Asset Prices, Business Cycles, and Markov-Perfect Fiscal Policy when Agents are Risk-Sensitive

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Abstract

We study a business cycle model in which a benevolent fiscal authority must determine the optimal provision of government services, while lacking credibility, lump-sum taxes, and the ability to bond finance deficits. Households and the fiscal authority have risk sensitive preferences. We find that outcomes are affected importantly by the household's risk sensitivity, but not by the fiscal authority's. Further, while household risk-sensitivity induces a strong precautionary saving motive, which raises capital and lowers the return on assets, its effects on fluctuations and the business cycle are generally small, although more pronounced for negative shocks. Holding the stochastic steady state constant, increases in household risk-sensitivity lower the risk-free rate and raise the return on equity, increasing the equity premium. Finally, although risk-sensitivity has little effect on the provision of government services, it does cause the fiscal authority to lower the income tax rate. An additional contribution of this paper is to present a method for computing Markov-perfect equilibria in models where private agents and the government are risk-sensitive decisionmakers.

Keywords: *Asset prices, business cycles, risk-sensitivity, Markov-Perfect fiscal policy.*

JEL Classification: E63, C61.

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1 Introduction

Risk-sensitive preferences are used increasingly to study questions related to consumption risk, welfare, and asset pricing. Often motivated in the context of the Ellsberg paradox, such preferences can be derived from ambiguity aversion or by concerns for robustness to model misspecification (Hansen and Sargent, 2008) and represent a special case of Epstein and Zin (1989) preferences. Placing risk-sensitive households in a real business cycle model, Tallarini (2000) shows that business cycle fluctuations can have large welfare effects and that the model can generate a low risk-free rate. Related work by Croce (2006) finds that stylized facts about the risk-free rate and the equity premium can be accounted for in a stochastic growth model containing capital adjustment costs when households have Epstein-Zin preferences.

In this paper, we consider the problem facing a benevolent fiscal authority that must formulate and conduct fiscal policy in an environment in which households and firms are optimizing. In this economy, the fiscal authority taxes linearly household income in order to finance the provision of government services, while lacking a commitment technology and the ability to bond-finance deficits. Households exhibit the standard aversion to risk in consumption and leisure (and government services), but they are additionally assumed to be risk-sensitive decisionmakers, averse to risk in expected future utility. We use this model as a laboratory to investigate the effects that risk-sensitive preferences have on the business cycle, asset prices, and the provision of government services in a Markov-perfect equilibrium.

We find that household risk-sensitivity, but not fiscal-authority risk-sensitivity, induces a strong precautionary saving motive, which raises importantly the stochastic steady state level of capital. Further, although household risk-sensitivity effects the stochastic steady state, its effects on economic volatility are small, a finding that is consistent with Tallarini (2000). At the same time, we find that although risk-sensitivity has little effect on how the economy responds to positive technology shocks, its effects for the case of negative technology shocks are more pronounced. In addition, due to its effect on capital's stochastic steady state, we show that increases in household risk-sensitivity serve to lower the risk-free rate, the return to equity, and the equity premium. However, holding the stochastic steady state constant, increases in household risk-sensitivity lower the risk-free rate slightly, raise the return on equity slightly, and increase the equity premium. But, the increase in the equity premium produced is small, suggesting that risk-sensitive preferences, in isolation, leave much of any observed equity premium unexplained, consistent with Croce (2006). Finally, although risk-sensitivity has little effect on the provision of government services, it does cause the fiscal authority to lower the income tax rate.

As noted above, the fiscal authority in our model lacks the ability to commit. For this reason, the equilibria that we study are Markov-perfect rather than Ramsey. In this respect, our study is related to work by authors such as Ortigueira (2006), Klein, Krusell, and Ríos-Rull (2008), Ambler and Pelgrin (2010), and Martin (2010), who also study Markov-perfect fiscal policy in balanced budget economies with capital.[1] Unlike Ortigueira (2006), Klein, Krusell, and Ríos-Rull (2008), and Martin (2010), however, the model we study is stochastic, allows for risk-sensitive preferences, and we focus on the business cycle and asset pricing rather than on steady state outcomes. And, where Ambler and Pelgrin (2010) do solve a model that is stochastic, they do not allow households to make a labor-leisure choice or allow for risk-sensitive preferences. Our analysis is most closely related to Tallarini (2000), who studies business cycles, asset prices, and welfare in a real business cycle model in which households are risk-sensitive. We differ from Tallarini (2000) in that our model contains a fiscal authority, and in that it considers Markov-perfect fiscal policy. We also differ from Tallarini (2000) in that our model allows both households and the fiscal authority to be risk-sensitive decisionmakers, in that we focus on transitory rather than permanent shocks, and in that we compute equilibria using a global method.[2]

One novel aspect of our model is that households and the fiscal-authority are each risk-sensitive decisionmakers. As is well-known (Hansen and Sargent, 2008), this risk-sensitivity can alternatively be motivated by ambiguity aversion or by robustness considerations, allowing us to remain agnostic about its precise motivation. There are very few papers that analyze models in which multiple agents are risk-sensitive, but Svec (2012) provides a notable example. Svec (2012) examines Ramsey equilibria in a Lucas and Stokey (1983) economy in which households are robust decisionmakers and in which the fiscal authority can be either "political" (maximize household welfare under the household's subjective probability model) or "paternalistic" (maximize household welfare under the true probability model).[3] Svec's political and paternalistic

[1] Our study is also related to the body of work that examines Markov-perfect and/or Ramsey equilibria in models that exclude capital (such as Stockman (2001), Chugh (2006), and Niemann, Pichler, and Sorger (2008, 2009)). And it is related to literature that examines the optimal financing of an exogenous stream of government spending (such as Benhabib and Rustichini (1997), Domínguez (2007), and Aguiar and Amador (2011)).

[2] Because there is no strategic interation in Tallarini's (2000) model, he is able to compute equilibrium using a linear-quadratic dynamic programming method.

[3] Karantounias (2013) performs a related exercise, also using a Lucas and Stokey (1983) model, focusing on Ramsey equilibria in which the fiscal authority is "paternalistic".

fiscal authorities are two special cases of our decision-making framework.

The remainder of this paper is organized as follows. Section 2 introduces the model and describes the decision problems facing the households, firms, and the fiscal authority. Section 3 discusses the model's parameterization and derives and discusses the system of equations that must be solved to compute equilibrium. Section 4 presents the main results while Section 5 examines the sensitivity of the main results to alternative parameterizations of the model. Section 6 concludes. An appendix presents the numerical method used to compute equilibrium.

2 The model

We consider a production economy populated by a unit-mass of identical atomistic households, a unit-mass of identical atomistic firms, and a fiscal authority. Firms rent capital and hire labor from households and use these inputs to produce goods that are sold to households and the fiscal authority. Goods sold to the fiscal authority are transformed costlessly into a government consumption good while those sold to households are either consumed or used to augment the capital stock. The fiscal authority taxes household income, using the revenue to finance the provision of the government consumption good. Markets are assumed to be perfectly competitive. Households and the fiscal authority are each assumed to be risk-sensitive decisionmakers.

2.1 Households

Households own the capital stock. They receive income by renting their capital and supplying their labor to firms at prices r_t^k and w_t, respectively. After paying income tax, households use their remaining income to purchase goods, which they use to offset capital-depreciation, to invest in their capital stock, and to consume. The representative household's lifetime utility function is summarized by

$$u_t = u\left(c_t, l_t, G_t\right) + \frac{\beta}{\theta^h} \ln\left[\mathrm{E}_t\left(\exp\left(\theta^h u_{t+1}\right)\right)\right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, $\theta^h \leq 0$ characterizes household risk-sensitivity, c_t denotes private consumption, l_t denotes leisure, G_t denotes consumption of government services, and the momentary utility function $u\left(c_t, l_t, G_t\right)$ is assumed to be strictly increasing, strictly concave, twice continuously differentiable, and to satisfy the Inada (1963) conditions. A simple application of L'Hôpitals' rule shows that equation (1) converges to the standard recursive formulation

$$u_t = u\left(c_t, l_t, G_t\right) + \beta \mathrm{E}_t\left(u_{t+1}\right),$$

in the limit as $\theta^h \uparrow 0$ while the effects of $\theta^h < 0$ are to distort the continuation value, a distortion arising from the household's aversion to risky future utility.

The capital owned by the representative household evolves over time to satisfy the flow budget constraint

$$k_{t+1} + c_t = k_t + (1 - \tau_t) \left[\left(r_t^k - \delta \right) k_t + w_t h_t \right], \tag{2}$$

where $\delta \in (0, 1)$ is the depreciation rate, $\tau_t > 0$ is the tax rate applied to household income (with a tax-allowance for capital-depreciation), k_t is the household's stock of capital as of the beginning of period t, and h_t is hours worked. Households maximize their expected lifetime utility, equation (1), subject to their flow-budget constraint, equation (2), and their time resource constraint

$$h_t + l_t = 1, \tag{3}$$

taking prices, the tax rate, and the provision of government consumption as given.

2.2 Firms

The stand-in aggregate firm employs capital and labor to produce output according to the neo-classical production technology

$$Y_t = e^{z_t} F\left(K_t, H_t\right), \tag{4}$$

where Y_t represents aggregate output, K_t denotes the aggregate capital stock as of the beginning of period t, H_t denotes aggregate hours worked, and z_t is an aggregate technology shock that obeys the law-of-motion

$$z_{t+1} = \rho z_t + \epsilon_{t+1}, \tag{5}$$

where $\rho \in (0, 1)$ and $\epsilon_t \sim i.i.d. \left[0, \sigma_\epsilon^2\right]$.

Markets for capital and labor are perfectly competitive and clear at the prices

$$\begin{aligned} r_t^k &= e^{z_t} F_K\left(K_t, H_t\right), && (6) \\ w_t &= e^{z_t} F\left(K_t, H_t\right) - e^{z_t} F_K\left(K_t, H_t\right) K_t, && (7) \end{aligned}$$

respectively, with the stand-in aggregate firm making zero-profits in equilibrium.

2.3 Fiscal authority

The fiscal authority cannot impose lump-sum taxes, but receives revenue by taxing household income at marginal rate $\tau_t > 0$. These tax revenues are used to purchase goods from firms that are transformed costlessly into government consumption goods (or government services) and provided to households at zero-unit-cost. The fiscal authority has no outstanding liabilities and cannot issue bonds. As a consequence, the fiscal authority's decisions about taxation and the provision of the government consumption good, decisions made to maximize

$$U_t = u\left(C_t, L_t, G_t\right) + \frac{\beta}{\theta^f} \ln\left[\mathrm{E}_t\left(\exp\left(\theta^f U_{t+1}\right)\right)\right], \tag{8}$$

where $\theta^f \leq 0$ characterizes the fiscal-authority's risk sensitivity, are constrained by the balanced-budget condition

$$G_t = \tau_t\left[\left(r_t^k - \delta\right) K_t + w_t H_t\right]. \tag{9}$$

Importantly, θ^f need not equal θ^h, which allows the fiscal authority and the representative household to differ in regard to their risk-sensitivity. If $\theta^f = \theta^h$, then the fiscal authority conducts policy in order to maximize the welfare of the representative household, accounting for the household's risk-sensitivity, (corresponding to Svec's (2012) "political" fiscal authority), while if $\theta^f = 0$, then the fiscal authority conducts policy in order to maximize the welfare of the representative household, ignoring the household's risk-sensitivity, (corresponding to Svec's (2012) "paternalistic" fiscal authority), but more generally the framework allows the fiscal authority to be either more or less risk-sensitive than the representative household.

2.4 Information, timing, and aggregation

With the current realization for the aggregate technology given by z_t, we denote the history of realizations for aggregate technology up to and including period t by $z^t = \{z_i\}_{i=0}^t$. Similarly, using $\mathbf{x}_t = \begin{bmatrix} z_t & k_t & K_t \end{bmatrix}'$ to denote the economy's state at the beginning of period t, we assume that at the beginning of period t all agents are endowed with the information set given by the history $\mathbf{x}^t$. After entering period t, and having observed $\mathbf{x}^t$, the fiscal authority makes its decision; households and firms make their decisions simultaneously, but subsequent to the fiscal authority. With this timing protocol, within the period, the fiscal authority has a first-mover advantage with respect to households and firms.[4] Our assumptions that households and firms are identical and that they are of unit-mass implies that $K_t = k_t$, $C_t = c_t$, $H_t = h_t$, and $L_t = l_t$ in aggregate.

[4] This timing protocol is used by Kydland and Prescott (1977), Ambler and Paquet (1997), Klein and Rios-Rull (2003), Klein, Quadrini, and Rios-Rull (2005), Ortigueria (2006), Klein, Krusell, and Rios-Rull (2008), Ortigueria, Pereira, and Pichler (2012), Anderson, Kim, and Yun (2010), and is ubiquitous in the literature on time-consistent monetary policy. The alternative timing protocol in which households, firms, and the fiscal authority all make their decisions simultaneously is considered by Cohen and Michel (1988) and Ortigueira (2006).

3 Solving the model

To simplify the exposition, we assume from the outset that the momentary utility function for the representative household is of the iso-elastic form

$$u\left(c_t, l_t, G_t\right) = \frac{c_t^{1-\sigma_c}}{1-\sigma_c} + \frac{\eta l_t^{1-\sigma_l}}{1-\sigma_l} + \frac{\gamma G_t^{1-\sigma_g}}{1-\sigma_g},$$

where $\{\sigma_c, \sigma_l, \sigma_G, \eta, \gamma\} > 0$ and that the production function is Cobb-Douglas

$$Y_t = e^{z_t} K_t^{\alpha} H_t^{1-\alpha},$$

where $\alpha \in (0,1)$, with the aggregate technology shock obeying equation (5).

With the fiscal authority having a first-mover advantage within each period, we begin by formulating the decision problem for the representative household. Using the household's time resource constraint to substitute for leisure, the representative household solves the decision problem described by the Bellman equation

$$v\left(z_t, k_t\right) = \max_{\{c_t, h_t, k_{t+1}\}} \left[\frac{c_t^{1-\sigma_c}}{1-\sigma_c} + \frac{\eta\left(1-h_t\right)^{1-\sigma_l}}{1-\sigma_l} + \frac{\gamma G_t^{1-\sigma_g}}{1-\sigma_g} + \frac{\beta}{\theta^h} \ln\left[\mathrm{E}_t \left(\exp\left(\theta^h v\left(z_{t+1}, k_{t+1}\right) \right) \right) \right] \right], \tag{10}$$

subject to the flow budget constraint

$$k_{t+1} = k_t + \left(1-\tau_t\right)\left[\left(r_t^k - \delta \right) k_t + w_t h_t \right] - c_t, \tag{11}$$

taking r_t^k, w_t, and τ_t as given, and the initial conditions $k_t = K_t > 0$, $z_t > 0$, known.

From this Bellman equation, and employing the Benveniste and Scheinkman (1979) condition, we obtain first-order necessary conditions, which after aggregating across identical households and employing equations (6)—(7), can be written as

$$\begin{aligned} C_t^{-\sigma_c} &= \frac{\beta}{\mathrm{E}_t\left[\exp\left(\theta^h v\left(z_{t+1}, K_{t+1}\right)\right)\right]} \\ &\quad \times \mathrm{E}_t\left[\exp\left(\theta^h v\left(z_{t+1}, K_{t+1}\right) \right) \left(1 + \left(1-\tau_{t+1}\right)\left(\alpha e^{z_{t+1}} K_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha} - \delta\right)\right) C_{t+1}^{-\sigma_c} \right] \qquad (12) \\ \eta\left(1-H_t\right)^{-\sigma_l} &= \left(1-\tau_t\right)\left(1-\alpha\right) e^{z_t} K_t^{\alpha} H_t^{-\alpha} C_t^{-\sigma_c}. \qquad (13) \end{aligned}$$

Equation (12) is, of course, an aggregate version of the consumption Euler equation while equation (13) summarizes aggregate labor supply. The aggregate capital stock evolves according to

$$K_{t+1} = \left(1-\delta\right) K_t + e^{z_t} K_t^{\alpha} H_t^{1-\alpha} - C_t - G_t. \tag{14}$$

Turning to the fiscal authority, its decision problem is described by the Bellman equation

$$V(z_t, K_t) = \max_{\{C_t, H_t, G_t, K_{t+1}\}} \left[\frac{C_t^{1-\sigma_c}}{1-\sigma_c} + \frac{\eta (1-H_t)^{1-\sigma_l}}{1-\sigma_l} + \frac{\gamma G_t^{1-\sigma_g}}{1-\sigma_g} + \frac{\beta}{\theta^f} \ln \left[\mathrm{E}_t \left(\exp \left(\theta^f V(z_{t+1}, K_{t+1}) \right) \right) \right] \right], \tag{15}$$

with the constraints given by equations (12)—(14) and by the government budget constraint,

$$G_t = \tau_t \left(e^{z_t} K_t^{\alpha} H_t^{1-\alpha} - \delta K_t \right), \tag{16}$$

taking next-period's policy function

$$G_{t+1} = G(z_{t+1}, K_{t+1}), \tag{17}$$

as given.

3.1 Equilibrium

A Markov-perfect Nash equilibrium for this model is a collection of household decision rules $\{c(z_t, k_t), h(z_t, k_t), k(z_t, k_t)\}$, a collection of aggregate decision rules, $\{C(z_t, K_t), H(z_t, K_t), G(z_t, K_t), K(z_t, K_t)\}$ and a collection of value functions, $\{v(z_t, k_t), V(z_t, K_t)\}$, such that

1. The collection $\{v(z_t, k_t), c(z_t, k_t), h(z_t, k_t), k(z_t, k_t)\}$ solves the household's decision problem described by the Bellman equation, equation (10), and the constraint, equation (11).

2. The collection $\{V(z_t, K_t), C(z_t, K_t), H(z_t, K_t), K(z_t, K_t), G(z_t, K_t)\}$ solves the fiscal authority's decision problem described by the Bellman equation, equation (15), and the constraints, equations (9), (12)—(14), and (17).

3. $k_t = K_t$, $C(z_t, K_t) = c(z_t, K_t)$, $H(z_t, K_t) = h(z_t, K_t)$, and $k(z_t, k_t) = K(z_t, K_t)$.

We apply value function iteration to the fiscal authority's problem to compute equilibrium, employing Chebyshev polynomials to approximate the unknown functions; this procedure is summarized in Appendix A.

3.2 Parameterization

Table 1 reports the model's benchmark parameterization, where the benchmark is a fiscal-policy model in which agents are not risk-sensitive. Accordingly, θ^h and θ^f each equal zero in the benchmark model. With the model parameterized to a quarterly frequency, the subjective discount factor, β, is set to 0.99, a standard value for real business cycle models in the absence of trending technological progress, while α is set to 0.36, implying a capital-share of output that is just over one-third.

Table 1: Benchmark parameterization of model

Parameter	Value	Interpretation
α	0.36	Capital-share of output
δ	0.05	Depreciation rate (value is annualized rate)
β	0.99	Subjective discount factor
σ_c	1.00	Utility curvature of private consumption
σ_l	1.00	Utility curvature of leisure
σ_g	1.00	Utility curvature of government services
η	1.15	Utility weight on leisure
γ	0.25	Utility weight on government services
θ^h	0.00	Household risk-sensitivity
θ^f	0.00	Fiscal risk-sensitivity
ρ	0.95	Persistence of technology shock
σ_ϵ	0.01	Standard deviation of technology shock

Turning to the remaining parameters, the values assigned to most parameters are quite standard. The elasticities of substitution in the momentary utility function are each assumed to equal 1.00, so that the momentary utility function is linear in logs (consistent with Tallarini (2000)), while the weights, η and γ, are chosen so that households spend about 40 percent of their available (i.e., non-sleeping) time working and so that government spending is around 20 percent of output. The value for the depreciation rate, δ, is chosen so that the capital-output ratio is about 12. Finally, the parameters that govern the shock process are also relatively standard, with the persistence parameter, ρ, set to 0.95 and the standard deviation for the technology innovations, σ_ϵ, set to 0.01.

Although the benchmark model has $\theta^h = \theta^f = 0.00$, these parameters are varied between 0.00 and -0.10 in the analysis with risk sensitivity that follows. While apparently small in magnitude, as we shall see, variations in θ^h and θ^f between 0.00 and -0.10 correspond to relatively large variations in risk aversion and produce a large precautionary-saving motive.

4 Results

In this section we solve for the Markov-perfect equilibrium of the benchmark model and for various specifications with risk-sensitive preferences. For each of specification we compute the stochastic steady state values, standard deviations, and correlations with output, and impulse responses for the key aggregate variables, and we compute the risk-free rate, the return to equity, and the equity premium (asset returns and the equity premium are reported in percentage points per quarter). Our main results are summarized in Tables 2 and 3 and in Figures 1 through 5.

Allowing for risk-sensitive preferences, the net-return on a risk-free one-period real bond is defined by

$$\frac{1}{1+r_{t+1}} = \frac{\beta \mathrm{E}_t \left[\exp\left(\theta^h v\left(z_{t+1}, K_{t+1}\right)\right) C_{t+1}^{-\sigma_c}\right]}{\mathrm{E}_t \left[\exp\left(\theta^h v\left(z_{t+1}, K_{t+1}\right)\right)\right] C_t^{-\sigma_c}}, \tag{18}$$

which illustrates the connection between risk-sensitivity and the discount factor, β. In the limit as $\theta^h \uparrow 0$, equation (18) simplifies to the standard expression with iso-elastic utility. Similarly, the one-period real return on equity is given by[5]

$$r_{t+1}^e = \alpha e^{z_{t+1}} K_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha} - \delta, \tag{19}$$

where the effects of risk-sensitivity and income-taxation enter through their impact on capital and labor. Equation (19) shows that the return on equity is a decreasing function of the capital-labor ratio.

The "deterministic" steady state results reported in Table 2 correspond to the steady state in a Markov-perfect equilibrium of a nonstochastic version of the benchmark model. However, because risk-sensitivity generates no risk-adjustment when the model is nonstochastic, the deterministic steady state is the same for all specifications. Allowing the model to be stochastic, but keeping the two risk-sensitivity parameters, θ^h and θ^f, equal to zero, leads to a stochastic steady state in which capital is slightly higher than in the deterministic model (comparing columns 2 and 3). Although capital is higher on average, this does not translate into higher output. Instead, output falls slightly as households lower their supply of labor. Looking at how output is allocated, investment remains relatively unchanged, but a larger share of output is allocated to personal consumption and a smaller share of output is allocated to government services. It follows from

[5] An alternative, equivalent, expression for the one-period real return on equity is $r_{t+1}^e = \frac{K_{t+1} + \left(\alpha e^{z_{t+1}} K_{t+1}^{\alpha} H_{t+1}^{1-\alpha} - I_{t+1}\right)}{K_{t+1}}$, where the term in brackets corresponds to the dividend payment.

Table 2 that the benevolent fiscal authority responds to uncertainty by lowering the income tax rate. The level of the equity premium is due almost entirely to the fact that the returns to equity are subject to income tax.

Table 2: The effects of risk-sensitivity on the steady state

Variable	Deterministic	$\theta^h = \theta^f = 0$	$\theta^h = -0.1, \theta^f = 0$	$\theta^h = \theta^f = -0.1$
Output	1.805	1.802	1.987	1.986
Consumption	1.183	1.189	1.256	1.257
Government	0.299	0.290	0.331	0.330
Investment	0.323	0.324	0.399	0.399
Labor	0.404	0.402	0.416	0.416
Tax rate	0.196	0.196	0.209	0.208
Capital	25.832	25.912	31.919	31.928
Risk-free return	1.010	1.010	0.785	0.784
Equity return	1.265	1.255	0.992	0.991
Equity premium	0.255	0.245	0.207	0.206

Introducing households with risk-sensitive preferences, but retaining $\theta^f = 0$ (corresponding to a paternalistic fiscal authority), leads to substantive changes in the stochastic steady state. In particular, comparing columns 3 and 4 in Table 2 it is evident that risk-sensitivity produces a strong precautionary-saving motive and raises capital in steady state from 25.912 to 31.919, an increase of almost 25 percent. Although hours-worked do rise, the large increase in capital raises considerably the capital-labor ratio, pushing down both the risk-free rate and the return to equity. With capital and labor both rising, the steady state value for output also rises, with this increase in output allocated disproportionately to consumption and, to a lesser extent, investment. Although government spending increases in steady state, it declines as a share of output, signalling a fall in the income tax rate. Thus, the paternalistic fiscal authority's response to risk-sensitive households is to lower the income tax rate, supporting the household's desire to increase capital and facilitating a substitution away from government services and toward private consumption. Turning to the case in which the fiscal authority is "political" ($\theta^f = \theta^h$), it is evident, through comparing columns 4 and 5, that the steady state outcomes are largely unaffected by the fiscal authority's risk-sensitivity.

Where the analysis above considered steady state outcomes, we now investigate the effect that risk-sensitivity has on decision rules, specifically on the household's decision rules for consumption and labor, and on the fiscal authority's decision rules for government spending and the tax rate. We report the decision rules in Figure 1, displaying them as a function of the capital stock only. This is achieved by numerically integrating with respect to the technology shock in order to compute the conditional expectations $\mathrm{E}[C(z_t, K_t)|K_t]$, $\mathrm{E}[G(z_t, K_t)|K_t]$, $\mathrm{E}[\tau(z_t, K_t)|K_t]$, and $\mathrm{E}[H(z_t, K_t)|K_t]$.

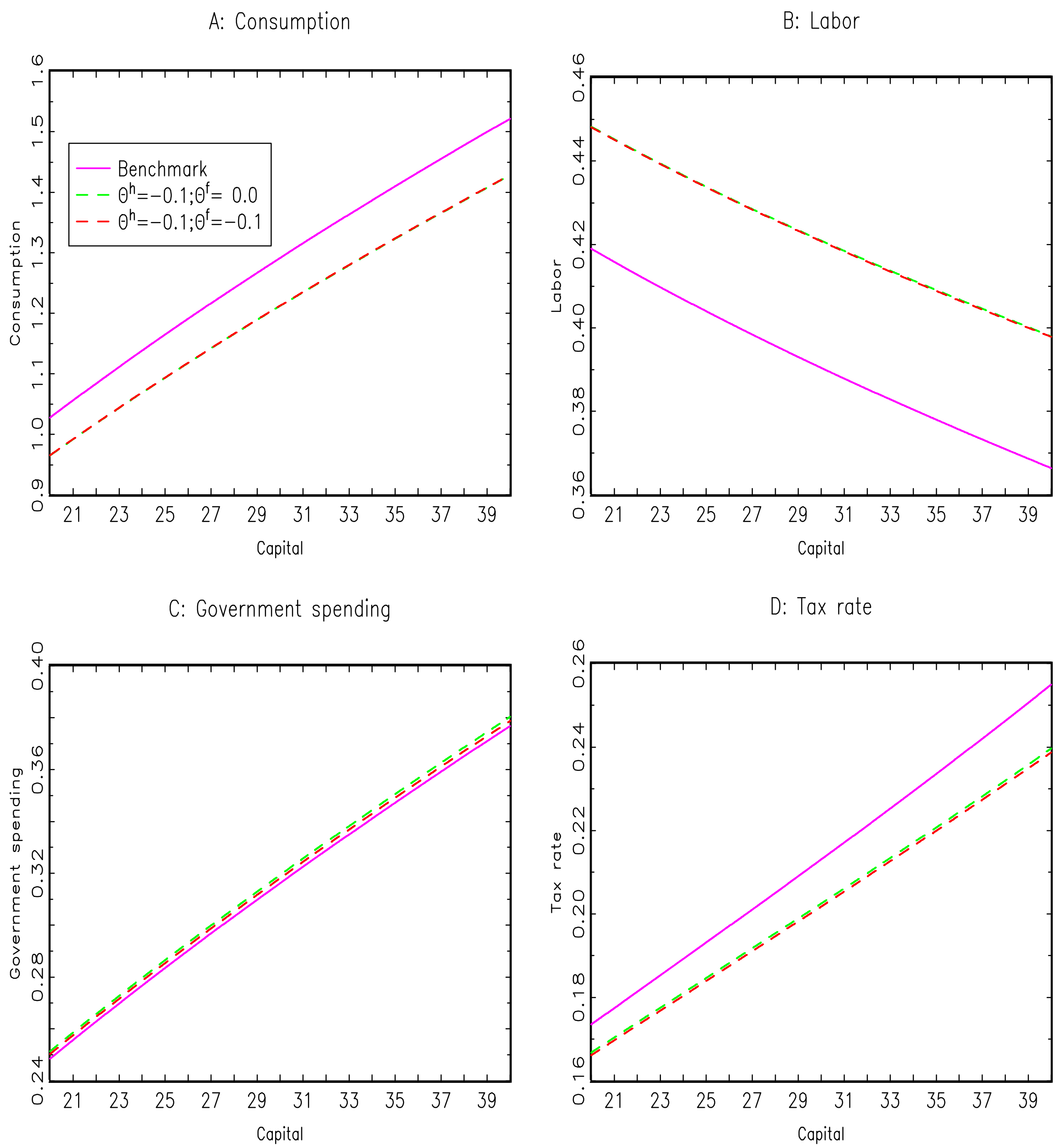

Figure 1: The effects of risk-sensitivity on decision rules

Looking at the household's consumption decision rule shown in Figure 1, panel A, two results are apparent. First, although risk-sensitivity has a large impact on the consumption decision rule, it is the household's risk-sensitivity that matters not the fiscal authority's. Second, the effect of risk sensitivity on the consumption decision rule is to lower the marginal propensity to consume out of wealth (capital), as reflected in the fact that the risk-sensitive decision rules have slightly flatter slopes than the benchmark decision rule. By consuming less from their wealth, a precautionary saving effect induced by risk-sensitivity, households increase their saving, and with the goods saved allocated to investment the outcome is higher capital in equilibrium (as shown in Table 1, above, and in Figure 2, below). The household's labor decision rule (panel B) is also affected materially by the household's risk-sensitivity (and not much by the fiscal authority's risk sensitivity), with the risk-sensitivity raising the household's supply of labor for each level of capital. Accordingly, the household's aversion to risky future utility leads it to increase hours-worked in order to increase savings and thereby boost expected future income.

Turning to the fiscal authority's decision rule for government spending (panel C), unlike consumption and labor, neither the household's risk-sensitivity nor the fiscal authority's risk-sensitivity has much effect on the decision rule for government spending. To the extent that there is an effect, it is to raise government spending at each level of capital. Thus, although muted, the fiscal authority seeks to address the decline in private consumption and the rise in hours-worked with an increase in government services. The response is greater for the "paternalistic" fiscal authority than for the "political" fiscal authority. Although the effects of risk-sensitivity on government spending appear to be small, the effects on the tax rate (Panel D) are somewhat larger, with increasing risk-sensitivity lowering the tax rate schedule. By lowering the tax rate schedule the government assists households in accumulating capital, which provides a buffer against the risk that households fear.

Where Table 2 presented information about (stochastic) steady state outcomes, Figure 2 presents the unconditional densities of the key variables. We construct these unconditional densities by simulating data from each model, linearly interpolating over capital and the technology shock.

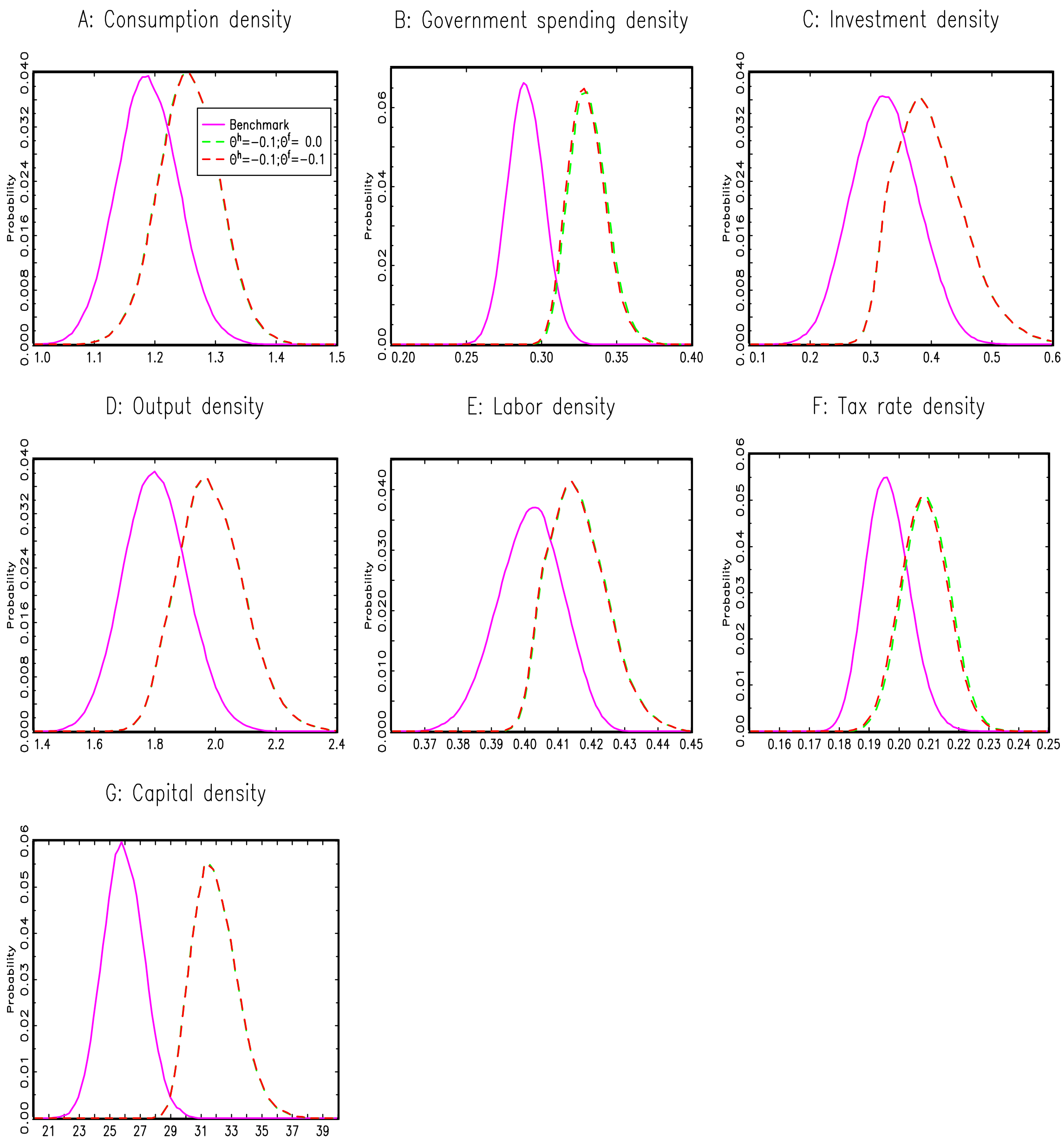

Figure 2: The effects of risk-sensitivity on the asymptotic equilibrium

The densities in Figure 2 reinforce the results in Table 2 and Figure 1. Not only does the fiscal authority's risk-sensitivity have little effect on decision rules or steady state outcomes, but it has little effect on the entire distribution of outcomes, with consumption and government services providing modest exceptions. Figure 2, panel A, shows that the fiscal authority's risk-sensitivity impacts the density for consumption when consumption is low while panels B and F show that the fiscal authority's paternalism shifts the densities for government spending and the tax rate systematically to the right. The household's risk-sensitivity, on the other hand, has a large impact on the densities of all variables. Notably, however, although some changes in volatility can be discerned (and are reported in Table 3 below), Figure 2 makes clear that the principle effect of the household's risk-sensitivity falls on the mean of each variable.

The effect that risk-sensitivity has on the business cycle is examined in Table 3, which focuses on volatilities and correlations, and in Figures 3 and 4, which focus on impulse responses to positive and negative technology shocks, respectively. The benchmark results in Table 3 indicate that private consumption is less volatile than output, that investment is considerably more volatile than output, that the volatility of government spending is about equal to that of private consumption, and that these variables, and labor, are all strongly positively correlated with output.

Table 3: The effects of risk-sensitivity on the business cycle

	Benchmark		$\theta^h = -0.1, \theta^f = 0$		$\theta^h = \theta^f = -0.1$	
Log-variable, X_t	$s.d.(\%)$	$\rho(Y_t, X_t)$	$s.d.(\%)$	$\rho(Y_t, X_t)$	$s.d.(\%)$	$\rho(Y_t, X_t)$
Output	0.0581	1.0000	0.0545	1.0000	0.0546	1.0000
Consumption	0.0428	0.9161	0.0404	0.9179	0.0404	0.9171
Government	0.0418	0.8922	0.0378	0.8867	0.0376	0.8851
Investment	0.1707	0.8724	0.1415	0.8882	0.1415	0.8877
Labor	0.0239	0.8755	0.0209	0.8694	0.0210	0.8688

Allowing households and/or the fiscal authority to be risk-sensitive has little effect on the volatilities and correlations reported in Table 3, a finding that is consistent with Tallarini (2000). To the extent that risk-sensitivity does impact these statistics it serves to lower the volatility of all variables, but most noticeably the volatility of government services and investment. This result makes intuitive sense, because risk-sensitive households are averse to volatility and their aversion induces them to employ decision rules that mitigate volatility.[6]

Figures 3 and 4 display how the key variables respond following a one-standard-deviation shock to aggregate technology; Figure 3 displays responses to a positive shock while Figure 4 displays the responses to a negative shock. In response to a positive technology shock, Figure 3 shows that the benchmark model responds to the improved technology with a rise in investment

[6] At the same time, risk-sensitivity also raises slightly each variable's correlation with output, which, particularly in regard to labor, worsens the model's ability to account for the correlations in the data (as reported in Hansen, 1985, for example). Mechanisms such as indivisable labor (Hansen, 1985) and/or variable capital utilization (Greenwood, Hercowitz, and Huffman, 1988) are likely to improve the model along this dimension.

(panel D) and labor (panel E), rises brought about through the higher marginal productivity of capital and labor, which raises output (panel A). With the technology shock lifting household income, households increase their demand for private consumption (panel B), government services (panel D), and leisure.[7] The tax rate (Panel F), declines immediately following the shock, with the fiscal authority allocating the benefits of increased income partly through higher government spending and partly through lower taxation. Relative to the responses for the benchmark model, the effects of risk sensitivity can be seen largely to lower the responses of private consumption and government services to the shock, consistent with the declines in volatility reported in Table 3.

[7] In equilibrium, however, the household's demand for increased leisure is dominated by the substitution effect by which households are induced by a higher real wage rate to increase their supply of labor. This result is standard in real business cycle models.

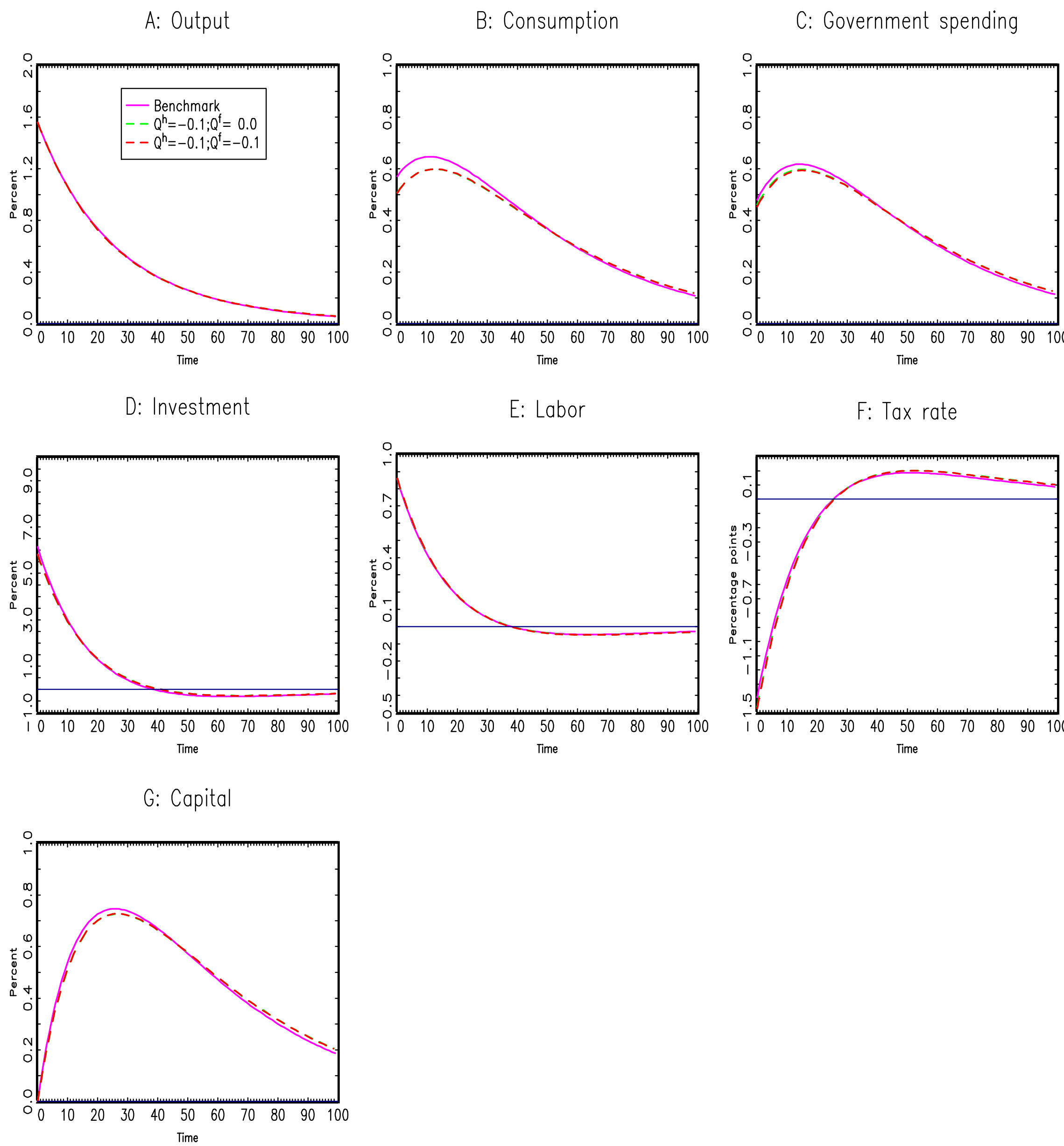

Figure 3: Responses to a positive technology shock (1 s.d.)

Of greater interest are the differences in how the various specifications respond to positive and negative technology shocks. From Figure 4 it is clear, first, that the economy responds asymmetrically to positive and negative technology shocks and, second, that the effects of risk-sensitivity on the model are considerably more pronounced when the technology shock is negative than when it is positive. The finding that risk-sensitivity matters more for negative shocks makes sense when the household's risk-sensitivity is interpreted from a robust control perspective, because this perspective argues that households and the political fiscal authority should design their decision rules in order to guard against a worst-case shock process.

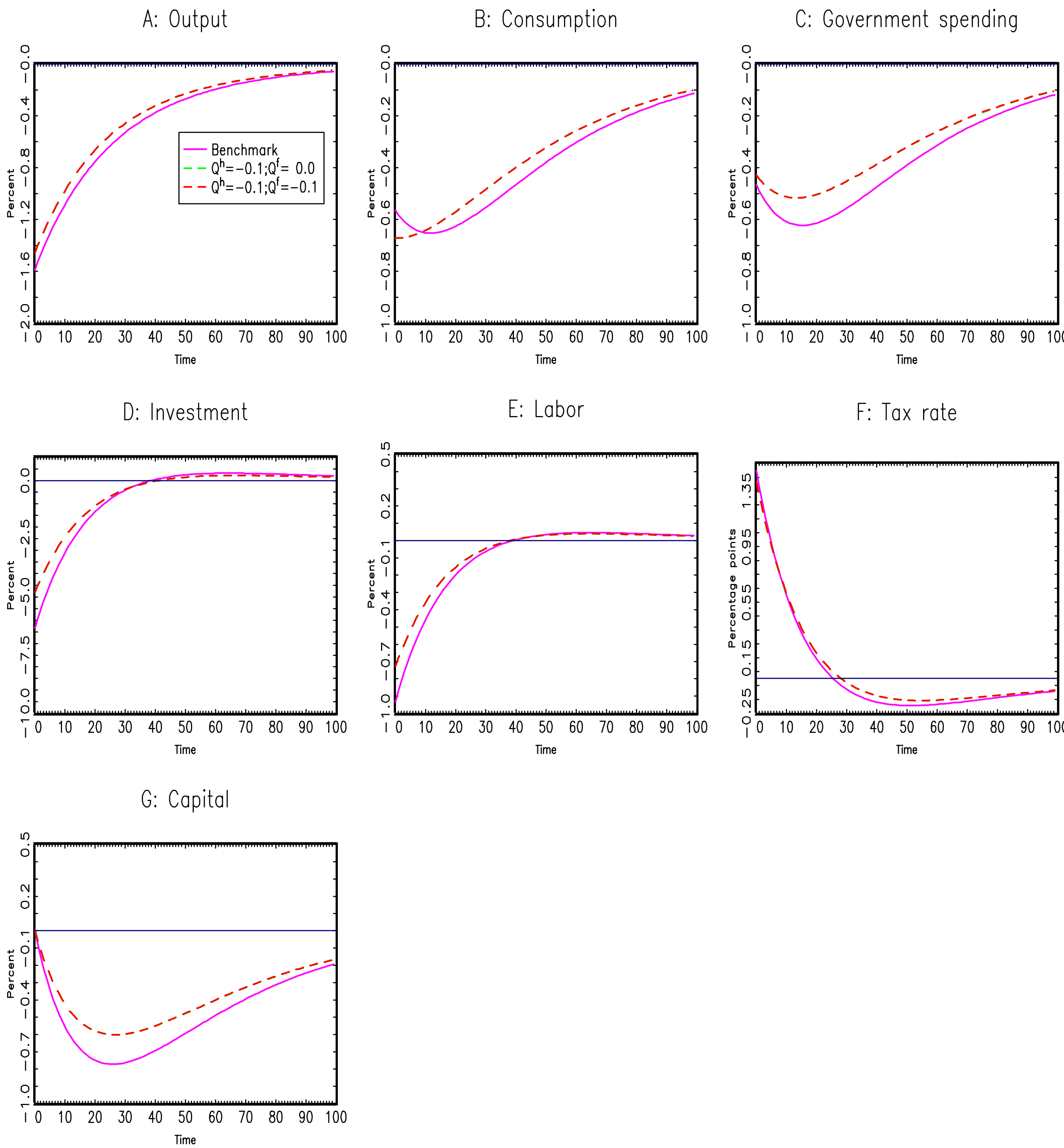

Figure 4: Responses to a negative technology shock (1 s.d.)

Interestingly, relative to the benchmark responses, with risk-sensitive preferences the declines in labor, investment, government spending, and output are all damped on the impact of the (adverse) technology shock. As a consequence, the medium-term decline in capital is also less than that for the benchmark model, which facilitates higher private consumption and higher government spending.

The remainder of this section explores the effect that risk-sensitive preferences have on the risk-free rate, on the return to equity, and on the equity premium, focusing on the case where the economy is populated by a "political" fiscal authority ($\theta^h = \theta^f$). The return on each asset, and the resulting equity premium, are calculated two ways. First we calculate returns holding the discount factor, β, constant at 0.99, while varying the risk-sensitivity parameters, $\theta^h = \theta^f$, between 0 and -0.1. Second, recognizing that the stochastic steady state varies with movements in the risk-sensitivity parameters, we next calculate returns varying $\theta^h = \theta^f$ between 0 and -0.1, while adjusting the discount factor, β, in order to keep the stochastic steady state for capital unchanged from its benchmark value (β adjusted). The results are shown in Figure 5, where the asset-returns and the equity premium are plotted against the absolute value of the risk-sensitivity parameters.

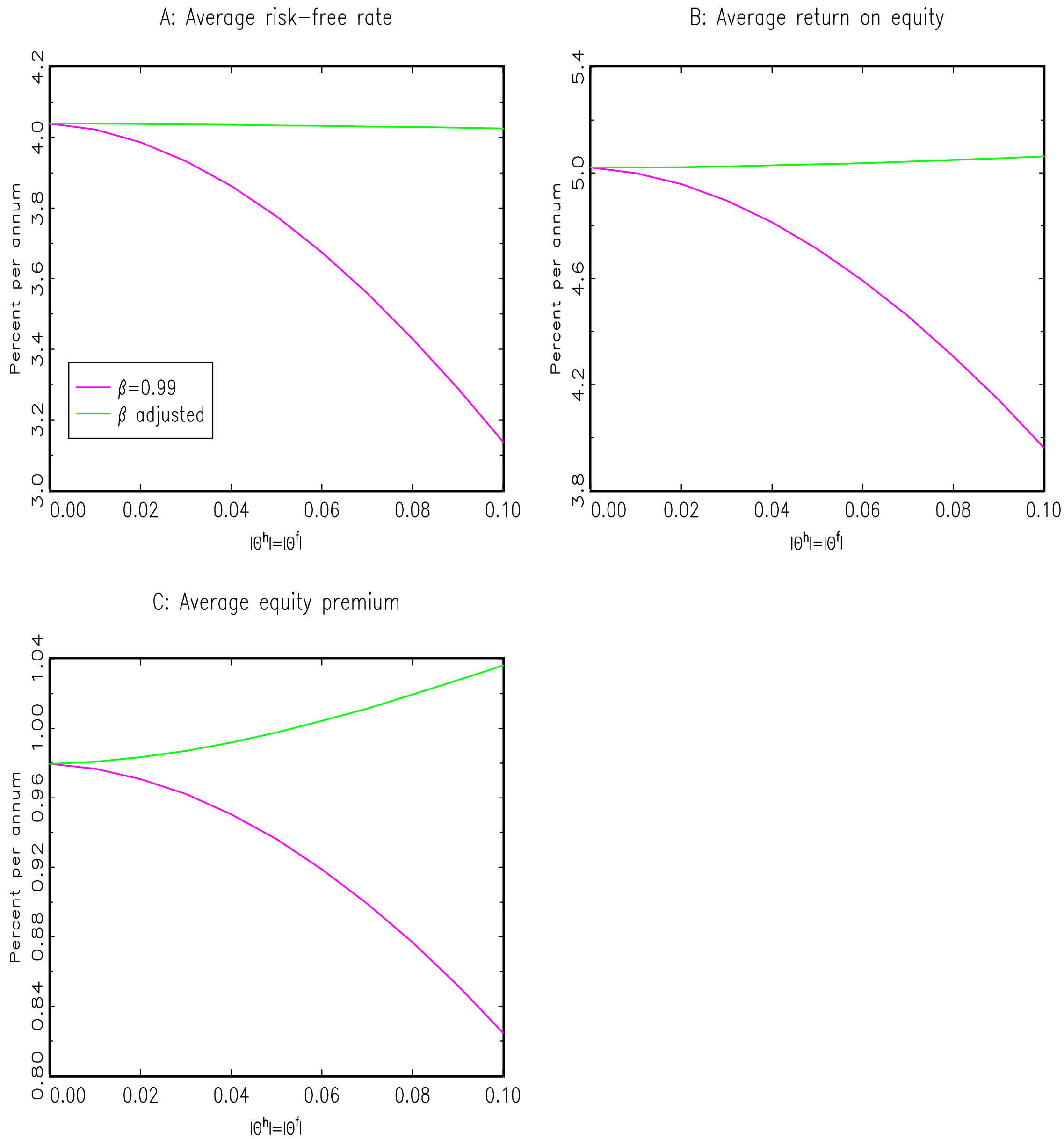

Figure 5: The effects of risk-sensitivity on annualized asset returns and the equity premium

Considering first the asset-returns calculated keeping the discount factor equal to 0.99, while the level of the equity premium is driven by the tax on capital income, it is clear from Figure 5 that the effect of risk-sensitivity is to lower the risk-free rate (panel A), lower the real return on equity (panel B), and to lower the equity premium (panel C). These results arise largely from the increase in the stochastic steady state value for capital that occurs as households become increasingly risk sensitive. As described above, we counteract this increase in steady state capital by adjusting the discount factor, and report the (β adjusted) results in Figure 5 alongside those where β is kept constant. When the discount factor is adjusted to keep unchanged the model's stochastic steady state for capital, the effects of risk sensitivity are to lower slightly the risk-free rate, raise slightly the return to equity, and to raise slightly the equity premium. The results in Figure 5 are consistent with Tallarini (2000) and they suggest that although risk-sensitive preferences may be able to account for a low risk-free rate, that additional mechanisms, such as long-run consumption risk arising through permanent technology shocks, or more importantly, and capital adjustment costs, as per Croce (2006), are needed to explain the equity premium.

5 Sensitivity analysis

The previous section examined the effects of risk-sensitivity on asset returns, the business cycle, and the conduct of fiscal policy under the assumption that the momentary utility function was linear in logs, i.e., under the assumption that $\sigma_c = \sigma_l = \sigma_g = 1$. In this section we briefly investigate whether the main findings of that section are sensitive to alternative values for these parameters. To be specific, we now consider the model's behavior in equilibria in which there is no risk-sensitivity ($\theta^h = \theta^f = 0$), in which the fiscal authority is paternalistic ($\theta^h = -0.1, \theta^f = 0$), and in which the fiscal authority is political ($\theta^h = \theta^f = -0.1$) for parameterizations in which, alternately, $\sigma_c = 5$, $\sigma_l = 2$, and $\sigma_g = 5$. Of interest is not whether the model behaves differently when the curvature parameters are changed, but whether these curvature paramters materially interact with risk-sensitivity. The results are summarized in Table 4, which reports the stochastic steady state outcomes for the various specifications and complements Table 2.

Table 4: Stochastic steady state outcomes: Sensitivity analysis									
	No risk-sensitivity			Paternalistic			Political		
Variable	$\sigma_c = 5$	$\sigma_l = 2$	$\sigma_g = 5$	$\sigma_c = 5$	$\sigma_l = 2$	$\sigma_g = 5$	$\sigma_c = 5$	$\sigma_l = 2$	$\sigma_g = 5$
Output	1.649	1.347	1.958	1.728	1.496	2.340	1.726	1.495	2.340
Consumption	1.037	0.888	0.947	1.047	0.942	1.105	1.047	0.943	1.106
Government	0.322	0.217	0.719	0.299	0.251	0.778	0.354	0.250	0.776
Investment	0.289	0.242	0.292	0.325	0.302	0.458	0.325	0.302	0.458
Labor	0.373	0.300	0.485	0.376	0.312	0.497	0.375	0.312	0.497
Tax rate	0.237	0.197	0.433	0.254	0.211	0.415	0.253	0.210	0.413
Capital	23.133	19.371	23.325	26.024	24.176	36.609	26.012	24.181	36.642
Risk-free return	1.006	1.010	1.009	0.852	0.772	0.617	0.852	0.772	0.617
Equity return	1.317	1.255	1.775	1.142	0.978	1.052	1.140	0.977	1.050
Equity premium	0.311	0.245	0.767	0.290	0.206	0.435	0.288	0.205	0.434

The findings in Table 4 are consistent with those in Table 2. In particular, for each value for the curvature parameters in the momentary utility function the introduction of risk-sensitive households has a large effect on the stochastic steady state outcomes, largely through its impact on capital accumulation. However, although household risk-sensitivity affects importantly the stochastic steady state, whether the fiscal authority is risk-sensitive or not is largely immaterial, as seen in the fact that the statistics for the political fiscal authority are almost identical to those for the paternalistic fiscal authority.

Focusing now on the specification for which $\sigma_c = 5$, Table 5 reports the volatilities and correlations with output of the key variables.

Table 5: The effects of risk-sensitivity on the business cycle when $\sigma_c = 5$						
	No risk-sensitivity		Paternalistic		Political	
Log-variable, X_t	$s.d.(\%)$	$\rho(Y_t, X_t)$	$s.d.(\%)$	$\rho(Y_t, X_t)$	$s.d.(\%)$	$\rho(Y_t, X_t)$
Output	0.0371	1.0000	0.0364	1.0000	0.0363	1.0000
Consumption	0.0121	0.9315	0.0118	0.9338	0.0118	0.9335
Government	0.0556	0.9025	0.0525	0.9050	0.0525	0.9042
Investment	0.1240	0.9357	0.1119	0.9395	0.1119	0.9396
Labor	0.0159	−0.3274	0.0152	−0.3368	0.0152	−0.3362

The results in Table 5 confirm those for the benchmark parameterization examined in section 4. Regardless of whether either households and/or the fiscal authority are risk-sensitive, the volatilities of the key variables and their correlations with output are largely unaffected. And to the extent that they are affected, the effect of risk-sensitivity is to lower volatility and raise the correlations with output. Clearly, in keeping with the benchmark parameterization, the presence of risk-sensitivity has important effects on the stochastic steady state, but negligible effects on the business cycle.

6 Conclusion

This paper examined the effects of risk-sensitivity on the business cycle, fiscal policy, and asset returns in a Markov-perfect equilibrium. Our model is one in which a fiscal authority must use a linear income tax to finance optimally a government consumption good, while lacking the ability to commit and the ability to bond-finance fiscal deficits. Further, our model assumes that households and the fiscal authority are risk-sensitive decisionmakers. This model extends the work of Ortigueira (2006), Klein, Krusell, and Ríos-Rull (2008), and Martin (2010) by introducing risk-sensitivity and aggregate uncertainty and it extends the work of Tallarini (2000) by introducing a risk-sensitive fiscal authority, and by focusing on Markov-perfect equilibria that are computed using a global method.

We find that household risk-sensitivity produces a powerful precautionary-saving motive, which raises importantly the stochastic steady state level of capital. However, although household risk-sensitivity effects the stochastic steady state, its effects on economic volatility are small. Furthermore, we find that while risk-sensitivity has relatively little effect on how the economy responds to positive technology shocks, its effects for the case of negative technology shocks are much larger. In addition, due to its effect on capital's stochastic steady state, we find that increases in household risk-sensitivity serve to lower the risk-free rate, the return to equity, and the equity premium. However, holding the stochastic steady state constant, increases in household risk-sensitivity lower the risk-free rate slightly, raise the return to equity slightly, and increase the equity premium. In regard to the fiscal authority, somewhat surprisingly, the key finding is that the model's behavior is qualitatively and quantitatively unaffected by whether the fiscal authority is risk-sensitive or not.

Although it extends existing work in important dimensions, the model that we have analyzed could feasibly be made still more sophisticated. In future work, we plan to examine whether risk-sensitivity matters more for fiscal policy, asset prices, and the business cycle in models containing features such as variable capital utilization, capital-adjustment costs, monopolistic competition, investment-specific technology shocks, and permanent technology shocks.

Appendix: Computing equilibrium

To compute the Markov-perfect equilibrium we approximate six objects: the process for technology; the decision rules for consumption, labor, and government spending, and the value functions for the household and the fiscal authority. Drawing on Tauchen (1986), we approximate the autoregressive process for technology with a finite-state Markov chain with n^z nodes, $\{z_j\}_{j=1}^{n^z}$. Unlike Tauchen (1986), however, our nodes for technology are not spaced uniformly, but determined as the roots of a Gauss-Hermite polynomial. To approximate the decision rules and value functions we employ Chebyshev polynomials. Thus, we form the approximations

$$\widehat{C}\left(z_t, K_t\right) = \sum_{j=1}^{n^z}\sum_{i=0}^{n^k} w_{ij}^{c}\Gamma_i\left(K_t\right), \tag{20}$$

$$\widehat{H}\left(z_t, K_t\right) = \sum_{j=1}^{n^z}\sum_{i=0}^{n^k} w_{ij}^{h}\Gamma_i\left(K_t\right), \tag{21}$$

$$\widehat{G}\left(z_t, K_t\right) = \sum_{j=1}^{n^z}\sum_{i=0}^{n^k} w_{ij}^{g}\Gamma_i\left(K_t\right), \tag{22}$$

$$\widehat{V}\left(z_t, K_t\right) = \sum_{j=1}^{n^z}\sum_{i=0}^{n^k} w_{ij}^{V}\Gamma_i\left(K_t\right), \tag{23}$$

$$\widehat{v}\left(z_t, K_t\right) = \sum_{j=1}^{n^z}\sum_{i=0}^{n^k} w_{ij}^{v}\Gamma_i\left(K_t\right), \tag{24}$$

where $\Gamma_i\left(K_t\right)$ represents the i'th order Chebyshev polynomial in aggregate capital, n^k represents the order of the Chebyshev polynomial in aggregate capital, and w_{ij}^c, w_{ij}^h, w_{ij}^g, w_{ij}^v, and w_{ij}^V represent the Chebyshev weights employed in each approximation. Notice that equations (20)—(22) exploit the discretization of the technology shock, treating technology as a shifter whose effects are reflected in the weights assigned to the polynomials.

Now, employing equations (20)—(22), we use Tauchen's method to compute the conditional expectation for each node (z_t, K_t), and solve for the fix-point of the system

$$\begin{aligned}
C_t^{-\sigma_c} &= \frac{\beta}{\mathrm{E}_t\left[\exp\left(\theta^h\widehat{v}\left(z_{t+1}, K_{t+1}\right)\right)\right]} \\
&\quad \times \mathrm{E}_t\left[\frac{\exp\left(\theta^h\widehat{v}\left(z_{t+1}, K_{t+1}\right)\right)}{\widehat{C}\left(z_{t+1}, K_{t+1}\right)^{\sigma_c}}\left(\begin{array}{c} 1+\left(1-\frac{\widehat{G}(z_{t+1},K_{t+1})}{e^{z_t}K_{t+1}^{\alpha}\widehat{H}(z_{t+1},K_{t+1})^{1-\alpha}-\delta K_{t+1}}\right) \\ \times\left(\alpha e^{z_{t+1}}K_{t+1}^{\alpha-1}\widehat{H}\left(z_{t+1},K_{t+1}\right)^{1-\alpha}-\delta\right)\end{array}\right)\right], \\
\eta\left(1-H_t\right)^{-\sigma_l} &= \left(1-\frac{G_t}{e^{z_t}K_t^{\alpha}H_t^{1-\alpha}-\delta K_t}\right)\left(1-\alpha\right)e^{z_t}K_t^{\alpha}H_t^{-\alpha}C_t^{-\sigma_c}, \\
K_{t+1} &= \left(1-\delta\right)K_t+e^{z_t}K_t^{\alpha}H_t^{1-\alpha}-C_t-G_t,
\end{aligned}$$

yielding the (approximate) aggregate reaction functions for consumption, $C\left(z_t, K_t, G_t\right)$, and labor, $H\left(z_t, K_t, G_t\right)$. With the aggregate reaction functions for consumption and labor in hand, we now turn to the Bellman equation for the fiscal authority, which using equation (23), is approximated by

$$V\left(z_t, K_t\right) = \max_{\{G_t, K_{t+1}\}} \left[\begin{array}{c} \frac{C(z_t, K_t, G_t)^{1-\sigma_c}-1}{1-\sigma_c} + \frac{\eta(1-H(z_t,K_t,G_t))^{1-\sigma_l}}{1-\sigma_l} + \gamma \frac{G_t^{1-\sigma_g}-1}{1-\sigma_g} \\ +\beta \mathrm{E}_t \left[\widehat{V}\left(z_{t+1}, K_{t+1}\right) \right] \end{array} \right], \tag{25}$$

subject to

$$K_{t+1} = (1-\delta) K_t + e^{z_t} K_t^{\alpha} H\left(z_t, K_t, G_t\right)^{1-\alpha} - C\left(z_t, K_t, G_t\right) - G_t. \tag{26}$$

Substituting equation (26) into the Bellman equation, we use Newton-Raphson to maximize $V\left(z_t, K_t\right)$ with respect to G_t, computing the conditional expectation in equation (25) using Tauchen's method, with this maximization giving rise to the policy rule for government spending $G\left(z_t, K_t\right)$ and the associated value function $V\left(z_t, K_t\right)$. Using $G\left(z_t, K_t\right)$ and the reaction functions for consumption and labor we compute the approximate decision rules $\widehat{C}\left(z_t, K_t\right)$ and $\widehat{H}\left(z_t, K_t\right)$. With these updated decision rules for consumption and labor and equation (24) we compute the household's value function according to the Bellman equation

$$v\left(z_t, K_t\right) = \frac{C\left(z_t, K_t\right)^{1-\sigma_c} - 1}{1-\sigma_c} + \frac{\eta\left(1 - H\left(z_t, K_t\right)\right)^{1-\sigma_l}}{1-\sigma_l} + \gamma \frac{G_t^{1-\sigma_g} - 1}{1-\sigma_g} + \beta \mathrm{E}_t\left[\widehat{v}\left(z_{t+1}, K_{t+1}\right)\right]. \tag{27}$$

By construction, $v\left(z_t, K_t\right) = v\left(z_t, k_t\right)$.

The weights in the Chebyshev polynomials are determined using Chebyshev-regression with capital constrained to the interval $K_t \in [20, 40]$ for the benchmark parameterization. To solve the model under its benchmark parameterization we set $n^z = 5$ and $n^k = 9$. We use 50 solution nodes for the capital stock where these solution nodes are determined from the roots of a Chebyshev polynomial.

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