Central Bank Instruments, Fiscal Policy Regimes, and the Requirements for Equilibrium Determinacy

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Abstract
In this paper we examine the local determinacy conditions for three monetary policy regimes in a business cycle model with staggered price setting. The central bank either controls the nominal interest rate, the money growth rate, or it conducts open market operations and controls the bond-to-money ratio herein. All instruments are set contingent on changes in current inflation. For the first two cases, equilibrium determinacy imposes strong restrictions on the endogenous response to changes in inflation, which depend on whether fiscal policy is Ricardian or non-Ricardian. In the case of open market policy, Ricardian equivalence does not hold and government solvency is guaranteed for finite bond-to-money ratios. The central bank can ensure determinacy by setting the latter not in an extremely reactive way. The equilibrium sequence of real financial wealth then exerts a stabilizing impact on prices and real activity such that equilibrium multiplicity as well as explosiveness is ruled out.

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1 Introduction

This paper aims at revealing if the central bank's instrument-choice alters the requirements for real equilibrium determinacy for different fiscal policy regimes. Since Poole's (1970) analysis, different monetary policy instruments are known to be relevant for the ability of monetary policy to stabilize real activity. Carlstrom and Fuerst (1995) have further shown that the choice between money growth and interest rate policy can substantially matter for welfare. Recent monetary business cycle theory, however, which considers some kind of nominal rigidity, mainly focuses on interest rate policy. One strand of this literature is concerned with the problem of multiple equilibria or unstable paths enabled by policy rules which are designed in an inappropriate way for a given structure of the economy and for a given fiscal policy regime (see, e.g., Benhabib et al., 2001a, Carlstrom and Fuerst, 2001, Dupor, 2001a, or, Meng, 2002). The interest in the interaction of monetary policy with the latter mainly emerged with the seminal work of Leeper (1991), Sims (1994), and Woodford (1994, 1995) followed by various contributions to the so-called 'Fiscal Theory of the Price Level' (FTPL),[2] which has revealed that the price level can be determined by the needs of fiscal solvency in cases where it is not pinned down by the central bank setting the nominal interest rate. Moreover, the particular fiscal policy regime is crucial for equilibrium determination, i.e., the restrictions on interest rate policy to ensure local determinacy, in an environment where prices are not fully flexible (see Benhabib et al., 2001a). The literature, however, leaves the question unanswered if fiscal policy is still decisive for the requirements for equilibrium determinacy when the central bank does not control the nominal interest rate.

In this paper we complement this line of research and examine how fiscal policy affects equilibrium determinacy when the central bank uses alternative instruments such that the nominal interest rate is endogenously determined. In particular, we consider regimes characterized by a central bank setting the money growth rate or the stance of open market operations contingent on changes in the current inflation rate. Following Benhabib et al. (2001a), fiscal policy regimes differ with regard to their ability to guarantee government solvency and, thus, the sequence of real wealth to ensure fulfillment of the households' transversality condition. As in the case of interest rate policy, Ricardian equivalence holds for a money growth policy, and fiscal policy is decisive for the way the central bank should set the money growth rate in response to changes in inflation in order to ensure equilibrium determinacy. When the fiscal policy regime guarantees government solvency (Ricardian policy), equilibrium determinacy requires the money growth rate not to rise with inflation by more than one for

[2] See Woodford (2001a) for a comprehensive discussion of the FTPL.

one. This restriction is, thus, exactly opposed to the well-known determinacy restriction on interest rate rules, i.e., the Taylor-principle (see Woodford, 2001b). A non-Ricardian policy fiscal regime can,[3] on the other hand, only lead to a uniquely determined equilibrium path when it is accompanied by accommodating money growth rates rising by more than one for one with changes in inflation, given that prices are not too rigid.

Turning to the case where the central bank controls the stance of open market operations, we obtain fundamentally different results regarding the requirements for equilibrium determinacy and the role of fiscal policy. Closely following Shreft and Smith (1998, 2000), this regime is specified by, first, restricting the supply of money and government bonds on open market operations, while, second, the central bank sets the ratio of outstanding bonds to money.[4] The latter induces Ricardian equivalence not to hold, which is of utmost importance for the interaction of monetary and fiscal policy. As money is linked to the outstanding stock of government bonds, the government's financing decision indirectly affects money supply and, thus, the willingness of households to consume. Hence, the equilibrium sequence of real financial wealth, which equals total government liabilities and is a predetermined variable, cannot be recursively determined as in the former monetary policy regimes. On the other hand, as the central bank relates the stocks of both government liabilities, government solvency and, thus, a Ricardian policy regime is ensured as long as the bond-to-money ratio takes finite values. Examining the local dynamics for the open market regime reveals that determinacy requires the bond-to-money ratio not to be extremely reactive to changes in inflation. Ricardian non-equivalence together with the induced government solvency is responsible for the equilibrium sequence of real financial wealth to exert a stabilizing impact on the economy, ruling out multiple equilibria and explosiveness. Hence, Shreft and Smith's (1998, 2000) open market regime is more appropriate than a money growth or an interest rate regime for a central bank which aims at avoiding the economy to be destabilized.

The remainder is organized as follows. Section 2 presents the model. In section 3 we derive the determinacy conditions for the three policy regimes. Section 4 concludes.

2 A sticky price model

In this section we develop a continuous time monetary business cycle model where prices are set in a staggered way. The model mainly differs from the one in Benhabib et al. (2001a) by allowing for endogenous labor supply and by considering three different monetary policy regimes: interest rate policy, money growth policy, and open market policy.

[3] In particular, we apply Dupor's (2001b) specification of open market operations as a non-Ricardian regime.
[4] Similar monetary policy regimes can be found in Wallace (1984) or in Battacharya and Kudoh (2002).

2.1 The household sector

Nominal variables are denoted by upper-case letters, while real variables are denoted by lower-case letters. The economy is populated by a continuum of identical households. A representative household is infinitely lived, with preferences given by the value of a discounted stream of instantaneous utility $u(.)$:

$$\int_0^\infty e^{-\theta t} u(c,l,m)\, dt, \qquad \text{with} \qquad \theta > 0, \tag{1}$$

where c denotes consumption, l leisure, $m = \frac{M}{P}$ real balances, M cash, P the aggregate price level, and θ the discount factor. Following Sidrauski (1967) we introduce real balances in the utility function as a short-cut for assuming that they provide transaction services. The utility function satisfies assumption 1 implying that it is separable with regard to all of its arguments.[5]

Assumption 1 *The utility function $u(c,l,m)$ is strictly increasing, strictly concave, twice continuously differentiable, and satisfies the usual Inada conditions and $u_{xy} = 0$ for $x \neq y$ with $x, y \in \{c, l, m\}$.*

We normalize the total time endowment equal to one, such that labor supply n is given by: $n = 1 - l$. Households are further endowed with financial wealth denoted by A, which consists of cash holdings M and holdings of government bonds $B : A = M + B$. The households' income comprises labor remuneration, interest payments on government bonds, and profits of firms which are owned by the households. The flow budget constraint of a representative household is given by

$$\dot{a} = (R - \pi)a - Rm + wn - c - \tau + \psi, \tag{2}$$

where $a \equiv A/P$, P, w, τ, R, π, and ψ denotes real financial wealth, the aggregate price level, the real wage, a lump sum tax, the nominal interest rate, the inflation rate, and the real profits of firms, respectively. Throughout the paper, we assume that the stock of government bonds is non-negative ($B \geq 0$). Imposing this restriction we rule out that households borrow funds from the public sector. The household maximizes (1) by choosing sequences for consumption, leisure, real balances, and real wealth, for a given initial stock of nominal financial wealth $A_0 > 0$ subject to the flow budget constraint (2) and to the no-Ponzi-game condition $\lim_{t\to\infty} a(t) \exp[-\int_0^t (R(v) - \pi(v))dv] \geq 0$, taking prices, public policy,

[5] Though, we are aware of the fact that non-separability of $u(.)$ matters for real determinacy, as shown in Matsuyama (1990), Matheny (1998), or Benhabib et al. (2001), separability is assumed for simplicity.

and profits of firms as given. The household's first order conditions are given by:

$$u_c = \lambda, \tag{3}$$

$$u_l = w\lambda, \tag{4}$$

$$u_m = R\lambda, \tag{5}$$

$$-\frac{\dot{\lambda}}{\lambda} = R - \pi - \theta, \tag{6}$$

In addition, the flow budget constraint (2) and the transversality condition

$$\lim_{t\to\infty} a(t) \exp\left[-\int_0^t (R(v) - \pi(v))\, dv\right] = 0, \tag{7}$$

must be satisfied in the household's optimum.

2.2 The production sector

There is a continuum of monopolistically competitive firms. Each firm $i \in [0,1]$ supplies a single differentiated good y_i aggregated to the composite final good; the latter can be thought of being produced by perfectly competitive production units. The aggregation technology for the final good y is assumed to be a CES aggregator of differentiated goods: $y^{(\varepsilon-1)/\varepsilon} = \int_0^1 y_i^{(\varepsilon-1)/\varepsilon} di$, where y_i and $\varepsilon > 1$ denotes the quantity of the i-th differentiated good and the elasticity of substitution between any two differentiated goods, respectively. As each firm i produces exactly one variant of the differentiated output good y_i, the relevant demand function for firm i, which is derived from cost minimization, can be written as:

$$y_i \leq \left(\frac{P_i}{P}\right)^{-\varepsilon} y, \qquad \text{with} \qquad P = \left[\int_0^1 P_i^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{8}$$

where P denotes the aggregate price index and P_i the price of the differentiated good indexed with i. As commonly assumed in the literature, firms have access to a production technology with labor as the single input. The production technology of a firm i, which is assumed to be linear in labor, reads:

$$y_i = n_i. \tag{9}$$

We introduce staggered prices as proposed by Calvo (1983). Firms set their prices to maximize a discounted stream of current and future real profits. They are assumed to be able to adjust prices only when they receive a random signal. Otherwise, they set their prices along with the steady state inflation rate $\overline{\pi}$. The time interval until the arrival of a random price-change signal is exponentially distributed such that the probability of not being allowed to adjust prices between dates t and $s > t$ is $\exp(-\delta[s-t])$, with $\delta > 0$. In period t a firm i receiving a price signal sets the price P_t^*, where the index i can be dropped from P_t^* as all firms receiving a signal will behave identically. The maximization problem is given by

$$\max_{P_t^*} \int_t^\infty e^{-(\delta+\theta)(s-t)}\lambda_s \left[\left(P_t^* e^{\overline{\pi}(s-t)} y_{is}(P_t^*) - MC_s y_{is}(P_t^*)\right)/P_s\right] ds, \tag{10}$$
$$\text{subject to } y_{is}(P_t^*) = (P_t^* e^{\overline{\pi}(s-t)})^{-\varepsilon} P_s^\varepsilon y_s.$$

where MC denotes the nominal marginal costs. Note that the term in square brackets in (10) gives real profits in s if the firm has last adjusted in time t, which is discounted with the probability of not adjusting and with the pricing kernel $\lambda_s e^{-\theta(s-t)}$ taken from the consumer's maximization problem. Maximization of (10) with respect to P_t^* for a given initial price level $P_0 > 0$ leads to the following first order condition:

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\int_t^\infty e^{-(\delta+\theta)(s-t)}\lambda_s \widetilde{P}_s^{\varepsilon-1} y_s \widetilde{MC}_s ds}{\int_t^\infty e^{-(\delta+\theta)(s-t)}\lambda_s \widetilde{P}_s^{\varepsilon-1} y_s ds}, \tag{11}$$

with $\widetilde{X}_s \equiv X_s/e^{\overline{\pi}(s-t)}$, for $X = P, MC$. The first order condition in (11), together with the price index given in (8), can be transformed into a linear differential equation in π.[6] After linearly approximating (8) and (11), we obtain the following aggregate supply constraint

$$\dot{\pi} = \theta(\pi - \overline{\pi}) - \delta(\delta + \theta)\frac{\varepsilon}{\varepsilon - 1}(mc - \overline{mc}), \tag{12}$$

where $\overline{x}$ denotes the steady state value of $x = \pi, mc$. The derivation of this continuous time version of the 'New Keynesian Phillips Curve' follows Benhabib et al. (2001a) and is given in the appendix. With perfect mobility of labor between different firms, profit maximization causes each firm to choose a labor demand schedule where real marginal costs $mc = MC/P$ are equal to the wage rate:

$$w = mc. \tag{13}$$

2.3 The public sector

The public sector consists of the fiscal authority and the central bank. The fiscal authority issues riskless bonds of immediate maturity B, pays interest RB on outstanding debt, collects lump-sum taxes τ from households, and receives a transfer τ^c from the central bank. Its budget constraint, thus, reads: $\dot{B} + P(\tau + \tau^c) = RB$. The central bank issues money and transfers the receipts from money creation to the fiscal authority: $\dot{M} = P\tau^c$. Hence, the

[6] An analogous procedure is presented in Yun (1996) for a discrete time model.

consolidated budget constraint reads

$$\dot{B} + \dot{M} + P\tau = RB. \quad (14)$$

Monetary policy regimes We consider three different monetary policy regimes. In order to treat them symmetrically, all instruments are set contingent on the current inflation rate. The *first* regime, which specified by closely following Shreft and Smith (1998, 2000), is characterized by the central bank supplying money via open market operations. Changes in the outstanding stock of money are restricted to be accompanied by inverse changes in the outstanding stock of government bonds less interest earnings

$$\dot{M} = -(\dot{B} - RB). \quad (15)$$

According to the exchange restriction for open market operations (15), the costs of money acquisition rises with the nominal interest rate. Furthermore, this monetary policy regime implies, $P\tau^c = -(\dot{B} - RB)$, such that taxes τ are equal to zero: $P\tau = 0$ (see 14). Seignorage exactly offsets the treasury's net earnings from bond issuance such that public sector net receipts $P\tau$, which ought to be transferred to balance the budget, are always equal to zero. In combination with the open market restriction (15), the central bank sets the ratio β of outstanding public liabilities contingent on the current inflation rate

$$\frac{B}{M} = \beta(\pi), \quad \text{with } \beta > 0. \quad (16)$$

We further assume that $\beta(\pi)$ has a unique solution for the steady state condition $\beta(\overline{\pi}) = \theta/\overline{\pi}$ for $\overline{\pi} > 0$. Defining total government liabilities $S \equiv M + B$ and using the policy rule (16), the open market restriction (15), which can be rewritten as $\dot{S} = RS - RM$, leads to the following restriction for the evolution of government liabilities

$$\dot{S} = \left[1 - (1 + \beta(\pi))^{-1}\right] RS \quad \Leftrightarrow \quad \dot{s} = \left(\left[1 - (1 + \beta(\pi))^{-1}\right] R - \pi\right) s. \quad (17)$$

Hence, total government liabilities S can only grow with the nominal interest rate if β converges to infinity. Otherwise, the evolution of real government liabilities $s \equiv S/P$, which equals real wealth in equilibrium, is restricted such that their growth rate is smaller than the real interest rate $(R - \pi)$ implying that the households's transversality condition (7) cannot be violated.

The *second* regime is characterized by the central bank setting the nominal interest rate on government bonds. Following related studies (see, Benhabib et al., 2001a, Dupor, 2001a, or, Meng, 2002), we assume that the nominal interest rate is set according to the following simple rule

$$R = \rho(\pi), \quad \text{with } \rho > 0. \tag{18}$$

To avoid multiple stationary equilibria, we assume that the policy rule (18) has exactly one solution for the steady state condition $\rho(\overline{\pi}) = \overline{\pi} + \theta$ for $\overline{\pi} > 0$.[7] As money supply is not further restricted, the fiscal authority is free to set the sequence of transfers $P\tau$.

This property also characterizes the *third* monetary policy regime where the central bank controls the growth rate of money. As in the former regimes, we allow for the money growth rate to depend on the current inflation rate

$$\frac{\dot{M}}{M} = \mu(\pi), \quad \text{with } \mu > 0. \tag{19}$$

We further assume that $\mu(\pi)$ has a unique solution for the steady state condition $\mu(\overline{\pi}) = \overline{\pi}$ for $\overline{\pi} > 0$.

Fiscal policy regimes In order to facilitate comparisons between all regimes we introduce a simple fiscal policy rule being sufficiently general to encompass those cases which will turn out to have different qualitative consequences for real determinacy. In particular, we assume that taxes are set according to

$$P\tau = \vartheta RS - RM. \tag{20}$$

A comparison of the government budget constraint (14) with (17) reveals that the open market policy implies $\vartheta = [1 + \beta(\pi)]^{-1}$. On the contrary, the policy parameter needs to be set by the fiscal authority in the case of interest rate or money growth policy. Following Benhabib et al. (2001a), we will distinguish two cases. The first case is characterized by a strictly positive value $\vartheta = \vartheta^R > 0$ implying that the public sector solvency is always satisfied:

$$\dot{s} = [(1 - \vartheta^R)R - \pi]s \quad \Rightarrow \quad \lim_{t\to\infty} s(t) \exp\left[-\int_0^t [R(v) - \pi(v)]dv\right] = 0. \tag{21}$$

Note that the solvency constraint, given on the right hand side of (21), equals the transversality constraint (7) in equilibrium. Following Benhabib et al. (2001a), this fiscal policy regime is called Ricardian. The alternative fiscal policy regime is given by $\vartheta = \vartheta^N = M/S \Rightarrow \tau = 0$ such that the evolution of government liabilities satisfies

$$\dot{s} = (R - \pi)s - Rm. \tag{22}$$

[7] See Benhabib et al. (2002a, 2002b) for the consequences of the zero bound on nominal interest rates for global and local determinacy.

As pointed out by Dupor (2001b), this particular tax policy does not ensure government solvency and can therefore actually lead to a non-Ricardian policy regime. Given that (22) holds, s can grow with the real interest rate $(R - \pi)$ such that government solvency is not guaranteed outside the equilibrium for $s_0 > 0$. In the case of interest rate or money growth policy, government solvency and, thus, the households' transversality condition (7) must be explicitly considered for equilibrium determination.

It is important to note that the first monetary policy regime is associated with the tax rule: $P\tau = 0$. Hence, fiscal policy is not sufficient to ensure government solvency. However, as the central bank controls the ratio of bonds to money to be constant in the long-run, public expenditures are not only financed by issuance of interest bearing assets (B) as long as β takes a finite value. Hence, for an open market regime the growth rate of real government liabilities will be smaller than the real interest rate $(R - \pi)$ for $\beta < \infty$ implying that the public sector is solvent (see 17).

2.4 Equilibrium

A *perfect foresight equilibrium* of this economy is a set of sequences of the model's endogenous variables $\{\lambda, c, n, \pi, mc, w, s, a, m, b, R\}$ characterized by i.) the first order conditions of the households, (3) to (6); ii.) the firms' first order conditions, (12) and (13), as well as the aggregate version of the production function (9); iii.) a monetary-fiscal policy regime characterized by a combination of policy rules given in table 1; iv.) the transversality condition of households (7), and v.) market clearance, such that the aggregate resource constraints of the goods and asset markets hold, $y = c$ and $a = s$, for a given initial value for real financial wealth $a_0 = A_0/P_0 > 0$, as well as for real balances $m_0 = M_0/P_0 > 0$ for the money growth policy regimes (MGR and MGN).

Table 1 Monetary-fiscal policy regimes

	Central bank instrument	Fiscal policy regime
OM	$b/m = \beta(\pi)$	$\dot{s} = (R - \pi)s - Rm$
IRR	$R = \rho(\pi)$	$\dot{s} = [(1 - \vartheta^R)R - \pi]s$
IRN	$R = \rho(\pi)$	$\dot{s} = (R - \pi)s - Rm$
MGR	$\dot{m}/m + \pi = \mu(\pi)$	$\dot{s} = [(1 - \vartheta^R)R - \pi]s$
MGN	$\dot{m}/m + \pi = \mu(\pi)$	$\dot{s} = (R - \pi)s - Rm$

3 Monetary-fiscal policy regimes and equilibrium determinacy

In this section we derive the conditions for local determinacy for the five monetary-fiscal policy regimes listed in table 1. Apparently, open market policy restricts fiscal policy in a particular way, whereas the other two monetary policy rules can be accompanied by either a Ricardian (R) or a non-Ricardian (N) fiscal policy regime. Hence, interest rate policy (IR) and money growth policy (MG) are considered with both fiscal policy regimes, whereas the open market policy (OM) regime is always associated with a consolidated public sector budget constraint satisfying: $\dot{s} = \left[\beta(1+\beta)^{-1}R - \pi\right] s$.

In order to examine the conditions for local determinacy we linearize the equilibrium conditions at the steady state. The latter, consisting of the endogenous variables $\{\overline{c}, \overline{\lambda}, \overline{m}, \overline{\pi}, \overline{R}\}$, is characterized by the following conditions

$$u_c(\overline{c}) = \frac{u_l(1-\overline{c})\varepsilon}{\varepsilon - 1} = \overline{\lambda} \tag{23}$$

$$u_m(\overline{m}) = u_c(\overline{c})\overline{R}, \tag{24}$$

$$\overline{R} = \overline{\pi} + \theta, \tag{25}$$

and either $\overline{R} = R(\overline{\pi})$ for an interest rate policy or $\mu(\overline{\pi}) = \overline{\pi}$ for a money growth policy. In these cases, the equilibrium sequence of real wealth does not directly affect the remainder of the economy. Hence, real wealth is not restricted to be constant in the steady state. In the case of an open market policy, a steady state, however, requires, by (16), a stationary value for real wealth given by $\overline{a} = (1+\beta(\overline{\pi}))\overline{m}$. As we further assumed that $b \geq 0$ and because real balances will be positive in the steady state, by (24), we can conclude that $\overline{a} > 0$ and $\dot{a} = 0$ implying $\overline{\pi} = \beta(\overline{\pi})\theta$ and $\overline{a} = \overline{m}\overline{R}/\theta$ by (17) and (25). It then follows from assumption 1, the conditions (23)-(25), and the prevailing policy rule that the steady state is unique and that all endogenous variables exhibit a positive value in the steady state.

We proceed the analysis with the case where the central bank sets the nominal interest rate, i.e., regime IRR and IRN, and derive determinacy results consistent with the findings in Benhabib et al. (2001a). We then examine money growth policy and show that the reactiveness, again, decides on determinacy for the MGR and the MGN regime. In the last part of this section we show that the OM regime behaves differently as Ricardian equivalence is obviously invalid given that money supply is, by (16), linked to the amount of government bonds outstanding. Hence, a change in public debt alters the shadow price of wealth, by (5), and, therefore, households' willingness to consume and their labor supply.

Remark 1 *Ricardian equivalence does not hold for the OM regime.*

To give a preview, the relevance of real financial wealth, which equals real government liabilities in equilibrium and is a predetermined state variable, will exert a stabilizing impact on the dynamics of the model for the OM regime such that the model is less prone to equilibrium multiplicity than for the other monetary policy regimes.

3.1 Interest rate policy

In the case where the central bank sets the nominal interest rate according to (18), reduction of the linearized equilibrium conditions leads to the following two conditions in π and λ

$$\dot{\lambda} = -\overline{\lambda}\left(\overline{\rho}_\pi - 1\right)\left(\pi - \overline{\pi}\right) - \left(\overline{R} - \overline{\pi} - \theta\right)\left(\lambda - \overline{\lambda}\right), \tag{26}$$

$$\dot{\pi} = \theta\left(\pi - \overline{\pi}\right) + \Phi_1(\lambda - \overline{\lambda}), \qquad \text{with} \quad \Phi_1 = \delta(\delta+\theta)\overline{u}_c^{-1}\left[1 + \sigma^l/\sigma^c\right] > 0, \tag{27}$$

with $\sigma^l \equiv -\frac{\overline{u}_{ll}\overline{l}}{\overline{u}_l}$ and $\sigma^c \equiv -\frac{\overline{u}_{cc}\overline{c}}{\overline{u}_c}$, and a non-linearized condition for the evolution of real wealth either given by

$$\dot{a} = [(1 - \vartheta^R)\rho(\pi) - \pi]a \tag{28}$$

for the Ricardian fiscal policy (IRR regime), or by

$$\dot{a} = (\rho(\pi) - \pi)a - \rho(\pi)m(\rho(\pi), \lambda) \tag{29}$$

for a non-Ricardian fiscal policy (IRN regime). The function $m(\rho(\pi), \lambda)$ in (29) follows from assumption 1 and (5). In principle, the rate of inflation and the shadow price of wealth are determined by (26) and (27), while the equilibrium sequence of real wealth can recursively be derived from (28) or (29). In particular, the evolution of real wealth is completely irrelevant for the equilibrium sequences for π and λ when fiscal policy is Ricardian ($\vartheta = \vartheta^A$) such that the government solvency constraint, which equals the transversality condition in equilibrium, is always satisfied. Hence, equilibrium determinacy for the IRR regime requires that the two eigenvalues of the 2×2 system (26) and (27) are unstable given that π and λ are jump variables.

In the case where the fiscal policy chooses a sequence of taxes, which does not ensure solvency (non-Ricardian fiscal policy), the evolution of real wealth, governed by (29), must be taken into account for determination of π and λ as real wealth can potentially violate the transversality condition (7). Hence, the latter, which is satisfied by public policy in the Ricardian case (28), now imposes an additional restriction on the equilibrium sequences of π, λ, and a for the IRN regime. However, the eigenvalue of the last equation (28) model can be separately determined. Given that real wealth is a predetermined variable (with $a_0 > 0$) and the condition (29) introduces a unstable eigenvalue, which equals $\rho(\overline{\pi}) - \overline{\pi} = \theta$, equilibrium determinacy now requires that the 2×2 system (26) and (27) exhibits one stable and one unstable eigenvalue. The requirements for the conduct of monetary policy, which ensure determinacy for the IIR and the IRN regime, are summarized in the following proposition.

Proposition 1 *Suppose that the central bank sets the nominal interest rate according to (18). Then there exists a unique equilibrium path converging to the steady state if*

1. $\overline{\rho}_\pi > 1$ *for the IRR regime, otherwise there exists a continuum of equilibrium paths,*

2. $\overline{\rho}_\pi < 1$ *for the IRN regime, otherwise there exists no stable equilibrium path.*

Proof. The claims made in the proposition can easily be verified by examining the 2×2 system (26) and (27) which determines the eigenvalues for λ and π independent of real wealth:

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\pi} \end{pmatrix} = A^\rho \begin{pmatrix} \lambda - \overline{\lambda} \\ \pi - \overline{\pi} \end{pmatrix}, \qquad \text{with} \quad A^\rho \equiv \begin{pmatrix} -\left(\overline{R} - \overline{\pi} - \theta\right) & -\lambda\left(\overline{\rho}_\pi - 1\right) \\ \Phi_1 & \theta \end{pmatrix}.$$

Using $\overline{R}-\overline{\pi}-\theta = 0$, the trace of the matrix A^ρ, which is given by trace$(A^\rho) = \theta > 0$, is strictly positive indicating that there is at least one positive eigenvalue. The sign of the determinant of A^ρ depends on the partial derivative of the interest rate rule: $\det(A^\rho) = \Phi_1\overline{\lambda}\left(\overline{\rho}_\pi - 1\right)$. When $\overline{\rho}_\pi > 1$, the determinant is positive and there are two unstable eigenvalues, while there is one stable and one unstable eigenvalue if $\overline{\rho}_\pi < 1$. Hence, the *IRR* regime exhibits a unique (continuum of) equilibrium path(s) if $\overline{\rho}_\pi > 1$ ($\overline{\rho}_\pi < 1$), whereas the *IRN* regime is associated with a unique (unstable) equilibrium path if $\overline{\rho}_\pi < 1$ ($\overline{\rho}_\pi > 1$). ■

The results summarized in proposition 1, which are consistent with the results in Benhabib et al. (2001a) and Woodford (2001), reveal that local determinacy crucially relies on the reactiveness of monetary policy measured by $\overline{\rho}_\pi$. When inflation is high and the central bank raises the nominal interest rate by less than one for one ($\overline{\rho}_\pi < 1$), the real interest rate and, therefore, the shadow price of wealth declines (see 26), so that households are willing to save less and to consume more exerting an upward pressure on prices. Hence, inflation expectations can be self-fulfilling when interest rate policy is passive ($\overline{\rho}_\pi < 1$). However, the conditions for equilibrium determinacy depend on the fiscal policy regime. The determinacy conditions are exactly opposed such that there is no robust strategy for the central bank to ensure determinacy. A Ricardian fiscal policy requires an active interest rate policy – also known as the Taylor-principle – for equilibrium uniqueness, whereas a non-Ricardian policy must be accompanied by a passive interest rate to avoid explosiveness. For any fiscal policy regime, there are multiple equilibrium paths for π and λ satisfying (26) and (27) which converge to the steady state for a passive interest rate policy. In the case where fiscal policy is non-Ricardian, however, fulfillment of the transversality condition imposes an additional restriction on the equilibrium values of π and λ leading to a uniquely determined equilibrium (see also Benhabib et al., 2001a).

3.2 Money growth policy

Now suppose that the central bank sets the money growth rate according to (19). The linearized model can not further be reduced than to a 3×3 system as the real value of money

m, which could be recursively determined for interest rate policy, is a predetermined state variable given that the central bank determines the future stock of nominal money and that prices are sticky. The linearized equilibrium conditions for π, λ, and m are given by

$$\dot{\lambda} = \overline{\lambda}(\pi - \overline{\pi}) - u_{mm}\left[m - \overline{m}\right] + \overline{R}\left[\lambda - \overline{\lambda}\right], \tag{30}$$

$$\dot{\pi} = \theta\left(\pi - \overline{\pi}\right) + \Phi_1(\lambda - \overline{\lambda}), \tag{31}$$

$$\dot{m} = \overline{m}\left(\overline{\mu}_{\pi} - 1\right)\left(\pi - \overline{\pi}\right), \tag{32}$$

and by an equilibrium condition for real wealth either given by (28) for the MGR regime, or by (29) for the MGN regime. Before we turn to the determinacy conditions we introduce, for simplicity, an additional assumption restricting real balances to enter the utility function at least in a logarithmic way; the latter is commonly used as a lower bound for the elasticity of marginal utility $-\frac{\overline{u}_{mm}\overline{m}}{\overline{u}_m}$ (see, e.g., Dupor, 2001a).

Assumption 2 *The intertempotal substitution elasticity of real balances* $1/\sigma^m \equiv -\frac{\overline{u}_m}{\overline{u}_{mm}\overline{m}}$ *satisfies* $\sigma^m \geq 1$.

Corresponding to the case where the central bank sets the nominal interest rate, real wealth does not affect the equilibrium sequences for the inflation rate, the shadow price of wealth, and real balances when fiscal policy is Ricardian (28). Thus, for the MGR regime, the equilibrium condition for real wealth (28) can again be neglected and equilibrium determinacy requires the 3×3 system (30)-(32) to exhibit one stable and two unstable eigenvalues given that it features exactly one predetermined variable (m). In contrast, the MGN regime additionally introduces an additional state variable (a) with an unstable eigenvalue (θ) by (29) such that determinacy requires (30)-(32) to have two stable and one unstable eigenvalues. Similar to the case of interest rate policy, the 3×3 system (30)-(32) exhibits multiple sets of equilibria paths for $\{\lambda, \pi, m\}$, from which fulfillment of the transversality condition selects a set associated with a stable path for real wealth. The following proposition summarizes the determinacy conditions for both regimes.

Proposition 2 *Suppose that the central bank sets the money growth rate according to (19). Then there exists a unique equilibrium path converging to the steady state if*

1. $\overline{\mu}_{\pi} < 1$ *for the MGR regime, otherwise there exists a continuum of stable equilibrium paths if* $\Phi_1 < \widetilde{\Phi}$ *and no stable equilibrium path if* $\Phi_1 > \widetilde{\Phi}$,

2. $\overline{\mu}_{\pi} > 1$ *and* $\Phi_1 < \widetilde{\Phi}$ *for the MGN regime, otherwise there is no stable equilibrium path,*

with $\widetilde{\Phi} \equiv \frac{\overline{R}\theta\left(\overline{R}+\theta\right)}{\overline{R}+\theta+\sigma^m(\mu_{\pi}-1)\overline{R}}\overline{u_c}^{-1}$.

Proof. In order to derive the conditions for determinacy, the system (30)-(32) is rewritten in matrix form

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\pi} \\ \dot{m} \end{pmatrix} = A^{\mu} \begin{pmatrix} \lambda - \overline{\lambda} \\ \pi - \overline{\pi} \\ m - \overline{m} \end{pmatrix}, \qquad \text{with} \quad A^{\mu} \equiv \begin{pmatrix} \overline{R} & \overline{\lambda} & -\overline{u}_{mm} \\ \Phi_1 & \theta & 0 \\ 0 & \overline{m}\left(\overline{\mu}_{\pi} - 1\right) & 0 \end{pmatrix}.$$

The trace of A^{μ} is positive and reads: trace(A^{μ})$= \overline{R} + \theta > 0$. The determinant, given by $\det = -\overline{u}_{mm}\Phi_1\overline{m}\left(\overline{\mu}_{\pi} - 1\right)$, is negative for $\overline{\mu}_{\pi} < 1$. This ensures the existence of one stable and two unstable roots, which leads to determinacy for the MGR regime. For the MGN regime the system (30)-(32) has to be characterized by two stable eigenvalues to avoid explosiveness. If $\overline{\mu}_{\pi} > 1$, A^{μ} can either have three or one positive (unstable) root. However, we know that when $-A^{\mu}$ exhibits three negative (stable) roots, A^{μ} has only positive (unstable) roots.

We use that $-A^{\mu}$ exhibits three stable roots if and only if the trace, trace$(-A^{\mu}) = -\left(\overline{R} + \theta\right) < 0$, the determinant, $\det(-A^{\mu}) = \overline{u}_{mm}\Phi_1\overline{m}\left(\overline{\mu}_{\pi} - 1\right)$ with $\det(-A^{\mu}) < 0$ if $\overline{\mu}_{\pi} > 1$, and the determinant of the following matrix $\mathcal{A}^{\mu}$ is negative (see Braun, 1993)

$$\mathcal{A}^{\mu} \equiv \begin{pmatrix} a_{11}^{\mu} + a_{22}^{\mu} & a_{23}^{\mu} & -a_{13}^{\mu} \\ a_{32}^{\mu} & a_{11}^{\mu} + a_{33}^{\mu} & a_{12}^{\mu} \\ -a_{31}^{\mu} & a_{21}^{\mu} & a_{22}^{\mu} + a_{33}^{\mu} \end{pmatrix} = -\begin{pmatrix} -\overline{R} - \theta & 0 & -\overline{u}_{mm} \\ -\overline{m}\left(\overline{\mu}_{\pi} - 1\right) & -\overline{R} & -\overline{\lambda} \\ 0 & -\Phi_1 & -\theta \end{pmatrix}.$$

Hence, for $\det(-\mathcal{A}^{\mu}) = -\left(\overline{R} + \theta\right)\overline{R}\theta + \left(-\overline{u}_{mm}\right)\overline{m}\left(\overline{\mu}_{\pi} - 1\right)\Phi_1 + \Phi_1\overline{\lambda}\left(\overline{R} + \theta\right) > 0$ the roots of $-A^{\mu}$ cannot all be stable, such that A^{μ} exhibits at least one stable root. This is the case if

$$\Phi_1 > \widetilde{\Phi}, \qquad \text{with} \qquad \widetilde{\Phi} \equiv \frac{\overline{R}\theta\left(\overline{R} + \theta\right)}{\overline{R} + \theta - \overline{u}_{mm}\frac{\overline{m}}{\overline{\lambda}}\left(\overline{\mu}_{\pi} - 1\right)}\overline{u}_c^{-1}.$$

As it has already been shown that A^{μ} either has two stable or only unstable roots, we can conclude that A^{μ} has exactly two stable roots for the case $\overline{\mu}_{\pi} > 1$, if prices are sufficiently flexible (high δ) such that $\Phi_1 = \delta(\delta + \theta)\overline{u}_c^{-1}\left[1 + \sigma^l/\sigma^c\right] > \widetilde{\Phi}$. In this case, a non-Ricardian fiscal policy regime is associated with a stable and uniquely determined equilibrium path. ■

As for the interest rate policy regimes, equilibrium determinacy critically hinges on the response of the money growth rate to changes inflation $\overline{\mu}_{\pi}$. In the case where fiscal policy is Ricardian (MGR) the inflation response ought to be less than one ($\overline{\mu}_{\pi} < 1$) to ensure determinacy. Hence, the prominent and most commonly applied constant money growth rule ($\overline{\mu}_{\pi} = 0$) also leads to a uniquely determined equilibrium for a Ricardian policy.[8] When the

[8] Similarly, Carlstrom and Fuerst (2000) show that a constant money growth policy ensures real determinacy (for plausible money demand elasticities) in a flexible price cash-in-advance model.

inflation response is larger than one $\overline{\mu}_\pi > 1$, a rise in the inflation rate will be accompanied by a rise in real balances, which stimulates the economy leading to an upward pressure on the price level. In this case the model can exhibit either multiple equilibrium paths or an unstable equilibrium path depending on the degree of price stickiness δ, the latter being a main component of the composite parameter Φ_1 with $\partial\Phi_1/\partial\delta > 0$. The condition in part 2 of proposition 2 reveals that the MGR regime allows for self-fulfilling inflation expectations, when prices are sufficiently flexible $\Phi_1 < \widetilde{\Phi}$ (with $\widetilde{\Phi} > 0$ given that $\overline{\mu}_\pi > 1$). Otherwise, a rise in money growth in response to higher inflation will be accompanied by a strong real stimulation (small λ), which further feeds inflation by (31) such that the economy will evolve on an explosive path. On the contrary, a non-Ricardian fiscal policy in a MGN regime demands money growth policy to be accommodating ($\overline{\mu}_\pi > 1$) and prices to be sufficiently flexible ($\Phi_1 < \widetilde{\Phi}$) to obtain a unique and stable equilibrium path for the set $\{\pi, \lambda, m, a\}$.

3.3 Open market policy

Now suppose that the central bank conducts open market operations according to (15), so that issuance of money and bonds is interrelated, and sets the ratio of bonds to money (16) herein. Analogous to the previous regimes, the policy instrument β is set contingent on the current inflation rate.[9] We linearize the model and eliminate the nominal interest rate with the first order condition for money (5). Money can further be replaced by applying the policy rule, $a = m[1 + \beta(\overline{\pi})]$, such that the model in λ, π, and a reads

$$\dot{\lambda} = \theta(1+\beta(\overline{\pi}))(\lambda - \overline{\lambda}) + \Phi_2(\pi - \overline{\pi}) - \overline{u}_{mm}(1+\beta(\overline{\pi}))^{-1}(a - \overline{a}), \tag{33}$$

$$\dot{\pi} = \theta\,(\pi - \overline{\pi}) + \Phi_1(\lambda - \overline{\lambda}), \tag{34}$$

$$\dot{a} = -\theta\beta(\overline{\pi})\sigma^m(a - \overline{a}) - \Phi_3(\lambda - \overline{\lambda}) - \Phi_4\,[\pi - \overline{\pi}]\,, \tag{35}$$

where the composite parameter Φ_2, Φ_3, and Φ_4 are defined as follows:

$$\Phi_2 \equiv \frac{\overline{u}_m}{(1+\beta(\overline{\pi}))\,\theta}(1 - \sigma^m\theta\overline{\beta}_\pi), \quad \Phi_3 \equiv \frac{\overline{m}\beta(\overline{\pi})}{\overline{u}_m}\,[\theta(1+\beta(\overline{\pi}))]^2 > 0, \tag{36}$$

$$\Phi_4 \equiv \overline{m}\left[(1+\beta(\overline{\pi})) - \overline{\beta}_\pi\theta\,(1+\beta(\overline{\pi})\sigma^m)\right].$$

It is crucial to note that the model substantially differs from the former cases, where real wealth does not affect the first two equations. Here, real wealth actually enters equation (33) such that the eigenvalue of real wealth cannot separately be determined. In contrast to the former cases, where the differential equation for $\dot{a}$, given by (28) or (29), was unambiguously

[9] As a monetary tightening corresponds to a rise in the bond-to-money ratio (see Shreft and Smith, 1998, 2000), the ratio β should satisfy $\beta_\pi \geq 0$ for a monetary policy regime aiming at stabilizing the economy.

associated with an unstable eigenvalue, we are now interested in deriving conditions for a negative eigenvalue which can be assigned to real wealth. The model further requires two unstable eigenvalues for the remaining variables λ and π. Analyzing the local dynamics of the model, we find that determinacy of the model can be ensured if the partial derivative $\overline{\beta}_\pi$ is small enough. The following proposition summarizes this finding.

Proposition 3 *Suppose that money supply is restricted by (15) and that the central bank sets the bond-to-money ratio according to (16). Then there exists a unique equilibrium path converging to the steady state if but not only if*

$$\overline{\beta}_\pi < \frac{1}{\sigma^m \theta} \tag{37}$$

Proof. The model (33)-(35) is rewritten in matrix form

$$\begin{pmatrix} \dot{\lambda} \\ \dot{\pi} \\ \dot{a} \end{pmatrix} = A^\beta \begin{pmatrix} \lambda - \overline{\lambda} \\ \pi - \overline{\pi} \\ a - \overline{a} \end{pmatrix}, \qquad \text{with} \quad A^\beta \equiv \begin{pmatrix} \theta(1+\beta(\overline{\pi})) & \Phi_2 & -\overline{u}_{mm}(1+\beta(\overline{\pi}))^{-1} \\ \Phi_1 & \theta & 0 \\ -\Phi_3 & -\Phi_4 & -\theta\beta(\overline{\pi})\sigma^m \end{pmatrix}.$$

The determinant of A^β is given by

$$\begin{aligned} \det(A^\beta) = & -(1+\beta(\overline{\pi}))\theta^3\beta(\overline{\pi})\sigma^m - \Phi_3\theta\overline{u}_{mm}(1+\beta(\overline{\pi}))^{-1} \\ & +\overline{u}_{mm}(1+\beta(\overline{\pi}))^{-1}\Phi_1\Phi_4 + \theta\beta(\overline{\pi})\sigma^m\Phi_1\Phi_2. \end{aligned} \tag{38}$$

Inserting $\Phi_3 \equiv \frac{\overline{m}\beta(\overline{\pi})}{\overline{u}_m}\left[\theta(1+\beta(\overline{\pi}))\right]^2$, the first line in (38) vanishes such that $\det(A^\beta)$ reduces to $\det(A^\beta) = \Phi_1\left(\theta\beta(\overline{\pi})\sigma^m\Phi_2 + \overline{u}_{mm}(1+\beta(\overline{\pi}))^{-1}\Phi_4\right)$. Further using the definitions for the composite parameter Φ_2 and Φ_4 (see 36) and simplifying gives:

$$\det(A^\beta) = \Phi_1\overline{u}_m\sigma^m\left(1+\beta(\overline{\pi})\right)^{-1}\left(\overline{\beta}_\pi\theta - 1\right).$$

For $\overline{\beta}_\pi < 1/\theta$, which will be assumed in what follows, $\det(A^\beta)$ is negative implying that there are either one or three stable eigenvalues. Otherwise, the model would exhibit two stable or only unstable roots, indicating either equilibrium indeterminacy or explosiveness. In order to identify the cases where there is exactly one stable eigenvalue, which ensures equilibrium determinacy, we further have to examine the trace of A^β, which is given by:

$$\text{trace}(A^\beta) = \theta\left(2 + \beta(\overline{\pi})\left(1-\sigma^m\right)\right).$$

For $\beta(\overline{\pi}) < 2/\left(\sigma^m - 1\right)$, the trace is positive such that there must be at least one unstable eigenvalue. Hence, we know that the 3×3 matrix A^β has one stable and two unstable eigenvalues if the monetary policy rule satisfies $\beta(\overline{\pi}) < 2/\left(\sigma^m-1\right)$ and $\overline{\beta}_\pi < 1/\theta$. However, we further have to consider the case where $\beta(\overline{\pi}) > 2/(\sigma^m - 1)$ implying that the trace is negative. Here, the model exhibits either one or three stable eigenvalues. The latter demands the following matrix $\mathcal{A}^\beta$

$$\mathcal{A}^\beta \equiv \begin{pmatrix} \theta(1+\beta(\overline{\pi}))+\theta & 0 & \overline{u}_{mm}(1+\beta(\overline{\pi}))^{-1} \\ -\Phi_4 & \theta(1+\beta(\overline{\pi}))-\theta\beta(\overline{\pi})\sigma^m & \Phi_2 \\ \Phi_3 & \Phi_1 & \theta-\theta\beta(\overline{\pi})\sigma^m \end{pmatrix},$$

to have a negative determinant (see proof of proposition 2), which is given by

$$\begin{aligned} \det(\mathcal{A}^\beta) = {} & \theta^3\left(2+\beta(\overline{\pi})\right)\left(1+\beta(\overline{\pi})\left(1-\sigma^m\right)\right)\left(1-\beta(\overline{\pi})\sigma^m\right) - \overline{u}_{mm}(1+\beta(\overline{\pi}))^{-1}\Phi_4\Phi_1 \\ & -\Phi_3\theta\left(1+\beta(\overline{\pi})\left(1-\sigma^m\right)\right)\overline{u}_{mm}(1+\beta(\overline{\pi}))^{-1} - \Phi_1\Phi_2\theta\left(2+\beta(\overline{\pi})\right). \end{aligned}$$

Replacing the composite parameter by their definitions in (36) and rearranging gives

$$\det(\mathcal{A}^\beta) = \left(\left(\sigma^m-1\right)\beta(\overline{\pi})-1\right)\left[\theta^3\left[\left(\sigma^m-1\right)\beta(\overline{\pi})-2\right] + \frac{\Phi_1\overline{u}_m(1-\sigma^m\overline{\beta}_\pi\theta)}{1+\beta(\overline{\pi})}\right] + \frac{\Phi_1\overline{u}_m(\sigma^m-1)}{1+\beta(\overline{\pi})}.$$

Given that $\sigma^m \geq 1$ and $\beta(\overline{\pi}) > 2/(\sigma^m - 1)$, the determinant of $\mathcal{A}^\beta$ exclusively consists of positive terms if $\overline{\beta}_\pi < 1/(\sigma^m\theta)$. In this case, the determinant will clearly be positive such that the model cannot exhibit three stable roots. Hence, $\overline{\beta}_\pi < 1/(\sigma^m\theta)$, which implies (by assumption 2) that $\overline{\beta}_\pi < 1/\theta$ is fulfilled, is sufficient to ensure that the model exhibits one stable and two unstable eigenvalues. ■

The sufficient condition (37) presents an upper bound for the reactiveness of the monetary policy instrument β to changes in inflation. This upper bound is certainly much larger than one given that the discount rate θ equals the steady state real interest rate. The result presented in proposition 3, thus, indicates that the central bank can ensure the existence of a stable and uniquely determined equilibrium path by setting the bond-to-money ratio not in an extremely reactive way. However, the upper bound on $\overline{\beta}_\pi$ actually depends on a utility parameter σ^m, whereas the determinacy conditions for money growth policy and interest rate policy refer to a numerical threshold, namely, one. Nevertheless, pegging the ratio of bond-to-money, as for example assumed in Shreft and Smith (1998, 2000), is a safe strategy for a central bank to ensure equilibrium determinacy.

The reason for open market policy to exert a stabilizing impact on the economy by ruling out self-fulfilling expectations can be rationalized as follows. Suppose that inflation rises. Given that the evolution of real wealth is restricted by open market operations (15), this tends to reduce the stock of real government liabilities and, thus, real financial wealth. Given that the latter is linked to money by the ratio β, changes in real wealth are passed through to proportional changes in real bonds and real balances, affecting marginal utility according to assumption 1. This exerts, by (33), an upward pressure on the shadow price of wealth in the subsequent periods, accompanied by a decline in consumption by (3). Forward looking price setters are thus willing to lower prices such that inflation expectations cannot be self-fulfilling in this economy. This stabilizing effect of real wealth, however, demands that the central bank sets the bond-to-money ratio not in an extremely reactive way ($\overline{\beta}_\pi < \frac{1}{\sigma^m \theta}$). Otherwise, a rise in inflation would lead to a strong rise in government bonds, which would heavily increase the interest payment obligations of the public sector. As this would lead to a further issuance of public liabilities, this can either lead to multiple equilibrium paths or to explosiveness.

4 Conclusion

It is shown in this paper that the requirements for monetary policy to ensure local determinacy critically hinge on, first, whether the fiscal policy regime is Ricardian or non-Ricardian and on, second, the particular central bank instrument. Similar to the case of interest rate policy, equilibrium determinacy for a money growth regime, where the central bank sets the growth rate of nominal money contingent on changes in inflation, depends on the magnitude of monetary policy reactiveness. Opposed to the well-known Taylor-principle for interest rate policy, the central bank should raise the money growth rate by less than one for one to changes in inflation when fiscal policy is Ricardian. A non-Ricardian fiscal policy regime, however, requires the central bank to increase the money growth rate by more than one for one to ensure determinacy, given that prices are not too rigid.

In contrast, it is shown that an open market policy regime, first, induces government solvency to be satisfied even outside the equilibrium, and, second, is much less prone to equilibrium multiplicity or explosiveness. In particular, local determinacy is ensured as long as the central bank sets the ratio of government bonds and money not extremely reactive to changes in inflation. The crucial feature is that Ricardian equivalence does not hold such that real activity and inflation is affected by the equilibrium sequence of government debt and, therefore, real financial wealth, whereas the latter can be recursively determined for the alternative monetary policy regimes, where Ricardian equivalence holds. Given that the open market policy restricts the issuance of public debt, real financial wealth, which is a predetermined variable, evolves in a non-explosive way and is shown to rule out self-fulfilling expectations. Our results, hence, indicate that a central bank, which aims at stabilizing the economy, should conduct monetary policy by implementing a moderately reactive open market policy.

5 Appendix: Derivation of the aggregate supply constraint

The firm's optimal price setting problem is

$$\max_{Q_t} \int_t^\infty e^{-(\delta+\theta)(s-t)} \lambda_s [(Q_t e^{\overline{\pi}(s-t)} y_{is}(Q_t) - MC_s y_{is}(Q_t))/P_s] ds.$$

subject to given initial prices and to (8) and (9). The first order condition is

$$\int_t^\infty e^{-(\delta+\theta)(s-t)} \frac{\lambda_s}{P_s} [(1-\varepsilon)(Q_t e^{\overline{\pi}(s-t)})^{-\varepsilon} P_s^\varepsilon y_s e^{\overline{\pi}(s-t)} +$$
$$\varepsilon \left(Q_t e^{\overline{\pi}(s-t)}\right)^{-\varepsilon-1} MC_s P_s^\varepsilon y_s e^{\overline{\pi}(s-t)}] ds = 0.$$

Simplifying and rearranging, this is equivalent to

$$\int_t^\infty e^{-(\delta+\theta)(s-t)} \lambda_s \widetilde{P}_s^{\varepsilon-1} y_s Q_t ds = \frac{\varepsilon}{\varepsilon-1} \int_t^\infty e^{-(\delta+\theta)(s-t)} \lambda_s \widetilde{P}_s^{\varepsilon-1} y_s \widetilde{MC}_s ds,$$

where we define $\widetilde{X}_s \equiv X_s / e^{\overline{\pi}(s-t)}, \quad X = P, MC$. Dividing both sides by P_t and letting $q_t \equiv Q_t/P_t$, we have

$$\int_t^\infty e^{-(\delta+\theta)(s-t)} \lambda_s \widetilde{P}_s^{\varepsilon-1} y_s q_t ds = \frac{\varepsilon}{\varepsilon-1} \int_t^\infty e^{-(\delta+\theta)(s-t)} \lambda_s \widetilde{P}_s^{\varepsilon-1} y_s \widetilde{MC}_s \frac{1}{P_t} ds.$$

Linearizing this expression around the steady state, we obtain

$$\int_t^\infty e^{-(\delta+\theta)(s-t)} \overline{\lambda}_s \overline{\widetilde{P}}_s^{\varepsilon-1} \overline{y}_s \overline{q}_t \left[\frac{\lambda_s - \overline{\lambda}_s}{\overline{\lambda}_s} + (\varepsilon-1) \frac{\widetilde{P}_s - \overline{\widetilde{P}}_s}{\overline{\widetilde{P}}_s} + \frac{y_s - \overline{y}_s}{\overline{y}_s} + \frac{q_t - \overline{q}_t}{\overline{q}_t} \right] ds$$
$$= \frac{\varepsilon}{\varepsilon-1} \int_t^\infty e^{-(\delta+\theta)(s-t)} \overline{\lambda}_s \overline{\widetilde{P}}_s^{\varepsilon-1} \overline{y}_s \overline{\widetilde{MC}}_s \frac{1}{\overline{P}_t} \left[\frac{\lambda_s - \overline{\lambda}_s}{\overline{\lambda}_s} + (\varepsilon-1) \frac{\widetilde{P}_s - \overline{\widetilde{P}}_s}{\overline{\widetilde{P}}_s} + \right.$$
$$\left. \frac{y_s - \overline{y}_s}{\overline{y}_s} + \frac{\widetilde{MC}_s - \overline{\widetilde{MC}}_s}{\overline{\widetilde{MC}}_s} - \frac{P_t - \overline{P}_t}{\overline{P}_t} \right] ds, \tag{39}$$

where, as usual, bars over variables denote the respective steady state values. Note that, in steady state, we have the following relations: $\overline{P}_s$ grows with the rate $\overline{\pi}$, whereas $\overline{\widetilde{P}}_s$ is constant (as are $\overline{\lambda_s}$ and $\overline{y_s}$). Further, the price chosen by an adjusting firm must equal the aggregate price index, such that $\overline{q}_t = 1$. The constant elasticity property of the demand function implies that the steady state price level is a constant markup over nominal marginal costs, $\overline{P}_s = \varepsilon/(\varepsilon-1)\overline{MC}_s$. Therefore, as $\overline{P}_s = \overline{P}_t e^{\overline{\pi}(s-t)}$, we have that $\varepsilon/(\varepsilon-1)\overline{\widetilde{MC}}_s/\overline{P}_t = 1$, and the coefficients on the left and right hand sides of (39) are the same. Hence, the equation simplifies to

$$\int_t^\infty e^{-(\delta+\theta)(s-t)}\frac{q_t-\overline{q}_t}{\overline{q}_t}ds = \int_t^\infty e^{-(\delta+\theta)(s-t)}\left[\frac{\widetilde{MC}_s-\overline{\widetilde{MC}}_s}{\overline{\widetilde{MC}}_s}-\frac{P_t-\overline{P}_t}{\overline{P}_t}\right]ds.$$

Noting that $(\widetilde{MC}_s-\overline{\widetilde{MC}}_s)/\overline{\widetilde{MC}}_s = (MC_s-\overline{MC}_s)/\overline{MC}_s$ and defining real marginal costs as $mc_s = MC_s/P_s$, this can be written as

$$\frac{q_t-\overline{q}_t}{\overline{q}_t} = (\delta+\theta)\int_t^\infty e^{-(\delta+\theta)(s-t)}\left[\frac{mc_s-\overline{mc}_s}{\overline{mc}_s}+\frac{P_s/P_t-\overline{P_s/P_t}}{\overline{P_s/P}_t}\right]ds. \tag{40}$$

The last term in square brackets in the preceding expression is a function of the deviations of the inflation rates between t and s from steady state inflation, as from $P_s/P_t = \exp(\int_t^s \pi_r dr)$ it follows that $(P_s/P_t-\overline{P_s/P_t})/\overline{P_s/P_t} = \int_t^s(\pi_r-\overline{\pi})dr$. Using this and differentiating (40) with respect to t we obtain by applying Leibnitz' rule:

$$\begin{aligned}\frac{d}{dt}\frac{q_t-\overline{q}_t}{\overline{q}_t} &= -(\delta+\theta)\frac{mc_s-\overline{mc}_s}{\overline{mc}_s}+\\ &\quad (\delta+\theta)\int_t^\infty(\delta+\theta)e^{-(\delta+\theta)(s-t)}\left[\frac{mc_s-\overline{mc}_s}{\overline{mc}_s}+\int_t^s(\pi_r-\overline{\pi_r})dr\right]\\ &\quad +e^{-(\delta+\theta)(s-t)}\left[-(\pi_t-\overline{\pi}_t)\right]ds\\ &= (\delta+\theta)\left[\frac{q_t-\overline{q}_t}{\overline{q}_t}-\frac{mc_s-\overline{mc}_s}{\overline{mc}_s}\right]-(\pi_t-\overline{\pi}_t).\end{aligned} \tag{41}$$

This can be converted into a differential equation in π by finding the relation between, respectively, the steady state deviations and the growth rates of inflation and the real reset price. First, the price index $P^{1-\varepsilon} = \left[\int_0^1 P_i^{1-\varepsilon}\mathrm{d}i\right]$ can be expressed as a function of past reset prices, where each historical reset price has to be weighted by the probability that a price set at time s is not adjusted in time t, which is given by $\delta\exp\{-\delta(t-s)\}$ (see Calvo, 1983, Benhabib et al., 2001a). Therefore, the price index can be written as

$$P_t^{1-\varepsilon} = \int_{-\infty}^t \delta e^{-\delta(t-s)}Q_s^{1-\varepsilon}ds.$$

Differentiating with respect to t, we get

$$\pi_t = \frac{\delta}{1-\varepsilon}(q_t-1),$$

which when linearized around the steady state implies

$$\pi_t-\overline{\pi_t} = \delta(q_t-\overline{q_t}). \tag{42}$$

Using (42) in (41) and noting that $\overline{q}_t = 1$ and $\overline{mc}_t = (\varepsilon - 1)/\varepsilon$, this finally results in

$$\dot{\pi} = \theta(\pi_t - \overline{\pi_t}) - \frac{\varepsilon\delta(\delta+\theta)}{\varepsilon - 1}(mc_t - \overline{mc}_t).$$

This is the linearized economy's aggregate supply constraint (12).

6 References

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