}\hat{g}^i_t + \frac{(1-\delta^p)k^g}{k^g}\hat{k}^g_t \tag{11.5.55}$$

$$\hat{k}^g_{t+1} = \delta^g \hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{11.5.56}$$

11.5.9 Linearized government budget constraint

$$(1-\tau^l_t)w^g_t n^g_t + g^i_t = \tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t. \tag{11.5.57}$$

Take natural logarithms from both sides to obtain

$$\ln\Bigg[(1-\tau^l_t)w^g_t n^g_t + g^i_t\Bigg] = \ln\Bigg[\tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t\Bigg]. \tag{11.5.58}$$

Totally differentiate with respect to time to obtain

$$\frac{d}{dt}\ln\Bigg[(1-\tau^l_t)w^g_t n^g_t + g^i_t\Bigg] = \frac{d}{dt}\ln\Bigg[\tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t\Bigg]. \tag{11.5.59}$$

or

$$\frac{1}{(1-\tau^l)w^g n^g + g^i}\frac{d\Bigg[(1-\tau^l_t)w^g_t n^g_t + g^i_t\Bigg]}{dt} = \frac{1}{\Bigg[\tau^k r k + \tau^l w^p n^p\Bigg]}\frac{d\Bigg[\tau^k_t r_t k_t + \tau^l_t w^p_t n^p_t\Bigg]}{dt} \tag{11.5.60}$$

Note that

$$(1-\tau^l)w^g n^g + g^i = \tau^k r k + \tau^l w^p n^p \tag{11.5.61}$$

Hence

$$\frac{d\left[(1-\tau_t^l)w_t^g n_t^g + g_t^i\right]}{dt} = \frac{d\left[\tau_t^k r_t k_t + \tau_t^l w_t^p n_t^p\right]}{dt}. \tag{11.5.62}$$

or

$$\begin{aligned}
&-w^g n^g \frac{d\tau_t^l}{dt}\frac{\tau^l}{\tau^l} + (1-\tau^l)n^g \frac{dw_t^g}{dt}\frac{w^g}{w^g} + (1-\tau^l)w^g \frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dg_t^i}{dt}\frac{g^i}{g^i} \\
&= rk\frac{d\tau_t^k}{dt}\frac{\tau^k}{\tau^k} + \tau^k k\frac{dr_t}{dt}\frac{r}{r} + \tau^k r\frac{dk_t}{dt}\frac{k}{k} + w^p n^p \frac{d\tau_t^l}{dt} + \tau^l n^p \frac{dw_t^p}{dt}\frac{w^p}{w^p} + \tau^l w^p \frac{dn_t^p}{dt}\frac{n^p}{n^p}
\end{aligned} \tag{11.5.63}$$

Pass to log-deviations to obtain

$$\begin{aligned}
&-\tau^l w^g n^g \hat{\tau}_t^l + (1-\tau^l)w^g n^g \hat{w}_t^g + (1-\tau^l)w^g n^g \hat{n}_t^g + g^i \hat{g}_t^i \\
&= \tau^k r k \hat{\tau}_t^k + \tau^k r k \hat{r}_t + \tau^k r k \hat{k}_t + \tau^l w^p n^p \hat{\tau}_t^l + \tau^l w^p n^p \hat{w}_t^p + \tau^l w^p n^p \hat{n}_t^p
\end{aligned} \tag{11.5.64}$$

11.5.10 Total hours/employment

$$n_t = n_t^g + n_t^p \tag{11.5.65}$$

Take logs from both sides to obtain

$$\ln n_t = \ln(n_t^g + n_t^p) \tag{11.5.66}$$

Totally differentiate to obtain

$$\frac{d\ln n_t}{dt} = \frac{d\ln(n_t^g + n_t^p)}{dt} \tag{11.5.67}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt} + \frac{dn_t^p}{dt}\right)\frac{1}{n} \tag{11.5.68}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \left(\frac{dn_t^g}{dt}\frac{n^g}{n^g} + \frac{dn_t^p}{dt}\frac{n^p}{n^p}\right)\frac{1}{n} \tag{11.5.69}$$

$$\frac{dn_t}{dt}\frac{1}{n} = \frac{dn_t^g}{dt}\frac{1}{n^g}\frac{n^g}{n} + \frac{dn_t^p}{dt}\frac{1}{n^p}\frac{n^p}{n} \tag{11.5.70}$$

Pass to log-deviations to obtain

$$\hat{n}_t = \frac{n^g}{n}\hat{n}_t^g + \frac{n^p}{n}\hat{n}_t^p \tag{11.5.71}$$

11.5.11 Linearized private wage rate

$$w_t^p = \alpha \frac{y_t}{n_t^p} \tag{11.5.72}$$

Take natural logarithms from both sides to obtain

$$\ln w_t^p = \ln \alpha + \ln y_t - \ln n_t^p \tag{11.5.73}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln w_t^p}{dt} = \frac{d \ln \alpha}{dt} + \frac{d \ln y_t}{dt} - \frac{d \ln n_t^p}{dt} \tag{11.5.74}$$

Simplify to obtain

$$\frac{dw_t^p}{dt}\frac{1}{w^p} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dn_t^p}{dt}\frac{1}{n^p} \tag{11.5.75}$$

Pass to log-deviations to obtain

$$\hat{w}_t^p = \hat{y}_t - \hat{n}_t^p \tag{11.5.76}$$

11.5.12 Linearized real interest rate

$$r_t = \alpha \frac{y_t}{k_t^p} \tag{11.5.77}$$

Take natural logarithms from both sides to obtain

$$\ln r_t = \ln \alpha + \ln y_t - \ln k_t^p \tag{11.5.78}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln r_t}{dt} = \frac{d \ln \alpha}{dt} + \frac{d \ln y_t}{dt} - \frac{d \ln k_t^p}{dt} \tag{11.5.79}$$

Simplify to obtain

$$\frac{dr}{dt}\frac{1}{r} = \frac{dy_t}{dt}\frac{1}{y} - \frac{dk_t^p}{dt}\frac{1}{k^p} \tag{11.5.80}$$

Pass to log-deviations to obtain

$$\hat{r}_t = \hat{y}_t - \hat{k}_t^p \tag{11.5.81}$$

11.5.13 Linearized government investment

$$g_t^i = g^{iy} y_t \tag{11.5.82}$$

Take natural logarithms from both sides to obtain

$$\ln g_t^i = \ln g^{iy} + \ln y_t \tag{11.5.83}$$

Totally differentiate with respect to time to obtain

$$\frac{d \ln g_t^i}{dt} = \frac{d \ln g^{iy}}{dt} + \frac{d \ln y_t}{dt} \tag{11.5.84}$$

or

$$\frac{dg_t^i}{dt}\frac{1}{g^i} = \frac{dy_t}{dt}\frac{1}{y} \tag{11.5.85}$$

Passing to log-deviations

$$\hat{g}_t^i = \hat{y}_t \tag{11.5.86}$$

11.5.14 Linearized government services

$$s_t^g = (n_t^g)^\gamma (k_t^g)^{(1-\gamma)} \tag{11.5.87}$$

Take natural logarithms from both sides to obtain

$$\ln s_t^g = \gamma \ln n_t^g + (1-\gamma) \ln k_t^g \tag{11.5.88}$$

Totally differentiate with respect to time to obtain

$$\frac{ds_t^g}{dt}\frac{1}{s^g} = \gamma \frac{dn_t^g}{dt}\frac{1}{n^g} + (1-\gamma)\frac{dk_t^g}{dt}\frac{1}{k^g} \tag{11.5.89}$$

$$\hat{s}_t^g = \gamma \hat{n}_t^g + (1-\gamma)\hat{k}_t^g \tag{11.5.90}$$

11.5.15 Linearized rent-seeking rule

$$rs_t = n_t^g \left[\frac{w_t^g}{w_t^p} - 1 \right] \tag{11.5.91}$$

Take natural logarithms from both sides to obtain

$$\ln rs_t = \ln n_t^g + \ln\left[\frac{w_t^g}{w_t^p} - 1\right] \tag{11.5.92}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln rs_t}{dt} = \frac{d\ln n_t^g}{dt} + \frac{d}{dt}\ln\left[\frac{w_t^g}{w_t^p} - 1\right] \tag{11.5.93}$$

$$\frac{drs_t}{dt}\frac{1}{rs} = \frac{dn_t^g}{dt}\frac{1}{n^g} + \frac{d\left[\frac{w_t^g}{w_t^p} - 1\right]}{dt}\frac{1}{\left[\frac{w^g}{w^p} - 1\right]} \tag{11.5.94}$$

$$\hat{rs}_t = \hat{n}_t^g + \hat{w}_t^g\frac{w^g}{w^p}\frac{n^g}{rs} + \hat{w}_t^p\frac{n^g}{rs}(w^g - 2) \tag{11.5.95}$$

11.5.16 Linearized technology shock process

$$\ln a_{t+1} \quad = \quad \rho^a \ln a_t + \epsilon_{t+1}^a \tag{11.5.96}$$

Totally differentiate with respect to time to obtain

$$\frac{d\ln a_{t+1}}{dt} \quad = \quad \rho^a\frac{d\ln a_t}{dt} + \frac{d\epsilon_{t+1}^a}{dt} \tag{11.5.97}$$

$$\frac{da_{t+1}}{dt} \quad = \quad \rho^a\frac{da_t}{dt} + \epsilon_{t+1}^a \tag{11.5.98}$$

where for $t = 1$ $\frac{d\epsilon_{t+1}^a}{dt} \approx \ln(e^{\epsilon_{t+1}^a}/e^{\epsilon^a}) = \epsilon_{t+1}^a - \epsilon^a = \epsilon_{t+1}^a$ since $\epsilon^a = 0$. Pass to log-deviations to obtain

$$\hat{a}_{t+1} \quad = \quad \rho^a\hat{a}_t + \epsilon_{t+1}^a \tag{11.5.99}$$

11.6 Log-linearized DCE system

$$0 \quad = \quad -\hat{y}_t + (1-\alpha)\hat{k}_t^p + \hat{a}_t + \alpha\hat{n}_t^p \tag{11.6.1}$$

$$k^p\hat{k}_{t+1}^p \quad = \quad y\hat{y}_t - c\hat{c}_t + (1-\delta^p)k^p\hat{k}_t^p - g^i\hat{g}_t^i \tag{11.6.2}$$

$$-\hat{c}_t = \hat{\lambda}_t \tag{11.6.3}$$

$$\hat{\lambda}_t \quad = \quad E_t\hat{\lambda}_{t+1} + \frac{\beta(1-\tau^k)(1-\alpha)y}{k^p}E_t\hat{y}_{t+1} - \frac{\beta(1-\tau^k)(1-\alpha)y}{k^p}E_t\hat{k}^p_{t+1} \tag{11.6.4}$$

$$\begin{aligned} \hat{c}_t = -\frac{1-n^g-\varphi(n^g)^2}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{n}^p_t - \frac{rs}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{rs}_t \\ -\frac{n^g(1+2\varphi n^g)}{1-n^p-n^g-\varphi(n^g)^2}\hat{n}^g_t + \hat{y}_t \end{aligned} \tag{11.6.5}$$

$$\begin{aligned} \hat{c}_t = -\frac{n^p}{1-n^p-rs-n^g-\varphi(n^g)^2}\hat{n}^p_t - \frac{rs}{1-n^p-rs-n^g-\varphi(n^g)^2}\hat{rs}_t \\ -\frac{n^g(1+2\varphi n^g)}{1-n^p-n^g-rs-\varphi(n^g)^2}\hat{n}^g_t + \hat{w}^g_t \end{aligned} \tag{11.6.6}$$

$$\hat{k}^p_{t+1} \quad = \quad \delta^p\hat{i}_t + (1-\delta^p)\hat{k}^p_t \tag{11.6.7}$$

$$\hat{k}^g_{t+1} \quad = \quad \delta^g\hat{g}^i_t + (1-\delta^g)\hat{k}^g_t \tag{11.6.8}$$

$$\begin{aligned} -\tau^l w^g n^g \hat{\tau}^l_t + (1-\tau^l)w^g n^g \hat{w}^g_t + (1-\tau^l)w^g n^g \hat{n}^g_t + g^i\hat{g}^i_t \\ = \tau^k rk\hat{\tau}^k_t + \tau^k rk\hat{r}_t + \tau^k rk\hat{k}_t + \tau^l w^p n^p \hat{\tau}^l_t + \tau^l w^p n^p \hat{w}^p_t + \tau^l w^p n^p \hat{n}^p_t \end{aligned} \tag{11.6.9}$$

$$\hat{g}^i_t = \hat{y}_t \tag{11.6.10}$$

$$\hat{rs}_t = \hat{n}^g_t + \hat{w}^g_t\frac{w^g}{w^p}\frac{n^g}{rs} + \hat{w}^p_t\frac{n^g}{rs}(w^g-2) \tag{11.6.11}$$

$$\hat{w}^p_t = \hat{y}_t - \hat{n}^p_t \tag{11.6.12}$$

$$\hat{r}_t = \hat{y}_t - \hat{k}^p_t \tag{11.6.13}$$

$$\hat{s}^g_t = \gamma\hat{n}^g_t + (1-\gamma)\hat{k}^g_t \tag{11.6.14}$$

$$\hat{a}_{t+1} \quad = \quad \rho^a\hat{a}_t + \epsilon^a_{t+1} \tag{11.6.15}$$

The model can be now solved by representing it in the following matrix form

$$\mathbf{A}E_{t+1}\hat{\mathbf{x}}_{\mathbf{t}} = \mathbf{B}\hat{\mathbf{x}}_{\mathbf{t}} + \mathbf{C}\varepsilon_{\mathbf{t}}, \tag{11.6.16}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are coefficient matrices, $\varepsilon_{\mathbf{t}}$ is a matrix of innovations, and $\hat{\mathbf{x}}_{\mathbf{t}}$ is the stacked vector of state (also called 'predetermined') variables, $\hat{\mathbf{s}}_{\mathbf{t}} = \begin{bmatrix} \hat{a}_t & \hat{k}_t^p & \hat{k}_t^g \end{bmatrix}'$, and control variables, $\hat{\mathbf{z}}_{\mathbf{t}} = \begin{bmatrix} \hat{y}_t & \hat{c}_t & \hat{i}_t & \hat{n}_t & \hat{n}_t^p & \hat{n}_t^g & \hat{w}_t^p & \hat{w}_t^g & \hat{\lambda}_t & \hat{rs}_t & \hat{g}_t^i & \hat{s}_t^g \end{bmatrix}'$. Klein's (2000) generalized eigenvalue ("Schur") decomposition algorithm was used to solve the model. The MATLAB function to solve the above linear system is *solab.m*. The inputs are matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ defined above and $nk = 3$, which is the number of state variables. The outputs are the coefficient matrices $\mathbf{M}$ and $\mathbf{\Pi}$ which solve the linearized system. A solution to an RBC model is in the form of (approximate) policy, or transition rule, which describes the evolution of each variable. In particular, the predetermined and non-predetermined variables can be represented in the following form:

$$E_t\hat{\mathbf{s}}_{\mathbf{t+1}} = \Pi\hat{\mathbf{s}}_{\mathbf{t}} \tag{11.6.17}$$

$$\hat{\mathbf{z}}_{\mathbf{t}} = M\hat{\mathbf{s}}_{\mathbf{t}} \tag{11.6.18}$$

To simulate the model, one requires a sequence of normally distributed disturbances, $\{\epsilon_t\}_{t=0}^{\infty}$ for the three exogenous shocks with sample size T, the initial values of the endogenous predetermined variables, $\{k_0^p, k_0^g, a_0\}$ ($a_0 = 1$), and the evolution of the endogenous non-predetermined variables in model solution form

$$\hat{\mathbf{s}}_{\mathbf{t+1}} = \Pi\hat{\mathbf{s}}_{\mathbf{t}} + \mathbf{D}\varepsilon_{\mathbf{t+1}} \tag{11.6.19}$$

$$\hat{\mathbf{z}}_{\mathbf{t}} = \mathbf{M}\hat{\mathbf{s}}_{\mathbf{t}}, \tag{11.6.20}$$

where

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{11.6.21}$$

Based on the above representation, MATLAB code was written to simulate the model. The computation of impulse responses using the linearized model solution is straightforward.