Fiscal Sustainability in a New Keynesian Model - Additional Appendix (Not for Publication)

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Abstract: Recent work on optimal monetary and fiscal policy in New Keynesian models suggests that it is optimal to allow steady-state debt to follow a random walk. Leith and Wren-Lewis (2012) consider the nature of the time-inconsistency involved in such a policy and its implication for discretionary policy-making. We show that governments are tempted, given inflationary expectations, to utilize their monetary and fiscal instruments in the initial period to change the ultimate debt burden they need to service. We demonstrate that this temptation is only eliminated if following shocks, the new steady-state debt is equal to the original (efficient) debt level even though there is no explicit debt target in the government's objective function. Analytically and in a series of numerical simulations we show which instrument is used to stabilize the debt depends crucially on the degree of nominal inertia and the size of the debt-stock. We also show that the welfare consequences of introducing debt are negligible for precommitment policies, but can be significant for discretionary policy. Finally, we assess the credibility of commitment policy by considering a quasi-commitment policy which allows for different probabilities of reneging on past promises. This on-line Appendix extends the results of this paper.

JEL Codes: E62, E63

Keywords: New Keynesian Model; Government Debt; Monetary Policy; Fiscal Policy; Credibility.

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1 Introduction

In Leith and Wren-Lewis (2012) we consider optimal monetary and fiscal policy in the face of shocks which raise the level of debt, where the policy maker can either act under full commitment, discretion or an intermediate case of quasi-commitment. This appendix complements the analysis of the paper, by presenting additional results. Firstly, Section 2 explores the implications of treating government spending as an exogenous stream that needs to be financed, rather than as a policy instrument which can be chosen optimally. In the main paper we conduct a robustness analysis by varying the degree of price stickiness and the initial debt-gdp ratio. Section 3 extends that robustness analysis by varying the remaining key model parameters, the intertemporal elasticity of substitution, $1/\sigma$, and the labor supply parameter, φ. Across all parameter variants the key results of the main paper are unaffected. Finally, section 4 provides the matrix representation of the model considered in the paper in a manner which enables us to nest the commitment, discretion and quasi-commitment descriptions of policy considered in the paper. We also outline the numerical algorithms of Himmels and Kirsanova (2012) which we use to solve the model under quasi-commitment.

2 Exogenous Government Spending

In the main paper we allowed government spending to be chosen optimally by the policy maker. We did so to reflect the fact that all the major fiscal consolidations analyzed by the IMF(2012) relied heavily on spending cuts to stabilize debt, as it was argued that electorates were increasingly resistant to tax increases. We found that, contrary to observed behavior, optimal policy would not imply significant adjustment of the government spending gap in an attempt to stabilize debt. While, some adjustment through tax revenues was possible, particularly at low debt-gdp ratios and high degrees of price stickiness, typically time-consistent fiscal stabilization relied on the reduction of debt service costs through a relaxation of monetary policy. In this section, for completeness, we reassess those results when we assume that government spending is not used as an instrument in the policy maker's problem. As would be expected from the earlier results, this does not significantly change the results presented in the main paper. Here we present the Figures analogous to Figures 1 and 2 in the main paper.

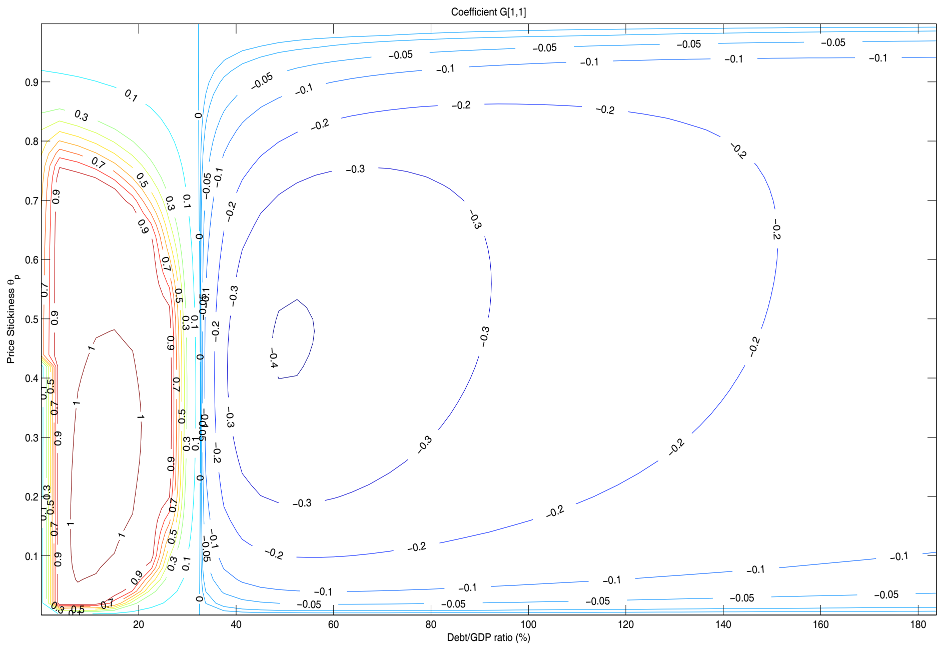

Figure 1: Evolution of Debt under Discretion as a function of Price Stickiness and the Debt/GDP Ratio - Exogenous G.

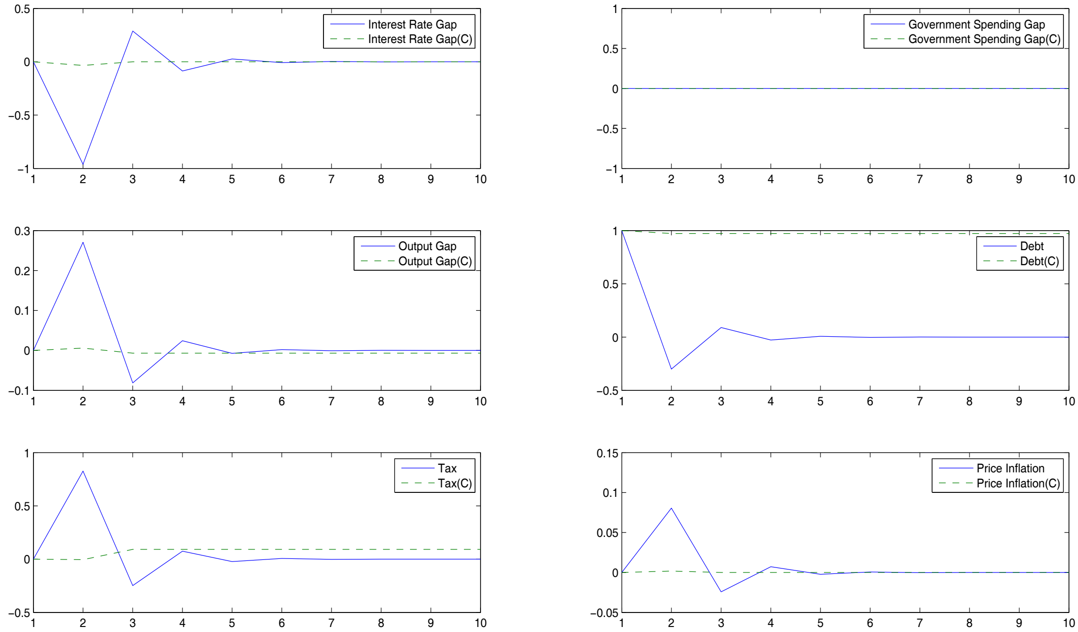

Figure 2: Response to Debt Shock under Commitment (C) and Discretion - Exogenous G.

3 Robustness Analysis

In this section we re-create some of the Figures from the main paper under alternative parameters. To begin with we allow for alternative values for the intertemporal elasticity of substitution, $1/\sigma$, specifically $\sigma = 1$ and 3, which covers the ranges of this parameter typically considered in the macro literature.

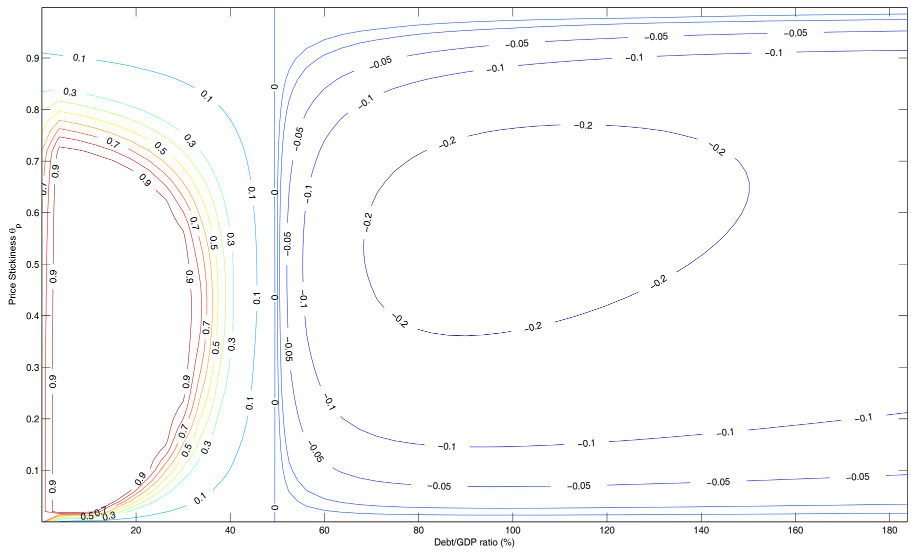

Figure 3: Evolution of Debt Under Discretion - $\sigma = 1$

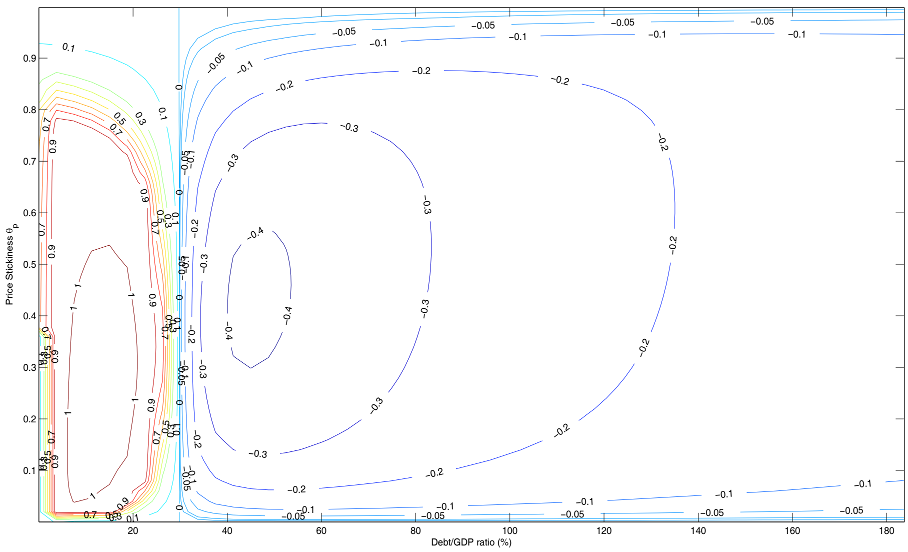

Figure 4: Evolution of Debt under Discretion - $\sigma = 3$

We now consider the same Figure, but with additional values of the labour supply parameter $\varphi = 2$ and 3, having returned to the benchmark calibration $\sigma = 2$.

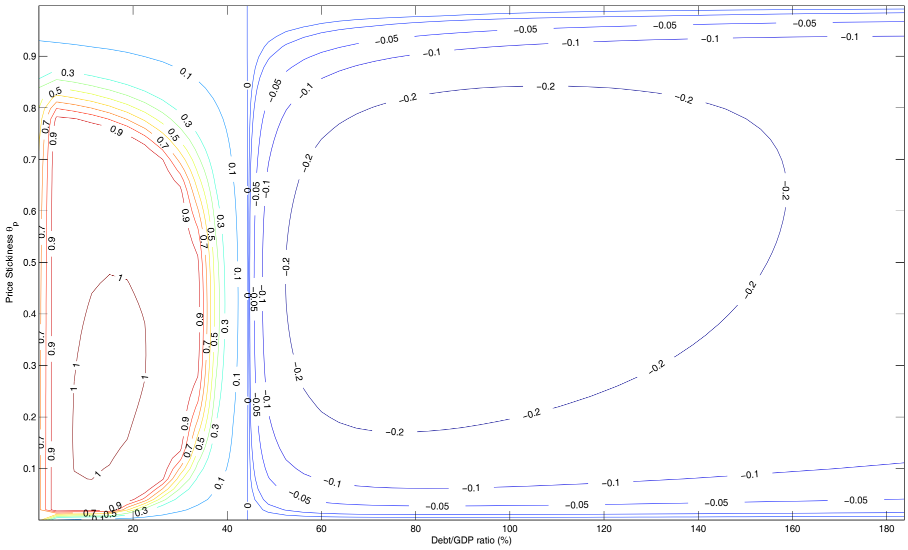

Figure 5: Evolution of Debt under Discretion - $\varphi = 2$

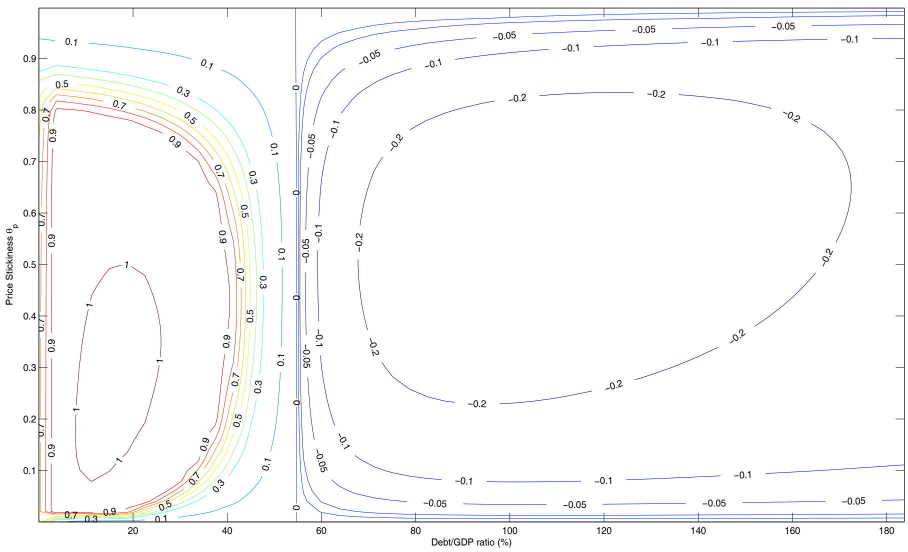

Figure 6: Evolution of Debt under Discretion - $\varphi = 3$

We now turn to Figure 4 in the paper which considers the relative contribution to debt stabilization under discretion across the same parameter permutations. Firstly, variations in the inverse of the intertemporal elasticity of substitution.

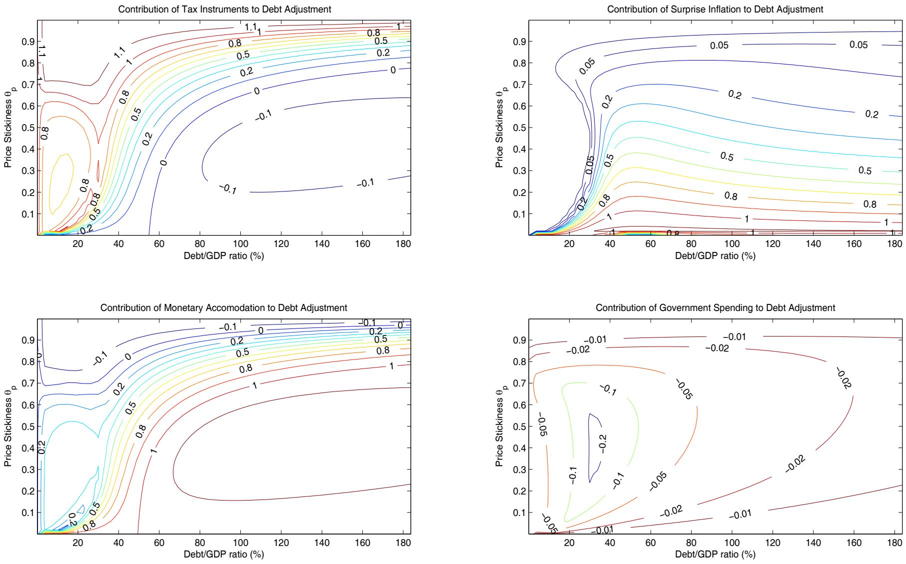

Figure 7: Contribution to Debt Stabilisation under Discretion - $\sigma = 1$

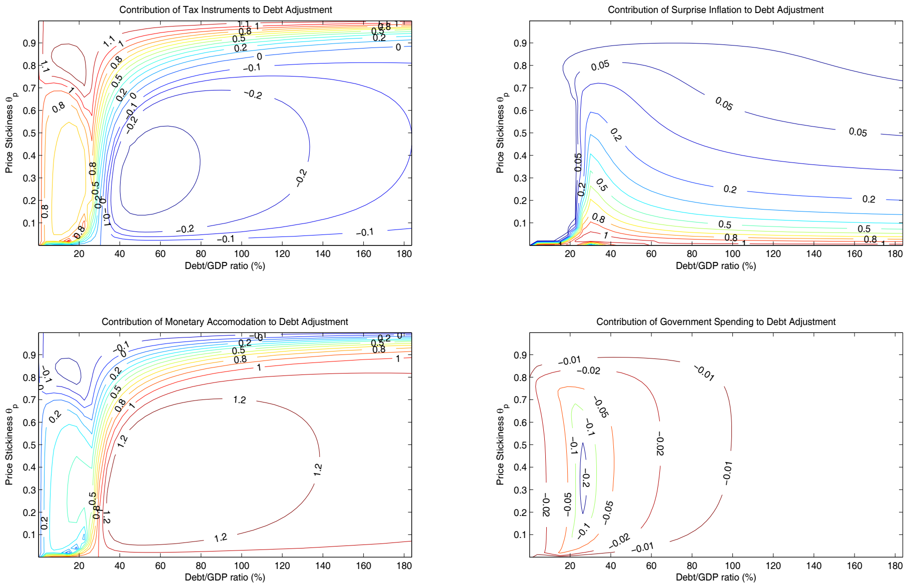

Figure 8: Contribution to Debt Stabilisation under Discretion - $\sigma = 3$

Similarly for the alternative values of the Frisch elasticity.

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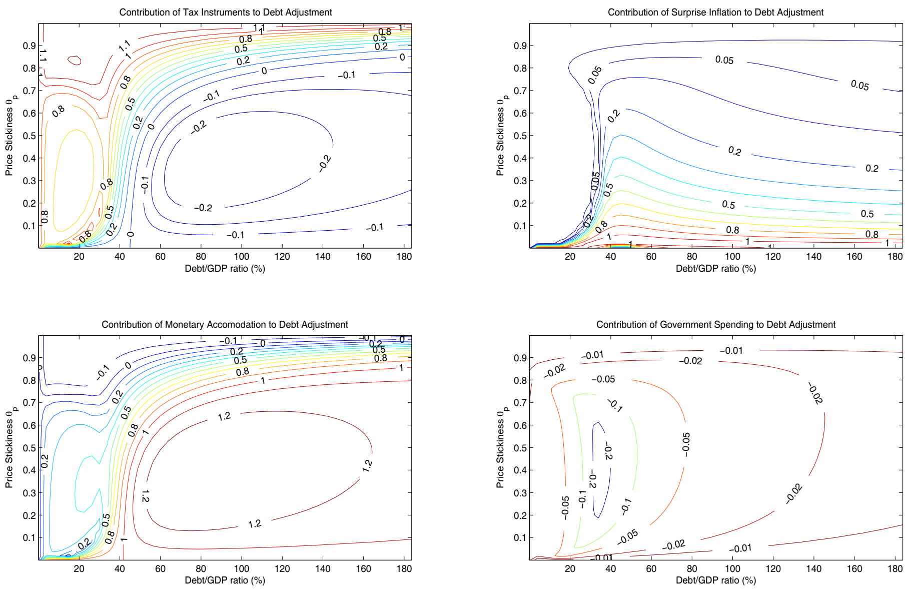

Figure 9: Contribution to Debt Stabilisation under Discretion - $\varphi = 2$

4 Quasi-Commitment

In this section we we follow Himmels and Kirsanova (2012) in recasting the quasi-commitment of Schaumburg and Tambalotti (2007) and Debertoli and Nunes (2010) in a general linear-quadratic form which can be solved using standard iterative techniques. This also nests the cases of commitment ($\alpha = 0$) and discretion ($\alpha = 1$) which can be solved using the computer codes of Soderlind (1999). We can write our model in the standard form as,

$$\begin{bmatrix} \boldsymbol{y}_{t+1} \\ E_t \boldsymbol{x}_{t+1} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \boldsymbol{y}_t \\ E_t \boldsymbol{x}_t \end{bmatrix} + \boldsymbol{B} \boldsymbol{u}_t + \boldsymbol{C} \boldsymbol{\xi}_t$$

where $\boldsymbol{y}_t = \begin{bmatrix} b_t \\ y^g_{t-1} \\ a_{t-1} \end{bmatrix}$, $\boldsymbol{x}_t = \begin{bmatrix} c^g_t \\ \pi_t \end{bmatrix}$, are the vectors of predetermined and jump variables respectively, while $\boldsymbol{u}_t = \begin{bmatrix} \widehat{\tau}_t \\ g^g_t \\ r^g_t \end{bmatrix}$ is the vector of controls. The coefficient matrices are defined as, $\boldsymbol{A} = [\boldsymbol{A}_0]^{-1} \boldsymbol{A}_1$, $\boldsymbol{B} = [\boldsymbol{A}_0]^{-1} \boldsymbol{B}_1$ and $\boldsymbol{C} =$

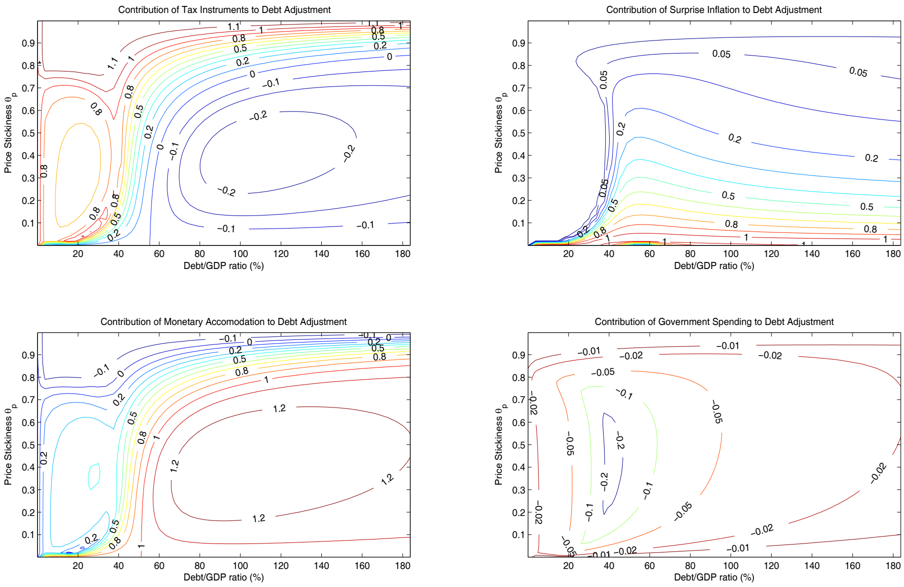

Figure 10: Contribution to Debt Stabilisation under Discretion - $\varphi = 3$

$[\boldsymbol{A}_0]^{-1}\boldsymbol{C}_1$ where,

$$\boldsymbol{A}_0 = \begin{bmatrix} \beta & \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(1+\vartheta) & f & -\sigma\beta & -\beta \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \sigma^{-1} \\ 0 & \gamma\varphi & 0 & 0 & \beta \end{bmatrix}, \boldsymbol{A}_1 = \begin{bmatrix} 1 & 0 & 0 & \sigma(\beta + \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}) & -1 \\ 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & \rho_a & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\gamma\sigma & 1 \end{bmatrix}$$

$$\boldsymbol{B}_1 = \begin{bmatrix} -\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}\frac{1}{1-\overline{\tau}} & \frac{\overline{G}}{\overline{b}} & 0 \\ 0 & 1-\theta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^{-1} \\ -\gamma\frac{\overline{\tau}}{1-\overline{\tau}} & 0 & 0 \end{bmatrix}$$

The objective function for the policy maker can be written as,

$$\begin{aligned} \Gamma &= -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{\sigma\theta(\widehat{c}_t^g)^2 + \sigma(1-\theta)(\widehat{G}_t^g)^2 + \varphi(\widehat{Y}_t^g)^2 + \frac{\epsilon}{\gamma}\pi_t^2\} \\ &= -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{x}_t \\ \boldsymbol{u}_t \end{bmatrix}'\boldsymbol{D}'\boldsymbol{Q}\boldsymbol{D}\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{x}_t \\ \boldsymbol{u}_t \end{bmatrix}\} \end{aligned}$$

where

$$\boldsymbol{Q} = \begin{bmatrix} \sigma\theta & 0 & 0 & 0 \\ 0 & \sigma(1-\theta) & 0 & 0 \\ 0 & 0 & \vartheta & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\lambda} \end{bmatrix} \text{ and } \boldsymbol{D} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \theta & 0 & 0 & (1-\theta) & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

This enables us to write the policy problem for the policy maker operating under quasi commitment as,

$$\min -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}((1-\alpha)\beta)^t\{\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{x}_t \\ \boldsymbol{u}_t \end{bmatrix}' \boldsymbol{D}'\boldsymbol{Q}\boldsymbol{D}\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{x}_t \\ \boldsymbol{u}_t \end{bmatrix} + \alpha\beta\boldsymbol{y}'_{t+1}\boldsymbol{S}^d\boldsymbol{y}_{t+1}\} \quad (1)$$

subject to,

$$\boldsymbol{y}_{t+1} = \boldsymbol{A}_{11}\boldsymbol{y}_t + \boldsymbol{A}_{12}\boldsymbol{x}_t + \boldsymbol{B}_1\boldsymbol{u}_t + \boldsymbol{C}\boldsymbol{\xi}_t \quad (2)$$

$$(1-\alpha)E_t\boldsymbol{x}_{t+1} + \alpha\boldsymbol{H}\boldsymbol{y}_{t+1} = \boldsymbol{A}_{21}\boldsymbol{y}_t + \boldsymbol{A}_{22}\boldsymbol{x}_t + \boldsymbol{B}_2\boldsymbol{u}_t \quad (3)$$

where $\boldsymbol{y}'_{t+1}\boldsymbol{S}^d\boldsymbol{y}_{t+1} = V(b_{t+1}, a_t)$ is the value function of the corresponding problem under discretion, and $\boldsymbol{H}$ is the link between jump and state variables for that same problem Therefore as α tends towards zero we move towards the commitment problem, and as α tends to 1 discretion. More generally, in the second line of the constraints, we can see that as we increase α the policy maker is forced to take expectations as given by the solution to the discretionary problem, while for lower values of α they are able to commit in a manner which successfully manipulates expectations. In the absence of any endogenous state variables, this would reduce to the case where expectations are increasingly taken as given by the policy maker as α increases. The policy problem is then solved using the computer codes developed in Himmels and Kirsanova (2012), which corresponds to those developed by Soderlind (1999) in the special cases of commitment ($\alpha = 0$) and discretion ($\alpha = 1$).

Given $\boldsymbol{H}$ and $\boldsymbol{S}^d$, the combination of the model and focs for the policy problem can be solved via Schur decomposition to generate a solution in the following form,

$$\begin{aligned} \begin{bmatrix} \boldsymbol{u}_t \\ \boldsymbol{x}_t \\ \boldsymbol{\psi}_t \end{bmatrix} &= \boldsymbol{X}\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{\varphi}_t \end{bmatrix}, \\ \begin{bmatrix} \boldsymbol{y}_{t+1} \\ \boldsymbol{\varphi}_{t+1} \end{bmatrix} &= \boldsymbol{M}\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{\varphi}_t \end{bmatrix}, \text{ and} \\ W(\boldsymbol{y}_{t+1}, \boldsymbol{\varphi}_t) &= \frac{1}{2}\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{\varphi}_t \end{bmatrix}' \mathbf{U}\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{\varphi}_t \end{bmatrix} \end{aligned} \quad (4)$$

where $\boldsymbol{\psi}_t$ and $\boldsymbol{\varphi}_t$ are the vectors of lagrange multipliers associated with the constraints in (2) and (3), respectively. Himmels and Kirsanova (2012) then suggest the following solution algorithm.

- Guess $\boldsymbol{M}$, $\boldsymbol{X}$ and $\mathbf{U}$ (which implicitly includes guesses of $\boldsymbol{H}$ and $\boldsymbol{S}^d$).
- Use these guesses to update $\mathbf{U}$ by using the guessed solution in (1).
- Obtain the solution to the model and FOCs conditional on the last iteration to update $\boldsymbol{M}$ and $\boldsymbol{X}$.
- Continue to iterate until there is no further updating of $\boldsymbol{M}$, $\boldsymbol{X}$ and $\mathbf{U}$.

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