Commitment vs. Discretion in the UK: An Empirical Investigation of the Monetary and Fiscal Policy Regime*

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Abstract

This paper investigates the conduct of monetary and fiscal policy in the post-ERM period in the UK. Using a simple DSGE New Keynesian model of non-cooperative monetary and fiscal policy interactions under fiscal intra-period leadership, we demonstrate that the past policy in the UK is better explained by optimal policy under discretion than under commitment. We estimate policy objectives of both policy makers. We demonstrate that fiscal policy plays an important role in identifying the monetary policy regime.

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1 Introduction

It has long been known in the literature on monetary policy that if policymakers can precommit to a stabilization plan then they can achieve a significant welfare gain. This is relative to the case of discretionary policy and in an environment where the current decisions of the forward-looking private sector are largely determined by their expectations.[1]

A policymaker who can commit chooses a policy plan once and then follows this policy at all dates in the future. This policy is the best from today's perspective, provided that the precommitment is credible. However, the policy is time-inconsistent and with the passage of time the policymaker will have an incentive to renege. Only a policymaker whose promises are perfectly credible can precommit.

In contrast, a discretionary policy is time-consistent and, as such, is perfectly credible. It is known that the policymaker reoptimizes every period. In the resulting equilibrium, given an opportunity to renege on the expected policy for the next period, the policymaker will find it optimal to choose the same policy for that period. The private sector believes all promises as there are no incentives to renege on them.

A credible commitment policy is able to take advantage of the forward-looking behavior of the agents by allowing them to understand how the policy will react to all circumstances in future periods. It can be formulated in terms of a contingent *intertemporal plan*, and the plan is linked to the initial date and has clear relations between consequent periods. This reflects the ability of the policymaker to manipulate the expectations in a desired way and convince the private sector to coordinate at the best possible intertemporal outcome, linked to the date of precommitment. In contrast, a discretionary policy can rather be described as a set of *intratemporal contingent rules*; the forward-looking private sector recognizes this feature and also

[1] See Kydland and Prescott (1977), Currie and Levine (1993), Woodford (2003a) among many others.

reacts optimally, but it reacts only to the current state, as past promises are ignored.

Although the different properties of these two benchmark policies having been known for decades, the issue of practical implementation of such policies remains controversial. Most discussions concern *monetary policy*. Although there is little doubt that major central banks are able to precommit to a target – as for example an inflation target – the way they actually manage the *expectations of policies* to achieve the target remains underexplored. The key problem with the acceptance of the theoretical concept of commitment policy as a practical option has always been its time-inconsistency. It is well understood that the policymaker will have an incentive to renege at *every* consequent period. This is because the private sector will have done part of the 'work' of the policymaker by setting its expectations in a particular way. From this perspective the policymaker would gain from exploiting the expectations of the private sector. Also, it is because the policymaker may have some additional and sudden 'distractions', like the task of maintaining financial stability. Issues with financial stability may require sudden monetary loosening regardless of the inflation record at the time.

Despite the well understood difficulties with the ability of a central bank to precommit to a policy plan, the statements of major central banks about their practices differ widely. The early statements do not suggest that banks precommit to a plan which is chosen once and forever. In particular, after the Bank of England gained its independence, King (1997) proclaimed a regime of 'constrained discretion'. In these statements the word 'discretion', which does not typically assume an ability to manipulate expectations over time, has rather been used to acknowledge inevitable 'distractions'. On the other hand, the word 'constrained' was meant to mean that the 'distractions' will not dominate. The Bank of England would, therefore, not pursue a short-term gain at the expense of mid-term inflation stability. This was meant to improve the credibility of the policy in the eyes of the private sector. Nothing in King (1997) suggests that the word 'discretion' is meant to exclude the possibility that the Bank of England would not be able to manipulate the private sector's expectations, and use information from a longer period of time, rather than just within the current period. Bernanke and Mishkin (1997) give similar arguments to describe the US monetary policy as discretionary. Givens (2012) and Coroneo, Corradi, and Santos Monteiro (2012) estimate that the Volker-Bernanke period in the US is best described by the discretionary monetary regime.

More recently, the statements of some European central banks have either described their current monetary policy as policy under commitment, or come very close to doing so. The intertemporal feature of a commitment policy is being communicated as a 'predictable response pattern'. See Bergo (2007) for the view of the Norges Bank and Svensson (2009) for policy recommendations for the Riksbank to follow in the footsteps of Norges Bank by generating policy projections as optimal projections. Using medium-scale macro models, Bache, Brubakk, and Maith (2010) for Norges Bank and Adolfson, Laseen, Linde, and Svensson (2011) for Riksbank, find that the past policy of these banks is better explained as optimal policy under commitment than as simple rules.

The recent documents may imply that the Bank of England takes a similar view on the issue (Tucker, 2006; Stockton, 2012). A clear target and a public commitment to anchor inflation expectations in line with this target, together with being understood to be willing to do whatever is necessary to achieve this goal, not just in the current period but in all periods, is critical to achieving credibility. Once credibility is achieved, a central bank that wants to maintain the credibility of its promises would then clearly recognize that reneging on past promises would lead to a loss of credibility. One might interpret this statement that, in effect, the Bank of England was able to *precommit to the policy* and then chose not to renege on its previously chosen *intertemporal* policy. It is an empirical question, however, whether the Bank of England was able to manage the private sector's expectations as if the Bank could not renege on its promises under any circumstances.

Fiscal policy arrangements are much less discussed in the literature, although they may play an important role in identifying the monetary policy regime as well as parameters of the model. Partly because of institutional arrangements, it is believed that fiscal policy is too inflexible to be used for active stabilization. However, recent developments in the world, recent episodes of using fiscal policy as a stabilization device, have shown that there might be a more active role for fiscal policy.[2] A more focussed discussion on the institutional design of stabilizing fiscal policy may not be too far into the future.

The main focus of this paper is the identification of the degree of policy precommitment in the UK. We work with the standard microfounded model of a small open economy.[3] We use the theoretical framework of *non-cooperative* monetary and fiscal discretionary interactions, as in Blake and Kirsanova (2011) and Fragetta and Kirsanova (2010), and we also develop the appropriate theoretical framework for non-cooperative commitment. The policymakers are assumed to minimize the microfounded social welfare loss function except that they can change the relative weight on inflation stabilization and introduce an additional penalty on the excess volatility of policy instruments. We estimate structural parameters of the model and weights of policy objectives under two alternative assumptions about the policymakers' degree of precommitment using the Bayesian approach (see e.g. An and Schorfheide, 2007).

We demonstrate that the monetary and fiscal policy regime in the UK under the assumption of fiscal leadership can best be described by a regime

[2] Recent examples include Economic Stimulus Act 2008, American Recovery and Reinvestment Act of 2009 in the US, and the establishment of the Office of Budget Responsibility in the UK. See Osborne (2008), see also Wyplosz (2005) and Kirsanova, Leith, and Wren-Lewis (2006) for more formal discussion.

[3] We build on models in Gali and Monacelli (2005, 2008), Lubik and Schorfheide (2005, 2007) and Justiniano and Preston (2010a,b).

of optimal policy under discretion: the probability that the actual data were generated by a model with optimal commitment policy, rather than by a model with optimal discretionary policy, is less than 1.0%. Both policymakers put a smaller weight on inflation stabilization than is socially optimal, and the fiscal policymaker pays much less attention to inflation stabilization than the monetary policymaker. We assess the empirical fit of an optimizing microfounded model based on first and second order moments and use DSGE-VAR methodology (Del Negro and Schorpfheide, 2004) to investigate the degree of misspecification of the model under different policies. In particular, we show that the DSGE model imposes useful restrictions to improve the in-sample predictive properties of the Bayesian VAR model. Finally, we demonstrate that the fiscal solvency constraint plays an important role as an identifying restriction for both fiscal and monetary policy reactions as well as model parameters: excluding the fiscal block from the system leads to greater degree of misspecification of the pure monetary model.

The focus of this paper is different from the one of Fragetta and Kirsanova (2010). We identify the degree of policy precommitment, while Fragetta and Kirsanova (2010) identify the degree of leadership and work with a discretionary model only.[4] Unlike Givens (2012) we use a microfounded model, and account for non-cooperative monetary and fiscal policy interactions which allows a more complete description of the UK macroeconomic policy regime. Finally, different from Fragetta and Kirsanova (2010) and Givens (2012) we process the data in a different way: following Lubik and Schorfheide (2005) we introduce a non-stationary world-wide technology shock which substantially reduces the model misspecification.

This paper is organized as follows. In the next section we outline the model and describe policy interactions under commitment and discretion. Section 3 explains the empirical methodology, the choice of priors and the

[4] In contrast to the model in Fragetta and Kirsanova (2010), our model accounts for habit persistence and inflation inertia.

data. The results are discussed in section 4 and section 5 concludes. Appendices contain details of derivations and the theoretical framework for the two policy regimes in a general rational expectations linear-quadratic framework.

2 The Model

We build on models by Gali and Monacelli (2005, 2008), Lubik and Schorfheide (2005, 2007), Justiniano and Preston (2010a,b) modified to include fiscal policy. The following section presents key structural equations of a small open economy model, which allows for habit formation and price indexation.

2.1 Households

The economy is populated by a unit-continuum representative household, by a unit-continuum monopolistically competitive firm, and by two policymakers: the government and the central bank.

Each household k maximizes the following objective:

$$W = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{(X_t^k / A_{Wt})^{1-\sigma}}{1-\sigma} + \chi \frac{(G_t / A_{Wt})^{1-\sigma}}{1-\sigma} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right). \tag{1}$$

Here $X_t^k = C_t^k - h\varkappa C_{t-1}$ is the habit-adjusted consumption, $C_{t-1} \equiv \int_0^1 C_{t-1}^k dk$ is the cross-sectional average of consumption, N_t is labour supply of a representative household and G_t is consumption of public goods. Parameter $0 \leq h \leq 1$ measures the degree of habit persistence, parameter β is the household discount rate, φ is the elasticity of labour supply and σ is the coefficient of relative risk aversion. Parameter χ is the scaling factor for the utility of consuming public goods. In order to guarantee that the model has a balanced growth path, we assume that households derive utility from effective consumption relative to the world-wide level of technology, A_{Wt}. The technology shock A_{Wt} is non-stationary, with growth rate $z_t = A_{Wt}/A_{Wt-1}$. Parameter $\varkappa$ is the steady state value of z_t.

Household k's consumption, C_t^k, is an aggregate of the continuum of goods $i \in [0,1]$ produced in the home country (indexed H) and abroad (indexed F)

$$C_t = \left((1-\alpha)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}},$$

where $0 \le \alpha < 1$ is the import share and $\eta > 0$ is the intratemporal elasticity of substitution between home and foreign consumption goods. C_H is a composite of domestically produced goods given by

$$C_{Ht} = \left(\int_0^1 C_{Ht}(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where z denotes the good's type or variety and ϵ is the intratemporal substitution between domestically produced goods. Similarly, the aggregate C_{Ft} is an aggregate across overseas countries i

$$C_{Ft} = \left(\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}},$$

where C_{it} is an aggregate similar to (2). Households allocate aggregate expenditures based on the demand functions:

$$C_{Ht} = (1-\alpha) \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t \text{ and } C_{Ft} = \alpha \left(\frac{P_{Ft}}{P_t} \right)^{-\eta} C_t,$$

where P_{Ht}, P_{Ft} are domestic and foreign goods price indices and

$$P_t = \left((1-\alpha) P_{Ht}^{1-\eta} + \alpha P_{Ft}^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

is the consumption-based price index.

Consumers face the following aggregated budget constraint

$$P_t C_t^k + \mathbb{E}_t\{Q_{t,t+1} A_{t+1}^k\} = A_t^k + (1-\tau_t) \left(W_t N_t^k + \Upsilon_t^k \right) + T_t$$

where A_{t+1}^k is the nominal payoff of portfolio held at the end of period t, W_t are wages, τ_t is the income tax rate and Υ are profits, T_t are lump-sum transfers paid by the government. $Q_{t,t+1}$ is the stochastic discount factor for one-period-ahead payoffs.

In the maximization problem households take the processes for C_{t-1}, W_t, T_t and the initial asset position A^k_{-1} as given. The optimization produces the standard first order conditions

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \left(\frac{P_t A_{Wt}}{P_{t+1} A_{Wt+1}} \left(\frac{X_t / A_{Wt}}{X_{t+1}/A_{Wt+1}} \right)^{\sigma} \right) \tag{3}$$

$$\frac{W_t}{P_t} = A_{Wt} \frac{\left(X_t / A_{Wt}\right)^{\sigma} N_t{}^{\varphi}}{(1-\tau_t)} \tag{4}$$

where $1+i_t = \left(\mathbb{E}_t\{Q_{t,t+1}\}\right)^{-1}$ is the gross return on a riskless one period bond paying off a unit of domestic currency in period $t+1$. We omit superscript k as all households are identical.

2.2 Firms

Domestic differentiated goods are produced by monopolistically competitive firms, which use labour as the only factor of production. The production technology is given by

$$Y_t\left(i\right) = A_{Wt} A_{Ht} N_t\left(i\right) \tag{5}$$

where $Y_t\left(i\right)$ is the amount of output produced by firm i in period t, $N_t\left(i\right)$ is the amount of labour employed by firm i in period t, and A_{Ht} is home-specific stationary technology shock.

We assume the familiar Calvo-type price setting (Calvo, 1983). A firm will not reset the price the next period with given probability θ. When firm i does not reset price, the price is costlessly adjusted with steady state rate of inflation Π. When firm i resets price, with probability $1-\zeta$ it chooses price P^f_{Ht} which maximizes

$$\max_{P^f_{Ht}(i)} \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(Y_{t+k}\left(i\right) P^f_{Ht}\left(i\right) \Pi^k - W_{t+k} N_{t+k}\left(i\right) \right)$$

subject to demand system

$$Y_{t+k}(i) = \left[\frac{p_{Ht}(i)\,\Pi^k}{P_{Ht+k}}\right]^{-\epsilon} Y_{t+k}$$

and production function (5). The solution to this optimization problem is given by

$$\frac{P^f_{Ht}}{P_{Ht}} = \frac{\epsilon}{\epsilon-1}\frac{K_{1t}}{K_{2t}}, \tag{6}$$

where K_{1t} and K_{2t} satisfy

$$\gamma\beta K_{1t+1}\left(\frac{P_{Ht+1}}{\Pi P_{Ht}}\right)^{\epsilon} = K_{1t} - \left(\frac{X^k_t}{A_{Wt}}\right)^{-\sigma}\frac{Y_t}{A_{Wt}}\frac{W_t}{P_t A_{Wt} A_{Ht}}, \tag{7}$$

$$\gamma\beta K_{2t+1}\left(\frac{P_{Ht+1}}{\Pi P_{Ht}}\right)^{\epsilon-1} = K_{2t} - \left(\frac{X^k_t}{A_{Wt}}\right)^{-\sigma}\frac{P_{Ht}}{P_t}\frac{Y_t}{A_{Wt}}. \tag{8}$$

When a firm resets price, then with probability ζ it chooses the new price P^b_{Ht} according to a simple rule of thumb

$$P^b_{Ht} = P^*_{Ht-1}\Pi_{Ht-1}, \tag{9}$$

where index of the reset prices P^*_{t-1} is given by

$$P^{*1-\epsilon}_{Ht} = (1-\zeta)\left(P^f_{Ht}\right)^{1-\epsilon} + \zeta\left(P^b_{Ht}\right)^{1-\epsilon}. \tag{10}$$

With share θ of firms keeping last period's price and share $(1-\theta)$ of firms setting a new price, the law of motion of aggregate price index P_{Ht} is

$$P^{1-\epsilon}_{Ht} = (1-\theta)\left(P^*_{Ht}\right)^{1-\epsilon} + \theta\left(\Pi P_{Ht-1}\right)^{1-\epsilon}. \tag{11}$$

Finally, the evolution of price dispersion $\Delta_t = \int_0^1 \left(\frac{p_{Ht}(i)}{P_{Ht}}\right)^{-\epsilon} di$ is given by

$$\Delta_t = (1-\theta)(1-\zeta)\left(\frac{P^f_{Ht}}{P_{Ht}}\right)^{-\epsilon} + (1-\theta)\zeta\left(\frac{P^b_{Ht}}{P_{Ht}}\right)^{-\epsilon} + \theta\left(\frac{P_{Ht}}{P_{Ht-1}}\right)^{\epsilon}\Delta_{t-1}. \tag{12}$$

2.3 Risk-sharing, Market Clearing and Private Sector Equilibrium

The bilateral terms of trade measure foreign country goods prices relative to home goods prices. The effective terms of trade S_t are given by

$$S_t = \frac{P_{Ft}}{P_{Ht}},$$

and the real exchange rate $\mathcal{Q}_t$ is defined as

$$\mathcal{Q}_t = \frac{P_t^* \mathcal{E}_t}{P_t},$$

where $\mathcal{E}_t$ is the nominal exchange rate. Assuming that the home country is small and the law of one price holds we obtain

$$S_t \equiv \frac{P_{Ft}}{P_{Ht}} = \frac{P_t^* \mathcal{E}_t}{P_{H,t}} = \frac{P_t}{P_{H,t}} \frac{P_t^* \mathcal{E}_t}{P_t} = \frac{P_t}{P_{H,t}} \mathcal{Q}_t.$$

The Euler equation for the rest of the world can be written as

$$\frac{1}{1+i_t^*} = \beta \mathbb{E}_t \left(\frac{X_t^*/A_{Wt}}{X_{t+1}^*/A_{Wt+1}} \right)^{\sigma} \left(\frac{P_t^*}{P_{t+1}^*} \right).$$

Combining two consumption Euler equations with the uncovered interest rate parity

$$\frac{1+i_t}{1+i_t^*} = \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t},$$

yields the international risk sharing relationship

$$\frac{X_t/A_{Wt}}{X_t^*/A_{Wt}} \mathcal{Q}_t^{-\frac{1}{\sigma}} = \frac{X_{t+1}/A_{Wt+1}}{X_{t+1}^*/A_{Wt+1}} \mathcal{Q}_{t+1}^{-\frac{1}{\sigma}}.$$

Goods market clearing requires

$$Y_t(j) = C_{Ht}(j) + \int_0^1 C_t^i(j) di + G_t(j),$$

where

$$C_{Ht}^i(j) = \alpha \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} \left(\frac{P_{Ht}}{\mathcal{E}_{it}P_t^i}\right)^{-1} C_t^i.$$

The allocation of government spending across goods is determined by the minimization of total costs,

$$G_t(j) = \left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} G_t.$$

Substituting everything into the market clearing condition yields

$$Y_t(j) = \left(\frac{P_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} \left[(1-\alpha)\frac{P_t C_t}{P_{Ht}} + \alpha \int_0^1 \frac{\mathcal{E}_{it}P_t^i C_t^i}{P_{Ht}} di + G_t\right].$$

Aggregation, using $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$ yields the aggregate demand equation

$$Y_t = (1-\alpha)\frac{P_t}{P_{Ht}} C_t + \alpha C_t^* + G_t. \tag{13}$$

Similarly, aggregation of production function (5) yields the aggregate production function

$$Y_t = A_{Wt} A_{Ht} \frac{N_t}{\Delta_t}. \tag{14}$$

We assume that all public debt consists of riskless one-period bonds. Therefore, the nominal value of end-of-period public debt $\mathcal{B}_t$ evolves according to the following law of motion:

$$\mathcal{B}_t = (1 + i_{t-1})\,\mathcal{B}_{t-1} + P_{Ht}G_t - \tau_t P_{Ht} Y_t. \tag{15}$$

For analytical convenience, we define $B_t = \frac{(1+i_{t-1})\mathcal{B}_{t-1}}{P_{t-1}A_{Wt}}$ as a measure of real government debt. Because A_{Wt} and B_t are observed at the beginning of period t, equation (15) can be rewritten as

$$B_{t+1}\frac{A_{Wt+1}}{A_{Wt}} = (1 + i_t)\left(B_t \frac{P_{Ht-1}}{P_{Ht}} - \tau_t \frac{Y_t}{A_{Wt}} + \frac{G_t}{A_{Wt}}\right). \tag{16}$$

Finally, the private sector equilibrium $\{\frac{X_t}{A_{Wt}}, \frac{W_t}{P_t}, N_t, Y_t, K_{1t}, K_{2t}, p_{Ht}^f, P_t^b, P_t^*, \Pi_t, \Delta_t, B_t\}$ is determined by equations (3), (4), (6), (7), (8), (9), (10), (11), (12), (13), (14), (16).

2.4 Linearization

We proceed by log-linearizing the model equations around the balanced growth path. Our model imposes common steady state real interest rates, inflation rates, growth rates and technologies. Because the model contains a non-stationary component, the world-wide technology shock A_{Wt}, we detrend the affected variables by their specific growth components beforehand.

We denote by lower-case letters the stationary transformation of corresponding variables, by dividing them by a common numeraire, A_{Wt}. Therefore, we denote $x_t = \frac{X_t}{A_{Wt}}$, $y_t = \frac{Y_t}{A_{Wt}}$, $g_t = \frac{G_t}{A_{Wt}}$, $c_t = \frac{C_t}{A_{Wt}}$. We present the model in a form where all variables are in log-deviations from the steady state, and for any variable u_t with steady state $\bar{u}$ we denote $\hat{u}_t = \log\left(\frac{u_t}{\bar{u}}\right)$.

The linearized system which describes the evolution of the economy can be written as

$$\begin{aligned}
\hat{\pi}_{Ht} &= \beta\frac{\theta}{\Phi}\mathbb{E}_t\hat{\pi}_{Ht+1} + \frac{\zeta}{\Phi}\hat{\pi}_{Ht-1} + \frac{\lambda}{\Phi}\Big(\sigma\hat{x}_t + \varphi\hat{y}_t + \alpha\hat{S}_t \\
&\quad - (\varphi+1)\hat{A}_{Ht}\Big) + \eta_{\pi t} \qquad (17)\\
\hat{x}_t &= \mathbb{E}_t\hat{x}_{t+1} - \frac{1}{\sigma}\left(\hat{\imath}_t - \mathbb{E}_t\hat{\pi}_{Ht+1} + \alpha\hat{S}_t - \alpha\mathbb{E}_t\hat{S}_{t+1} - \mathbb{E}_t\hat{z}_{t+1}\right) \qquad (18)\\
\hat{S}_t &= \frac{\sigma}{(1-h)(1-\alpha)}\left(\hat{c}_t - h\hat{c}_{t-1} - \hat{c}^*_t + h\hat{c}^*_{t-1}\right) \qquad (19)\\
\hat{x}_t &= \frac{1}{(1-h)}(\hat{c}_t - h\hat{c}_{t-1}) + \frac{h}{(1-h)}\hat{z}_t \qquad (20)\\
\hat{y}_t &= \alpha(2-\alpha)\eta\hat{S}_t + (1-\alpha)\hat{c}_t + \alpha\hat{c}^*_t + \frac{\bar{g}}{\bar{c}}\hat{g}^y_t \qquad (21)\\
\tilde{b}_{t+1} &= \frac{1}{\beta}\left(\tilde{b}_t - \frac{\bar{B}}{\bar{y}\Pi}\hat{\pi}_{Ht} + \frac{\bar{g}}{\bar{y}}\hat{g}^y_t - \left(\tau - \frac{\bar{g}}{\bar{y}}\right)\hat{y}_t\right) + \frac{\bar{B}}{\bar{y}\Pi}\hat{\imath}_t \qquad (22)\\
&\quad -\frac{\bar{B}}{\bar{y}\Pi}\mathbb{E}_t\hat{z}_{t+1}\\
\Psi_t &= \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \qquad (23)\\
\Omega_t &= \hat{g}^y_t - \hat{g}^y_{t-1} + \eta_{gt} \qquad (24)
\end{aligned}$$

Here $\hat{g}^y_t = \hat{g}_t - \hat{y}_t$ is spending to output ratio, or the government share, and $\tilde{b}_t = \frac{\bar{B}}{\bar{y}\Pi}\log\left(\frac{B_t}{\bar{B}}\right)$ is a measure of real debt. Variables Ψ_t and Ω_t are the growth rate of output and of the government share correspondingly. Parameters are $\Phi = \zeta + \theta - \zeta\theta + \theta\beta\zeta$, $\lambda = (1-\theta)(1-\theta\beta)(1-\zeta)$. All microfounded shocks are assumed to follow AR(1) process:

$$\hat{A}_{Ht} = \rho_a \hat{A}_{Ht-1} + \eta_{at} \tag{25}$$

$$\hat{c}_t^* = \rho_c \hat{c}_{t-1}^* + \eta_{ct} \tag{26}$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \eta_{zt} \tag{27}$$

where η_{at}, η_{ct} and η_{zt} are i.i.d. Note that additional to three microfounded shocks, $\hat{A}_{Ht}$, $\hat{z}_t$, and $\hat{c}_t^*$, we have added shocks $\eta_{\pi t}$ and η_{gt} into the final specification of the system. Shock $\eta_{\pi t}$ captures inefficient variations in mark ups. It is assumed to be an i.i.d. to allow easier identification of the degree of inflation inertia. Shock η_{gt} captures the non-systematic part of fiscal policy, the discrepancy between the observed government share and the unobservable policy instrument. As a measurement error, η_{gt} is assumed to be i.i.d.

2.5 Policy

2.5.1 Social Welfare

The aggregated household utility (1) implies the following social welfare loss function

$$\begin{aligned} W &= \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\pi_{Ht}^2 + \frac{\zeta}{\theta(1-\zeta)} (\pi_{Ht} - \pi_{Ht-1})^2 \right. \\ &\quad \left. + L\left(\hat{x}_t, \hat{c}_t, \hat{y}_t\ , \hat{g}_t^y, \hat{A}_{Ht}, \hat{c}_t^*, \hat{c}_{t-1}^*\right)\right) \end{aligned}$$

where $L\left(\hat{x}_t, \hat{c}_t, \hat{y}_t\ , \hat{g}_t^y, \hat{A}_{Ht}, \hat{c}_t^*, \hat{c}_{t-1}^*\right)$ is a collection of quadratic terms which can be rearranged to describe 'gap' targets, see Appendix A.

2.5.2 Policy Objectives

It is often suggested that realistic policymakers are not benevolent. There are some theoretical reasons for introducing additional objectives and for distorting social weights of a *discretionary* policymaker. A discretionary monetary policymaker can reduce the 'stabilization bias'[5] in several ways. For example, Woodford (2003b) demonstrates that if the discretionary monetary policymaker adopts an additional interest rate smoothing target then the policy becomes 'history-dependent' and the dynamics of the economy under discretion is similar to the one under commitment policy, with higher level of social welfare attained. A similar result is demonstrated in Vestin (2006): introducing the price level target into the monetary policymaker's objectives improves the social welfare too. Also, following the famous result of Barro and Gordon (1983), Clarida, Galí, and Gertler (1999) show that the discretionary monetary authority which puts higher than socially optimal weight on inflation stabilization target can achieve the same level of welfare as under the optimal precommitment-to-rules policy.[6]

The policymakers may not be benevolent because of some institutional restrictions, both under commitment and discretion. For example, fiscal instrument smoothing may result from fiscal policy being 'delayed'. All spending decisions should pass the parliament scrutiny. In order to avoid large changes in inappropriate times, it may be optimal to propose only relatively small changes. In our framework, such policy can be described by introducing a penalty on the change of fiscal instrument. Similarly, a debt target can reflect some international agreements. It can also help to avoid large risk premium, which cannot be described by this model.

To account for possible delegation schemes and institutional restrictions, we assume a more general form of the monetary policymakers' objective function in our empirical part of the paper

$$\begin{aligned} W^M &= \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\Phi_{\pi M} \left(\hat{\pi}_{Ht}^2 + \frac{\zeta}{\theta (1-\zeta)} \left(\hat{\pi}_{Ht} - \hat{\pi}_{Ht-1} \right)^2 \right) \right. \\ & \left. + L \left(\hat{x}_t, \hat{c}_t, \hat{y}_t \, , \hat{g}_t^y, \hat{A}_{Ht}, \hat{x}_t^*, \hat{c}_t^* \right) + \Phi_{\Delta I} \left(\Delta \hat{\imath}_t \right)^2 \right). \end{aligned}$$

[5]See Svensson (1997).

[6]For other policy delegation proposals see e.g. Svensson (1997), Walsh (2003) and Woodford (2003b).

Here we add additional interest rate smoothing weight $\Phi_{\Delta I}$ and allow for 'inflation conservatism' $\Phi_{\pi M}$.

We adopt the same general form of policy objectives for fiscal policy:

$$\begin{aligned} W^F &= \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\Phi_{\pi F} \left(\hat{\pi}_{Ht}^2 + \frac{\zeta}{\theta(1-\zeta)} \left(\hat{\pi}_{Ht} - \hat{\pi}_{Ht-1} \right)^2 \right) \right. \\ & \left. + L\left(\hat{x}_t, \hat{c}_t, \hat{y}_t\ , \hat{g}_t^y, \hat{A}_{Ht}, \hat{x}_t^*, \hat{c}_t^* \right) + \Phi_{\Delta G} \left(\Delta \hat{g}_t^y \right)^2 + \Phi_B b_t^2 \right), \end{aligned}$$

where we account for 'inflation conservatism' $\Phi_{\pi F}$ and instrument smoothing $\Delta \hat{g}_t^y$, but also add debt target Φ_B. Both policymakers are not assumed to modify social 'gap' targets $L\left(\hat{x}_t, \hat{c}_t, \hat{y}_t\ , \hat{g}_t^y, \hat{A}_{Ht}, \hat{x}_t^*, \hat{c}_t^* \right)$; their change might move us too far from the original microfounded criterion and might not allow us to make a simple interpretation of results.[7]

2.5.3 Strategic Interactions

We assume that the monetary policymaker uses the nominal interest rate, i_t, as its instrument, and the fiscal policymaker uses the government share, g_t^y.[8] Both policymakers act *non-cooperatively* in order to stabilize the economy against shocks.

We assume that the fiscal policymaker acts as an intraperiod leader and the monetary authority acts as an intraperiod follower. (Fragetta and Kirsanova (2010) show that the model of fiscal leadership gives better fit to the UK data than the model of simultaneous moves under discretion.) This assumption implies that the leader, the fiscal authority, knows the reaction function of the monetary authority and takes it into account when formulating policy.

If both policy authorities are benevolent, and the steady state level of debt is not too high, then in the resulting equilibrium the optimizing mon-

[7]See Dennis (2006), Ilbas (2010) and Givens (2012) who directly estimate the relative weights of the authority's *ad hoc* loss function.

[8]We follow Gali and Monacelli (2005, 2008) in the choice of fiscal instrument. The empirical evidence in Favero and Monacelli (2005), Taylor (2000) and Auerbach (2003), for example, also suggests that government spending does move.

etary policymaker reacts to inflation nearly in the same way as if the debt accumulation problem were not the part of the problem, and the optimizing fiscal policymaker adjusts the fiscal instrument to keep debt under control, see e.g. Blake and Kirsanova (2011). Moreover, the level of debt is optimally brought back only slowly – under commitment it is just not allowed to explode – so the optimal volatility of fiscal instrument is relatively small. This optimal 'division of responsibilities', where the burden of economic stabilization is optimally carried by the monetary policymaker, describes the standard case of benevolent policymakers.[9]

Once the policy objectives are made distinct, the optimizing policymakers may engage into a fight, each trying to offset the harm done by the other. This fight, however, is more likely to happen if the authorities move simultaneously, see Dixit and Lambertini (2003) and Blake and Kirsanova (2011). The regime of fiscal leadership considered here is likely to mitigate the conflict. If the fiscal policymaker has lesser need to stabilize inflation but prefers faster stabilization of output gap than the monetary policymaker does, it knows that an increase in government spending will not produce higher output but just higher interest rate, so the fiscal policymaker optimally refrains from moving the fiscal instrument excessively, and concentrates on debt stabilization. If the fiscal authority is able to conduct itself as an intraperiod leader, then it will willingly allow the monetary authority to carry out almost all of the required macroeconomic stabilization.

Policymakers who act under commitment are able to manipulate expectations of the private sector along the whole dynamic path. The monetary policymaker takes the state of fiscal policy as given and precommits to policy taking into account the forward-looking behavior of the private sector and the evolution of predetermined states. The fiscal policymaker also takes into account the forward-looking behavior of the private sector and the evolution

[9] See the discussion in Kirsanova, Leith, and Wren-Lewis (2009). If both policymakers are benevolent and there is a unique equilibrium then the leadership does not matter, of course.

of predetermined states, but it also takes into account the reaction function of the monetary policymaker. Both policymakers precommit once, in the initial period, and then follow their (initially optimal) plans. The private sector's expectations are among the policymakers' choice variables so that the best-at-point-of-precommitment outcome is achieved. The formal treatment of non-cooperative optimization under commitment is relegated into Appendix B.

In contrast, discretionary policymakers reoptimize (or change office) every period, and the forward-looking sector knows this. As a result, all agents choose their optimal reactions as functions of current predetermined states only.[10] The optimal reaction rule of the private sector feeds back on all observed states, including policy. The monetary policymaker takes into account this reaction function of the private sector, as well as predetermined states and fiscal policy. The fiscal policymaker takes into account reaction rules of the monetary policymaker and the private sector. Although the policymakers cannot affect expectations of the private sector to the extent available under commitment, they can influence the endogenous predetermined state, which evolution is taken into account by the private sector. Similarly, the expectations of the monetary policymaker are affected by actions of the fiscal policymaker through their effect on endogenous predetermined states. The formal treatment of non-cooperative optimization under discretion is presented in Blake and Kirsanova (2011).

Solving the optimization problem yields the following policy reaction functions:

$$\begin{aligned} \hat{\imath}_t &= r_z\eta_{z,t} + r_a\eta_{a,t} + r_{y^*}\eta_{y0^*,t} + r_{y1^*}\eta_{y^*,t-1} + r_r\hat{\imath}_{t-1} + r_g\hat{g}^y_{t-1} && (28)\\ &\quad + r_\pi\hat{\pi}_{Ht-1} + r_c\hat{c}_{t-1} + r_b\tilde{b}_t + r_\Lambda\Lambda_{mt} \\ \hat{g}^y_t &= g_z\eta_{z,t} + g_a\eta_{a,t} + g_{y^*}\eta_{y0^*,t} + g_{y1^*}\eta_{y^*,t-1} + g_r\hat{\imath}_{t-1} + g_g\hat{g}^y_{t-1} && (29)\\ &\quad + g_\pi\hat{\pi}_{Ht-1} + g_c\hat{c}_{t-1} + g_b\tilde{b}_t + g_\Lambda\Lambda_{ft} \end{aligned}$$

[10] We assume no memory.

where all coefficients are non-linear functions of structural and policy parameters. Terms $r_\Lambda \Lambda_{mt}$ and $g_\Lambda \Lambda_{ft}$ are linear functions of predetermined Lagrange multipliers and are only included in case of commitment, see Appendix B, Proposition 1.

3 Estimation Strategy and Empirical Implementation

3.1 Empirical Specification and Data

Using DYNARE toolkit (Juillard, 2005) we estimate the model using Bayesian techniques that have been developed to estimate and evaluate DSGE models (see e.g. An and Schorfheide, 2007).

The empirical specification of the system for estimation consists of equations (17)-(27) and two policy rules in the form of (28)-(29). The observable variables are domestic inflation, $\hat{\pi}_{Ht}$, the growth rate of output, Ψ_t, nominal interest rate, $\hat{\imath}_t$, terms of trade, $\hat{S}_t$, and the growth rate of government share, Ω_t. Government debt is treated as unobservable variable.[11] In case of commitment the set of unobservable endogenous variables also includes predetermined Lagrange multipliers. We keep very tight restrictions on the number of shocks being equal to the number of observed variables; this allows us to asses possible misspecifications using the DSGE-VAR approach as in Del Negro and Schorpfheide (2004). The estimation of commitment assumes that the commitment policy was announced at some point that predates the sample, at a date which we do not have to identify in estimation. Therefore, we chose to initialize the predetermined Lagrange multipliers to their steady state values when we start the Kalman filter.[12]

In order to approximate variables $\hat{\pi}_{Ht}, \hat{\imath}_t, \hat{S}_t, \Psi_t$ and Ω_t we use seasonally

[11] The model is formulated in terms of quarterly debt. Because of the data availability on the very short-term debt we treat this variable as unobservable.

[12] Alternatively, Ilbas (2010) and Adolfson et al. (2011) use presample to initialize the Lagrange multipliers. Both approaches give very similar results.

adjusted quarterly data on real GDP, the GDP deflator, nominal interest rates on three month Treasury bills, current government spending on goods and services, and the data on exports and imports of goods and services at current market prices and as chained volume measures. All data series are obtained from the Office of National Statistics Database.[13] Home inflation rates are defined as log differences of the GDP deflator and multiplied by 100 to obtain quarterly percentage rates. The data on the terms of trade are constructed as the relative price of export and import. The estimation is based on demeaned data.

We study the post-ERM period 1992:1-2008:2. During this period the UK maintained flexible exchange rate regime, with explicit inflation targeting in post-1997 period. We, therefore, effectively assume that during the whole sample period the monetary authorities, whether independent or as part of the government, were trying to maintain stability of the economy as described by low and stable inflation and low unemployment. Although the period covers several governments, we nevertheless estimate the fiscal regime 'on average'. Being the intraperiod leader, the fiscal policymaker is expected to remain relatively inactive, concentrating on keeping the debt accumulation under control.

3.2 Priors

We keep a number of parameters fixed, as some of them are related to steady state values and cannot be estimated from a log-linearized demeaned model. We calibrate the discount factor, β, to be 0.99, which implies an annual steady state interest rate of about 4%. The steady state tax rate is set to 0.35, the steady state government share is set to 0.2 and the steady state debt to GDP ratio is set to 0.1, as the UK data suggest.[14] We set the intratemporal

[13] The ONS codes for the data series are GZSN, YBHA, AJRP, YBGB, IKBI, IKBH, IKBL, IKBK.

[14] We work with one-period debt stock, its proportion in the total debt stock is relatively small over the observed period.

substitution elasticity $\eta = 1$, as did Lubik and Schorfheide (2007).

Table 1 summarizes prior distributions for structural parameters, policy objectives and shocks. They are consistent with priors used in e.g. Smets and Wouters (2003) and Lubik and Schorfheide (2005). We set relatively wide priors for all parameters which affect persistence of the propagation mechanism: the price indexation parameter ζ is beta-distributed with mean 0.5 and standard error of 0.25, and the habit persistence parameter h is beta-distributed with mean 0.5 and standard error or 0.10. Priors for elasticities σ and φ are consistent with e.g. Liu and Mumtaz (2011), Justiniano and Preston (2010a), Adjemian, Paries, and Moyen (2008).

Parameters $\Phi_{\pi M}$, $\Phi_{\Delta I}$, $\Phi_{\pi F}$, $\Phi_{\Delta G}$ and Φ_B measure the extent of deviation of the empirical policy objectives from those microfounded. We set gamma distributions for the 'conservatism' parameters, $\Phi_{\pi M}$ and $\Phi_{\pi F}$. The mean prior of $\Phi_{\pi M}$ is set to 1.0, and the mean prior of $\Phi_{\pi M}$ is set to 0.5 to reflect our belief that the monetary policymaker is likely to prioritize inflation stabilization, while the fiscal policymaker may have less of priorities to stabilize inflation. However, both prior distributions are very wide and allow both posterior means to exceed one.

The interest rate smoothing target weight $\Phi_{\Delta I}$ can be interpreted as a measure of importance of this target relative to the inflation stabilization target. It is widely accepted that the monetary authorities find the inflation target as most important, and it might be difficult to justify the mean posterior of $\Phi_{\Delta I}$ if it exceeds one. Note, however, that $\Phi_{\Delta I}$ directly affects the instrument inertia in the implied policy reaction function. The empirical reaction function may not be fully determined by either commitment or discretion, it may have some non-strategic components which we cannot identify within this framework. The presence of such non-strategic components may imply a large estimate of $\Phi_{\Delta I}$. Following Dennis (2006) and Ilbas (2010) we do not constrain $\Phi_{\Delta I}$ to be less than one. We choose gamma distribution with mean 0.75 and standard error of 0.25; this gives us a very wide prior.

There is no wide agreement that the policy weight on instrument smoothing in fiscal objectives should not dominate the inflation target. However, because we remove the stochastic trend from the government share over the observed period, we do not expect to find a great deal of the fiscal instrument smoothing. We do not expect to find an important debt stabilization target either, given the observed high and persistent debt to output ratio in the UK. To reflect these beliefs we choose gamma distribution with mean 0.1 and standard error of 0.09 for $\Phi_{\Delta G}$ and choose gamma distribution with mean 1×10^{-3} and standard error of 9×10^{-4} for Φ_B.

All shock variances are assumed to be distributed as inverted Gamma distribution. Their means are taken from similar studies, predominantly from Lubik and Schorfheide (2005), Dennis (2006), Ilbas (2010) and Givens (2012).

Finally, some priors are more dispersed than others. Note that more diffuse priors do not necessarily deliver higher marginal data density. While the in-sample fit improves slightly, wider priors relax some of the parameter restrictions and this leads to a larger penalty for model complexity. The second effect can outweigh the first one and this leads to an overall fall in the marginal data density. We therefore, make a prior more concentrated when such effect is evident.

4 Empirical Results

4.1 Parameter Estimates

Markov Chain Monte Carlo methods, described e.g. in Lubik and Schorfheide (2005), are used to generate draws from the posterior distribution of model parameters. We present the summary statistics in Tables 2-4.

Overall, the estimates of the structural parameters fall within plausible ranges, consistent with the most of literature, and are similar for commitment and discretion, see Table 2.

The estimate of the Calvo parameter θ implies that prices remain fixed between two and three quarters. The price indexation parameter ζ is estimated rather moderate: when firms adjust the price, less than half of them change prices optimally, rather than adopt a rule of thumb and index the growth rate of prices to the past observed inflation rate. These estimates are consistent with those obtained in Lubik and Schorfheide (2005).

We do not find evidence of substantial habit persistence, measured by parameter h; this is similar to findings in Liu and Mumtaz (2011) for the UK and in Justiniano and Preston (2010b) for New Zealand. The estimate of the preference parameter α is lower than the UK import share, and this is consistent with much of the open economy literature, see e.g. Lubik and Schorfheide (2007) and Justiniano and Preston (2010a). The inverse of the intertemporal elasticity of substitution, σ, is consistent with those obtained in the literature, e.g. Justiniano and Preston (2010a,b).

These results are interrelated and are the consequence of fitting equation (19) to the data. If we treat the terms of trade as a non-observable variable then the tension between the prior and posterior for α, σ and h is greatly reduced.

The marginal data density is relatively flat in the inverse elasticity of labour supply, ϕ, as the posterior distribution of ϕ is not much different from the prior.

All priors for policy parameters do not conflict with the data, see Table 3. The mean posterior of $\Phi_{\pi M}$ is only slightly less than the mean prior, and the posterior distribution is only slightly more concentrated than the prior distribution. It implies that the weight on inflation stabilization is consistent with the microfounded weight, and is relatively large. There is no tension between the prior and posterior of $\Phi_{\Delta I}$: although the mean posterior is slightly higher than the mean prior, it remains below one and the confidence interval is not too wide.

The policy priorities of the fiscal policymaker are described by relative weights $\Phi_{\pi F}, \Phi_{\Delta G}$ and Φ_B. The posterior of the conservatism weight, $\Phi_{\pi F}$, is more concentrated than the (very wide) prior, with the mean shifted towards zero. This implies that the relative weight on inflation target is lower than the microfoundations suggest. The fiscal smoothing weight $\Phi_{\Delta G}$ is very small, this is likely to be a consequence of the chosen detrending method. We did not find any evidence that the fiscal policymakers have the debt target, Φ_B.

The estimates of standard errors of structural shocks are in line with those obtained in most of the literature, see e.g. Smets and Wouters (2003), Lubik and Schorfheide (2007), for equally stylized models. Standard errors of technology and cost push shocks are relatively low and, similar to results in Lubik and Schorfheide (2007), the standard error of the foreign demand shock is relatively high. The foreign demand shock is likely to accumulate various misspecifications of our simple model.

4.2 Commitment vs. Discretion

If we allow for policy delegation, then the dynamics of the economy under discretion can be made very similar to the one under commitment.[15] Additionally, if the private sector is predominantly backward-looking then the difference between the dynamics of the economy under commitment and discretion is small. Our model has both these features. First, we have estimated some degree of habit persistence and inflation inertia. Second, we have also estimated different parameters of policy objectives of the two policymakers. It might become difficult to distinguish between the two policy regimes.

Nevertheless, there are some differences between the estimated parameters under discretion and commitment. First, to fit the same data the price-setters under commitment reset prices more frequently, but most of these changes are based on the rule of thumb rather than on optimality. The mean

[15]Interest rate smoothing, price level targeting, speed-limit policy are all designed to approximate the commitment equilibrium in a simple New Keynesian model. In models with optimizing fiscal policy such result may be less clear if the policymakers choose to engage into a fight.

share of the rule of thumb setters under commitment is 10% greater than under discretion. Second, the monetary policymaker under discretion has a bigger penalty on the interest rate smoothing target than the policymaker under commitment. In this model, given the same policy objectives and parameters of the model for commitment and discretion, the optimal policy under commitment generates lower volatility of inflation and higher volatility of interest rate. At the same time, inflation is found to be more sensitive to interest rate changes under commitment than under discretion. Therefore, in order to fit both models to the same data on interest rate and inflation, we have to have a lower weight on interest rate smoothing under commitment. An increase in this weight under commitment generates lower volatility of interest rate and much higher volatility of inflation which is rejected by the data.

In order to improve our understanding of the dynamics of economy under the two policies we compute impulse response functions. Figure 1 reports the responses of endogenous variables to one-standard-deviation shocks. Each subplot plots results for commitment and discretion together and shows mean responses of observable variables together with 5th and 95th percentiles. We only plot first ten quarters, as all variables are converging back to their base lines in the long run.[16]

A positive home technology shock A_H reduces the marginal cost and drives inflation down. The monetary policymaker reduces interest rate so consumption and output rise. The real exchange rate depreciates. Fiscal policymaker increases spending such that the government share rises. Under commitment, interest rate is reduced by more, which leads to higher consumption, inflation overshooting and to a reduction in government debt.

An increase in the world output η_{y^*} increases foreign demand for both home and foreign goods. This results in an appreciation of the real exchange

[16]Because the debt target $\Phi_B \neq 0$, although very small, the debt under commitment is not a unit-root variable.

rate. Because of the risk sharing assumption consumers increase consumption of foreign goods which leads to an initial reduction in domestic output. Inflation falls and the interest rate is reduced. Interest rate under commitment is reduced by more, this results in inflation overshooting and lower debt. Fiscal policy increases spending and the government share.

A positive cost-push shock η_π increases inflation, optimal monetary policy rises interest rate in response. This leads to lower consumption and output and a consequent reduction in inflation. Fiscal policy reduces spending. The real exchange rate appreciates. Under commitment the interest rate is raised by more which leads to bigger appreciation of the real exchange rate and bigger fall in consumption. The resulting reduction in marginal cost is insufficiently strong to outweigh the inflation persistence and deliver the same speed of reduction in inflation as under discretion. The government share rises because of the bigger fall in consumption and output.

A positive world-wide productivity shock A_W results in the real exchange rate depreciation. The productivity-adjusted output rises. The initial impact on habit-adjusted consumption is positive because of the increase in real wage following the shock. The higher marginal cost drives inflation up. The optimal interest rate is raised. Government spending have to be lower to control the accumulation of debt. Interest rate under commitment rises higher than under discretion. This ensures quick reduction in inflation with overshooting.

4.3 Model Fit

Table 4 reports the marginal data density for both policy regimes. This is a measure of relative fit, and allows one to compare different specifications of the model. A comparison of marginal data densities leads to the conclusion that the regime of fiscal leadership in the UK can be best described as discretion. The difference between the log marginal data densities can be interpreted as log posterior odds under the assumption that the two specifications have equal prior probabilities. Our finding suggests that the probability that the actual data was generated by a model with optimal commitment policy, rather than by a model with optimal discretionary policy, is less than 1.0%. In interpretation of Kass and Raftery (1995), there is a 'substantial' evidence in favor of discretion over commitment.

While the data density provides a measure of relative fit, we also present RMSEs an second-order moments which measure absolute fit, see e.g. Justiniano and Preston (2010b), Rabanal and Tuesta (2010). Table 5 reports RMSEs and second order moments for the data and the corresponding statistics implied by the estimated models. We report means with standard errors in parentheses.

The DSGE model under discretion produces good fit of standard deviations of all variables, in particular of interest rate and inflation. The volatility of the output growth rate Ψ_t is slightly overestimated under both discretion and commitment. The volatility of interest rate is substantially overestimated under commitment. Namely these properties produce most of differences in impulse responses between the two models in Figure 1.

Empirical autocorrelation of inflation is best captured by the commitment model, while the discretion model underestimates it. Both models overestimate the autocorrelation of the growth rate of output and underestimate the autocorrelation of the government share. The autocorrelation of the terms of trade is captured reasonably well.

Further, Tables 6-7 report autocorrelations up to the fifth order. The autocorrelations generated by the model of discretion are close to the data autocorrelations for all variables. However, both and discretion and commitment models are able to match empirical autocorrelations closely.

To assess the degree of model fit we also report marginal data densities for reduced form vector autoregressions with four lags, estimated under different Minnesota-type priors. More specifically, following Del Negro and Schorpfheide (2004), Lubik and Schorfheide (2005) and Adjemian et al.

(2008) we populate the original data sample with additional artificial data generated by the DSGE model. The relative importance of the prior information can be measured by parameter $\hat{\lambda} = N/T$ where N is the size of artificial sample and T is the size of data sample. We estimate the optimal weight, $\hat{\lambda}$, of either commitment or discretion DSGE prior in the BVAR model. Following Del Negro and Schorpfheide (2004) we call it DSGE-VAR or BVAR model interchangeably. The relative importance of the prior information is a measure of the degree of misspecification of the model. If $\hat{\lambda}$ is estimated to be high then it means that the DSGE model imposes useful restrictions to improve the (in sample) predictive properties of the BVAR model. Conversely, if $\hat{\lambda}$ is estimated to be low then the DSGE model is not coherent with the data. Finding $\hat{\lambda} \simeq 1.0$ suggests that the DSGE models do impose some useful restrictions: Del Negro et al. (2007) and Adjemian et al. (2008) demonstrate that $\hat{\lambda} = 0.35$ is close to the point where the DSGE provides no useful information, while $\hat{\lambda} > 0.6$ demonstrates some coherence of the DSGE model with the data.

Notably, the marginal data densities for both BVARs are almost identical, see Table 4. Both commitment and discretion DSGE models impose similarly useful restrictions. A comparison of DSGE and BVARs can help us to identify the tightest restrictions imposed by the DSGE models. Table 5 demonstrates that both BVARs improve the fit of the growth rate of output and interest rate but at the expense of the fit of other variables. Figure 2 compares impulse responses of DSGE and DSGE-VAR models under discretion. It is apparent that the dynamics of inflation, interest rate and the terms of trade is less volatile as implied by the BVAR, but at the same time we do not observe any big differences in direction of responses and in their persistence which is implied by this correction.

Finally, Figure 3 reports the historic and the one-step-ahead predicted data under the two policy regimes. Both policy regimes result in very similar estimates.

4.4 Role of Fiscal Policy

We have estimated fiscal policy to be relatively inactive: although there is a general consistency of the data with microfounded policy objectives, fiscal policy does very little to stabilize the economy. Under our assumption of fiscal intraperiod leadership this is the optimal outcome as the fiscal policymaker leaves the stabilization work to the monetary policymaker. The monetary policymaker, however, observes fiscal variables and takes them into account when formulating policy. In this section we argue that the state of fiscal stance does play a role in identification of the model.

The evolution of the government debt and fiscal spending are among the identifying restrictions for the model. To assess the importance of these restrictions we re-estimate the model excluding the government solvency constraint and treating the government share as following AR(1) process with coefficient ρ_g. The monetary policymaker is assumed to act either under discretion or commitment. The results of estimation are given in Table 2, in the last two columns.

Some of the key structural parameters appear to be different when the fiscal problem is excluded. In particular, the Calvo reset probability, the degree of inflation inertia, and volatility of home technology shocks are larger when the fiscal block is ignored; the difference is particularly large for the preferred specification of the optimal discretion. The monetary policy parameters are affected too once the fiscal block becomes exogenous: the monetary policymaker is less inflation conservative and operates with greater interest rate smoothing. All these changes in estimated parameters are required to generate greater endogenous persistence observed in the data.

Table 4 demonstrates that the monetary model leads to lower marginal data density. (There is also much less difference between commitment and discretion.) The absolute fit is assessed in Table 5. Standard deviations of interest rate, terms of trade and the growth rate of output is substantially overestimated. The autocorrelation of the growth rate of output is underestimated and there is a big increase in RMSE for this variable. At the same time, the autocorrelation of the government share is much less underestimated.

To understand these results we look at the role of the government debt accumulation equation in the model.

First, the debt accumulation process is highly persistent both under commitment and discretion. Its persistence propagates through the whole system; in particular, in case of discretion where all reaction functions are time-invariant and can be written as linear functions of predetermined states, the speed of convergence of all variables, including debt, is the same. If we remove the government budget constraint from the system, then in order to fit the same persistent data we require more inflation inertia and higher penalty on policy instrument movements. This role of debt process as persistent process might be played by some other 'slow' processes, like the capital accumulation process, which are omitted from our simple model.

Second, the debt accumulation process is potentially explosive. All economic agents are aware of this and should take decisions which are compatible with non-explosiveness of debt dynamics. In particular, the fiscal policymaker may optimally prefer to feed back on debt strong enough in order to allow the monetary policymaker to concentrate on inflation stabilization tasks. The monetary-fiscal model estimation results are consistent with non-explosiveness of debt. Once the government budget constraint is removed and the dynamics of fiscal instrument is approximated by an AR(1) exogenous process, the monetary policymaker does not take into account whether the problem of debt stabilization is resolved or not. This yields higher volatility of interest rate to fit the volatility of inflation in the data. Higher penalty on the instrument smoothing terms would result in higher volatility of inflation, inconsistent with the data. This role of the debt accumulation process as potentially explosive process may not be played by an intrinsically stable process like the capital accumulation process.

One can argue that the monetary policymaker alone can precommit to the chosen plan, while if we estimate monetary and fiscal interactions jointly then discretionary policy of fiscal policymakers results in the overall dominance of discretion. Indeed, we find that the gap between discretion and comment reduced once we excluded the fiscal sector from the economy. However, the smaller difference can also be a result of higher estimated persistence of the economy and the smaller role of expectations.

Finally, results from corresponding DSGE-VAR models suggest that there is a reduction in $\hat{\lambda}$ so the monetary DSGE model imposes tighter restrictions on the data. The BVAR marginal data density values improve by about 80 units and are closer to those obtained in more general monetary-fiscal DSGE-VAR models. Also, the first and second order moments are not as closely matched as in the monetary-fiscal model.

5 Conclusions

This paper identifies the degree of precommitment in monetary and fiscal policy interactions in the UK. We specify a small-scale structural general equilibrium model of a small open economy and estimate it using Bayesian methods. Unlike most of the existing empirical research we explicitly take into account the solvency constraint faced by the fiscal authorities. We also assume that the authorities act non-cooperatively, and may have different objectives.

We find that the model of discretionary policy explains the data better than the model of commitment policy. We find that both policymakers put smaller weight on inflation stabilization than is socially optimal. The fiscal policymaker pays much less attention to inflation stabilization than the monetary policymaker. The presence of fiscal block in the model plays the important role in identification of monetary policy and structural parameters.

A Social welfare

Social welfare is written as

$$W = \sum_{t=0}^{\infty} \beta^t \left(\frac{x_t^{1-\sigma}}{1-\sigma} + \chi \frac{g_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

Linearization yields

$$\begin{aligned} W &= \sum_{t=0}^{\infty} \beta^t \left(x^{1-\sigma} \left(\hat{x}_t + \frac{1}{2}(1-\sigma)\hat{x}_t^2 \right) + \chi g^{1-\sigma} \left(\hat{g}_t + \frac{1}{2}(1-\sigma)\hat{g}_t^2 \right) \right. \\ & \left. -N^{1+\varphi} \left(\hat{N}_t + \frac{1}{2}(1+\varphi)\hat{N}_t^2 \right) \right) + tip(3). \end{aligned}$$

where $tip(3)$ includes terms independent of policy of third order and higher. Production function (14) yields the exact relationship $\hat{N}_t = \hat{\Delta}_t + \hat{y}_t - \hat{A}_{Ht}$. We substitute $\hat{N}_t$ out and use

$$\sum_{t=0}^{\infty} \beta^t \hat{\Delta}_t = \frac{\theta}{1-\theta\beta} \hat{\Delta}_{-1} + \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \frac{\epsilon\theta(1-\zeta)}{\lambda} \left(\pi_{Ht}^2 + \frac{\zeta}{\theta(1-\zeta)} (\pi_{Ht} - \pi_{Ht-1})^2 \right)$$

to yield

$$\begin{aligned} W &= \sum_{t=0}^{\infty} \beta^t \left(x^{1-\sigma} \left(\hat{x}_t + \frac{1}{2}(1-\sigma)\hat{x}_t^2 \right) + \chi g^{1-\sigma} \left(\hat{g}_t + \frac{1}{2}(1-\sigma)\hat{g}_t^2 \right) \right. \\ & \left. -\chi_N N^{1+\varphi} \left(\begin{array}{c} \hat{y}_t + \frac{1}{2} \frac{\epsilon\theta(1-\zeta)}{\lambda} \left(\pi_{Ht}^2 + \frac{\zeta}{\theta(1-\zeta)} (\pi_{Ht} - \pi_{Ht-1})^2 \right) \\ + \frac{1}{2}(1+\varphi)\hat{N}_t^2 \end{array} \right) \right) + tip(3). \end{aligned}$$

The linearized up to second order national income identity and the international risk sharing condition yield

$$
\begin{aligned}
\hat{y}_t + \frac{1}{2}\hat{y}_t^2 &= (1-\alpha)\frac{c}{y}\left(\hat{c}_t + \alpha\eta\hat{S}_t + \frac{1}{2}\alpha\eta\left(-\eta + 2\alpha\eta + 1 - \alpha\right)\hat{S}_t^2\right. \\
&\left. + \frac{1}{2}\hat{c}_t^2 + \alpha\eta\hat{S}_t\hat{c}_t\right) + \frac{g}{y}\left(g_t + \frac{1}{2}g_t^2\right) \\
&+ \alpha\frac{c}{y}\left(\eta\hat{S}_t + \hat{c}_t^* + \frac{1}{2}\eta^2\hat{S}_t^2 + \frac{1}{2}\hat{c}_t^{*2} + \eta\hat{S}_t\hat{c}_t^*\right) \\
\sigma\hat{x}_t + \frac{1}{2}\sigma^2\hat{x}_t^2 &= (1-\alpha)\hat{S}_t + \sigma\hat{x}_t^* + \frac{1}{2}\frac{(\eta\alpha - 2\alpha + 1)}{(1-\alpha)}(1-\alpha)^2\hat{S}_t^2 \\
&+ \frac{1}{2}\sigma^2\hat{x}_t^{*2} + \sigma(1-\alpha)\hat{S}_t\hat{x}_t^*
\end{aligned}
$$

Combining them allows us to substitute out the terms of trade:

$$
\begin{aligned}
\hat{y}_t &= \frac{c}{y}(1-\alpha)\hat{c}_t + \frac{g}{y}g_t + \frac{c}{y}\frac{\alpha\eta\,(2-\alpha)\,\sigma}{(1-\alpha)}\hat{x}_t + \frac{c}{y}\frac{1}{2}\frac{\alpha\eta\,(2-\alpha)\,\sigma^2}{(1-\alpha)}\hat{x}_t^2 \\
&+ \frac{c}{y}\frac{1}{2}\alpha\eta\left(\alpha\left(\eta - \alpha\eta + 1\right) - (1-\alpha)^2\right)\frac{\sigma^2}{(1-\alpha)^2}\left(\hat{x}_t - \hat{x}_t^*\right)^2 \\
&- \frac{c}{y}\frac{\alpha\eta\,(2-\alpha)\,\sigma^2}{(1-\alpha)}\left(\hat{x}_t - \hat{x}_t^*\right)\hat{x}_t^* + (1-\alpha)\frac{c}{y}\frac{\alpha\eta\sigma}{(1-\alpha)}\left(\hat{x}_t - \hat{x}_t^*\right)\hat{c}_t \\
&+ \frac{c}{y}\frac{\alpha\eta\sigma}{(1-\alpha)}\left(\hat{x}_t - \hat{x}_t^*\right)\hat{c}_t^* + \frac{g}{y}\frac{1}{2}g_t^2 - \frac{1}{2}\hat{y}_t^2 + (1-\alpha)\frac{c}{y}\frac{1}{2}\hat{c}_t^2
\end{aligned}
$$

Using

$$
\sum_{t=0}^{\infty}\beta^t\hat{x}_t = -\frac{h}{(1-h)}\hat{c}_{-1} + \frac{(1-\beta h)}{(1-h)}\sum_{t=0}^{\infty}\beta^t\hat{c}_t
$$

we arrive to

$$
W = \sum_{t=0}^{\infty}\beta^t\mathcal{W}_t
$$

where

$$\begin{aligned}
\mathcal{W}_t &= \left(-\frac{x^{1-\sigma}}{N^{1+\varphi}}\frac{(1-\beta h)}{(1-h)}+\frac{\alpha\eta(2-\alpha)\sigma(1-\beta h)}{(1-\alpha)(1-h)}\frac{c}{y}+(1-\alpha)\frac{c}{y}\right)\hat{c}_t \\
&+\left(\frac{g}{y}-\frac{\chi g^{1-\sigma}}{N^{1+\varphi}}\right)\hat{g}_t+\frac{1}{2}\left(\frac{c}{y}\frac{\alpha\eta(2-\alpha)\sigma^2}{(1-\alpha)}-\frac{x^{1-\sigma}}{N^{1+\varphi}}(1-\sigma)\right)\hat{x}_t^2 \\
&+\frac{1}{2}\frac{\epsilon\theta(1-\zeta)}{\lambda}\left(\pi_{Ht}^2+\frac{\zeta}{\theta(1-\zeta)}(\pi_{Ht}-\pi_{Ht-1})^2\right) \\
&+\frac{c}{y}\frac{1}{2}\frac{\left(\alpha(\eta-\alpha\eta+1)-(1-\alpha)^2\right)\alpha\eta\sigma^2}{(1-\alpha)^2}(\hat{x}_t-\hat{x}_t^*)^2 \\
&+\frac{c}{y}\alpha\eta\sigma(\hat{x}_t-\hat{x}_t^*)\hat{c}_t-\frac{c}{y}\frac{(2-\alpha)\alpha\eta\sigma^2}{(1-\alpha)}(\hat{x}_t-\hat{x}_t^*)\hat{x}_t^* \\
&+\frac{c}{y}\frac{\alpha\eta\sigma}{(1-\alpha)}(\hat{x}_t-\hat{x}_t^*)\hat{c}_t^*+\frac{1}{2}(1+\varphi)\left(\hat{y}_t-\hat{A}_{Ht}\right)^2 \\
&+\frac{1}{2}(1-\alpha)\frac{c}{y}\hat{c}_t^2+\frac{1}{2}\left(\frac{g}{y}-\frac{\chi g^{1-\sigma}}{N^{1+\varphi}}(1-\sigma)\right)\hat{g}_t^2-\frac{1}{2}\hat{y}_t^2+tip(3).
\end{aligned}$$

We are interested in comparing stabilization performance of different policies, therefore we assume that a time-invariant labour subsidy offsets monopolistic distortions, $\frac{x^{1-\sigma}}{N^{1+\varphi}}=\frac{c}{y}\left(\frac{\sigma\alpha\eta(2-\alpha)}{(1-\alpha)}+\frac{(1-h)(1-\alpha)}{(1-\beta h)}\right)$. We chose χ so that $\frac{g}{y}=\chi\frac{g^{1-\sigma}}{N^{1+\varphi}}$. This yields the quadratic approximation to the social welfare loss in the form

$$\begin{aligned}
\mathcal{W}_t &= \pi_{Ht}^2+\frac{\zeta}{\gamma(1-\zeta)}(\pi_{Ht}-\pi_{Ht-1})^2+\frac{\lambda\sigma(1-\theta)}{\epsilon\gamma(1-\zeta)}\hat{g}_t^2 \\
&+\frac{\lambda\varphi}{\epsilon\gamma(1-\zeta)}\left(\hat{y}_t^2-\frac{(1+\varphi)}{\varphi}\hat{A}_{Ht}\right)^2 \\
&+\frac{2\theta\lambda}{\epsilon\gamma(1-\zeta)}\left(\frac{1}{2}\Psi_x\hat{x}_t^2+(1-\alpha)\hat{c}_t^2-\frac{\alpha\eta\sigma^2(\alpha\eta-\alpha^2\eta+1)}{(1-\alpha)^2}\hat{x}_t\hat{x}_t^*\right. \\
&\left.+\alpha\eta\sigma\hat{x}_t\hat{c}_t+\frac{\eta\alpha\sigma}{(1-\alpha)}\hat{x}_t\hat{c}_t^*-\alpha\eta\sigma\hat{x}_t^*\hat{c}_t\right)+tip(3)
\end{aligned}$$

where $\Psi_x=\frac{\sigma\alpha\eta}{(1-\alpha)}\left((2-\alpha)(2\sigma-1)+\frac{((1-\alpha)\eta+1)\alpha\sigma}{(1-\alpha)}\right)-\frac{(1-h)(1-\alpha)(1-\sigma)}{(1-\beta h)}-\alpha\eta\sigma^2$.

B Theoretical Framework

Our model belongs to the class of nonsingular linear stochastic rational expectations models of the type described by Blanchard and Kahn (1980), augmented by a vector of control instruments.

We label the two policymakers as leader (L) and follower (F), and denote them with index i, $i \in \{L, F\}$. (In this paper the leader is the fiscal policymaker and the follower is the monetary policymaker.)

The evolution of the economy is explained by the following system:

$$\begin{bmatrix} \mathsf{y}_{t+1} \\ \mathcal{E}_t \mathsf{x}_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \mathsf{y}_t \\ \mathsf{x}_t \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1} \\ 0 \end{bmatrix}. \quad (30)$$

where y_t is a vector of predetermined variables with initial conditions y_0 given, $\mathsf{y}_t = [a_t, y_t^*, \varepsilon_t^\pi, b_t]'$, x_t is a vector of non-predetermined (or jump) variables, $\mathsf{x}_t = [\pi_{Ht}, x_t]'$ where $x_t \equiv y_t - g_t$. u_t^F and u_t^L are the two vectors of policy instruments of the two policymakers, named F and L. $\mathsf{u}_t^F = i_t$ and $\mathsf{u}_t^L = g_t$ in the model. ϵ_t is a vector of i.i.d. shocks.

Each of the two policymakers has the following loss functions:

$$J_t^j = \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} (G_s' \mathcal{Q}^j G_s), \quad (31)$$

where $j = \{L, F\}$ and G_s^j is a vector of goal variables of policymaker i; which is a linear function of state variables and instruments, $G_s^j = \mathcal{C}^j \left[\mathsf{y}_s', \mathsf{x}_s', \mathsf{u}_s^{L\prime}, \mathsf{u}_s^{F\prime}\right]'$.

Commitment policy means that each policymaker is able to commit, with full credibility, to a policy *plan* (Currie and Levine, 1993). Thus, the policy plan has to specify the desired levels of the target variables (e.g. inflation, the output gap, etc.) at all current and future dates and states of nature.

Assumption 1 *At each time t the follower observes the current decision of the leader u_t^L. The private sector observes both decisions u_t^L and u_t^F.*

Assumption 2 *At any time t both policymakers know Assumption 1.*

The follower observes policy of the leader and reacts to it. The leader knows the follower reacts to its policy and is able to exploit it.

Assumption 3 *Suppose at time t the private sector and the policymakers only responds to the current state*

$$\begin{bmatrix} \mathsf{x}_t \\ \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{bmatrix} = - \begin{bmatrix} \mathsf{N}^d \\ F_1^L \\ F_2^F \end{bmatrix} \mathsf{y}_t \tag{32}$$

Assumption 4 *At each time t the private sector observes the current policy decisions $\mathsf{u}_t^L, \mathsf{u}_t^F$ and expects that future policymakers will reoptimize, and will apply the same decision process and implement decision $[F_1^{L\prime}, F_2^{F\prime}]'$. At each time t the follower observes the current policy decision of the leader u_t^L and expects that future leader will reoptimize, and will apply the same decision process and implement decision F_1^L.*

Problem 1 (Leadership under commitment) *Under commitment policy the follower solves*

$$\min_{\{u_s^F\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(G_s^{F\prime}\mathcal{Q}^F G_s^F) \tag{33}$$

subject to constraint (30).

The leader solves

$$\min_{\{u_t^L\}_{s=t}^{\infty}} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(G_s^{L\prime}\mathcal{Q}^L G_s^L) \tag{34}$$

subject to constraint (30) and to the system of first order conditions of the follower's optimization problem.

Policy determined by $[F_1^{L\prime}, F_2^{F\prime}]'$ is commitment policy if both policymakers find it optimal to follow $[F_1^{L\prime}, F_2^{F\prime}]'$ each time $s > t$ given Assumptions 1-2.

Discretionary policy means that the policymaker treats its optimal policy problem, as described above, as one of 'sequential optimization', i.e. without committing to any future course of action it makes the decision that is optimal within that period only.

Problem 2 (Leadership under discretion) *Under discretion the follower solves*

$$\min_{u_t^F} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(G_s^{F\prime}\mathcal{Q}^F G_s^F) \tag{35}$$

subject to constraint (30).

The leader solves

$$\min_{u_t^L} \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(G_s^{L\prime}\mathcal{Q}^L G_s^L) \tag{36}$$

subject to constraint (30) and to the system of first order conditions of the follower's optimization problem.

Policy determined by $[F_1^{L\prime}, F_2^{F\prime}]'$ is discretionary if both policymakers find it optimal to follow $[F_1^{L\prime}, F_2^{F\prime}]'$ each time $s > t$ given Assumptions 1-4.

Finally, we prove the following proposition.

Proposition 1 (First order conditions) *A solution to both commitment and discretionary leadership problem can be written in the following dynamic form:*

$$\begin{aligned} \mathsf{z}_{t+1} &= \mathsf{M}\mathsf{z}_t, & (37)\\ \mathsf{v}_t &= -\mathsf{N}\mathsf{z}_t, & (38) \end{aligned}$$

where variable $\mathsf{z}_t \equiv \mathsf{y}_t$ under discretion and $\mathsf{z}_t \equiv [\mathsf{y}_t', \lambda_t']'$ under commitment and λ_t are predetermined Lagrange multipliers; variable $\mathsf{v}_t = [\mathsf{x}_t', \mathsf{u}_t^{L\prime}, \mathsf{u}_t^{F\prime}]'$. Matrices M and N are functions of policy objectives $\mathcal{Q}^L$ and $\mathcal{Q}^F$ and of the system matrices A and B.

Proof. We first solve the problem under commitment. Minimization problem of the Follower can be presented by the following intraperiod Lagrangian:

$$\begin{aligned} H_s^F = \frac{1}{2}\beta^{s-t}(&\mathsf{x}_s' Q_{22}^F \mathsf{x}_s + 2\mathsf{y}_s' Q_{12}^F \mathsf{x}_s + \mathsf{y}_s' Q_{11}^F \mathsf{y}_s + 2\mathsf{u}_s^L P_{11}^{F\prime} \mathsf{y}_s + 2\mathsf{u}_s^L P_{21}^{F\prime} \mathsf{x}_s \\ &+ 2\mathsf{u}^F P_{12}^{F\prime} \mathsf{y}_s + 2\mathsf{u}_s^F P_{22}^{F\prime} \mathsf{x}_s + \mathsf{u}_s^L R_{11}^F U_s^L + 2\mathsf{u}_s^L R_{12}^F \mathsf{u}_s^F + \mathsf{u}_s^F R_{22}^F v_s^F \\ &+ \lambda_{s+1}^{Fy\prime}(A_{11}\mathsf{y}_s + A_{12}\mathsf{x}_s + D_1 \mathsf{u}_s^L + B_1 \mathsf{u}_s^F - \mathsf{y}_{s+1}) \\ &+ \lambda_{s+1}^{Fx\prime}(A_{21}\mathsf{y}_s + A_{22}\mathsf{x}_s + D_2 \mathsf{u}_s^L + B_2 \mathsf{u}_s^F - \mathsf{x}_{s+1})) \end{aligned}$$

where Lagrange multipliers λ^{Fy} are non-predetermined (as those on predetermined variables) with terminal conditions and λ^{Fx} are predetermined with initial conditions. The first order conditions are:

$$
\begin{aligned}
0 &= Q_{22}^F \mathsf{x}_s + Q_{21}^F \mathsf{y}_s + P_{21}^F \mathsf{u}_s^L + P_{22}^F \mathsf{u}_s^F + \beta A_{12}' \lambda_{s+1}^{Fy} + \beta A_{22}' \lambda_{s+1}^{Fx} - \lambda_s^{Fx} \\
0 &= Q_{12}^F \mathsf{x}_s + Q_{11}^F \mathsf{y}_s + P_{11}^F \mathsf{u}_s^L + P_{12}^F \mathsf{u}_s^F + \beta A_{11}' \lambda_{s+1}^{Fy} + \beta A_{21}' \lambda_{s+1}^{Fx} - \lambda_s^{Fy} \\
0 &= P_{12}^{F\prime} \mathsf{y}_s + P_{22}^{F\prime} \mathsf{x}_s + R_{12}^{F\prime} \mathsf{u}_s^L + R_{22}^F \mathsf{u}_s^F + \beta B_1' \lambda_{s+1}^{Fy} + \beta B_2' \lambda_{s+1}^{Fx}
\end{aligned}
$$

The minimization problem of the Leader or Fiscal policymaker

$$
\begin{aligned}
H_s^L = \frac{1}{2}\beta^{s-t} & (\mathsf{x}_s' Q_{22}^L \mathsf{x}_s + 2Y_s' Q_{12}^L \mathsf{x}_s + \mathsf{y}_s' Q_{11}^L \mathsf{y}_s + 2\mathsf{u}^L P_{11}^{L\prime} \mathsf{y}_s + 2\mathsf{u}^L P_{21}^{L\prime} \mathsf{x}_s \\
& + 2\mathsf{u}^F P_{12}^{L\prime} \mathsf{y}_s + 2\mathsf{u}_s^F P_{22}^{L\prime} \mathsf{x}_s + \mathsf{u}_s^L R_{11}^L \mathsf{u}_s^L + 2\mathsf{u}_s^L R_{12}^L \mathsf{u}_s^F + \mathsf{u}_s^F R_{22}^L \mathsf{u}_s^F \\
& + \lambda_{s+1}^{Ly\prime} (A_{11} \mathsf{y}_s + A_{12} \mathsf{x}_s + D_1 \mathsf{u}_s^L + B_1 \mathsf{u}_s^F - \mathsf{y}_{s+1}) + \nu_s^{Lx\prime} \Big(\beta A_{12}' \lambda_{s+1}^{Fy} \\
& + \beta A_{22}' \lambda_{s+1}^{Fx} - \lambda_s^{Fx} + \mathcal{Q}_{22}^F \mathsf{x}_s + \mathcal{Q}_{21}^F \mathsf{y}_s + P_{21}^F \mathsf{u}_s^L + P_{22}^F \mathsf{u}_s^F \Big) \\
& + \lambda_{s+1}^{Lx\prime} (A_{21} \mathsf{y}_s + A_{22} \mathsf{x}_s + D_2 \mathsf{u}_s^L + B_2 \mathsf{u}_s^F - \mathsf{x}_{s+1}) + \nu_s^{Ly\prime} \Big(A_{11}' \beta \lambda_{s+1}^{Fy} \\
& + A_{21}' \beta \lambda_{s+1}^{Fx} - \lambda_s^{Fy} + \mathcal{Q}_{12}^F \mathsf{x}_s + \mathcal{Q}_{11}^F \mathsf{y}_s + P_{11}^F \mathsf{u}_s^L + P_{12}^F \mathsf{u}_s^F \Big) \\
& + \nu_s^{Lu\prime} \Big(P_{12}^{L\prime} \mathsf{y}_s + P_{22}^{L\prime} \mathsf{x}_s + R_{12}^{F\prime} \mathsf{u}_s^L + R_{22}^F \mathsf{u}_s^F + \beta B_1' \lambda_{s+1}^{Fy} + \beta B_2' \lambda_{s+1}^{Fx} \Big))
\end{aligned}
$$

where Lagrange multipliers λ^{Ly} are non-predetermined (as those on predetermined variables) with terminal conditions and λ^{Lx} are predetermined with initial conditions. The first order conditions are:

$$
\begin{aligned}
0 &= Q_{22}^{L}\mathsf{x}_s + Q_{21}^{L}\mathsf{y}_s + P_{21}^{L}\mathsf{u}_s^{L} + P_{22}^{L}\mathsf{u}_s^{F} + \mathcal{Q}_{12}^{F\prime}\nu_s^{Ly} + \mathcal{Q}_{22}^{F\prime}\nu_s^{Lx} + P_{22}^{F}\nu_s^{Lu\prime} \\
&\quad + \beta A_{12}^{\prime}\lambda_{s+1}^{Ly} + \beta A_{22}^{\prime}\lambda_{s+1}^{Lx} - \lambda_s^{Lx}, \\
0 &= Q_{12}^{L}\mathsf{x}_s + Q_{11}^{L}\mathsf{y}_s + P_{11}^{L}\mathsf{u}_s^{L} + P_{12}^{L}\mathsf{u}_s^{F} + \mathcal{Q}_{11}^{F\prime}\nu_s^{Ly} + \mathcal{Q}_{21}^{F\prime}\nu_s^{Lx} + P_{12}^{F}\nu_s^{Lu} \\
&\quad + \beta A_{11}^{\prime}\lambda_{s+1}^{Ly} + \beta A_{21}^{\prime}\lambda_{s+1}^{Lx} - \lambda_s^{Ly}, \\
0 &= P_{11}^{L\prime}\mathsf{y}_s + P_{21}^{L\prime}\mathsf{x}_s + R_{11}^{L}\mathsf{u}_s^{L} + R_{12}^{L}\mathsf{u}_s^{F} + P_{11}^{F\prime}\nu_s^{Ly} + P_{21}^{F\prime}\nu_s^{Lx} + R_{12}^{F}\nu_s^{Lu} \\
&\quad + \beta D_1^{\prime}\lambda_{s+1}^{Ly} + \beta D_2^{\prime}\lambda_{s+1}^{Lx}, \\
0 &= P_{12}^{L\prime}\mathsf{y}_s + P_{22}^{L\prime}\mathsf{x}_s + R_{12}^{L\prime}\mathsf{u}_s^{L} + R_{22}^{L}\mathsf{u}_s^{F} + P_{12}^{F\prime}\nu_s^{Ly} + P_{22}^{F\prime}\nu_s^{Lx} + R_{22}^{F\prime}\nu_s^{Lu} \\
&\quad + \beta B_1^{\prime}\lambda_{s+1}^{Ly} + \beta B_2^{\prime}\lambda_{s+1}^{Lx}, \\
0 &= A_{11}\nu_s^{Ly} - \nu_{s+1}^{Ly} + A_{12}\nu_s^{Lx} + B_1\nu_s^{Lu}, \\
0 &= A_{21}\nu_s^{Ly} + A_{22}\nu_s^{Lx} - \nu_{s+1}^{Lx} + B_2\nu_s^{Lu}, \\
0 &= A_{11}\mathsf{y}_s + A_{12}\mathsf{x}_s + D_1\mathsf{u}_s^{L} + B_1\mathsf{u}_s^{F} - \mathsf{y}_{s+1}, \\
0 &= A_{21}\mathsf{y}_s + A_{22}\mathsf{x}_s + D_2\mathsf{u}_s^{L} + B_2\mathsf{u}_s^{F} - \mathsf{x}_{s+1}.
\end{aligned}
$$

The system of first order conditions to both optimization problems can be written as:

$$
G\begin{bmatrix} K_{s+1} \\ L_{s+1} \end{bmatrix} = D\begin{bmatrix} K_s \\ L_s \end{bmatrix}
$$

where $K_s = (\mathsf{y}_s^{\prime}, \mu_s^{x\prime})^{\prime}$ is predetermined variable, and $L_s = (\mathsf{u}_s^{\prime}, \mathsf{x}_s^{\prime}, \mu_s^{y\prime})^{\prime}$ is non-predetermined variable and $\mu_s^{y\prime}$ and $\mu_s^{x\prime}$ collect corresponding Lagrange multipliers. Matrix G can be singular, so using singular form decomposition (see Söderlind, 1999) we find the solution of the system in the form:

$$
\begin{bmatrix} Y_{s+1} \\ \mu_{s+1}^{x} \end{bmatrix} = Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}\begin{bmatrix} Y_s \\ \mu_s^{x} \end{bmatrix}, \begin{bmatrix} \mathsf{u}_s \\ X_s \\ \mu_s^{y} \end{bmatrix} = Z_{21}Z_{11}^{-1}\begin{bmatrix} Y_s \\ \mu_s^{x} \end{bmatrix} \tag{39}
$$

where matrices Z, S and T are obtained when solving the generalized eigenvalue problem. Note that S_{11} and Z_{11} have to be invertible, but T_{11} does not need to be. Moreover, if D has any zero roots, then any transformations with Hermitian matrices leave them zero, and they are collected in the top left part of T, such that $T_{ii}/S_{ii} < 1$.

Denote $\mathsf{M} = Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1}$, $N = -Z_{21}Z_{11}^{-1}$, and matrix N contains first $k_F + k_F + n_2$ rows of matrix N. Then, system (39) is written in form of (37)-(38).

We now prove the proposition for discretion.

Solution to a discretionary problem in any time t gives a value function for each policymaker i, $i \in \{L, F\}$, which is quadratic in the state variables,

$$W_t^i = \frac{1}{2}\mathsf{y}_t' S^i \mathsf{y}_t \tag{40}$$

a linear relation between the forward-looking variables

$$\mathsf{x}_t = -N\mathsf{y}_t \tag{41}$$

and a linear policy reaction function

$$\mathsf{u}_t^F = -F^F\mathsf{y}_t - L\mathsf{u}_t^L, \tag{42}$$

$$\mathsf{u}_t^L = -F^L\mathsf{y}_t, \tag{43}$$

where $L = -\partial\mathsf{u}_t^F/\partial\mathsf{u}_t^L$: in a leadership equilibrium the follower treats the leader's policy instrument parametrically.

We seek solution in the class of matrices with time-invariant coefficients. Given y_0 and system matrices A and B, matrices N, F^F, F^L and L, define the trajectories $\{\mathsf{y}_s, \mathsf{x}_s, \mathsf{u}_s\}_{s=t}^{\infty}$ in a unique way and vice versa: if we know that $\{\mathsf{y}_s, \mathsf{x}_s, \mathsf{u}_s\}_{s=t}^{\infty}$ solve the discretionary optimization problem then, by construction, there are unique time-invariant linear relationships between them which we label by N, F^F, F^L and L. Matrix S^i defines the cost-to-go for a policymaker i along a trajectory. Given the one-to-one mapping between equilibrium trajectories and $\{\mathsf{y}_s, \mathsf{x}_s, \mathsf{u}_s\}_{s=t}^{\infty}$ and the sextuple of matrices $\mathcal{T} = \{N, F^F, F^L, L, S^F, S^L\}$, it is convenient to continue with definition of policy equilibrium in terms of $\mathcal{T}$, not trajectories. This approach has become standard since Oudiz and Sachs (1985) and Backus and Driffill (1986).

The first order conditions on $\mathcal{T}$ are derived in Blake and Kirsanova (2011). We substitute (43) into (42) and obtain equation (38) with $\mathsf{N} = [N', \left(F^F - LF^L\right)', F^{L\prime}]'$. We substitute (41),(43) into (42) into equation 30 and obtain equation (37) with $M = A_{11} - A_{12}N - B_{11}F^L - B_{12}\left(F^F - LF^L\right)$.
■

C System Matrices

System (17)-(27) can be brought to the form

$$
\begin{aligned}
\hat{\pi}_{Ht} &= \beta\frac{\theta}{\Phi}\mathbb{E}_t\hat{\pi}_{Ht+1} + \frac{\zeta}{\Phi}\hat{\pi}_{Ht-1} + \frac{\lambda}{\Phi}\left(\pi_c\hat{c}_t - \pi_{c1}\hat{c}_{t-1} + \pi_g\hat{g}_t^y + \pi_{c^*}\hat{c}_t^*\right. \\
&\quad \left. + \pi_{c1^*}\hat{c}_{t-1}^* + \frac{\sigma h}{(1-h)}\hat{z}_t - (\varphi+1)\,\hat{a}_{Ht}\right) \\
\hat{c}_t &= \frac{1}{(1+h)}\mathbb{E}_t\hat{c}_{t+1} + \frac{h}{(1+h)}\hat{c}_{t-1} - \sigma_c\left(\hat{r}_t - \mathbb{E}_t\hat{\pi}_{Ht+1}\right) + c_{c^*}\hat{c}_t^* \\
&\quad - \frac{h\alpha}{(1+h)}\hat{c}_{t-1}^* - c_z\hat{z}_t \\
\hat{b}_{t+1} &= \frac{1}{\beta}\left(\hat{b}_t - \frac{b}{y}\hat{\pi}_{Ht} + b_g\hat{g}_t^y + b_c\hat{c}_t - b_{c1}\hat{c}_{t-1} + b_{c^*}\hat{c}_t^* + b_{c1^*}\hat{c}_{t-1}^*\right) \\
&\quad + \frac{b}{y}\hat{r}_t - \frac{b}{y}\rho_z\hat{z}_t \\
\hat{y}_t &= y_c\hat{c}_t - y_{c1}\hat{c}_{t-1} + y_{c^*}\hat{c}_t^* + y_{c1^*}\hat{c}_{t-1}^* + \frac{g}{c}\hat{g}_t^y
\end{aligned}
$$

where

$$\begin{aligned}
\pi_c &= \frac{\sigma}{(1-h)(1-\alpha)}\left(1+\varphi\frac{(1-h)}{\sigma_\alpha}\right), \pi_{c1} = \frac{\sigma h\left(1+(2-\alpha)\alpha\eta\varphi\right)}{(1-h)(1-\alpha)} \\
\pi_g &= \varphi\frac{g}{c}, \pi_{c^*} = \left(\varphi\left(1-\frac{(2-\alpha)\eta\sigma}{(1-h)(1-\alpha)}\right)-\frac{\sigma}{(1-h)(1-\alpha)}\right)\alpha \\
\pi_{c1^*} &= \frac{\alpha\sigma h\left(1+(2-\alpha)\eta\varphi\right)}{(1-h)(1-\alpha)}, b_g = (1-\tau)\frac{g}{c}, b_c = \frac{\left(\frac{g}{y}-\tau\right)\sigma}{(1-\alpha)\sigma_\alpha} \\
b_{c1} &= \left(\frac{g}{y}-\tau\right)\frac{(2-\alpha)\alpha\eta\sigma}{(1-h)(1-\alpha)}h, b_{c^*} = \left(\frac{g}{y}-\tau\right)\alpha\left(1-\frac{(2-\alpha)\eta\sigma}{(1-h)(1-\alpha)}\right) \\
b_{c1^*} &= \left(\frac{g}{y}-\tau\right)\frac{\alpha(2-\alpha)\eta\sigma}{(1-h)(1-\alpha)}h, \sigma_\alpha = \frac{\sigma}{(1-\alpha+\alpha\omega)} \\
\omega &= \frac{(2-\alpha)\sigma\eta}{(1-h)}-(1-\alpha), \sigma_c = \frac{(1-h)(1-\alpha)}{(1+h)\sigma}, c_{c^*} = \left(1-\frac{\rho_{c^*}}{(1+h)}\right)\alpha \\
c_z &= (1-\rho_z)\frac{(1-\alpha)}{(1+h)}h, y_c = \frac{\alpha(2-\alpha)\eta\sigma}{(1-h)(1-\alpha)}+(1-\alpha), \\
y_{c1} &= \frac{\alpha(2-\alpha)\eta\sigma h}{(1-h)(1-\alpha)}, y_{c^*} = \alpha-\frac{\alpha(2-\alpha)\eta\sigma}{(1-h)(1-\alpha)}, y_{c1^*} = \frac{\alpha(2-\alpha)\eta\sigma h}{(1-h)(1-\alpha)}
\end{aligned}$$

Therefore, system (30) can be written in the following matrix form

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta\frac{\theta}{\Phi} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_c & \frac{1}{(1+h)}
\end{bmatrix}
\begin{bmatrix}
z_{t+1} \\ \hat{a}_{Ht+1} \\ \hat{c}^*_{t+1} \\ \hat{c}^*_t \\ \hat{r}_t \\ \hat{g}_{H,t} \\ \hat{b}_{t+1} \\ \pi_{Ht} \\ \hat{c}_t \\ \mathbb{E}_t\pi_{Ht+1} \\ \mathbb{E}_t\hat{c}_{t+1}
\end{bmatrix} =$$

$$
\begin{bmatrix}
\rho_z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \rho_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_{c^*} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{b\rho_z}{y} & 0 & \frac{b_{c*}}{\beta} & \frac{b_{c1*}}{\beta} & 0 & 0 & \frac{1}{\beta} & 0 & -\frac{b_{c1}}{\beta} & -\frac{b}{\beta y} & \frac{b_c}{\beta} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-\frac{\lambda\sigma h}{\Phi(1-h)} & \frac{\lambda(\varphi+1)}{\Phi} & -\frac{\lambda\pi_{c^*}}{\Phi} & -\frac{\lambda\pi_{c1^*}}{\Phi} & 0 & 0 & 0 & -\frac{\zeta}{\Phi} & \frac{\lambda\pi_{c1}}{\Phi} & 1 & -\frac{\lambda\pi_c}{\Phi} \\
c_z & 0 & -c_{c^*} & \frac{\alpha h}{(1+h)} & 0 & 0 & 0 & 0 & -\frac{h}{(1+h)} & 0 & 1
\end{bmatrix}
$$

$$
\times \begin{bmatrix} z_t \\ \hat{a}_{Ht} \\ \hat{c}_t^* \\ \hat{c}_{t-1}^* \\ \hat{r}_{t-1} \\ \hat{g}_{H,t-1}^y \\ \hat{b}_t \\ \pi_{Ht-1} \\ \hat{c}_{t-1} \\ \pi_{Ht} \\ \hat{c}_t \end{bmatrix}
+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ \frac{b}{y} \\ 0 \\ 0 \\ 0 \\ \sigma_c \end{bmatrix} \hat{r}_t
+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ \frac{1}{\beta} b_g \\ 0 \\ 0 \\ -\frac{\lambda}{\Phi}\pi_g \\ 0 \end{bmatrix} \hat{g}_t^y;
$$

Vector of goal variables

$$
G_t = \begin{bmatrix} x_t^* & \hat{c}_t^* & \hat{g}_t & x_t & \hat{c}_t & \hat{y}_t - \hat{A}_{Ht} & \hat{y}_t & \pi_{Ht} - \pi_{Ht-1} & \pi_{Ht} \end{bmatrix}'
$$

$$
= T \begin{bmatrix} z_t & \hat{a}_{Ht} & \hat{c}_t^* & \hat{c}_{t-1}^* & r_{t-1} & \hat{g}_{t-1}^y & b_t & \pi_{Ht-1} & \hat{c}_{t-1} & \pi_{Ht} & \hat{c}_t & \hat{g}_t^y & r_t \end{bmatrix},
$$

where

$$
T = \begin{bmatrix}
\frac{h}{(1-h)} & 0 & \frac{1}{(1-h)} & -\frac{h}{(1-h)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & y_{c^*} & y_{c1^*} & 0 & 0 & 0 & 0 & -y_{c1} & 0 & y_c & \frac{y}{c} & 0 \\
\frac{h}{(1-h)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{h}{(1-h)} & 0 & \frac{1}{(1-h)} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & y_{c^*} & y_{c1^*} & 0 & 0 & 0 & 0 & -y_{c1} & 0 & y_c & \frac{g}{c} & 0 \\
0 & 0 & y_{c^*} & y_{c1^*} & 0 & 0 & 0 & 0 & -y_{c1} & 0 & y_c & \frac{g}{c} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
$$

and the social welfare objective matrix $\mathcal{Q}^{L,F} =$

$$\begin{bmatrix} 0 & 0 & 0 & q & -\frac{c}{y}\alpha\eta\sigma & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{c}{y}\frac{\eta\alpha\sigma}{1-\alpha} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{g}{y}\sigma & 0 & 0 & 0 & 0 & 0 & 0 \\ q & \frac{c}{y}\frac{\eta\alpha\sigma}{1-\alpha} & 0 & \frac{c}{y}\Psi_x & \frac{c}{y}\alpha\eta\sigma & 0 & 0 & 0 & 0 \\ -\frac{c}{y}\alpha\eta\sigma & 0 & 0 & \frac{c}{y}\alpha\eta\sigma & (1-\alpha)\frac{c}{y} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\varphi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\epsilon\zeta}{\lambda} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\epsilon\theta(1-\zeta)}{\lambda} \end{bmatrix}$$

where $q = -\frac{c}{y}\frac{\alpha\eta\sigma^2\left(\alpha\eta-\alpha^2\eta+1\right)}{(1-\alpha)^2}$.

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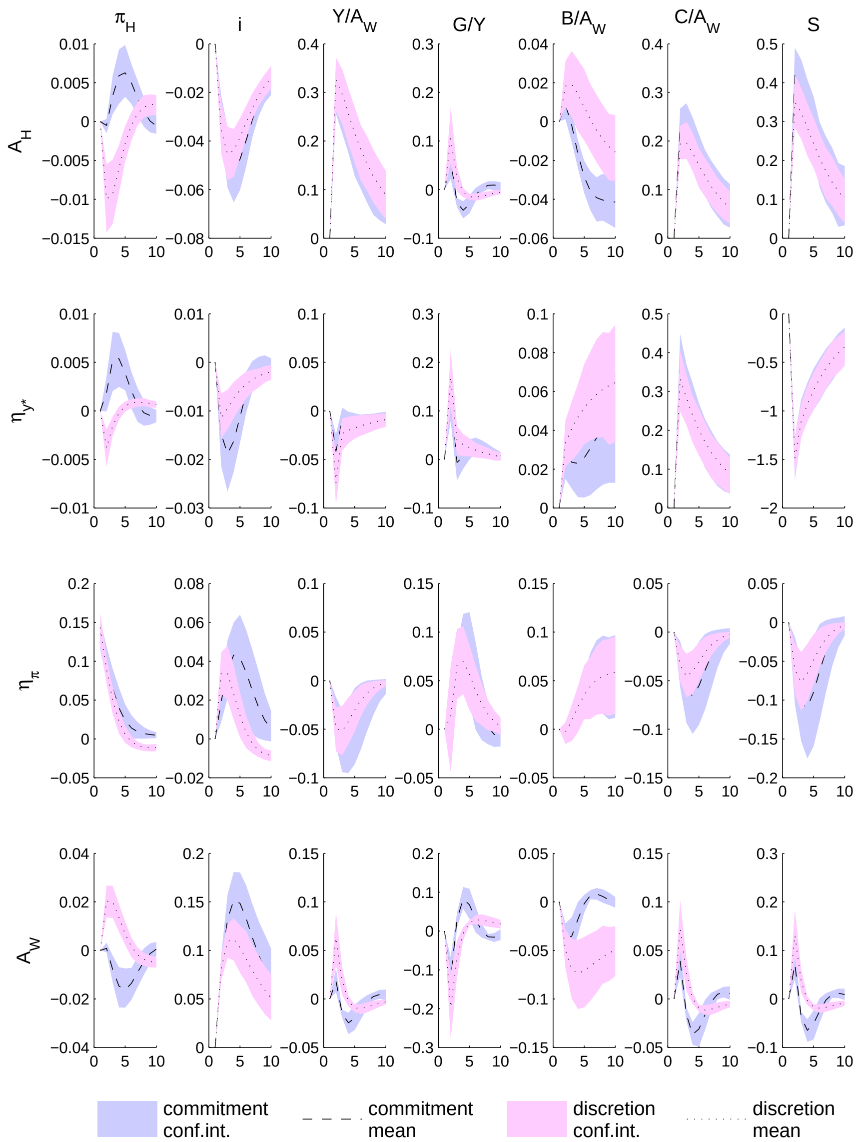

Figure 1: Impulse responses to one-standard-deviation structural shocks.

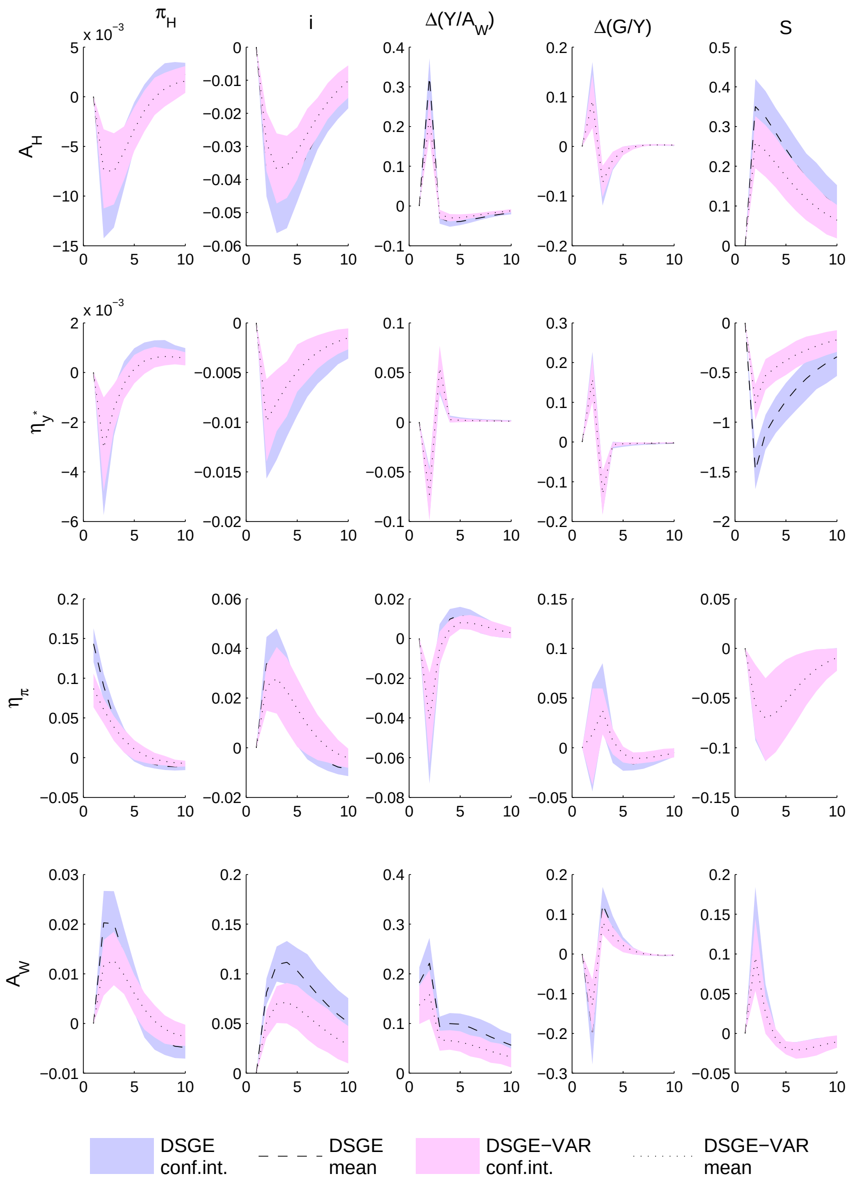

Figure 2: Impulse responses under discretion

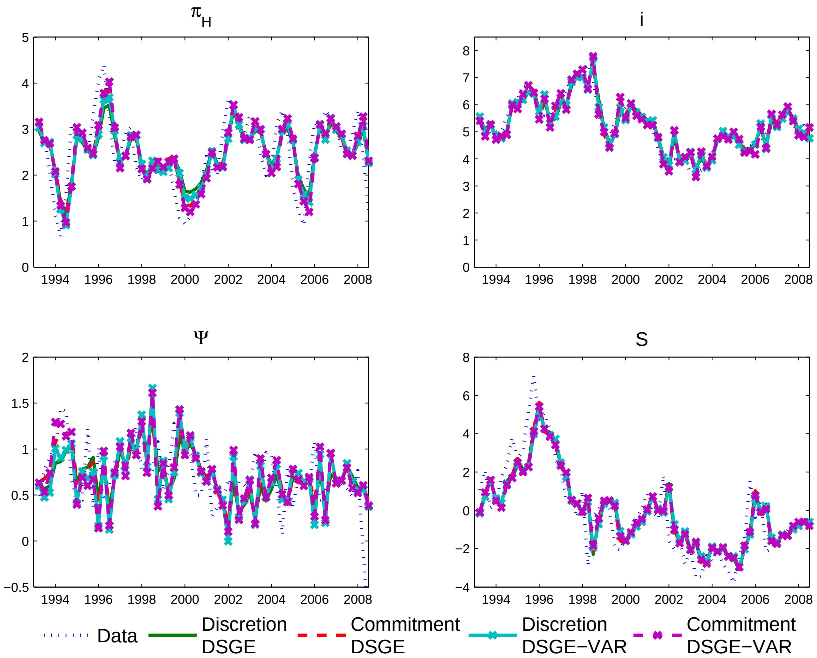

Figure 3: The data and one-step-ahead forecast

Table 1: Priors for Structural Parameters and Policy Weights

		domain	dist.	p(1)	p(2)
Structural Parameters					
home techn.	ρ_a	[0,1)	B	0.8	0.05
world output	ρ_{y^*}	[0,1)	B	0.8	0.05
world techn.	ρ_z	[0,1)	B	0.8	0.05
Calvo param.	θ	[0,1)	B	0.6	0.05
Habit persist.	h	[0,1)	B	0.5	0.10
Inflation inertia	ζ	[0,1)	B	0.5	0.25
Openness	α	[0,1)	B	0.3	0.03
CRRA	σ	$\mathbb{R}$	G	1.5	0.10
Inverse Frisch	φ	$\mathbb{R}$	G	2.0	0.25
Monetary Policy Parameters					
Conservatism	$\Phi_{\pi M}$	$\mathbb{R}^+$	G	1	0.25
Smoothing	$\Phi_{\Delta I}$	$\mathbb{R}^+$	G	0.75	0.25
Fiscal Policy Parameters					
Conservatism	$\Phi_{\pi F}$	$\mathbb{R}^+$	G	0.5	0.40
Debt	Φ_B	$\mathbb{R}^+$	G	1×10^{-3}	9×10^{-4}
Smoothing	$\Phi_{\Delta G}$	$\mathbb{R}^+$	G	0.1	0.09
Smoothing	ρ_g	[0,1)	B	0.8	0.05
Standard Deviation of Shocks					
world techn.	σ_z	$\mathbb{R}^+$	I	0.2	4
home techn.	σ_a	$\mathbb{R}^+$	I	0.5	4
world output	σ_{y^*}	$\mathbb{R}^+$	I	0.5	4
cost-push	σ_π	$\mathbb{R}^+$	I	0.1	4
spending	σ_g	$\mathbb{R}^+$	I	2.0	4

Notes: B stands for Beta, G Gamma and I Inverted Gamma distributions. Parameters p(1) and p(2) list the means and standard deviations for all distributions.

Table 2: Estimated Structural Parameters and Shocks

		monetary-fiscal model		monetary model	
		discretion	commitment	discretion	commitment
Structural Parameters					
home techn.	ρ_a	0.83 [0.77,0.89]	0.84 [0.77,0.90]	0.87 [0.82,0.92]	0.87 [0.81,0.93]
world output	ρ_{y^*}	0.84 [0.79,0.90]	0.83 [0.78,0.89]	0.83 [0.77,0.90]	0.82 [0.75,0.88]
world techn.	ρ_z	0.87 [0.82,0.92]	0.88 [0.83,0.93]	0.85 [0.79,0.91]	0.85 [0.79,0.91]
Calvo param.	θ	0.64 [0.58,0.70]	0.62 [0.54,0.67]	0.70 [0.64,0.76]	0.63 [0.57,0.70]
Habit persist.	h	0.09 [0.05,0.12]	0.10 [0.05,0.14]	0.08 [0.05,0.11]	0.08 [0.05,0.11]
Inflation inertia	ζ	0.64 [0.52,0.77]	0.75 [0.61,0.88]	0.79 [0.69,0.90]	0.76 [0.60,0.91]
Openness	α	0.26 [0.21,0.30]	0.26 [0.22,0.31]	0.27 [0.22,0.32]	0.27 [0.22,0.32]
CRRA	σ	1.23 [1.10,1.36]	1.22 [1.09,1.35]	1.19 [1.06,1.32]	1.19 [1.05,1.32]
Inverse Frisch	φ	2.03 [1.64.2.42]	2.03 [1.63,2.42]	1.87 [1.50,2.15]	1.92 [1.54,2.26]
Standard Deviation of Shocks					
world techn.	σ_z	0.18 [0.15,0.22]	0.23 [0.18,0.28]	0.25 [0.19,0.30]	0.29 [0.23,0.36]
home techn.	σ_a	0.38 [0.32,0.45]	0.41 [0.34,0.49]	0.92 [0.75,1.08]	0.89 [0.73,1.04]
world output	σ_{y^*}	1.15 [0.97,1.34]	1.17 [0.97,1.37]	1.30 [1.10,1.50]	1.30 [1.10,1.51]
cost-push	σ_π	0.14 [0.12,0.16]	0.14 [0.12,0.16]	0.14 [0.12,0.16]	0.14 [0.12,0.16]
spending	σ_g	2.25 [1.92,2.59]	2.28 [1.92,2.61]	2.23 [1.90,2.57]	2.24 [1.90,2.57]

Notes: Mean and posterior percentiles are from 8 chains of 100,000 draws generated using Random Walk Metropolis algorithm, where we discard initial 50,000 draws. Convergence diagnostics were assessed using trace plots.

Table 3: Estimated Policy Parameters and Weights

		monetary-fiscal model		monetary model	
		discretion	commitment	discretion	commitment
Monetary Policy Parameters					
Conservatism	$\Phi_{\pi M}$	0.91 [0.55,1.27]	0.92 [0.56,1.25]	0.78 [0.45,1.11]	0.90 [0.52,1.26]
Smoothing	$\Phi_{\Delta I}$	0.90 [0.49,1.27]	0.80 [0.43,1.14]	1.09 [0.64,1.52]	0.92 [0.53,1.25]
Fiscal Policy Parameters					
Conservatism	$\Phi_{\pi F}$	0.39 [0.01,0.79]	0.35 [0.04,0.66]	–	–
Debt	Φ_B	0.00 [0.00,0.00]	0.00 [0.00,0.00]	–	–
Smoothing	$\Phi_{\Delta G}$	0.01 [0.00,0.03]	0.01 [0.00,0.03]	–	–
Smoothing	ρ_g	–	–	0.85 [0.80,0.90]	0.85 [0.80,0.90]

Table 4: Data Density

		type of policy regime	type of BVAR prior	data density	hyper-parameter $\hat{\lambda}$
Monetary	DSGE	discretion	–	−226.25	–
-Fiscal	DSGE	commitment	–	−231.65	–
Model	BVAR	–	discretion	−183.37	0.98 [0.70,1.24]
	BVAR	–	commitment	−182.20	0.98 [0.72,1.23]
Monetary	DSGE	discretion	–	−288.37	–
Model	DSGE	commitment	–	−289.52	–
	BVAR	–	discretion	−203.45	0.76 [0.60,0.92]
	BVAR	–	commitment	−205.05	0.74 [0.72,1.23]

Table 5: RMSEs and moments implied by the data and models

	data	monetary-fiscal model				monetary model			
		DSGE with type of policy		BVAR with type of prior		DSGE with type of policy		BVAR with type of prior	
		discr	comm	discr	comm	discr	comm	discr	comm
RMSE									
π_H		0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
i_t		0.08	0.08	0.10	0.10	0.08	0.08	0.11	0.11
Ψ_t		0.39	0.37	0.43	0.42	0.63	0.63	0.75	0.77
S_t		1.28	1.27	1.28	1.25	1.27	1.26	1.25	1.24
Ω_t		2.27	2.30	2.30	2.32	2.33	2.33	2.34	2.33
Standard deviations									
π_H	*0.20*	0.18	0.20	0.12	0.15	0.20	0.20	0.13	0.12
		(0.01)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
i_t	*0.25*	0.31	0.41	0.20	0.26	0.37	0.46	0.20	0.23
		(0.03)	(0.04)	(0.02)	(0.03)	(0.03)	(0.04)	(0.02)	(0.02)
Ψ_t	*0.37*	0.53	0.59	0.37	0.39	1.15	1.20	0.48	0.51
		(0.03)	(0.05)	(0.03)	(0.04)	(0.06)	(0.07)	(0.03)	(0.03)
S_t	*2.40*	2.68	2.66	1.41	1.38	3.38	3.31	1.41	1.40
		(0.19)	(0.20)	(0.18)	(0.17)	(0.19)	(0.18)	(0.15)	(0.15)
Ω_t	*2.22*	2.28	2.29	1.37	1.39	2.32	2.32	0.94	0.94
		(0.16)	(0.17)	(0.15)	(0.15)	(0.18)	(0.18)	(0.10)	(0.09)
Autocorrelations									
π_H	*0.69*	0.61	0.71	0.69	0.80	0.69	0.68	0.78	0.73
		(0.00)	(0.00)	(0.0027)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
i_t	*0.92*	0.94	0.96	0.93	0.95	0.95	0.96	0.93	0.94
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Ψ_t	*0.28*	0.44	0.60	0.38	0.52	0.07	0.11	0.14	0.19
		(0.05)	(0.05)	(0.07)	(0.07)	(0.03)	(0.04)	(0.06)	(0.06)
S_t	*0.82*	0.82	0.82	0.79	0.79	0.84	0.83	0.81	0.80
		(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)
Ω_t	*-0.16*	-0.01	-0.00	-0.01	-0.01	-0.07	-0.07	-0.10	-0.10
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 6: Autocorrelations implied by the data and models

lag	data	DSGE		BVAR	
		type of policy		type of prior	
		discretion	commitment	discretion	commitment
		inflation, π_H			
1	0.69	0.61 (0.00)	0.71 (0.00)	0.69 (0.00)	0.80 (0.00)
2	0.23	0.42 (0.00)	0.41 (0.00)	0.57 (0.00)	0.50 (0.00)
3	0.01	0.21 (0.00)	0.22 (0.00)	0.40 (0.00)	0.33 (0.00)
4	0.03	0.05 (0.00)	0.11 (0.00)	0.25 (0.00)	0.21 (0.00)
5	0.07	-0.06 (0.00)	0.06 (0.00)	0.14 (0.00)	0.14 (0.00)
		output growth rate, Ψ_t			
1	0.28	0.44 (0.05)	0.60 (0.05)	0.38 (0.07)	0.52 (0.07)
2	0.08	0.06 (0.02)	0.10 (0.03)	0.10 (0.03)	0.14 (0.04)
3	0.09	0.06 (0.02)	0.10 (0.03)	0.10 (0.03)	0.13 (0.04)
4	0.04	0.06 (0.02)	0.09 (0.03)	0.09 (0.03)	0.12 (0.04)
5	-0.01	0.05 (0.02)	0.09 (0.03)	0.08 (0.03)	0.11 (0.03)
		interest rate, i_t			
1	0.92	0.94 (0.00)	0.96 (0.00)	0.93 (0.00)	0.95 (0.00)
2	0.80	0.85 (0.00)	0.86 (0.00)	0.81 (0.00)	0.82 (0.00)
3	0.64	0.74 (0.00)	0.74 (0.00)	0.68 (0.01)	0.67 (0.01)
4	0.51	0.63 (0.00)	0.62 (0.00)	0.55 (0.01)	0.52 (0.01)
5	0.42	0.53 (0.00)	0.52 (0.00)	0.43 (0.01)	0.41 (0.01)

Table 7: Autocorrelations implied by the data and models - continued

lag	data	DSGE		BVAR	
		type of policy		type of prior	
		discretion	commitment	discretion	commitment
		terms of trade, S_t			
1	0.82	0.82 (0.00)	0.82 (0.00)	0.79 (0.00)	0.79 (0.00)
2	0.76	0.71 (0.01)	0.70 (0.01)	0.68 (0.00)	0.67 (0.00)
3	0.64	0.61 (0.00)	0.59 (0.01)	0.58 (0.00)	0.56 (0.00)
4	0.57	0.52 (0.01)	0.49 (0.01)	0.48 (0.00)	0.46 (0.01)
5	0.53	0.44 (0.01)	0.42 (0.01)	0.41 (0.00)	0.39 (0.01)
		growth rate of government share, Ω_t			
1	-0.16	-0.01 (0.00)	-0.00 (0.00)	-0.01 (0.00)	-0.01 (0.00)
2	-0.00	-0.06 (0.00)	-0.06 (0.00)	-0.08 (0.00)	-0.08 (0.00)
3	-0.02	-0.06 (0.00)	-0.05 (0.00)	-0.06 (0.00)	-0.06 (0.00)
4	0.42	-0.05 (0.00)	-0.04 (0.00)	-0.05 (0.00)	-0.05 (0.00)
5	-0.06	-0.04 (0.00)	-0.04 (0.00)	-0.04 (0.00)	-0.04 (0.00)