The Life-Cycle-Permanent-Income Model: A Reinterpretation and Supporting Evidence

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December 2002

ABSTRACT

It is generally agreed that the consumption path implied by the standard stochastic life-cycle version of the permanent-income model follows a random walk. The failure of the latter to conform to data, however, undermines the suitability of the framework within which the random walk path is obtained. We propose an alternative interpretation of Friedman's revision rule which implies that consumption follows an *ARIMA*(1,1,0) path. We show that this path is compatible with the solution to a life-cycle optimising problem with habit formation and precautionary saving motives. Evidence, obtained by applying the Kalman filter technique to U.S. data for 1929-2001, strongly supports the proposed approach.

KEYWORDS: permanent-income; excess sensitivity; excess smoothness; habit formation; precautionary saving; Kalman filter

JEL CLASSIFICATION: E21

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1. INTRODUCTION

The life-cycle framework continues to be one of the most popular behavioural frameworks within which micro-based models are developed to study a variety of macroeconomic phenomena. The flexibility and richness of the framework have made it possible to examine the role of a number of factors crucial to understanding consumer behaviour. For example, the choice of the objective function has allowed exploring questions regarding the effects of 'impatience', 'attitude towards risk', 'non-separabilities'[1] and 'precautionary saving'. Moreover, a range of relevant constraints and time horizons have been employed to investigate, for example, the effects of 'capital market imperfections', 'Ricardian equivalence', 'rationality of expectations', 'bequests', 'age', etc. on consumption. Combined with advanced computational techniques which enable the handling of complicated dynamic optimisation problems, these features of the life-cycle framework have also made it possible for researchers to find explanations for key phenomena – e.g. the asset pricing and equity premium puzzle (Abel, 1990; Constantinides, 1990); the response of output to monetary and fiscal policy shocks (Fuhrer, 2000; Ljungqvist and Uhlig, 2000); the positive correlation between saving and growth (Carroll, Overland and Weil, 2000); etc.

Parallel to these developments, the relevance of agents' heterogeneous behaviour and their asymmetric access to information – and hence the importance of aggregation – have been increasingly recognised in macroeconomic analysis (see, for instance, Lewbel, 1994; Goodfriend, 1992; Clarida, 1991; Galí, 1990; Pischke, 1995). Nevertheless, the micro-based models, which have been developed on the basis of a representative agent's optimal intertemporal behaviour, continue to play a crucial role in providing intuitive explanations for various macroeconomic phenomena. One of the best known amongst these is a version of the

[1] Both, over time (as, for instance, implied by habit persistence) as well as across typical factors entering the temporal utility function, e.g. categories of consumption (durables, nondurables, services, etc), leisure, public goods and consumption of other relevant agents (as in "catching up with Joneses").

permanent-income model proposed by Friedman (1957), hereafter referred to as the LC–PI model.

Two features of the LC–PI model attracted researchers' attention immediately. First, it approximates the representative agent's consumption path by a rule-of-thumb smoothing or revision process which states that at any point in time the agent sets (planned) consumption equal to the annuity associated with the present value of the total – human and non-human – wealth. More importantly, this path can also be derived, within the life-cycle framework, by solving a utility maximisation problem that explicitly incorporates the structure of intertemporal preferences and budget constraints. Second, the LC–PI model yields a relationship between consumption and income which has theoretically interpretable parameters and is empirically superior to those implied by the earlier, somewhat *ad hoc*, rival models – namely the 'absolute income' and the 'relative income' hypotheses. However, a glance through the literature on the consumption function over the last two decades raises severe doubts in one's mind about the ability of the LC–PI model to deliver a robust empirical relationship between consumption and income (see Deaton, 1992, for details). Briefly, while for most time series data sets the existence of a unit root in the level of *per-capita* real consumption cannot be rejected and the change in *per-capita* real consumption can be safely regarded as a stationary stochastic process, the latter series tend to exhibit a rather strong first order autoregressive pattern. This has led to the main empirical objections to the LC–PI model on the grounds that consumption exhibits 'excess sensitivity' and 'excess smoothness' with respect to income. These were originally discussed, respectively, by Flavin (1981) and Deaton (1987) in connection with testing the random walk model which was implied by Hall's (1978) interpretation of the LC–PI model. Clearly, these findings, which were also confirmed by other studies, cannot be disregarded when the LC–PI model is used to approximate the intertemporal consumption decision of a representative agent in micro-based models that are designed to study macroeconomic phenomena. Nevertheless, given its intuitively appealing foundations, it would be desirable to generalise the LC–PI model so that its implications cohere with the empirical regularities of the relationship between consumption and income reported in the literature.

A number of studies have already questioned the way evidence is interpreted in connection with the framework originally proposed by Friedman (1957) and later elaborated in Friedman (1963) (see Carroll, 2001a, for an example). This paper is another attempt in this direction. We re-examine the rule-of-thumb smoothing implied by the LC–PI model and show that it is more plausible to interpret this rule within the life-cycle framework if the representative agent's preferences exhibit some degree of habits persistence in consumption. This is in contrast to the existing practice in the literature on the optimising version of the permanent-income model where the representative agent's preferences are always assumed to be fully separable over time. It is now well established that consumption series generated by the solution to a life-cycle optimisation problem under habit persistence exhibit strong autocorrelation properties (for details see Muellbauer, 1988; Campbell and Cochrane, 1995; Alessie and Lusardi, 1997; Carroll, 2000; Guariglia and Rossi, 2002). We show that this property matches with a reinterpretation of a rule-of-thumb smoothing or revision scheme of the kind originally proposed by Friedman and it reconciles the theory with the evidence; the theoretical consumption path is an *ARIMA*(1,1,0) process and evidence, obtained by applying the Kalman filter technique to the recently revised U.S. aggregate data for 1929-2001, strongly supports the proposed approach.

The rest of the paper is organised as follows. Section 2 sets out the theoretical issues by: (i) briefly reviewing the life-cycle framework and showing how the relevant version of the permanent-income model fits into that framework; (ii) explaining the 'excess sensitivity' and 'excess smoothness' problems; (iii) showing that the permanent-income model is a smoothing rule that is consistent with the optimal plan of a life-cycle model with habit persistence; and finally, (iv) arguing that the 'excess sensitivity' and 'excess smoothness' problems need not arise under the new interpretation. Section 3 uses the recently revised U.S. data to test the empirical relevance of the model developed in Section 2. Section 4 concludes the paper.

2. THEORY

It is convenient to start by restating the standard definitions which are commonly used in the literature and which will also be used throughout this paper. Using discrete time denoted by subscript t and an infinite time horizon, the life-cycle framework that is relevant to our analysis can be summarised in the following way. At the beginning of any period t, the agent chooses C_{t+j} for all $j \geq 0$ in order to maximise the expected value of the objective function $E_t[U_t]$ – where $U_t = U(C_t, C_{t+1}, \ldots, C_{t+j}, \ldots)$ and E_t denotes the expectations operator conditional on the information at the beginning of period t – subject to the constraint

$$A_{t+j+1} = \left(1 + r_{t+j}\right)A_{t+j} + X_{t+j} - C_{t+j}\,. \tag{1}$$

Equation (1) is the budget constraint that should hold for all $j \geq 0$, C is consumption, X is real (after tax) labour income, A is the real value of stock of non-human wealth, and r is the real (after tax) interest rate between two adjacent periods. Note that A_{t+j} is measured at the beginning of period and $r_{t+j}A_{t+j}$, C_{t+j} and X_{t+j} are payments which are assumed to take place at the end of period. In the absence of any capital market imperfections, with an infinite planning horizon and suitable transversality condition, the solution to this problem yields a smoothing rule for the expected marginal utility of consumption, i.e.

$$E_t\left(\frac{\partial U_t}{\partial C_{t+j-1}} - (1 + r_{t+j})\frac{\partial U_t}{\partial C_{t+j}}\right) = 0; \quad j \geq 1\,. \tag{2}$$

It is easily shown that if (i) the utility function is time separable and satisfies certain standard properties, and (ii) the agent regards the real interest rate as a constant and uses it to discount both the future income and the future utility of consumption (i.e. if the constant discount rate provides an accurate measure of the rate of time preference), then the rule in (2) implies that agent's expected consumption remains constant, i.e. $E_t C_{t+j} = E_t C_t$ for all $j \geq 1$. As time passes, the only revisions in the previously formulated plans, i.e. $\left(E_{t+1} C_{t+j} - E_t C_{t+j}\right)$, are due to the unexpected factors which affect agent's income.

2.1. The LC–PI model

Assuming that the real rate of interest remains constant and letting $\rho \equiv 1/(1+r)$, the lifetime version of the budget constraint in equation (1) that can be utilised in the absence of liquidity constraints (where agents can lend or borrow at a constant rate in a perfect capital market against their future income) is

$$\sum_{j=0}^{\infty} \rho^{j+1} C_{t+j} = A_t + \sum_{j=0}^{\infty} \rho^{j+1} X_{t+j} . \tag{3}$$

Within the above framework, permanent income, Y^P, is defined as the annuity associated with the present value of the human and non-human wealth, i.e. the right-hand-side of (3). Thus,

$$Y_t^P = r\left(A_t + \sum_{j=0}^{\infty} \rho^{j+1} E_t X_{t+j} \right). \tag{4}$$

Using equation (3), Y^P also satisfies the following

$$r \sum_{j=0}^{\infty} \rho^{j+1} E_t C_{t+j} = Y_t^P , \tag{5}$$

and, hence,

$$Y_t^P = (1/\rho) Y_{t-1}^P - ((1-\rho)/\rho) C_{t-1} + V_t , \tag{6}$$

where V is the annuity associated with the present value of the revisions in future income due to news between two adjacent periods (see Flavin, 1981, for details),

$$V_t = r \sum_{j=0}^{\infty} \rho^{j+1} \left(E_t X_{t+j} - E_{t-1} X_{t+j} \right). \tag{7}$$

Note that V will behave as an unpredictable disturbance term if expectations are formed rationally. Thus, because $E_{t-1}V_t = 0$, it follows that an agent who consumes its permanent income will also expect it to remain constant in the future. In other words, if we let $C_{t-1} = Y_{t-1}^P$, then $E_{t-1}Y_t^P = Y_{t-1}^P$ follows. This simple rule-of-thumb consumption revision scheme, which is consistent with the solution to the intertemporal utility maximisation described above, lies at the heart of Friedman's contribution[2]. However, Friedman's actual account deviated from this simple framework and resulted in some confusion which was later noted by other writers[3]. The latest version of the LC–PI model which, following Hall (1978), is commonly known as the random walk model in the literature, is derived from Friedman's model when the rational expectations hypothesis is used to revise permanent income. To illustrate this here we follow Campbell and Deaton (1989) and assume that labour income X can be approximated by an *ARIMA*(1,1,0) process

$$\Delta X_t = \lambda \Delta X_{t-1} + \varepsilon_t , \tag{8}$$

where Δ is the first difference operator, $0<\lambda<1$ is a constant parameter and ε is an independently distributed random disturbance. Given that equations (7) and (8) also imply the following, respectively,

$$V_t = \sum_{j=0}^{\infty} \rho^j \left(E_t \Delta X_{t+j} - E_{t-1} \Delta X_{t+j} \right), \tag{7'}$$

and

$$E_t \Delta X_{t+j} - E_{t-1} \Delta X_{t+j} = \lambda^j V_t ; \quad j \geq 0 , \tag{8'}$$

we can substitute from (8') into (7') to obtain

[2] See Lettau and Uhlig (1999) for details on the rule-of-thumb nature of the revision and how it relates to an optimisation rule. Rodepeter and Winter (2000) use numerical analysis to compare utility levels under rules-of thumb and optimisation rules.

[3] See Friedman (1957, 1960 and 1963). Johnson (1971) and Darby (1974) explain the theoretical issues. Sargent (1979) discusses the specification of the process for updating permanent income. Zellner and Geisel (1970) examine the econometric specification of the model with a transitory consumption. Carroll (2001a) provides an account of the way the literature has deviated from Friedman's intended model.

$$V_t = \pi\varepsilon_t\,, \tag{9}$$

where $\pi = (1-\lambda\rho)^{-1} > 1$. The optimal intertemporal path of consumption can now be obtained as the reduced form of equations (6) and (9) and the assumption that the *marginal propensity to consume out of permanent income is unity*, that is $C_{t+j} = Y^P_{t+j}$ since, as explained above, within the life-cycle version agents are assumed to consume all their permanent income. These yield the random walk model,

$$\Delta C_t = \pi\varepsilon_t\,. \tag{10}$$

2.2. Conflict between theory and evidence

A version of the random walk model was originally derived and tested by Hall (1978). Afterwards, two studies, Flavin (1981) and Deaton (1987), raised severe doubts about the empirical validity of this model. Flavin showed that the cross equation restrictions between generalisations of (10) and (8) are violated empirically since past changes in actual income turn out to be significant when they are included as additional regressors in (10). Deaton compared the sample variances of ε_t and ΔC_t and illustrated that the data implied $Var(\Delta C_t) < Var(\varepsilon_t)$ hence violating the theoretical requirement that $\pi > 1$ should hold in (10). Many other studies have examined these issues empirically for data sets from various countries. Overall, the accumulated evidence supports the joint proposal by Flavin and Deaton that consumption exhibits an excessive degree of sensitivity and smoothness with respect to income beyond that implied by the random walk version of the LC–PI model[4].

2.3. A reinterpretation of the LC–PI model

We now explain that an alternative interpretation of Friedman's smoothing rule yields a path for consumption which is different from the random walk model outlined above. The crucial

[4] See Pesaran (1992) and Deaton (1992) for further details on both theoretical and empirical aspects. For further aspects, see West (1988), Quah (1990), Caballero (1990), Campbell and Mankiw (1991), Flavin (1993) and Carroll (1994).

point in our departure is to note that in the life-cycle version of Friedman's permanent-income model the role of Y^P as a catalyst is no longer needed. Hence, rather than introducing Y^P and then restricting the marginal propensity to consume out of it to unity, we simply solve out Y^P from the model and use the result to specify directly an updating rule for consumption[5]. To do so, we first substitute from equations (5) and (9) into (6) to obtain

$$(1-\rho)\sum_{j=0}^{\infty}\rho^j E_t C_{t+j} = \big((1-\rho)/\rho\big)\sum_{j=0}^{\infty}\rho^j E_{t-1}C_{t+j-1} + \big((1-\rho)/\rho\big)C_{t-1} + \pi\varepsilon_t\,,$$

which we rearrange as follows

$$\sum_{j=0}^{\infty}\rho^j\left(E_t\Delta C_{t+j} - E_{t-1}\Delta C_{t+j}\right) = \pi\varepsilon_t\,. \tag{11}$$

Equation (11) states that the present value of the revision in the consumption plan should be proportional to the present shock to income. The simplest revision rule consistent with (11) is one based on exponentially declining weights (suggested by Friedman for updating permanent income), namely

$$E_t\Delta C_{t+j} - E_{t-1}\Delta C_{t+j} = \beta^j k\pi\varepsilon_t; \quad j \geq 0\,, \tag{12}$$

where β is a constant parameter reflecting the weight used to smooth the path of ΔC_t and $k = 1-\beta\rho$ ensures that the path in (12) remains consistent with the budget constraint in (11)[6]. Clearly, as long as $\beta<1$, equation (12) is simply a revision rule for updating the change in consumption between periods t and t-1 once the news about income at t embodied in ε_t is revealed, namely

$$\Delta C_t = \beta\Delta C_{t-1} + k\pi\varepsilon_t\,. \tag{13}$$

The generalised, empirical, version of (13) may be written in the following way

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5 See Deaton (1991) and Carroll (2001b) for different explanations of why the marginal propensity to consume out of permanent income can be less than unity.

6 Galí (1991) uses a generalisation of this process and derives restrictions to test the relative smoothness of consumption.

$$\Delta C_t = \alpha_t + \beta \Delta C_{t-1} + \gamma \xi_t + u_t , \tag{14}$$

where α_t is a drift parameter representing any 'autonomous' factors that agents may use in their revision, ξ_t is the 'empirical' version of income innovation term, u_t is an $iid(0,\sigma^2)$ disturbance term, and β and γ are constant parameters representing, respectively, the extent of habit formation and the 'marginal propensity to consume out of transitory income' as elaborated by Friedman (1960, 1963).

It is a straightforward exercise to show that equation (14) also corresponds to the solution to the life-cycle optimisation problem described at the beginning of this section, provided that the utility function is not fully time separable and satisfies certain other standard conditions. A number of studies have addressed the implications, as well as the empirical validity, of the intertemporal separability assumption. These studies explore the possibility and consequences of allowing for intertemporally non-separable preferences due to various behavioural phenomena, e.g. rational addiction, habit persistence, seasonality, subjective discounting and aversion to intertemporal trade-offs. Winder and Palm (1991) and Deaton (1992) provide detailed explanations of the technical and behavioural aspects of the problem[7]. Evidence has also been emerging which shows that allowing for habit persistence enables one to find an explanation for problems that could not be explained with fully time separable preferences – see, for example, Abel (1990), Fuhrer (2000), Ljungqvist and Uhlig (2000) and Carroll, Overland and Weil (2000).

Building on the framework used in Caballero (1990), Alessie and Lusardi (1997) focus on obtaining a closed form solution to the life-cycle problem outlined at the beginning of this section with habit persistence introduced to a more general class of (temporal) utility

[7] For other details, see Iannaccone (1986), Becker and Murphy (1988), Muellbauer (1988), Constantinides (1990), Loewenstein and Prelec (1992), Heaton (1993), and Dockner and Feichtinger (1993).

functions. Allowing for some non-separability by letting $U_t = \sum_{j=0}^{\infty}(1+\delta)^{-j}u(C_{t+j}^*)$ – where $0<\delta<1$ is the subjective rate of time preference, $u(C_{t+j}^*)$ is the temporal utility, $C_{t+j}^* = C_{t+j} - \beta C_{t+j-1}$, and β captures the extent of habit persistence[8] – they show that a negative exponential utility function (i.e. CARA) implies that C_{t+j}^* follows a martingale with drift, namely,

$$C_t^* = a_t + C_{t-1}^* + \eta\varepsilon_t . \tag{15}$$

In the above equation, a_t is the drift term capturing the extent of the precautionary saving motives induced by the uncertainty about future income, ε_t is again the income innovation term as before and η is a parameter measuring the marginal propensity to consume out of transitory income. Following Caballero (1990), they show that with a negative exponential utility function the latter is related to the (time-varying) conditional variance of consumption. Upon substitution from $C_{t+j}^* = C_{t+j} - \beta C_{t+j-1}$, equation (15) yields an expression similar to (14). Guariglia and Rossi (2002) point out the disadvantages of using a negative exponential utility function and argue in favour of adopting a CRRA utility function. They show that, subject to minor modifications, the corresponding regression equation will not be different from (14).

2.4. Empirical implications

In this section we have argued that, unlike what is taken as granted in the literature, the LC–PI model does not need to imply that consumption should be modelled as a random walk process with drift. It is of course true that the latter would provide an accurate representation of the consumption path if the marginal propensity to consume out of permanent income were unity,

[8] See Deaton (1992) for details of this specification of habit formation. Recently, Carroll (2000) has proposed a richer specification by letting $C_{t+j}^* = C_{t+j} / C_{t+j-1}^{\beta}$.

planned consumption were to remain constant, and the only revisions were due to surprises in income. But as we have shown, a more plausible interpretation of Friedman's smoothing process within the life-cycle framework implies that consumption has an *ARIMA*(1,1,0) representation with drift. Furthermore, not only does our reinterpretation not change the consistency of the permanent-income model with the life-cycle framework, the *ARIMA* path of consumption can also be shown to correspond to the solution to a life-cycle optimisation problem when consumption habits persist. In other words, the new interpretation provided in this paper suggests that the LC–PI model is more relevant for explaining the consumption decision of a representative life-cycle optimising agent whose preferences exhibit habit formation. It is now becoming increasingly clear in the literature that the latter feature is rather important and, as pointed out in the introduction, it has been used to provide explanations for a number of phenomena such as the asset pricing and equity premium puzzle (Abel, 1990; Constantinides, 1990), the response of output to monetary and fiscal policy shocks (Fuhrer, 2000; Ljungqvist and Uhlig, 2000) and the positive correlation between saving and growth (Carroll, Overland and Weil, 2000). It is also clear that the excess smoothness problem outlined above is no longer so acutely relevant under the new interpretation. To see this, we compare equation (13) with its predecessor, equation (10). As noted above, the latter has been used in the literature to show how the LC–PI implied that changes in consumption are more volatile than income innovations since $Var(\Delta C_t) = (1-\lambda\rho)^{-2} Var(\varepsilon_t) > Var(\varepsilon_t)$ always follows from (10) for all plausible values of the parameters λ and ρ. However, using equation (13) instead, we see that the unconditional variance of ΔC_t will not exceed that of ε_t as long as λ, ρ and β satisfy the condition $(1-\beta\rho)^2 - (1-\beta^2)(1-\lambda\rho)^2 < 0$ [9].

[9] This follows since (10) implies $\Delta C_t = k\pi \sum_{s=0}^{\infty} \beta^s \varepsilon_{t-s}$ where $k = 1-\beta\rho$ and $\pi = (1-\lambda\rho)^{-1} > 1$. See the Appendix for the more general case in (14), i.e. $\Delta C_t = \beta \Delta C_{t-1} + (\alpha_t + \gamma_t \varepsilon_t)$ where α_t and γ_t follow *AR*(1).

Following the formulation offered by Hall (1978), much of the empirical evidence on the permanent-income hypothesis reported in the literature rejects the explicit 'infinite-horizon liquidity-unconstrained life-cycle rational-optimising representative-agent' version of the model. The above analysis, however, suggests a different specification of the model and is sufficiently persuasive to invite another attempt at checking the empirical consistency of the LC–PI model formulated above. This task is taken up in the next section.

3. EVIDENCE

In this section we confront equation (14) with data to test the empirical consistency of the underlying theory. To choose an appropriate data set, which is in accordance with the relevant features of the theory, we recall that equation (14) is derived on the basis of the intertemporal consumption decisions of an agent with the following qualifications. First, the agent is a *'representative consumer'* who: (i) has an infinite planning horizon with no concern for any specific terminal conditions (such as bequests); and (ii) does not face any liquidity constraints. The most appropriate measure therefore is *per-capita* aggregate consumption. Second, *'consumption'* in this case should be defined as outlay corresponding to all items that cannot be classified as *physical* assets. Hence, it would have to exclude the expenditure on durable goods and mortgage payments. The measure closest to this is aggregate consumers' expenditure on non-durable goods and services. Third, the frequency of data should match the implications of the underlying theory. Whilst the life-cycle framework is, in general, totally flexible in accommodating intertemporal planning at all frequencies (see Browning and Crossley, 2001, for discussion and evidence), the version of the permanent-income hypothesis described above is more relevant in the context of decisions concerning medium term, or year to year, plans. This is because, by construction, the focus of (this maximising version) of Friedman's original hypothesis is not on how consumption is allocated within a year, but on how the annuity of total (human and nonhuman) wealth is allocated over the medium term (see

Carroll, 2001a, for further details). Given this, and recalling the lag structure assumed for the habit formation mechanism, it is more appropriate to use annual frequency series[10] to test the empirical consistency of equation (14).

We shall use the U.S. annual series for the period 1929-2001 on (personal) consumers' expenditure and disposable income[11]. The added advantage of this data set, in connection with testing the model developed above, is its coverage of several crucial episodes – i.e. the years following the Great Depression; World War II, the Korean War; the stable growth over the 1960s; the shocks over 1970s; and the depressions of early 1980s and 1990s – which allow us to test the evolution of habit formation as well as changes in the extent of any effects due to precautionary saving motives. In the rest of this section we describe the statistical features of the series, present our estimates of equation (14) and use a number of tests to detect if the theory, as summarised in equation (14), is empirically robust.

3.1. Statistical features of the series

Table 1 gives the relevant summary sample statistics for the levels of *per-capita* consumption and income (C and Y) and their changes (ΔC and ΔY), whose sample behaviours are depicted in Figure 1[12]. From these, it is clear that: (i) income has, in general, been more volatile than consumption; (ii) the volatility in both series reduced drastically in the 1950-1970 interval; (iii) both C and Y have a unit root whilst their first differences, ΔC and ΔY, are stationary AR

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[10] In the model discussed here the momentary utility is $u(C^*_{t+j})$ where $C^*_{t+j} = C_{t+j} - \beta C_{t+j-1}$. Unless the aim is to explain habit formation over very short frequencies such as weekly, the model should take account of seasonal nature of habits for monthly or quarterly frequencies. For instance, in the latter case we should postulate $C^*_{t+j} = C_{t+j} - \beta C_{t+j-4}$.

[11] Although theory requires the use of labour income, data for the latter does not exist and a reliable measure for the period 1929-2001 cannot be constructed. We follow the literature and approximate income innovation by the unanticipated component of personal disposable income.

[12] All data were obtained from the U.S. National Income and Product Accounts (NIPA), Bureau of Economics Analysis, U.S. Department of Commerce. All series (except population) are measured in billions of chained 1996 dollars. Nominal non-durable and services expenditure were obtained from NIPA Table 1.1 (Gross Domestic Product). The implicit price deflators for these consumption components were obtained from NIPA Table 7.1 (Quantity and Price Indexes for Gross Domestic Product). Real personal disposable income (i.e. nominal income deflated by the implicit price deflator for personal consumption expenditures) and population were obtained from NIPA Table 2.1 (Personal Income and Its Disposition).

processes; and (iv) the gap between Y and C has steadily increased over time. This pattern is also confirmed statistically by extensive co-integration tests allowing for various specifications of the deterministic term[13].

Table 1. Summary Sample Statistics for C, ΔC, Y and ΔY

Volatility of Consumption and Income

	1930-2001		1930-1949		1950-1969		1970-2001	
	MEAN	S.D.	MEAN	S.D.	MEAN	S.D.	MEAN	S.D.
ΔC	0.120	0.175	0.077	0.183	0.194	0.100	0.280	0.164
ΔY	0.251	0.322	0.092	0.412	0.255	0.187	0.348	0.292

Unit Root Tests (excluding a linear deterministic trend)

C			ΔC			Y			ΔY		
WS	ADF	PP	WS	ADF	PP	WS	ADF	PP	WS	ADF	PP
0.274	2.248	1.378	-3.127	-4.936	-29.85	0.350	1.054	1.133	-4.250	-5.485	-47.08
[0.995]	[0.999]	[0.995]	[0.008]	[0.000]	[0.001]	[0.996]	[0.995]	[0.992]	[0.000]	[0.000]	[0.000]

The Autocovariance Structure of Stationary Variables

	Order	AC	S.E.	L-B
	1	0.548	0.118	22.6 [0.000]
ΔC	2	0.274	0.149	28.3 [0.000]
	3	0.078	0.156	28.8 [0.000]
	1	0.315	0.118	7.43 [0.006]
ΔY	2	0.215	0.129	11.0 [0.004]
	3	-0.089	0.134	11.6 [0.009]

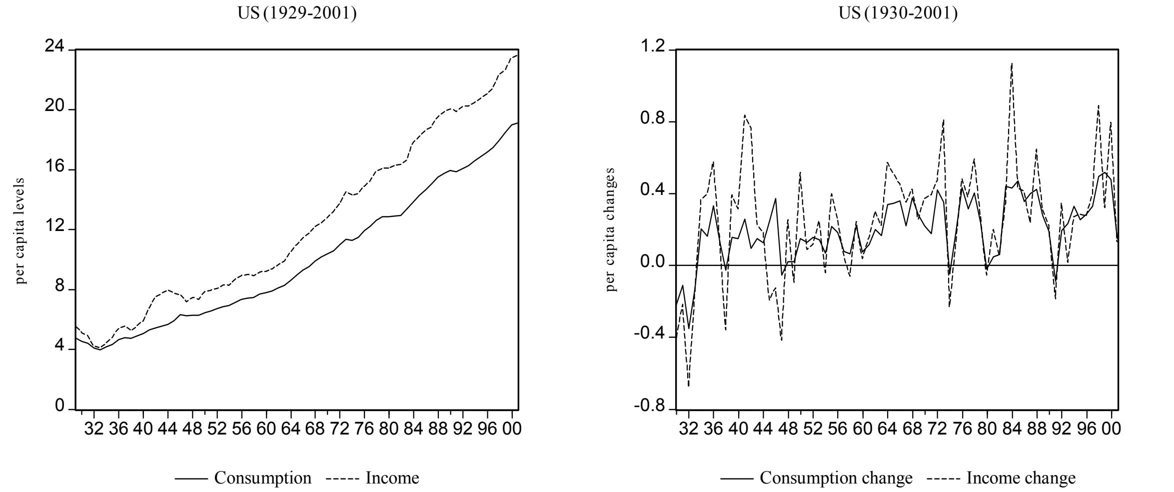
- Given that the power of univariate unit root tests can vary considerably (see, for instance Pantula *et al.*, 1994), several alternative tests are presented: WS, ADF and PP are the Weighted Symmetric (see Pantula *et al.*, 1994), the Augmented Dickey-Fuller (see Dickey and Fuller, 1979, 1981) and the Phillips-Perron (see Phillips and Perron, 1988) tests for unit roots.
- Numbers in square brackets are the *p-values* and are calculated using the tables reported in MacKinnon (1994).
- The optimal lag length, determined by the Akaike Information Criterion (max lag=2 and min lag=0) was found to be 2 for all cases. These tests yield consistent conclusions, which remain unaltered when a linear deterministic trend is added to the testing equations. To preserve space these results are not reported here but will be made available upon request.
- AC, S.E., and L-B are the Autocorrelation Coefficient, Standard Error of the Autocorrelation Coefficient, and Ljung-Box statistics for the corresponding lag. The latter is distributed $\chi^2_{(n)}$ where n is the number of lags. The numbers in square brackets are *p-values*.

Clearly, the evidence that ΔC is correlated with its own past is sufficiently strong to reject the hypothesis that the level of consumption follows a random walk process, and to proceed to estimating and testing equation (14). To do so, however, we require a measure of

[13] The results are not reported here but are available on request.

income innovation. Following common practice, we approximate the income process by a univariate *ARIMA* model. The results are reported in Table 2 below.

Figure 1. Levels and Changes in Consumers' Expenditure on Non-durable Goods and Services and Personal Disposable Income



Unless otherwise stated all the calculations in the Tables and Figures which follow make use of the entire sample period, 1929-2001.

Table 2. OLS Estimation of the Income Generating Process, $\Delta Y_t = \varphi + \lambda \Delta Y_{t-1} + \xi_t$

Coeff. Estimates: $\hat{\varphi}$= 0.181 (3.998); $\hat{\lambda}$=0.315 (2.850)

Diagnostic Tests: S_1=2.389 [0.122]; S_2=1.487 [0.222]; S_3=0.657 [0.720]; S_4=0.102 [0.950]

Volatility of Income Innovation, $\hat{\xi}_t$

1930-2001		1930-1949		1950-1969		1970-2001	
MEAN	S.D.	MEAN	S.D.	MEAN	S.D.	MEAN	S.D.
0.000	0.297	-0.095	0.361	0.000	0.190	0.057	0.305

Autocovariance Structure of $\hat{\xi}_t$

Order	AC	S.E.	L-B
1	-0.071	0.119	0.38 [0.539]
2	0.109	0.119	1.26 [0.532]
3	-0.146	0.121	2.89 [0.409]

Numbers in square bracket are *p-values*; S_1 is the Lagrange multiplier $\chi^2_{(1)}$ statistic for residual first-order serial correlation; S_2 is the Ramsey RESET $\chi^2_{(1)}$ test for functional form misspecification (based on the square of fitted values); S_3 is the $\chi^2_{(2)}$ test for the normality (based on a test of skewness and kurtosis of the residuals); and S_4 is the $\chi^2_{(1)}$ statistic for heteroscedasticity (based on a regression of squared residuals on squared fitted values). For AC, S.E., and L-B see the notes in Table 1.

3.2. *Empirical consistency of the ARIMA(1,1,0) version of the LC–PI model*

Preliminary estimates of the modified LC–PI model, i.e. equation (14) with a time invariant intercept, and other related specifications are given in columns (I)-(IV) of Table 3 below. In column (I) we give estimates of the random walk version of the LC–PI model, which allows for 'excess sensitivity' of consumption with respect to income, as defined by Flavin (1981), i.e. $\Delta C_t = \alpha + \phi \Delta Y_{t-1} + \gamma \xi_t + u_t$. It is clear that the random walk model fails due to exhibiting the strong 'excess sensitivity' symptom as the coefficient of ΔY_{t-1} is positive and significant. Next, in column (II) we give estimates of the random walk model which is augmented with both ΔY_{t-1} and ΔC_{t-1}. The results show that the further addition of ΔC_{t-1} renders the coefficient of ΔY_{t-1} insignificant. This finding throws doubt on interpreting the significance of ΔY_{t-1} in the random walk model as a sign of 'excess sensitivity'. If the theory discussed in the previous section is relevant, then ΔY_{t-1} in column (I) was merely capturing the omitted effect of habit formation, which is better embodied in ΔC_{t-1}. We therefore drop ΔY_{t-1} and in column (III) give estimates of the *ARIMA*(1,1,0) version of the LC–PI model. We also provide the relevant diagnostic as well as nested and non-nested test statistics (see the notes to Table 3) to compare the statistical performance of the specifications in columns (I)-(III).

These results on the whole support the specification in column (III). In order to check if the latter exhibits any symptoms of omitted dynamics – due, as suggested in the literature, to information lags and/or aggregation over different cohorts with a finite horizon – in column (IV) we include ΔC_{t-2} and $\hat{\xi}_{t-1}$ as additional regressors and find that neither plays a significant role; the value of the $\chi^2_{(2)}$ Wald statistic for the joint significance of the coefficients of these variables was 0.149 with *p-value* of 0.928 (ΔY_{t-1} was also included in addition but was found to be statistically irrelevant). Finally, we also note that the estimates in column

(III) do not raise concerns for excess smoothness as β =0.466 and λ =0.315 (see Table 2) imply that the condition explained at the end of Section 2 (and footnote 8) holds for all r < 0.5.

Table 3. OLS estimates of $\Delta C_t = \alpha + \beta \Delta C_{t-1} + \gamma \xi_t + u_t$
and various related specifications

Regressors	coefficient estimates			
	(I)	**(II)**	**(III)**	**(IV)**
Intercept	0.152 (8.62)	0.114 (7.04)	0.112 (6.89)	0.1220 (5.02)
ΔC_{t-1}	---	0.401 (2.99)	0.466 (7.01)	0.404 (2.50)
$\hat{\xi}_t$	0.369 (7.51)	0.334 (6.23)	0.328 (6.59)	0.333 (5.93)
ΔY_{t-1}	0.212 (4.05)	0.045 (0.52)	---	---
ΔC_{t-2}	---	---	---	0.027 (0.240)
$\hat{\xi}_{t-1}$	---	---	---	0.033 (0.372)
Statistics				
$\overline{R}^2$	0.577	0.643	0.645	0.616
σ	0.110	0.1009	0.1006	0.1025
S_1	9.79 [0.002]	1.63 [0.201]	2.10 [0.147]	1.46 [0.227]
S_2	4.67 [0.031]	6.00 [0.014]	6.18 [0.013]	5.84 [0.016]
S_3	9.78 [0.008]	3.21 [0.201]	2.75 [0.252]	2.78 [0.249]
S_4	4.41 [0.354]	15.4 [0.018]	3.62 [0.461]	15.21[0.055]
S_5	3.57 [0.000]	---	0.75 [0.453]	-0.22[0.824]
S_6	12.8 [0.000]	---	0.56 [0.456]	0.114[0.893]

Numbers in parentheses below coefficient estimates are *t-ratios* adjusted for heteroscedasticity; σ is the standard error of the regression; S_1 to S_4 are defined as in Table 2 and S_5 and S_6 are non-nested tests for the model in the corresponding column against its rival model. S_5 is JA test statistic proposed by Fisher and McAleer (1981) and has a *t-distribution*, whereas S_6 is the Encompassing test statistic suggested by Mizon and Richard (1986) and has an F(1,66) distribution. Numbers in square brackets are the *p-values* corresponding to S_j.

3.3. Liquidity constraint and precautionary saving

Given the results in Table 3, the empirical performance of the *ARIMA* model is encouraging. It performs quite well against the random walk alternative, resolves the ‘excess sensitivity’ and ‘excess smoothness’ problems, and does not exhibit any symptoms of omitted dynamics that can typically arise when the underlying model is based on inadequate assumptions. Moreover, it supports the significant role of habit formation, which is becoming increasingly relevant in explaining various macroeconomic phenomena. We therefore proceed by subjecting the *ARIMA* specification to two further tests. First, we recall that the version of the LC–PI model explored here describes a smoothing rule that can only apply in the absence of liquidity constraints. We therefore should test whether the *ARIMA* specification remains robust against a more general alternative in which liquidity constraints are binding for some agents. We follow the approach by Campbell and Mankiw (1991) who, on the assumption that constrained agents set $\Delta C_t = \Delta Y_t$, suggest estimating a weighted average of the latter and the original regression equation describing the behaviour of the unconstrained agents. Denoting the weights by θ and (1-θ) respectively – and noting that θ can be interpreted as the proportion of agents for whom constraints are binding – this approach implies estimating

$$\Delta C_t = \alpha + (1-\theta)\beta \Delta C_{t-1} + (1-\theta)\gamma \xi_t + \theta \Delta Y_t + u_t , \tag{16}$$

and testing whether θ is statistically significant. The results are provided in Table 4 below and show no sign of significant presence of liquidity constraints.

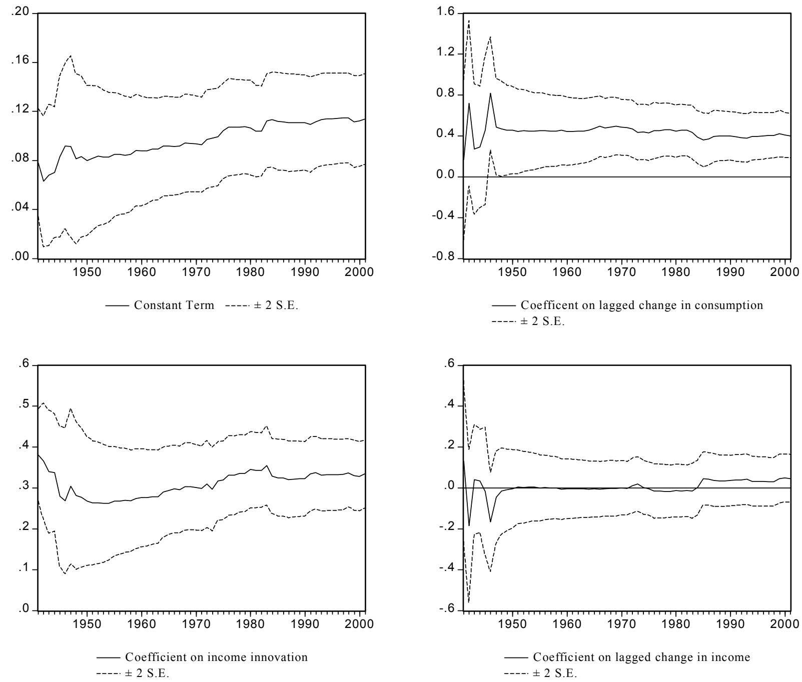
Table 4. Testing for the presence of liquidity constraint

Estimation Method	Coefficient Estimates				Diagnostic Statistics[(i)]			
	α	β	γ	θ	$\bar{R}^2$	σ	RSS	D-W
LS[(ii)]	0.0875 (2.40)	0.4687 (5.78)	0.2220 (1.32)	0.1443 (0.77)	0.6426	0.1008	0.6816	1.997
GMM[(iii)]	0.0864 (1.66)	0.4669 (3.86)	0.3691 (1.33)	0.1528 (0.413)	0.5646	0.1092	0.7874	2.161
LS[(iv)]	0.1136 (6.16)	0.4202 (4.38)	0.3501 (6.73)	0.0455 (0.77)	0.6426	0.1008	0.6816	1.997

Numbers in parentheses below coefficient estimates are *t-ratios* adjusted for heteroscedasticity; **(i)** σ, RSS and D-W, denote the standard error of the regression, residual sum of squares and Durbin Watson statistic for 1st order autocorrelation; **(ii)** LS is non-linear least squares; **(iii)** the instrument set included a constant term, ΔY_{t-1}, $\hat{\xi}_{t-1}$ and ΔC_{t-2}. The calculated value of the J-statistic is 0.00; **(iv)** This regression replaces ΔY_t with ΔY_{t-1} on the grounds that liquidity constrained agents may set $\Delta C_t = \Delta Y_{t-1}$.

Next, we examine whether there is any significant evidence for the presence of 'precautionary saving motive' effects. As pointed out above, the optimising version of the LC–PI model captures the latter by a drift term in the *ARIMA*(1,1,0) model. Thus, if precautionary savings affect the path of consumption, the constant intercept in the regressions reported in columns of Table 3 should be replaced with a time-varying intercept. However, before doing so we examine the extent to which the parameters of the model are, in general, time-invariant by re-estimating the regression equation in column (II) of Table 3 recursively. Figure 2 below shows the parameter estimates and their confidence intervals.

Figure 2. Recursive Coefficient Estimates for $\Delta C_t = \alpha + \beta \Delta C_{t-1} + \delta \Delta Y_{t-1} + \gamma \xi_t + u_t$



It is clear from these recursive estimates that, once the degrees freedom become sufficiently adequate, (i) the intercept exhibits a strong time-varying pattern that resembles fluctuations around a 'mild' deterministic trend; (ii) the coefficient of ΔC_{t-1} rapidly settles around a positive constant value implying that the pattern and extent of habit formation has remained constant and stable over the sample; (iii) the coefficient of $\hat{\xi}_t$ tends to fluctuate around a positive constant indicating that long but stable cycles might be inherent in the marginal propensity to consume out of transitory income; and finally, (iv) excess sensitivity is remarkably absent throughout the whole sample as the coefficients of ΔY_{t-1} rapidly settles around 0. These findings support the presence of precautionary saving effects which are embodied in a time-varying intercept. They also explain why diagnostic tests, especially Ramsy's RESET and heteroscedasticity tests, did not indicate 'clean residuals' in Table 3. We therefore re-estimated the regressions in Table 3 by adding a deterministic time trend as an additional regressor. The results are reported in Table 5 below.

Table 5. OLS estimates of $\Delta C_t = \alpha_o + \alpha_1 t + \beta \Delta C_{t-1} + \gamma \xi_t + v_t$
and its various generalisations

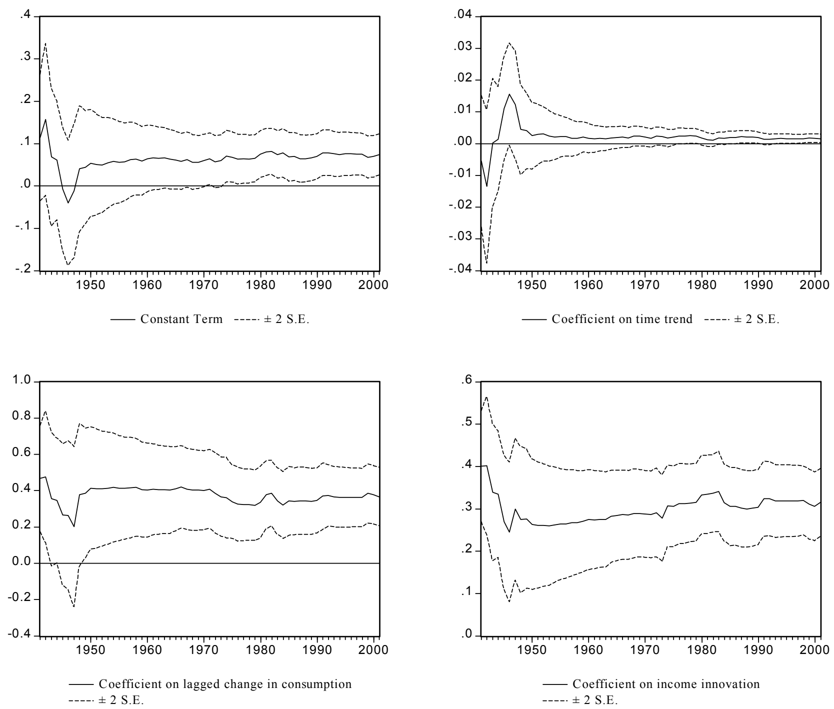
Regressors	**coefficient estimates**			
	(I)	**(II)**	**(III)**	**(IV)**
Intercept	0.078 (2.91)	0.074 (3.05)	0.074 (3.09)	0.083 (3.06)
Time Trend, *t*	0.0024 (3.38)	0.0016 (2.04)	0.0016 (2.07)	0.0018 (2.13)
ΔC_{t-1}	---	0.274 (1.77)	0.367 (4.65)	0.331 (1.98)
$\hat{\xi}_t$	0.336 (6.74)	0.322 (6.19)	0.315 (6.40)	0.318 (5.72)
ΔY_{t-1}	0.158 (3.16)	0.061 (0.71)	---	---
ΔC_{t-2}	---	---	---	-0.044 (-0.393)
$\hat{\xi}_{t-1}$	---	---	---	0.038 (0.440)
Statistics				
$\bar{R}^2$	0.644	0.665	0.665	0.641
σ	0.1007	0.0976	0.0977	0.0992
S_1	4.890 [0.027]	0.701 [0.402]	0.177 [0.674]	1.954 [0.162]
S_2	0.457 [0.499]	3.088 [0.079]	3.997 [0.046]	4.081 [0.043]
S_3	18.94 [0.000]	6.139 [0.046]	3.654 [0.161]	4.122 [0.127]
S_4	3.870 [0.694]	13.90 [0.084]	4.955 [0.550]	13.50 [0.197]
S_5	2.263 [0.024]	---	1.042 [0.297]	-1.012[0.308]
S_6	5.121 [0.027]	---	1.086 [0.301]	0.710 [0.495]

All the notes in Table 3 apply to this Table. However, the Encompassing test statistic S_6 in this Table has an F(1,65) distribution for columns I and III and F(2,63) for column IV.

The estimates reported in Tables 5 indicate that the coefficient of the time trend is positive (although very small) and significant and that this modification does not change any of the previous conclusions regarding the robustness of the model in column (III) against the 'excess sensitivity' problem and omitted dynamics. As shown in Figure 3 below, which gives the recursive estimates of the coefficients of the model with deterministic trend, adding the latter appears to have explained the pattern in the intercept in Figure 2. Diagnostic tests however do not indicate improvement when comparing the estimates in column (III) of Tables 3 and 5.

In view of drastic changes in volatility in the economy over the sample period 1929-2001, it is unlikely that a deterministic time trend can uniformly capture the extent to which the consumption path reflects the impact of precautionary saving motives. Furthermore, adding a deterministic trend does not change the behaviour of the estimates of the marginal propensity to consume out of transitory income which, as shown in Figure 3, still exhibit the cyclical pattern as before.

Figure 3. Recursive Coefficient Estimates for $\Delta C_t = \alpha_o + \alpha_1 t + \beta \Delta C_{t-1} + \gamma \xi_t + v_t$



The Kalman filter estimation method provides a more satisfactory approach to including a time-varying intercept in the *ARIMA*(1,1,0) model by allowing the drift term itself to have an $AR(1)$ or a random walk generating process, i.e. $\alpha_t = \mu\alpha_{t-1} + \zeta_t$ where $0 < \mu \leq 1$ is a constant parameter and ζ_t is an $iid(0,\sigma^2)$ random disturbance term. The advantage of this stochastic, rather than deterministic, specification of the intercept term is to let α_t be 'optimally' estimated subject to the sample information and any prior belief regarding the size of the coefficient μ (see Harvey, 1989 for details). In addition, given that the recursive estimates reported above indicate that the coefficient of income innovation exhibits a cyclical pattern over the sample period, we can also exploit the flexibility offered by the Kalman filter approach to explore whether the coefficient capturing the marginal propensity to consume out of transitory income has in fact responded to the degree of uncertainty surrounding the future of the economy. Such cyclical fluctuations are more in line with Friedman's characterisation of the precautionary savings, which are induced by uncertainty about the future income[14]. We can therefore allow this coefficient to also evolve according to an $AR(1)$ or random walk generating process, i.e. replace γ with $\gamma_t = \nu\gamma_{t-1} + \psi_t$, where $0 < \nu \leq 1$ is a constant parameter and ψ_t is an $iid(0,\sigma^2)$ random disturbance term.

Given the pattern of the recursive estimates in Figures 2 and 3, it is plausible to experiment with μ and ν close, or equal, to unity. Our estimates suggest that, based on the values of the log-likelihood (LL) and the Akaike information criterion (AIC), the stationary cases with μ and ν close to unity are statistically preferable[15] to the random walk specifications for α_t and γ_t. In Table 6 below we report the estimates for the values of μ and ν, which maximises the LL and minimise the AIC in a grid search between 0.9 and 1. The

[14] See Friedman (1960 and 1963). Going through his explanations, it becomes difficult, at times, to distinguish clearly between the discount rate, the inverse of planning horizon, the subjective rate of time preference, marginal propensity to consume out of the transitory component of income (or cash windfalls), and the weight in the adaptive expectations scheme that is used to revise permanent income, i.e. the coefficient ϕ in $\Delta Y_t^P = \phi\left(Y_t - Y_{t-1}^P\right)$. In one way or another, all these factors are affected by, and hence should reflect, the degree of uncertainty about the future.

Table only gives estimates of the final states α_T and γ_T, but Figure 4 plots the smoothed estimates of α_t and γ_t over the sample.

Table 6. Kalman Filter Estimation of $\Delta C_t = \alpha_t + \beta \Delta C_{t-1} + \gamma_t \xi_t + u_t$

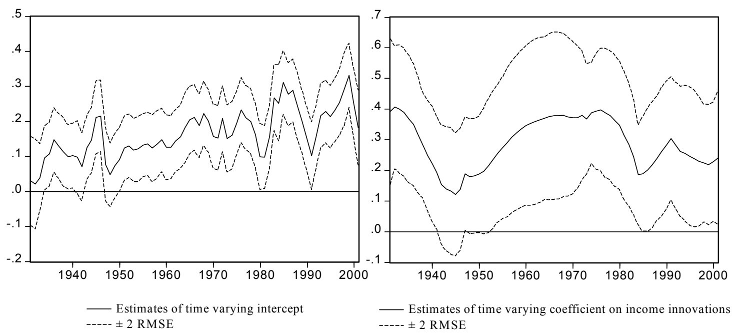
$\alpha_t = \mu\alpha_{t-1} + \zeta_t$; $\gamma_t = \nu\gamma_{t-1} + \psi_t$; $\mu = \nu = 0.97$; $u_t \sim iid(0, \sigma^2)$

Coefficient	Estimate	Std. Error(1)	z-Statistic	Prob.
β	0.213	0.883	2.411	0.0159
Final State α_T	0.177	0.084	2.10	0.0356
Final State γ_T	0.233	0.125	1.86	0.0624
Log likelihood:	50.30	Akaike info. criterion:		-1.361
σ:	0.0664(2)			

(1) For estimates of, α_T and γ_T, we report the Root MSE rather than Std Errors.

(2) Note that $\sigma = \sqrt{e^{\hat{p}}}$, *where* $\hat{p} = -5.425$ with an estimated standard error of 0.152.

Figure 4: Smoothed State Estimates of α_t and γ_t



It is clear that the sample behaviour of α_t reflects a response to changes in the volatility of the economy at large, with troughs occurring over periods when the economy has been depressed. The sample behaviour of γ_t also shows that it responds to the degree of uncertainty in the economy, although γ_t exhibits a much smoother pattern than α_t. Clearly, the patterns of fluctuations in α_t and γ_t are in line with the essence of precautionary savings and the evidence provided here puts the LC–PI model in a new light which enables us to better

[15] The LL and AIC for $\mu = \nu = 1$; $\mu = \nu = 0.95$; and $\mu = \nu = 0.90$ are LL=33.86, AIC=–0.897; LL=50.02, AIC=-1.35; and LL=47.27; AIC=-1.28 respectively.

interpret the determinants of the change in consumption, namely: (i) an ‘autonomous’ component, i.e. α_t, which responds to shocks in a manner consistent with precautionary savings motives such that the path of ΔC_t shifts down and up as the economy goes through depression, recovery and boom; (ii) a component which determines the evolution of consumption according to the way habits are formed, i.e. $\beta \Delta C_{t-1}$; and finally, (iii) $\gamma_t \xi_t$ which corresponds to what Friedman described as consumption of a ‘cash windfall’, and whose fluctuations also shift the path of ΔC_t. The pattern found in γ_t can be explained as follows. If, as it is usually the case in the literature, the realisation of cash windfalls or transitory income, as defined by ξ_t, does not (fully) reflect changes in the degree of uncertainty about the future in the economy, then precautionary behaviour requires that the rate at which the latter is capitalised into wealth, i.e. (1-γ_t), should exhibit a positive correlation with the degree of uncertainty in the economy. This is exactly what we find.

4. CONCLUSION

In the existing literature, the theoretical path of consumption, which is associated with the standard stochastic life-cycle-optimising version of the permanent-income model, is commonly agreed to follow a random walk with drift. However, the persisting failure of the random walk model to conform to data casts doubt on the suitability of the specific framework within which the random walk model is developed. In this paper we propose an alternative interpretation of the rule-of-thumb revision scheme associated with the permanent-income hypothesis and show that it implies an *ARIMA*(1,1,0) path for consumption. We use U.S. data for 1929-2001 to examine the empirical consistency of the model and find that evidence supports this generalisation of the consumption path. In particular, the main objections which are raised in the literature against the random walk model – i.e. that it exhibits both “*excess smoothness*” and “*excess sensitivity*” to income – are no longer found to have any empirical grounds. The *ARIMA* model also passes the robustness tests against biases caused by liquidity constraints and dynamic misspecifications (that are typically argued to be due to incomplete information and/or aggregation over cohorts).

An appealing feature of the *ARIMA* version of the permanent-income model is that it can also be derived as a solution to a life-cycle optimising problem with habit formation and precautionary saving motives. Our results suggest that the habit formation effect, captured by the coefficient of ΔC_{t-1}, is strongly present and remarkably constant over the whole sample. Furthermore, the consumption path which incorporates some degree of habit formation does not exhibit any excess sensitivity to income. Recall that the existence of excess sensitivity can be tested by checking whether past income affects the path of consumption significantly beyond that indicated by the underlying theory. Starting with the random walk model, we find that the coefficient of ΔY_{t-1} is positive and statistically significant when the latter is included as an additional explanatory variable in the original regression equation. However, we note adding ΔC_{t-1} completely undermines that role of ΔY_{t-1} in explaining variations in ΔC_t; when both ΔC_{t-1} and ΔY_{t-1} are included as regressors the coefficient of the latter drops to zero and becomes statistically insignificant. This leads to the conclusion that, rather than being an indication of excess sensitivity symptoms, the significance of coefficient of ΔY_{t-1} in the random walk model was merely capturing the habit formation effect, which is better encapsulated in ΔC_{t-1}. It is worth recalling that Duesenberry (1952) proposed his relative-income hypothesis to explain the so-called ratchet effect of factors such as habits and/or 'keeping up with Joneses'[16] and found the past peak income to serve as a good proxy in capturing the effects empirically.

[16] Modigliani (1949) also explained similar phenomenon. For a more recent modelling of the 'keeping up with Joneses' phenomena and its similarity to the habit formation, see Ljungqvist and Uhlig (2000).

Our results also show strong effects associated with precautionary saving motives which manifest themselves in variations in both the intercept and the coefficient capturing the marginal propensity to consume from transitory income. The former reflects agents attitude towards risk (that can be explicitly modelled using a suitable utility function – see above) and the latter captures the essence of ‘precautionary behaviour’ described by Friedman when he considers how agents capitalise a ‘cash windfall’. We use the Kalman filter approach to allow for time-varying parameters to capture the corresponding drifts in the *ARIMA* path and find strong support for the existence of such effects precautionary motive effects within the sample period.

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Appendix: The excess smoothness condition when consumption follows an *ARIMA*(1,1,0) path with drift.

We are interested in comparing the unconditional variances of the change in consumption and the income innovation, $V(\Delta C_t)$ and $V(\varepsilon_t)$ respectively, when the theoretical path of consumption is given by the following

$$\Delta C_t = \beta \Delta C_{t-1} + (\alpha_t + \gamma_t \varepsilon_t);\ 0 < \beta < 1 \text{ and } \varepsilon_t \sim iid(0, \sigma_\varepsilon^2), \tag{A1}$$

$$\alpha_t = \mu \alpha_{t-1} + \zeta_t;\ 0 < \mu \le 1 \text{ and } \zeta_t \sim iid(0, \sigma_\zeta^2), \tag{A2}$$

$$\gamma_t = v \gamma_{t-1} + \psi_t;\ 0 < v \le 1 \text{ and } \psi_t \sim iid(0, \sigma_\psi^2). \tag{A3}$$

Rewriting the *AR*(1) expressions as infinite *MA* series, the above imply

$$\Delta C_t = \sum_{s=0} \mu^s \sum_{j=0} \beta^j \zeta_{t-s-j} + \sum_{s=0} v^s \sum_{j=0} \beta^j \varepsilon_{t-j} \psi_{t-s-j}, \tag{A4}$$

which can now be rearranged as

$$\Delta C_t = \frac{\zeta_t}{(1-\mu L)(1-\beta L)} + \frac{\psi_t \varepsilon_t}{(1-vL)(1-\beta L)}, \tag{A5}$$

where L is the lag operator. Hence, given that all random variables appearing in the numerator of the right-hand-side of (A5) are independently distributed, we have

$$V(\Delta C) = \frac{V(\zeta)}{(1-\mu^2)(1-\beta^2)} + \frac{V(\psi)V(\varepsilon)}{(1-v^2)(1-\beta^2)}, \tag{A6}$$

where $V(\cdot)$ denotes the variance operator and the subscript t is dropped since variances are unconditional.

We are interested to know if the result in (A6) can be consistent with the empirical observation that $V(\Delta C) < V(\varepsilon)$. Letting μ = v and $V(\zeta) = V(\psi) = \sigma^2$, as imposed in our estimates, (A6) can be written as

$$V(\Delta C) = \frac{\sigma^2}{(1-\mu^2)(1-\beta^2)}\left(1 + V(\varepsilon)\right). \tag{A7}$$

Thus, $V(\Delta C) < V(\varepsilon)$ hols if

$$V(\Delta C) - V(\varepsilon) = K - (1-K)V(\varepsilon) < 0, \tag{A8}$$

where $K = \dfrac{\sigma^2}{(1-\mu^2)(1-\beta^2)}$. This condition requires 0<K<1 and $\dfrac{K}{1-K} < V(\varepsilon)$. From our estimates:

μ	β	σ	K	$K/(1-K)$	$V(\varepsilon) \cong V(\hat{\xi})$ in Table 2
0.97	0.213	0.0664	0.07815	0.084772	0.08821

which satisfy the condition.