On Policy Implications of Endogenous Technological Progress*

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Abstract

One of the most well-known empirical regularities in the R&D-productivity literature is the existence of substantial under-investment in R&D. This strongly supports government policy to actively promote research activities. However, the so-called quality-ladders models of endogenous technological progress are inconsistent with this observation. In an extreme case, Grossman and Helpman (1991, Ch.4) suggest that R&D should always be taxed irrespective of the size of quality improvement. This paper attempts to reconcile these empirical and theoretical ndings by showing that the normative results of Grossman and Helpman are not robust.

Key Words: Endogenous technological progress, R&D, policy, subsidies, patent breadth

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A distinguishing feature of R&D-based models of endogenous technological change, as opposed to the neo-classical growth models, is that public policy is a crucial determinant of the rate of technological advance, hence long-run growth.[1] This *positive* policy implication has a substantial influence on policy makers. For example, HM Treasury (1999) states that "the Government is interested in calculating the economy's trend growth rate ... to monitor the ***effectiveness*** of its policies in raising the level of trend output and the ***trend*** growth rate" (p.1; emphasis added).

However, the *normative* policy conclusions of these R&D-based models are inconsistent with the received wisdom that there exists substantial under-investment in R&D. This frequent finding in the voluminous R&D-productivity literature (see Griliches, 1995) strongly supports extensive government involvement in promoting knowledge creation activities in developed economies. In stark contrast, R&D-based growth models suggest that R&D *should* be taxed, since R&D over-investment can occur.[2] Arguably this is a major weakness in these influential models. Such an empirically inconsistent prediction begs a closer look at those models, given their considerable influence on policy design.

Motivated by this observation, this paper extends the model of Grossman and Helpman (1991, Ch.4) (hereafter GH) to account for the results of the empirical studies. The choice reflects its popularity in the growth literature. To set the stage of our analysis, we list the main normative policy implications of the GH model.[3] (i) R&D aimed at *radical* technological breakthroughs (with sufficiently large quality improvement) should be *taxed*;[4] (ii) the optimal *tax* rate for such radical innovation is monotonically *increasing* in the size of quality improvement; (iii) R&D subsidies should be used only when the size

[1] See Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990).
[2] This is true for both improving-quality and expanding-variety models, as Benassy (1998) shows.
[3] See Figure 4.5 of Grossman and Helpman (1991, p.105).
[4] It is also optimal to tax innovation of small quality improvement.

of innovation falls into an intermediate range; and (iv) in some cases, policy implications (i) and (iii) are replaced with a stronger result that *all* R&D activities should be *taxed*.[5]

This paper argues that these inconsistent normative conclusions arise due to two simplifying restrictions adopted in the GH model, i.e. (1) Cobb-Douglas consumer preferences and (2) the lack of interactions between industrial and patent policies. We relax these restrictions as plausible generalisations of the in uential model. More importantly, we show that such generalisations are sufficient to eliminate the puzzling policy implications of the GH model by making it consistent with the received wisdom of substantial R&D under-investment in the market economy.[6]

Cobb-Douglas consumer preferences impose a unit elasticity of substitution between any two consumption goods. Here preferences are generalized by adopting the Dixit-Stiglitz-type CES utility function, allowing the elasticity of substitution to be greater than one. This extension may seem trivial. However, results obtained under a unit elasticity can sometimes change dramatically when the elasticity of substitution is greater than one. This is exempli ed by the Solow growth model. If the production function takes a Cobb-Douglas form in labour and capital without technical progress, endogenous growth is not possible. But growth can be endogenous if the elasticity of substitution is sufficiently large.[7]Under the assumption of the Dixit-Stiglitz-type CES utility function, we establish that (a) radical technological innovations should *always* be subsidized, and (b) the optimal subsidy for radical innovation is monotonically increasing in the size of quality enhancement. These results overturn the policy implications of GH mentioned above.

[5]In Figure 4.5 of Grossman and Helpman (1991, p.105), it occurs when $\lambda/\ln\lambda$ lies above $L/a\rho + 1$.

[6]Our theoretical approach complements Jones and Williams (1999) who address a similar issue on the basis of calibration exercises.

[7]See Barro and Sala-i-Martin (1995, pp.42-46).

The main reasons for these results lie in the balancing of opposing external effects. Positive externalities (e.g. knowledge spillovers) tend to make technology advance too slowly, whereas negative externalities have the opposite effect. In particular, the business-stealing effect plays a crucial role. This negative externality arises because private agents do not care about the adverse effects of their successful innovation on monopoly rents of others. In the GH model, this externality gets stronger with the size of quality improvement and manifests itself in GH s results (i)∼(iv) above. However, in our generalised models, once the size of innovation reaches a threshold level, the business-stealing effect reamins constant, while the positive externalities become greater. This is the key mechanism of our results.

As the second generalisation of the GH model, we introduce the consideration of patent breadth. In our context, patent breadth de nes the extent of quality improvement to which a product is protected from the infringement of its patent by lower-quality goods producers. A key question is whether R&D subsidies become more or less important when the government can set patent breadth.[8] We will establish that the optimal policy *always* involves R&D subsidies for incremental as well as radical innovation (i.e. irrespective of the size of quality improvement). Thus, when the patent policy is available, the importance of R&D subsidies become more pronounced in achieving the social optimum.

There is a further implication of our results. There are many studies which use the GH framework and derive policy implications based on a normative analysis.[9] Our study suggests that their analysis should be treated cautiously, as the results are not robust to the generalisations we make in this paper.

Next we brie y touch on widely found empirical results of R&D under-investment.

[8] See Hall and van Reenen (1999) and Lerner (1994) for effects on R&D of subsidies and patent breadth.
[9] For example, Grossman and Helpman (1991, Ch 6) and Glass and Saggi (1999).

Scherer (1982) and Terleckyj (1980) show that once spillovers within and across industries are taken into account, the social rate of return from R&D is about 100%. In summarising this class of studies, Griliches (1995, p.72) wrote that ... there has been a signi cant number of reasonably well done studies all pointing in the same direction: R&D spillovers are present, their magnitude may be quite large, and social rates of return remain signi cantly above private rates. Moreover, Jones and Williams (1998) demonstrate robustness of these empirical ndings on the basis of new growth theory. In the open economy context, substantial positive knowledge spillovers among countries are documented in Coe and Helpman (1995) and Coe, Helpman and Hoffmeister (1997). These empirical ndings strongly support government policy to actively encourage research activities, e.g. R&D tax credits and inter-governmental collaboration such as EU-wide Framework Programmes.

The present paper is structured as follows. Section 1 presents a generalized GH model, allowing the elasticity of substitution to be greater than one. In Section 2, the market equilibrium is compared with the social optimum. Section 3 introduces the patent breadth policy with the endogenous size of innovation. Section 4 concludes.

1 The Model

1.1 Consumers

There are a continuum of nal goods industries indexed by $j \in [0,1]$. $q_m(j)$ denotes quality level of each consumption good after it is upgraded m times $(m = 0, 1, 2...)$ due to technological innovation. We assume $q_m(j) = \lambda q_{m-1}(j)$ where $\lambda > 1$ is the size of innovation. Radical technological breakthroughs are those with a sufficiently large λ.

A representative consumer has the intertemporal utility function $\int_t^{\infty} e^{-\rho(\tau-t)} \ln D(\tau)\, d\tau$. ρ is the rate of time preference and consumption index $D(.)$ is de ned as

$$D(t) = \left\{ \int_0^1 \left[\sum_m q_m(j)\, x_{mt}(j) \right]^{\alpha} dj \right\}^{\frac{1}{\alpha}}, \qquad 1 > \alpha \geq 0 \tag{1}$$

where $x_{mt}(j)$ denotes consumption good of quality $q_m(j)$ in variety j at time t. Equation (1) shows that the elasticity of substitution between any two varieties is $\varepsilon = 1/(1-\alpha)$. The Cobb-Douglas preferences used in GH are a special case where $\alpha = 0$.

Note that goods of different quality levels in variety j are perfect substitutes. Thus, in equilibrium, consumers purchase goods of the lowest quality-adjusted price, which are equivalent to the state-of-the-art goods in industry j (see below). Given the CES structure, (1) gives rise to the demand function for the highest quality product:

$$x_{mt}(j) = \frac{q_m(j)^{\varepsilon-1}\, p_{mt}(j)^{-\varepsilon}}{\displaystyle\int_0^1 \left[q_m(j') / p_{mt}(j') \right]^{\varepsilon-1} dj'}. \tag{2}$$

In (2), aggregate consumers nominal spending is normalised to one, so the intertemporal utility maximization implies that the nominal interest rate always equals ρ.

1.2 Product Markets

One unit of variety goods is produced with one unit of labour services. With this technology, rms producing different quality goods in industry j are engaged in Bertrand price competition. This assumption makes sure that only varieties of the lowest quality-adjusted price are consumed. To verify this, substitute (2) into (1) to express the consumption index in terms of prices: $D(p_{mt}) = \left\{ \int_0^1 \sum_m \left[p_{mt}(j) / q_m(j) \right]^{-(\varepsilon-1)} dj \right\}^{1/(\varepsilon-1)}$. Clearly, $x_{mt}(j)$ gives the highest utility in j if and only if $p_{mt}(j) / q_m(j) \,\square\, p_{m-1t}(j) / q_{m-1}(j)$ or

$$p_{mt}(j) \,\square\, \lambda w(t) \tag{3}$$

where $w(t)$ is the wage rate and $p_{m-1t}(j) = w(t)$ due to Bertrand competition.[10]

Now, since the demand function (2) has a price elasticity of $-1/(1-\alpha)$, a top-quality firm sets $p_{mt}(j) = w(t)/\alpha$ as long as (3) holds or $1/\alpha \leq \lambda$. This case is called "drastic innovation" in the sense that firms' price decisions are not constrained by potential competition from incumbent producers of lower-quality goods.[11] On the other hand, if $p_{mt}(j) = w(t)/\alpha$ violates condition (3), the firm charges a limit price $p_{mt}(j) = \lambda w(t)$. This is the case of "non-drastic" innovation. In summary,

$$p_{mt}(j) = \frac{w(t)}{\theta} \quad \text{where } \theta \equiv \begin{cases} \alpha \quad \text{for } \lambda \geq 1/\alpha \qquad \text{(drastic innovation)} \\ 1/\lambda \quad \text{for } \lambda < 1/\alpha \qquad \text{(non-drastic innovation).} \end{cases} \tag{4}$$

Note that GH assume $\alpha = 0$, which precludes drastic innovation. This difference has important implications for welfare analysis below.

It is now easy to verify that the quality leader earns

$$\pi_{mt}(j) = (1-\theta)\frac{q_m(j)^{\varepsilon-1}}{Q(t)} \tag{5}$$

$$Q(t) = \int_0^1 q_{mt}(j')^{\varepsilon-1} dj' \tag{6}$$

where $Q(t)$ is the average quality across industries. To look ahead, let us identify two kinds of the business-stealing effects within and across industries following innovation in industry $j' \neq j$. The former quality leader in the same industry loses all of her profits. At the same time, profits in all other industries fall due to an increase in $Q(t)$. The within-industry business-stealing has a profound impact on the efficiency properties of the model. In addition, intuition suggests that the within-industry effect strengthens as profits increase. To explore the implication of this further, let us hold the market share

[10] If consumers are indifferent between the top and second-highest products on the quality ladder (i.e. $p_m = \lambda/w$), they are assumed to purchase the state-of-the-art goods.

[11] "Radical", another term used to capture a sufficiently large quality improvement in this paper, has nothing to do with pricing behaviour.

term $q_m(j)^{\varepsilon-1}/Q(t)$ in (5) constant. Then, the within-industry externality gets larger, as θ falls. That is, the externality effect expands as the size of innovation rises for non-drastic innovation when $\theta = 1/\lambda$. However, this effect does not increase further, once innovation becomes drastic, since $\theta = \alpha$. Thus, for a sufficiently high λ, the business-stealing effects are relatively weak compared to other positive externalities. This is the underlying mechanism of our results and is consistent with ndings of Jones and Williams (1998).

1.3 R&D

Quality improvement follows a stochastic process. If $R_{m+1t}(j)$ workers are used in R&D in industry j, the $(m+1)$th innovation is brought about with a Poisson arrival rate of[12]

$$\iota_{m+1t}(j) = \frac{R_{m+1t}(j)\,Q(t)}{aq_m(j)^{\varepsilon-1}}, \qquad a > 0. \tag{7}$$

Note that this is decreasing in the quality level $q_m(j)$. This captures the fact that R&D becomes increasingly difficult as the technological frontier advances. An example is silicon chips. Note that this assumption introduces a new source of negative externalities which tend to make the growth rate excessively high. It is because entrepreneurs do not internalize the detrimental effect of their successful innovation on the arrival rate of future technological breakthroughs.

This negative externality is offset at least partially by the knowledge spillover effect which is captured by $Q(t)$ in (7). A higher $Q(t)$ increases the probability of research success in industry j during an R&D race (during which $q_m(j)$ is xed), as other industries

[12] ι depends on α, a consumer utility parameter, to ensure *constant* long-run growth of utility. But, the homothetic utility function (1) can be taken as a production function with $u(.)$ and $x(.)$ denoting nal and intermediate products (see Grossman and Helpman (pp.49-50)). Thus, this assumption is justi ed if production technology is somehow related to innovation technology. In fact, such assumption is used in Barro and Sala-i-Martin (1995, p249-250).

experience R&D successes. Note that these positive effects are not confined to a given industry but extended across industries.[13] This is consistent with empirical studies which found strong inter-industry externalities (see Griliches (1995) for example). This feature is absent in the GH model.[14]

Entrepreneurs finance the up-front costs of R&D by issuing equity, and all profits are distributed as dividends to investors if R&D turns out successful. Let $v_{m+1t}(j)$ denote the stockmarket value if the $(m+1)$th invention is made in industry j. At each moment, entrepreneurs maximise the expected benefit of R&D $v_{m+1t}(j)\,\iota_{m+1t}(j) - (1-s)\,w(t)\,R_t(j)$ where s is the rate of R&D subsidies for $1 > s > 0$ and taxes for $s < 0$. The optimal choice of $R_{m+1t}(j)$ satisfies the first-order condition:

$$v_{m+1t}(j) = \frac{(1-s)\,aq_m(j)^{\varepsilon-1}\,w(t)}{Q(t)} \tag{8}$$

for $R_{m+1t}(j) > 0$.

The value of innovation is defined by the asset equation

$$\frac{\pi_{mt}(j)}{v_{mt}(j)} + \frac{\dot{v}_{mt}(j)}{v_{mt}(j)} - \iota_{m+1t}(j) = \rho. \tag{9}$$

The left-hand side gives the rate of return from equity investment, which is equated to that of safe assets. This condition ensures that investors are indifferent between the risky and safe financial assets. Now use (5) and (8) to rewrite the asset equation (9) as $\frac{(1-\theta)\lambda^{\varepsilon-1}}{(1-s)aw(t)} + \frac{\dot{w}(t)}{w(t)} - \frac{\dot{Q}(t)}{Q(t)} - \iota_{m+1t}(j) = \rho$. This implies $\iota_{m+1t}(j) = \iota(t)$, i.e. the arrival rate of innovation is identical in all industries.

In order to express the asset equation in terms of $\iota(t)$ and $w(t)$ only, note that quality improvement $q_{m+1}(j)^{\varepsilon-1} - q_m(j)^{\varepsilon-1}$ occurs with the arrival rate of $\iota(t)$. Therefore, the

[13] In (7), the exponent of $Q(t)$ is one, which is required for endogenous growth. See Li (1999).

[14] An important omission in (7) is the congestion externality, which causes over-investment in R&D. It is not included partly because we wish to develop discussion within the original GH model.

law of large numbers implies that $Q(t)$ changes according to

$$\dot{Q}(t) = \int_0^1 \iota(t) \left[q_{m+1}(j)^{\varepsilon-1} - q_m(j)^{\varepsilon-1} \right] dj = \left(\lambda^{\varepsilon-1} - 1 \right) \iota(t) Q(t) \tag{10}$$

where the second equality uses (6). Thus, (5), (8) and (10) enables us to express the asset equation (9) as

$$\frac{(1-\theta)\lambda^{\varepsilon-1}}{(1-s)aw(t)} + \frac{\dot{w}(t)}{w(t)} - \lambda^{\varepsilon-1}\iota(t) = \rho. \tag{11}$$

1.4 The Labor Market

There are two sources of labour demand. First, the R&D sector employs $R_t(t) = \int_0^1 R_t(j)\, dj = a\iota(t)$ which uses (6) and (7). Second, employment in the manufacturing sector is derived from (2): $M(t) = \int_0^1 x_{mt}(j)\, dj = \theta / w(t)$. Equating them to the xed labour supply L gives the full-employment condition:

$$L = a\iota(t) + \frac{\theta}{w(t)}. \tag{12}$$

2 Equilibrium

2.1 Decentralised Equilibrium

For the rest of analysis, we focus on steady state where $\dot{w} = 0$. The laissez-faire equilibrium is de ned by the system of two equations (11) and (12), and solving them gives

$$\iota^*(\lambda) = \frac{1}{1-s\theta} \left[(1-\theta)\frac{L}{a} - \theta \frac{(1-s)\rho}{\lambda^{\varepsilon-1}} \right]. \tag{13}$$

To compare our generalised model with the GH model, its solution reproduced:

$$\iota^{\mathrm{GH}}(\lambda) = \left(1 - \frac{1}{\lambda}\right)\frac{L}{a} - \frac{\rho}{\lambda} \tag{14}$$

which is obtained by setting $\varepsilon = 1$, $\theta = 1/\lambda$ and $s = 0$ in (13). Note that as λ gets arbitrarily large, ι^{GH} approaches L/a. On the other hand, ι^* converges to $(1-\alpha)\, L/a$ as $\lambda \to \infty$ for $s = 0$. That is, ι^* does not rise as much as ι^{GH} when innovation becomes sufficiently radical. This fact leads to the following observation.

Recall that ι^{GH} is monotonically increasing purely because λ determines the monopoly markup due to non-drastic innovation, hence a greater λ leads to higher profits. This generates a strong within-industry business-stealing effect when λ is large, creating empirically inconsistent policy implications, e.g. there is too much R&D incentives for radical innovation. The key assumption for this result is the Cobb-Douglas consumer preferences which make innovation non-drastic irrespective of the value of λ. On the other hand, ι^* does not rise as much as ι^{GH}, because innovation turns from non-drastic to drastic at $\lambda = 1/\alpha$. As a result, monopoly profits do not rise with λ above this threshold (with the market share term $q_m(j)^{\varepsilon-1}/Q(t)$ in (5) being fixed), creating an upper bound on the within-industry business stealing effect in our generalised model. This difference translates into a sharp contrast in the policy implications of the two models.

2.2 Social Optimum and Industrial Policy

Next we examine the social optimum. To economize on space, mathematical details are relegated to Appendix. It shows that the optimal R&D intensity is given by

$$\iota^s(\lambda) = \frac{L}{a} - \frac{\varepsilon - 1}{\lambda^{\varepsilon-1} - 1}\rho. \tag{15}$$

This expression is identical to the first-best solution of the GH model when $\varepsilon = 1$. Like the GH solution, the optimal intensity $\iota^s(\lambda)$ monotonically increases in λ. However, note $\frac{\varepsilon-1}{\lambda^{\varepsilon-1}-1} < \lim_{\varepsilon\to 1} \frac{\varepsilon-1}{\lambda^{\varepsilon-1}-1} = 1/\ln\lambda$ for $\lambda > 1$. This means that the GH model under-predicts the socially optimal R&D intensity.

There are several differences between (13) and (15). They are due to externalities which exist in the market economy but which are internalised by the social planner: (a) the positive consumer-surplus effects; (b) the positive knowledge spillover effects within and across industries; (c)the negative business-stealing effects within and across industries; (d) the negative intertemporal spillover effects due to increasing R&D difficulty; and (e) the absence of a monopoly distortion effect given the CES-type preferences.

To evaluate normative results, we evaluate the difference between $\iota^s(\lambda)$ and $\iota^*(\lambda)$:

$$\iota^s(\lambda) - \iota^*(\lambda) = \begin{cases} [A(\lambda) - n(\lambda)]\,\rho/\lambda & \text{for } \lambda < 1/\alpha \\ [A(\lambda) - d(\lambda)]\,\rho\alpha & \text{for } \lambda \geq 1/\alpha \end{cases} \tag{16}$$

where $A(\lambda) = L/a\rho + \lambda^{-(\varepsilon-1)}$, $n(\lambda) = \frac{(\varepsilon-1)\lambda}{\lambda^{\varepsilon-1}-1}$ and $d(\lambda) = \varepsilon/\left(\lambda^{\varepsilon-1}-1\right)$. This shows that the sign of $\iota^s(\lambda) - \iota^*(\lambda)$ depends on that of the terms inside the square brackets. Note that since $s = 0$ in (16), the optimal industrial policy is subsidies ($s > 0$) when technology advances too slowly ($\iota^s(\lambda) > \iota^*(\lambda)$) or taxes ($s < 0$) in the reverse situtation ($\iota^s(\lambda) < \iota^*(\lambda)$). To facilitate presentation, we distinguishes two cases; (a) a high elasticity of subsitutiton, $\varepsilon \in [2, \infty)$, and (b) a low elasticity of substitution, $\varepsilon \in (1, 2)$.

2.2.1 A High Elasticity of Substitution: $\varepsilon \in [2, \infty)$

Fig. 1 depicts (16) for $\varepsilon \in [2, \infty)$.[15]First consider $n(\lambda)$ which is relevant for non-drastic innovation. It monotonically falls up to $\lambda = 1/\alpha$. An intuition lies in the balance of the externalities identi ed above. As λ increases, the magnitude of the positive and negative externalities are changing. In particular, the combined effects of the positive externalities are *increasing* faster than the combined negative externality effects. This leads to a

[15] $\underline{\lambda}^s$ denotes the minimum level of λ for $\iota^s > 0$. It is greater than the minimum λ required for $\iota^* > 0$.

monotonic decline of $n(\lambda)$. As regards $d(\lambda)$ for drastic innovation, the same intuition holds, in addition to the fact that the magnitude of the within-industry business stealing effect does not change with λ.

Now, we are in a position to establish the rst result:

Result 1: *The set of innovations which should be subsidised is not empty.*

The result should be clear from Fig. 1. Recall that this set can be empty in the GH model (result (iv) mentioned in the introduction). Moreover, in Fig. 1, all innovations of quality $\lambda > \widehat{\lambda}$ should be subsidised. If radical technological innovation is taken as one with a sufficiently large size of quality improvement, we can state the following result:

Result 2: *Radical innovations should be subsidised.*

In Fig. 1, $\widehat{\lambda}$ happens to be smaller than $1/\alpha$, and we can have $\widehat{\lambda} \geq 1/\alpha$. But the point is that innovation of greater quality improvement is more likely to require R&D subsidies to achieve the Pareto optimum. Note that innovation of incremental quality improvement $\lambda < \widehat{\lambda}$ should be taxed. However, this result is modi ed once λ is endogenised (see below).

Next we examine how the optimal rate of subsidy changes. In Fig. 1, the sign of $\iota^s(\lambda) - \iota^*(\lambda)$ depends on the vertical distance between $A(\lambda)$ and $n(\lambda)$ or $d(\lambda)$. In fact, the distance governs the extent to which R&D should be subsidised for $\iota^s > \iota^*$ or taxed for $\iota^s < \iota^*$. Now consider the case of drastic innovation. From (16), we can easily establish $A(\lambda) - d(\lambda) = L/a\rho - \left(\varepsilon - 1 + \lambda^{-(\varepsilon-1)}\right) / \left[\lambda^{\varepsilon-1}\left(\lambda^{\varepsilon-1} - 1\right)\right]$, which is rising in λ. In Fig. 1, the vertical distance between $A(\lambda)$ and $d(\lambda)$ is increasing as λ gets larger.

Result 3: *The rate of the optimal subsidy for radical innovation is monotonically increasing in the size of innovation.*

As regards non-drastic innovation, the optimal subsidy (and tax) can change in either direction in general.

2.2.2 A Low Elasticity of Substitution: $\varepsilon \in (1, 2)$

This case is closer to the GH model in that the elasticity of substitution is relatively small. $n(\lambda)$ now takes a U shape, as Fig. 2 shows. It is increasing for large λ, since an increase in the within-industry business-stealing effect becomes so strong in that range. A noticeable feature is that the range of the optimal subsidy can be interrupted by the optimal tax in the intermediate range of $\lambda \in (\lambda', \lambda'')$. However, it is also possible that the kink at $1/\alpha$ is located below $A(\lambda)$ so the range of the optimal R&D subsidy is continuous. In any case, Results 1∼3 derived above still hold true for this case as well.

In the GH model, R&D subsidies should be applied to innovation with the intermediate size of quality improvement (result (iii) mentioned in the introduction). Why is this? In the GH model, $A(\lambda)$ is horizontal at $L/a\rho + 1$ in Fig. 2, since $\varepsilon = 1$, and the kinked local maximum of $n(\lambda)$ at $\lambda = 1/\alpha$ goes to in nity and $n(\lambda)$ extends along the dotted curve. Moreover, the $n(\lambda)$ curve can be entirely located above $L/a\rho + 1$, so it is always optimal to tax R&D. Clearly these GH results are overturned in our generalised model, highlighting the crucial role of the unit elasticity of substitution in their welfare analysis.

3 Industrial and Patent Policies

In the analysis conducted above, it is optimal to subsidise radical innovation but tax incremental innovation. In this section, this result is modi ed when patent policy is introduced with the endogenous size of innovation. We will establish that the optimal policy always involves R&D subsidies irrespective of λ. Analysis is conducted for $\varepsilon = 1$ to isolate the effect on R&D subsidies of introducing patent policy and endogenous λ.[16]

[16] In this section, we drop the argument j and subscript m unless ambiguity may arise.

3.1 Patent Breadth

The statutory duration of patent life is still assumed to be infinite. But the government can now determine the breadth of patent. Suppose that q_m is the highest quality in an industry. The patent breadth permits its holder to prohibit the producer of the second-highest quality goods from producing quality above q_{m-1}/b where $b > 0$ measures the patent breadth.[17] $b = 1/\lambda$ means no patent protection, and GH impose $b = 1$.[18]

Recall that only the product of the lowest quality-adjusted price is consumed. That is, for the state-of-the-art product of quality q_m to be consumed, we must have $p_m/q_m \leq p_{m-1}b/q_{m-1}$ or $p_m \leq b\lambda w$ (see Section 1.2). Therefore, since $\alpha = 0$ is assumed, the profit-maximising price is $p_m = b\lambda w$. The associated profits are given by (5) with $\varepsilon = 1$ and $\theta = b\lambda$. Note that an increase in patent breadth (a rise in b) increases profits. By implication, growth is promoted by a greater patent breadth, as one expects.

Let us briefly consider the effect of introducing the patent policy on the industrial policy for an exogenous λ. When technology advances too fast ($\iota^s < \iota^*$) due to too strong private R&D incentives, the social optimum requires R&D taxes which reduce R&D profitability. Extending this reasoning, the social optimum can in fact be achieved by narrowing patent breadth (i.e. reducing b) without R&D taxes. Similarly, when $\iota^s > \iota^*$ (too little R&D incentives), the government can achieve the social optimum by widening patent breadth that raises R&D profitability, without R&D subsidies.[19] Thus, the patent policy makes the industrial policy less important. This should not come as surprise,

[17] We are implicitly assuming that any quality level between q_m and q_{m-1} can be produced once q_m is invented.

[18] The patent literature distinguishes leading and lagging breadth of a patent (see O'Donoghue, *et al.* (1998)). The former protects a patented product from superior goods, while the latter from inferior goods. The distinction does not matter in our model due to symmetric nature of patent breadth assumed.

[19] This requires $b > 1$, i.e. property rights of lower quality goods should be transferred to a patent holder of the top-quality goods. It cannot be socially optimal when λ is endogenous (see below).

because the government tries to hit a single target of the optimal R&D intensity with two policy tools. But the story changes when the government has twin goals of the optimal R&D intensity and size of innovation.

3.2 Endogenous Size of Innovation

Given $\alpha = 0$, the Poisson arrival rate of innovation is given by $\iota = R/a(\lambda)$ where $a'(\lambda) > 0 > a''(\lambda)$. The arrival rate falls as researchers aim at a greater quality improvement. As before, entrepreneurs maximise the expected bene t of R&D $\Pi = v\iota - (1-s)wR$ to determine the optimal R. This gives the R&D free entry condition (8) with $\varepsilon = 1$ and a being replaced with $a(\lambda)$. In choosing λ, entrepreneurs again maximise Π, which can be rewritten as $\Pi = (1 - 1/b\lambda) R/a(\lambda) C - (1-s) wR$ where $C = \rho + \iota - \dot{v}/v$, using (5) and (9) after imposing $\varepsilon = 1$. Taking C as given, entrepreneurs choose the pro t-maximising size of innovation λ^*, which is de ned by

$$\frac{\lambda^* a'(\lambda^*)}{a(\lambda^*)} = \frac{1}{b\lambda^* - 1} \tag{17}$$

where $b > 1/\lambda^*$ is assumed. Note that λ^* is independent of s.

Fig. 3 depicts (17), assuming that $\lambda a'(\lambda)/a(\lambda)$ is non-decreasing in λ to exclude multiple equilibria for simplicity. Note that a narrower patent breadth (a fall in b) increases λ^*. An intuition is simple. The marginal bene t of increasing λ is an increase in future pro ts, and the marginal cost is a reduction of the probability of an R&D success. A lower b increases the marginal bene t but reduces the marginal cost. Putting it differently, as patent breadth becomes narrow, pro ts fall, but to offset this, entrepreneurs raise the size of the quality step. Note that λ^* is independent of R&D subsidies. Therefore, the primary objective of the patent policy is to induce the rm to achieve the socially optimal λ.

The market outcome (17) is now compared with the social optimum. Appendix shows that the socially optimal size of quality step λ^s is de ned by

$$\frac{\lambda^s a'(\lambda^s)}{a(\lambda^s)} = \frac{1}{\ln \lambda^s} \tag{18}$$

Given $b = 1$, Fig. 3 shows that $\lambda^* < \lambda^s$, since $\ln \lambda < \lambda - 1$. The result is due to an externality known as the appropriability effect. Entrepreneurs decision of λ are purely motivated by pro ts, whereas maximising the social surplus is the objective of the social planner. Since pro ts are always smaller than the social surplus, given the downward sloping demand curve, the private incentive is always too small.

Interestingly, to achieve the social optimum, the patent breadth should fall (i.e. b should drop from $b = 1$). Note that this reduces instantaneous pro ts $1 - 1/b\lambda$ (see (5)), hence an R&D incentive. Achieving the optimal size of innovation comes at the expense of falling R&D efforts. Moreover, differences between λ^s and λ^* are caused by the fact that a private incentive is too small. Despite this, the optimal policy is to *reduce* pro ts by narrowing the patent breadth. This somewhat counter-intuitive result is the underlying mechanism of our key result that the industrial policy becomes more important when λ is endogenous.

3.3 The Optimal Policy Mix

The government has two goals of achieving (1) the optimal intensity of innovation and (2) the optimal size of innovation with the aid of the industrial and patent policy. First consider the size of innovation. Using (17) and (18), the rst-best solution $\lambda^s = \lambda^*(b^s)$ can be re-expressed as

$$b^s = \frac{1 + \ln \lambda^s}{\lambda^s} \tag{19}$$

where λ^s is defined in (18). This condition defines the optimal patent policy b^s required for the optimally sized innovation. Note $b^s \in (1/\lambda^s, 1)$, as b^s decreases in λ^s. Not surprisingly, the optimal patent breadth is greater than $1/\lambda^s$, leaving profit incentives for R&D.

Next, it should be clear that the private R&D intensity (13) is not affected by the assumption of endogenous λ, except for a being replaced with $a(\lambda)$. Appendix also shows that the social optimum still requires (15) with a being replaced with $a(\lambda)$. Given these results, the first-best R&D intensity is achieved when $\iota^s = \iota^*(s^s)$ (for $\varepsilon = 1$), i.e.[20]

$$s^s = \frac{\Delta - 1}{\Delta - (b^s\lambda^s)^{-1}}, \qquad \Delta = \left(\frac{L}{a\rho} + 1\right)\left(1 - \frac{1}{b^s\lambda^s}\right). \tag{20}$$

This defines the combination of the industrial and patent policy required for the optimal R&D intensity. We are interested in the sign of s^s, which depends on the value of Δ. In fact, we can easily verify that $\Delta > 1$ as long as $\iota^* > 0$ where ι^* is defined by (13) with $\varepsilon = 1$.[21] Moreover, since $(b^s\lambda^s)^{-1} < 1$ (see (19)), we also have $\Delta > (b^s\lambda^s)^{-1}$. Therefore, we established the following result.

Result 4: *It is always optimal to subsidise R&D when the size of innovation is endogenous and the government can set the patent breadth.*

To develop an intuition, let us first consider the GH model which imposes $b = 1$. In this case, the optimal size λ^s cannot be achieved, as λ^* is independent of R&D subsidies (see (17)). The second-best policy, which ensures $\iota^* = \iota^s$ only, is to subsidise or tax R&D, depending on parameter values. Starting from this situation, suppose that the government can now set the patent breadth. To achieve the socially optimal λ^s, the government decreases b from $b = 1$ such that (19) is satisfied. In turn, it tends to reduce profits and private incentives for R&D, resulting in $\iota^* < \iota^s$. To restore the socially optimal

[20] (20) is obtained by equating (13) and (15) for $\varepsilon = 1$, then using (19) to eliminate $\ln \lambda^s$.
[21] We can write $\Delta - 1 = \iota^*/\rho > 0$.

R&D intensity, R&D is subsidised to the extent that Result 4 holds. In developing this intuition, we established the final key result.[22]

Result 5: *R&D subsidies become more important in attaining the first-best outcome when the government has the twin goals of achieving the first-best R&D intensity and size of innovation with the industrial and patent policy tools.*

4 Conclusion

To reconcile the policy implications of the influential GH model with the widely-accepted empirical finding of R&D under-investment, this paper extended the model in two important ways; the unit elasticity of substitution of goods in consumption is dropped, and the interactions of industrial and patent policies are introduced. These generalisations revealed that the policy implications of GH are not robust. We established that R&D subsidies should play a more important role in achieving the Pareto optimum than the GH model suggests. We believe that our results represent a step forward to a firmer understanding of the role of public policy towards R&D which is clearly required in modern economies where technological progress is increasingly important.

Appendix: The Social Optimum

Static Optimisation: We first examine the problem of the static labor allocation across manufacturing industries, taking total workers in manufacturing as constant. Dropping the time argument, the social planner solves $\max_{x_m(j)} \ln D$ s.t. $M = \int_0^1 x_m(j)\,dj$ where D is given in (1). With δ denoting the Lagrangian multiplier, the first-order conditions

[22] Result 5 is not affected by the introduction of finite patent life. In fact, it will reinforce the result, because finite patent duration reduces an R&D incentive, increasing the optimal rate of subsidies.

are given by $\delta \int_0^1 [q_m(j') x_m(j')]^\alpha dj' = q_m(j)^\alpha x_m(j)^{\alpha-1}$, which gives rise to $x_m(j') = [q_m(j')/q_m(j)]^{\varepsilon-1} x_m(j)$ for $j \neq j'$. Substituting this expression into the left-hand side of the rst-order conditions yields $x_m(j) = q_m(j)^{\varepsilon-1}/Q\delta$. Plugging this back into $M = \int_0^1 x_m(j) dj$ gives $M = 1/\delta$, which enables us to rewrite $x_m(j) = q_m(j)^{\varepsilon-1}/Q\delta$ as $x_m(j) = Mq_m(j)^{\varepsilon-1}/Q$. Substituting this into (1) gives $D = MQ^{\frac{1}{\varepsilon-1}}$.

Dynamic Optimisation: The social planner maximises $\int_0^\infty e^{-\rho t} \ln MQ^{\frac{1}{\varepsilon-1}} dt$ subject to $\dot{Q} = \left(\lambda^{\varepsilon-1} - 1\right) \iota Q$ and $L = a\iota + M$. The Hamiltonian is

$$\mathcal{H} = \ln MQ^{\frac{1}{\varepsilon-1}} + \xi \frac{\left(\lambda^{\varepsilon-1} - 1\right)(L - M)}{a} Q \tag{21}$$

where ξ is a costate variable. By Pontryagin s maximum principle,

$$\frac{1}{M} = \xi \frac{\lambda^{\varepsilon-1} - 1}{a} Q, \tag{22}$$

$$\dot{\xi} = \rho\xi - \frac{1}{(\varepsilon - 1) Q} - \xi \frac{\left(\lambda^{\varepsilon-1} - 1\right)(L - M)}{a}. \tag{23}$$

In steady state where $\dot{M} = 0$, (22) implies $\dot{\xi}/\xi = -\dot{Q}/Q$. Using this result and (22), equation (23) becomes $\xi Q = \frac{1}{(\varepsilon-1)\rho}$. Substituting this into (22) gives $M^s = (\varepsilon - 1) \rho a / \left(\lambda^{\varepsilon-1} - 1\right)$. Using this and the labour market condition (12), one can easily derive (15).

Endogenous Size of Innovation: Now $a(\lambda)$ is a function of λ. The Hamiltonian (21), the rst-order conditions (22) and (23) and the optimal R&D intensity (15) do not change, except for a being replaced with $a(\lambda)$. As regards the optimal size of innovation, it is obtained by maximising (21) with respect to λ. The rst-order condition is $\lambda a'(\lambda)/a(\lambda) = (\varepsilon - 1)/\left(\lambda^{\varepsilon-1} - 1\right)$, which is reduced to (18) for $\varepsilon \to 1$.

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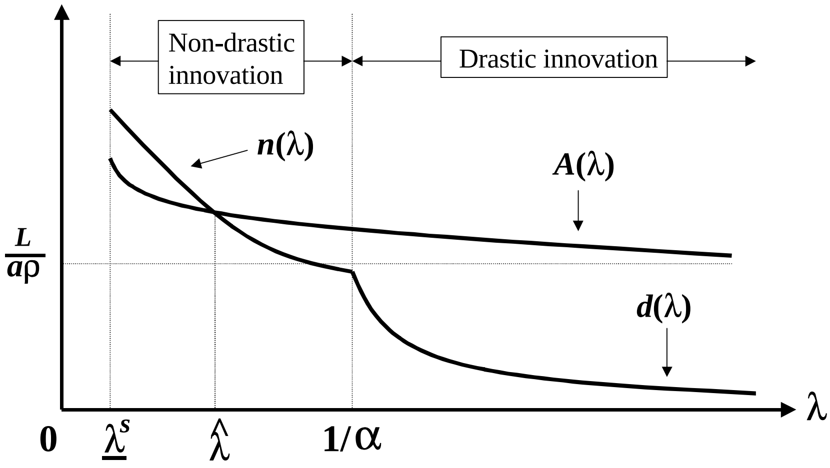

Figure 1: The case of $\varepsilon \in [2, \infty)$.

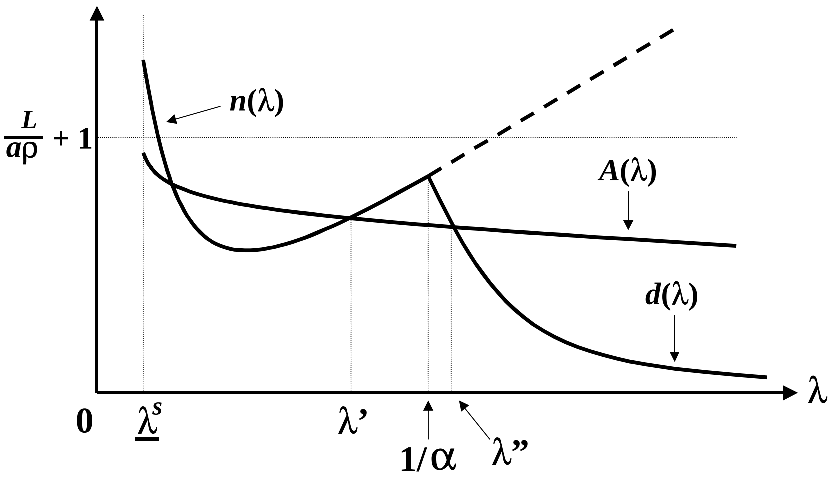

Figure 2: The case of $\varepsilon \in (1, 2)$.

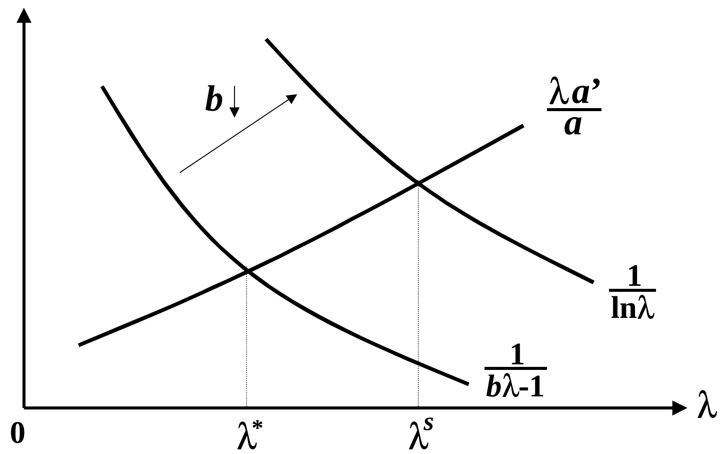

Figure 3: The effect of a narrower patent breadth.