Linking Current and Future Negotiations

by

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Abstract

By means of two-player multi-issue sequential bargaining models, this paper investigates negotiation processes in which current and future negotiations are linked. In particular, the probability of game continuation after the initial agreement depends on the initial agreement. We show that the division of one cake à la Rubinstein can be subgame perfect for a sufficiently small discount factor. Moreover, multiple equilibria can exist, either the Rubinsteinian equilibrium and a two-stage equilibrium co-exist (for specific values of the discount factor) or different two-stage equilibria can be sustained (even for a sufficiently high discount factor). Finally, we show that there are equilibria in which players compromise future beneficial negotiations.

1 Introduction

In many bargaining situations the possibility of future negotiations is affected by the initial agreement. For instance, in peace processes, where issues are, in general, very controversial, once an agreement has been reached on an initial issue this does not imply that the negotiation process will continue with certainty. Frictions and difficulties, due to either external factors or aspects related to the negotiations (such as, the timing of the initial agreement and the agreement itself), can impede bargaining over a new issue.

In this paper, we focus on the case in which the probability of game continuation depends on the agreement reached over the initial issue. This problem is interesting since it is a common aspect of many real-life bargaining processes, moreover, it has not received great attention in bargaining theory. Some related papers belong to the literature of third parties (see for instance, Manzini and Ponsatì, 2002). In these cases there is a third party who can give additional resources to the bargaining parties, who therefore receive not only part of the initial surplus, but also some additional contributions. However, these contributions are independent of the precise agreement reached, moreover, the third party is affected by these contributions (i.e., the game has three players, two bargaining parties and a stakeholder). Also the literature on agenda formation (see, Flamini, 2001) is related to the problem of linked negotiations since two bargaining stages are considered. However, the probability of re-starting the negotiations after the initial agreement is constant (and often equal to 1, see, for instance, Busch and Horstmann, 1997 and Inderst, 2000). In many real-life negotiations (e.g., peace processes), the initial agreement matters and it is perceived as the initial stage for future agreements. Moreover, the characteristics of this agreement are also important. For instance, if the initial agreement is fairly 'equal' then it is very likely that a second agreement can be settled. However, if the initial agreement is 'unfair' then future settlement can be less likely.

The aim of this paper is to answer the question: what happens when the first agreement can affect the probability of future negotiations? Our investigation is based on an alternating-offer bargaining game with two players, who attempt to find an agreement over two issues sequentially. That is, once an initial agreement has been reached, parties can start bargaining over the second issue. The bargaining procedure is further characterised by the sequential implementation of agreements. The distinguishing feature of this model is that the probability of game continuation (or alternatively, the between-cake discount factor[1]) is a function of the initial agreement and can assume different forms. These can be classified as one of two basic types. The first type has the property that the more equal the first division the

[1] In other words, as in Muthoo (1995, 1999), we can assume that there is an interval of time between an acceptance and a new proposal rather than a probability of game continuation after the first agreement. However, differently from Muthoo, the length of this interval is now a function of the initial agreement.

more negotiations are likely to extend to a second issue. This represents the case in which whenever a party gets less than his rival, he appears weak and finds it more profitable to search for other partners. Therefore, the probability of game breakdown increases when the division is more unequal. In the second type, unequal divisions make it more likely that the negotiation will proceed. The player who obtains large concessions in the first stage possesses continuation power, in the sense that he will pursue the negotiation process.

Even though it would be interesting to analyse the case of a general continuation probability function, it is instructive to focus on the solution of specific simple cases, as they cover a large range of possible continuation probabilities. We also find that a number of our results are robust across these specifications. In particular, interrupting the negotiation process after agreeing on one issue can be subgame perfect. When only one issue is negotiated, the equilibrium strategies are as in the standard Rubinstein bargaining model (RBM), Rubinstein (1982). Moreover, in terms of agenda formation, we show that, in equilibrium, a party will make large concessions in the initial negotiations if his more important issue is postponed in the bargaining process, even though these concessions reduce the probability of game continuation. That players make concessions on the initial issue so as to negotiate on a second more important issue is not surprising (see, for instance, Flamini, 2001). However, as far as we know, the dramatic result that players compromise future beneficial negotiations in equilibrium is new.

Finally, we prove that multiple equilibria can arise: either the Rubinsteinian equilibrium co-exists with equilibria in which the two cakes are shared (for specific values of the discount factor δ) or different two-stage equilibria can co-exist (for a sufficiently high value of δ).

The paper is organised as follows. In section 2 the bargaining model is presented. We then solve the model for the cases in which the continuation probability is maximised when an equal partition is implemented at the first stage (in section 3). The opposite case, in which unequal divisions ensure that the second stage is played with certainty, is investigated in section 4. Finally, section 5 concludes the paper.

2 Two-Player Two-Cake Bargaining Game

Before discussing the forms that the probability of continuation can take, we briefly describe the model, which is based on the RBM. There are two bargaining stages, in each stage two players, named 1 and 2 attempt to share a surplus (named cake i, with $i = 1, 2$), as in the RBM (i.e., an alternating-offer bargaining game). Time is discrete ($t = 0, 1, 2...$), at t = 0, player 1 proposes a partition of cake 1. Players' time preferences are represented by the common discount factor δ. Only once the agreement over the division of cake 1 is reached may parties bargain over the division of cake 2. We assume that the first mover at the second stage is the responder to a successful proposal in the previous stage. This assumption is not restrictive, since the only case we need to exclude is the trivial case in which the first mover at the second stage is the proposer to a successful proposal in the previous stage. The implementation of the agreements is sequential and if the agreement is not reached on the partition of a cake, players get zero payoffs at that stage (disagreement).

The probability of game continuation, $\alpha(\cdot)$, is a function of the agreement at the first stage. For instance, a buyer and a seller who agreed the price of an item, may not start bargaining over the price of another item if a player perceived the first agreement as imbalanced or unfair. Alternatively, as pointed out in the introduction, the factor $\alpha(\cdot)$ represents a between–cake discount factor. Since in this case, there are intervals of time of different length, t indicates the name of a time interval.

As in the literature of agenda (e.g., Busch and Horstmann, 1997), players are assumed to have different valuations of the cake sizes. The non-negative parameters λ represent the relative importance of cake i to player i, with $i = 1, 2$ (see (1) and (2) below). That is, if λ differs from 1, players have opposite preferences over issues. However, since the focus of this paper is on the way in which current and future negotiations are linked, we often consider the case of equally valued cakes, i.e., $\lambda = 1$. When this assumption is relaxed and the effect of opposite valuations of the cake size is introduced, similar results can be established.

Players' utilities are linear functions of the shares obtained at each stage. For instance, if at time t, with $t = 0, 1, 2, ...$ player 1 proposes a successful division $(x_1, 1 - x_1)$, while only after n period, player 2 makes a successful proposal $(1 - y_2, y_2)$, then player i obtains v_i with $i = 1, 2$, specified below.

$$v_1 = \delta^t(\lambda_1 x_1 + \alpha_1(x_1)\delta^n(1 - y_2)) \tag{1}$$

$$v_2 = \delta^t(1 - x_1 + \alpha_1(x_1)\delta^n \lambda_2 y_2) \tag{2}$$

The focus is on subgame perfect equilibria (SPE). Obviously, the last stage is played as in the RBM, that is, the equilibrium demand is $1/(1+\delta)$. To define the equilibrium demand at the first stage, we consider some specific functions for the probability of continuation. In the following section, the focus is on cases where a more equal first division makes negotiations over a second issue more likely.

3 When Equal Division Ensures the Continuation Game

In this section three games are analysed, in the first two, only the equal division of the first cake ensures the continuation game. In the first game the probability of continuation is piecewise linear (section 3.1), while in the second it is an extremely simple function: always null except for the division $(\frac{1}{2}, \frac{1}{2})$ (section 3.2). This case is then generalised (in section 3.3) to include the possibility that the probability of continuation is null except for a range of divisions in which it is one.

We show that when the probability of continuation is equal to zero only at extreme divisions, the division that allows players to play the second stage with probability one is not always an equilibrium, even when the common discount factor tends to one ($\delta \to 1$). The SPE strategies depend on players' valuations of the cake size. If the second cake represents a relatively more important issue to player 2, player 1 cannot do any better than make large concession at the first stage, even though these reduce the probability of reaching the second stage. Similarly, when the first cake is the most important to player 1, he is able to extract large concessions from his rival.

On the other hand, when the probability of continuation is zero for a continuum range of divisions, the solution to the RBM is robust. Players do not negotiate on the second issue. These results have important implications for the agenda formation problem: in equilibrium, parties do not attempt to maximise the probability of playing future beneficial negotiations.

3.1 Players with Equal Continuation Power

In this section we consider a probability of continuation, $\alpha(x_1)$, which is piecewise linear with the following specification:

$$\alpha(x_1) = \begin{cases} 2x_1 \text{ if } x_1 \leq 1/2 \\ 2(1 - x_1) \text{ if } x_1 > 1/2 \end{cases} \tag{3}$$

where x_1 is the share player 1 obtains at the first stage. Then, the maximum probability of continuation, $\alpha(x_1) = 1$, is reached when the first cake is shared equally ($x_1 = \frac{1}{2}$). If, in agreement, a player receives a share smaller than his rival's, then the probability of continuation is smaller than 1. In this sense, players have the same bargaining continuation power.

It is intuitive that the equilibrium outcome in the first stage should be close to the partition $(\frac{1}{2}, \frac{1}{2})$, since this increases the probability of bargaining over the future issue. However, this is not always an equilibrium, as shown in the following proposition and corollary. The interpretation of the result follows.

Proposition 1 *If $\delta \to 1$, it is subgame perfect to share the first cake equally, only if λ assumes intermediate values (i.e., $(\frac{\sqrt{7}-2}{3} \leq \lambda \leq 2+\sqrt{7})$. If λ is sufficiently small ($\lambda < (\frac{\sqrt{7}-2}{3})$), player 1 asks for less, and player 2 for more than half a cake. Conversely, when λ is sufficiently large ($\lambda > 2+\sqrt{7}$), player 1 asks for a share bigger than a half and player 2 smaller than a half.*

Proof. See Appendix. ■

Corollary 2 *Let $\lambda_u = \frac{5+3\delta^2+\sqrt{\Delta}}{4(\delta^2+\delta-1)}$ and $\lambda_l = \frac{-(5+3\delta^2)+\sqrt{\Delta}}{4(\delta^2+\delta-1)}$ with $\Delta = 9+46\delta^2+25\delta^4+32\delta^3$. If $\lambda > \lambda_u$, at the first stage the SPE demands are as follows:*

$$ {}_d x_1 = -\frac{4\lambda\delta^2 + 2\delta^2 - \lambda\delta - 2\lambda^2\delta + 2\delta + 3\lambda - 2\lambda^2 + 2}{-3\lambda\delta^2 + 2\lambda^2\delta^2 - 2\delta^2 + 2\lambda\delta + 2\lambda^2\delta + 2\delta - 3\lambda + 2\lambda^2 - 2} > 1/2 \quad (4) $$

$$ {}_d y_2 = \frac{\lambda(\delta + 2\lambda + 1)}{-3\lambda\delta^2 + 2\lambda^2\delta^2 - 2\delta^2 + 2\lambda\delta + 2\lambda^2\delta + 2\delta - 3\lambda + 2\lambda^2 - 2} < 1/2 \quad (5) $$

If $(1+\delta)/2\delta < \lambda < \lambda_u$, then the SPE demands are

$$ {}_e x_1 = \frac{2 + \delta + 4\lambda - \delta^2(1 + 2\lambda)}{2(1 + \delta + 2\lambda)} > 1/2, \ {}_e y_2 = 1/2 \quad (6) $$

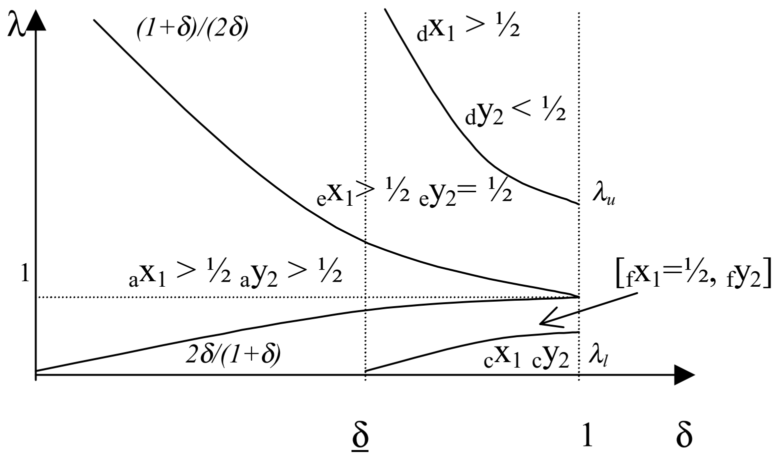

Figure 1: Representation of the SPE in the space (δ,λ)

If $2\delta/(1+\delta)<\lambda<(1+\delta)/2\delta$, *then the SPE demands are as follows:*

$$ {}_ax_1 = \frac{2\delta^3 - 4\lambda\delta^3 + 3\lambda\delta^2 - 4\lambda\delta - 2\delta + 5\lambda + 2\lambda^2 + 2}{2\lambda^2\delta^3 + 2\delta^3 - 5\lambda\delta^3 + 3\lambda\delta^2 - 3\lambda\delta + 2\lambda^2 + 5\lambda + 2} > 1/2 \quad (7)$$

$$ {}_ay_2 = \frac{-4\lambda\delta^3 + 2\lambda^2\delta^3 + 3\lambda\delta^2 - 4\lambda\delta - 2\lambda^2\delta + 2\lambda^2 + 5\lambda + 2}{2\lambda^2\delta^3 + 2\delta^3 - 5\lambda\delta^3 + 3\lambda\delta^2 - 3\lambda\delta + 2\lambda^2 + 5\lambda + 2} > 1/2 \quad (8)$$

If $\lambda_l < \lambda < (1+\delta)/2\delta$, *players' demands become*

$$ {}_fx_1 = 1/2 \text{ and } {}_fy_2 = \frac{4 + 2\lambda + \delta\lambda - \delta^2(\lambda - 2)}{2(\lambda(1+\delta) + 2)} > 1/2 \quad (9)$$

Finally, for $\lambda < \lambda_l$, *the SPE demands are*

$$ {}_cx_1 = \frac{\lambda\delta + 2 + \lambda}{2\delta^2 - 3\lambda\delta^2 - 2\lambda^2\delta^3 + 2\lambda\delta + 2\delta - 2\lambda^2\delta + 2 - 3\lambda - 2\lambda^2} \leq 1/2 \quad (10)$$

$$ {}_cy_2 = \frac{2 - 2\lambda^2\delta^2 - 4\lambda\delta^2 + 2\delta + \lambda\delta - 2\lambda^2\delta - 2\lambda^2 - 3\lambda}{2\delta^2 - 3\lambda\delta^2 - 2\lambda^2\delta^3 + 2\lambda\delta + 2\delta - 2\lambda^2\delta + 2 - 3\lambda - 2\lambda^2} > 1/2 \quad (11)$$

The equilibrium demands are represented in Figure 1 above. As shown in the proof of Proposition 1, the relevant curves $(1+\delta)/(2\delta)$ and $(2\delta)/(1+\delta)$ describe the spaces for λ and δ in which the optimal demand of a player is 1/2 rather than 1. While the curves λ_u and λ_l limit the space for δ and λ in which the indifference conditions define SPE demands. Note that except in these curves, the SPE defined above are unique since the game has a stationary structure as the RBM (i.e., at any node where player i proposes, the game is the same) and the proof follows standard arguments (see for instance Sutton, 1986). A discussion of the SPE of this game follows.

The assumption of infinitely patient players allows us to focus on the role of the probability of breakdown and ratios representing the relative importance of the cakes. When players are infinitely patient, the key aspect driving the result is that players are allowed to disagree over the importance of the issues. It is intuitive that there is an equilibrium in which the probability of playing the second stage is maximised. Indeed, the partition $(\frac{1}{2}, \frac{1}{2})$ in the first stage allows another division $(\frac{1}{2}, \frac{1}{2})$ in the second stage. However, this holds only when the relative importance ratio, λ, does not assume extreme values. When λ is large ($\lambda > 2 + \sqrt{7}$), that is, the first issue is relatively more important to player 1, such a player can obtain more than half of the first cake. Even though this reduces the probability of future bargaining, player 1 can enjoy a larger share on his more important cake. This is sustainable as an equilibrium, in spite of the fact that player 2 faces the risk of not bargaining over his more important issue. The reason is that, by leaving a large share to the opponent, player 2 avoids the cost of rejection, even if infinitely small (the first mover advantage in the second stage is not a crucial effect as shown below). Conversely, when cake 2 is relatively more important to player 1, he concedes more than half of the first cake to the opponent at the first stage (even if this decreases the probability of continuation). If he asked for more, player 2 would reject his proposal, given that the second cake is not relevant to him and rejections are not very costly.

To sum up, when players are infinitely patient and have sufficiently different preferences over issues, they play equilibrium strategies in which the probability of continuation is not maximised. The player who values the issue negotiated first more can obtain large concessions from his opponents, even though impatience is not a predominant force in the bargaining process. This has an interesting implication for the theory of agenda formation. If an issue that matters to a party is postponed in the bargaining process, the party cannot do any better than make large concessions at the first stage of the negotiations, even though these concessions reduce the possibility of continuation. This not only enforces the already-known result in the literature of agenda, first things first (e.g., Flamini 2001), but also implies something new, namely, players can compromise future beneficial negotiations in equilibrium.

In classic bargaining games when the discount factor is small, players focus only on the exploitation of all the immediate gains (RBM). In this game this applies only if issues are valued similarly $(2\delta/(1+\delta) < \lambda < (1+\delta)/(2\delta)$, with $\delta \leq \underline{\delta}$, as shown in Figure 1). Indeed, in the extreme case of infinitely impatient players ($\delta \to 0$), a proposer asks for the entire cake, even though this reduces to zero the possibility of future bargaining (the game is reduced to a RBM).

However, when players' preferences over issues are strongly differentiated (and $\delta < \underline{\delta}$), there is a player who attempts to maximise the probability of game continuation. In particular, if player 2 strongly prefers the second cake, ($\lambda > (1+\delta)/2\delta$), he attempts to maximise the probability of game continuation. Then player 1 is able to obtain a large share at the beginning of the game. On the other hand, when player 2's most important issue is represented by cake 1 ($\lambda < 2\delta/(1+\delta)$), he demands a large share at the first stage. However, player 1, who moves first can obtain half of the first cake so as to maximise the probability of reaching the second stage. The outcome is asymmetric since the structure of the game is asymmetric (in both cases player 1 is the first mover).

The demands defined when the discount factor is low are subgame perfect also when the discount factor is large ($\delta > \underline{\delta}$, see Figure 1). Moreover, in this case, two new equilibria exist. If λ is large ($\lambda > \lambda_u$), then player 2 makes large concessions at the first stage to reach the second even if these concessions reduce the probability of game continuation ($y_2 < 1/2$). Similarly, player 1 makes large concessions at the first stage when λ is small ($\lambda < \lambda_l$).

An interesting case is represented by players with similar valuations of the importance of the issues ($\lambda = 1$). In this case regardless of the value of the discount factor ($\delta < 1$), the division ensuring the continuation game is never subgame perfect. Players always ask for large shares (i.e., $x_1 = y_2 = \frac{3-2\delta}{3-\delta} > 1/2$), since the advantage of having a second bargaining stage with certainty is outweighed by the loss of gain in playing the partition $(\frac{1}{2}, \frac{1}{2})$. Only when the cost of rejection tends to zero ($\delta \to 1$), players propose the partition $(\frac{1}{2}, \frac{1}{2})$.

To conclude, although splitting the game in half is optimal, in the sense that it ensures that the second bargaining stage is played with certainty, it is not Pareto-superior. It is preferred only by player 1 when λ is small ($\lambda < 0.21$, for $\delta \to 1$) and only by player 2 when λ is large ($\lambda > 4.65$). However, if there were a utilitarian external regulator able to select among partitions, he would always choose the partition $(\frac{1}{2}, \frac{1}{2})$.

3.2 Equality or Nothing

In the previous section, we have shown that the optimal partition, in the sense that it ensures that the second stage is played with certainty, is not an equilibrium for a large range of players' relative ratios λ even when the cost of rejection tends to zero ($\delta \to 1$). This might be due to the fact that players have a high probability of continuing bargaining when they ask for a share close to a half. To check whether or not this is the feature that explains why the continuation game is not ensured with certainty, we consider the extreme case in which the probability of continuation is always null except in the case of equality (i.e., $\alpha(x_1) = 1$ iff $x_1 = 1/2$, 0 otherwise). In this case there are two main results. First, the intuitive equilibrium in which players first share equally the first cake and then bargain as in RBM is sustained only for sufficiently high values of the discount factor (Proposition 3). Second, when rejections are too costly the second stage is not played regardless of the relative importance of the cakes. Instead, the Rubinsteinian division is played at the first stage (Proposition 4).

To simplify the analysis, we first assume that players have the same valuation of the cake size ($\lambda = 1$). At the end of this section, we show that similar results can also be established when this assumption is relaxed. The focus is firstly on the case of patient players.

Proposition 3 *: If $\delta > 1/3$ in the unique SPE players divide the first cake equally, the second bargaining stage is reached with certainty and is played as in the RBM. This is also an SPE for $\delta = 1/3$.*

Proof. When $\delta \geq 1/3$, the SPE strategies are the following: in the first stage player i demands 1/2 and accepts any share not smaller than $\delta(1 + 3\delta)/2(1 + \delta)$, unless equal to 1/2, while in the second the RBM strategies are played. To show that these strategies are SPE is straightforward, and the proof is therefore omitted. We now turn to show the uniqueness of the SPE strategies. The argument is standard.

Suppose that M_i (m_i) is the supremum (infimum) SPE payoff to player i in any subgame beginning with an offer by player i with $i = 1, 2$. We prove that if $M_1 > (1+3\delta)/2(1+\delta) > m_1$, then there are profitable deviations. Similar arguments apply for player 2. If in an SPE, the payoff to player 1 is M with $M_1 \geq M > (1+3\delta)/2(1+\delta)$, then only one cake will be shared, since for two cakes only the division $(\frac{1}{2}, \frac{1}{2})$, followed by the Rubinstein game can be played. Player 2 is better off in rejecting such a proposal and offering the division $(\frac{1}{2}, \frac{1}{2})$. This is accepted by player 1 and it is profitable to Player 2 if $\delta(1/2+\delta/(1+\delta)) > 1-M$ (that is, $M > (2+\delta-3\delta^2)/2(1+\delta)$). This always hold if $\delta > 1/3$, since $M > (1+3\delta)/2(1+\delta) > (2+\delta-3\delta^2)/2(1+\delta)$. Then if $\delta > 1/3$, player 1 cannot obtain more than $M_1 = (1+3\delta)/2(1+\delta)$.

We now consider the case in which $m_1 < (1+3\delta)/2(1+\delta)$. In this case, player 1 would be better off in proposing the division $(\frac{1}{2}, \frac{1}{2})$, player 2 would accept this since $1/2+1/(1+\delta) > \delta M_2 = \delta(1+3\delta)/(1+\delta)$, and player 1 would get $1/2+\delta/(1+\delta) > m_1$ by assumption. Therefore, the equality $m_1 = (1+3\delta)/(1+\delta)$ must hold. ■

The SPE specified in Proposition 2 is intuitive, players first equally divide the first cake and then they divide the second as in the RBM. In this SPE, the payoff to player 1 is not smaller than the payoff to the first mover in the RBM, since the discount factor is sufficiently large ($\delta \geq 1/3$). Moreover, the responder is always better off when two cakes are shared. However, when the discounting is strong, this cannot hold. Next we consider the case of small discount factors. Often in the literature of bargaining the attention is focused on infinitely patient players (for instance Muthoo, 1999), however not uniquely (Merlo and Wilson, 1998). Indeed, in some cases a small discount factor is the most appropriate assumption (see, for instance, Manzini and Mariotti, 2000, where the corresponding p is the probability of breakdown after a rejection, where one party is an alliance of heterogeneous players). By avoiding constraints over δ, we show that less intuitive equilibria exist. In particular, the Rubinsteinian equilibrium can be re-established for δ sufficiently small. The intuition behind this result is the following: since rejections are costly (δ is small), a proposer can ask for a large share at the first stage. Moreover, the status of responder at the second stage is not attractive when δ is small (however, the switch of roles of proposer and responder after an acceptance is not crucial, as discussed after Proposition 4). Then, player 1 is better off in avoiding the division of two cakes.

Proposition 4 *When players are sufficiently impatient ($\delta \leq 1/3$), the game is played à la Rubinstein, and only the first cake is shared. This is the unique SPE for $\delta < 1/3$.*

Proof. Consider the following strategies, $\delta \in [0, 1/3]$ player i proposes $1/(1+\delta)$ and accepts any share not smaller than $\delta/(1+\delta)$ with $i = 1, 2$. Given that $1/(1+\delta) \neq 1/2$, for δ in $[0, 1/3]$, only one cake is shared. It is straightforward to show that these strategies are SPE. The proof of the uniqueness of these strategies (for $\delta \in [0, 1/3)$) is omitted since it follows standard arguments. ■

Even in the extreme case in which all the partitions except one imply the end of the game, the optimal partition, that is $(\frac{1}{2}, \frac{1}{2})$, is not always played. Players must be sufficiently patient to play the second stage. Moreover, differently from the case studied in section 3.1, where almost all the partitions ensure that the second stage is played with a positive probability, players are unable to extract a large gain at the first stage, when the cost of rejection is low. Indeed, given the rigid structure of the probability of game continuation, when the discount factor is large, future beneficial negotiations are never compromised.

These results can be re-established also under different assumption. For instance, when the cakes are valued differently ($\lambda \neq 1$), the subgame perfection of the solution of the RBM can be re-obtained for $\delta \leq \lambda/(\lambda + 2)$ and $\delta \leq 1/(1 + 2\lambda)$. The first condition is the restrictive one when the first cake is relatively more important to player 2 than to his opponent ($\lambda < 1$) and vice-versa. Moreover, the upper bound for the discount factor can be larger than 1/3, though not larger than 1. Then the results obtained in Propositions 3 and 4 can be re-established under less restrictive conditions depending on the values assumed by λ.

Also the assumption on the switch of roles between proposer and responder after an acceptance is not crucial. For instance, a similar result could be obtained if players' roles at the second stage were random, as long as the discount factor is sufficiently small and the probability that player 2 proposes at the second stage is sufficiently large (if this is called p, then what is required is that $p > (1 + \delta)/2(1 - \delta)$ with

$\delta < 1/3$).

When two equilibria, namely, the Rubinsteinian and the partition $(\frac{1}{2}, \frac{1}{2})$, co-exist, the latter is Pareto-superior. However, these multiplicity is not very interesting since it can only be sustained for a specific value of the discount factor ($\delta = 1/3$).

3.3 Almost Equality or Nothing and Modifications

The case presented in the section 3.2 is an extreme example of bargaining in which the parties, on the one hand tend to negotiate on one issue, but on the other are strongly constrained in terms of the division that allows bargaining continuation. In this section, the analysis is generalised to the case in which parties can reach the second stage (with probability one) if the partition agreed is close to $(\frac{1}{2}, \frac{1}{2})$. In this generalised case, it is found that players still focus on the first stage if they are sufficiently impatient, however, when they are patient, they cannot always obtain the largest share which ensures the second stage. This is possible in equilibrium only if the cost of rejection is not extremely low (Proposition 5). To simplify, in this section we assume that players have the same valuation of the size of the cakes ($\lambda = 1$).

Finally, in this section the probability of continuation is assumed to be equal to 1 for an interval of x_1, wlog, say, $[a \pm r]$ with $a \in [0, 1]$, $0 \leq r < \frac{1}{2}$, and null for the other values of x_1 (a generalisation of the previous section consists in setting $a = 1/2$). For this case, we find that similar result can be established, depending on the identity of the 'powerful' player.

We now turn to discuss the case in which players can enjoy the division of two cakes if at the first stage they agree on a partition close to fifty-fifty ($a = 1/2$). This assumption implies that if the RBM is robust to this specification of the probability of continuation, then it must hold under stronger conditions. As before, the intuition is that for small δ, the gain player 1 obtains by the division of the second cake is outweighed by the reduced demand on the first cake which allows the continuation game. This is shown formally in the following Proposition. Moreover, we also prove that a proposer can obtain the highest share allowing the continuation game (i.e., $a + r$) if the discount factor is not extremely high. Only when the cost of rejection is very low, will a proposer make some concessions to ensure that the second stage is played with certainty (and his demand is smaller than $a + r$).

Proposition 5 *In this game there are 3 kinds of SPE, which are:*

1) If players are sufficiently patient (i.e., $\delta \geq \delta_1 = \frac{3-2r}{3+2r}$), the SPE strategies are the following. At the first stage players demand $x_1 = y_2 = \frac{2-\delta}{1+\delta}$ and accept any share not smaller than $\delta(x_1 + \frac{\delta}{(1+\delta)})$. The second stage is played as in the RBM. This SPE is unique for $\delta > \delta_1$.

2) If players are sufficiently impatient ($\delta \leq \delta_2 = \frac{1-2r}{3+2r}$), the SPE strategies are to demand $x_1 = y_2 = \frac{1}{1+\delta}$ and accept any share not smaller than $\frac{\delta}{1+\delta}$. Then, the second stage cannot be reached. This SPE is unique for $\delta < \delta_2$.

3) In all the other cases ($\delta_2 \leq \delta \leq \delta_1$) players' SPE strategies are to demand $x_1 = y_2 = r + 1/2$ and accept any share not smaller than $\delta(r + \frac{1}{2} + \frac{\delta}{1+\delta})$ unless equal to $1/2 - r$. The second stage is played as in the RBM. This SPE is unique for $\delta_2 < \delta < \delta_1$.

Proof. In Appendix. ■

In conclusion, players divide either one cake, if they are sufficiently impatient, or two cakes if they are sufficiently patient. When the probability of game continuation is maximised only for the division $(\frac{1}{2}, \frac{1}{2})$ (i.e., $r \to 0$), the results stated in Propositions 3 and 4 are re-established. The new equilibrium which arises in this section (the first kind in Proposition 5) is characterised by the division of two cakes in which a proposer makes some concessions to the opponent at the first stage to reach the second stage. This equilibrium is defined by the indifference conditions and is sustained by large discount factors.

A proposer is able to obtain the largest share at the first stage which allows the continuation game. This is true only when the discount factor assumes intermediate values. Indeed, if the cost of rejection is high ($\delta < \delta_2$), a deviating demand in which a player asks for more would be accepted in spite of the fact that only one cake would be shared.

To conclude, in anticipation of the following section (negotiations with powerful players), the game is now extended to include the case in which the second stage will take place with certainty, when at the first stage a player, say 1, obtains a share x_1 in $[a-r, a+r]$, with $a \in [0,1]$ but different from 1/2 and $r < min(a, 1-a)$. Then, if a is close to 0 or 1, this example represents a bargaining process in which a player is powerful, in the sense that when a party enjoys a profitable agreement on the first issue, negotiations at the second stage are more likely to take place. Suppose, for instance, that player i, with $i = 1, 2$ represents a country that suffered long periods of conflicts. Then, if in the first agreement country i obtains a positive outcome, then it will be very willing to pursue the continuation of the negotiation process and the second stage is reached with a high probability. The results are summarised in the following Corollary.

Corollary 6 *If* $\frac{2-a-r}{1+a+r} \leq \delta \leq \frac{2-a+r}{1+a-r}$, *in equilibrium two cakes are divided and the proposer will make some concessions at the first stage (SPE of the first kind defined in Proposition 5). For smaller values of the discount factor* ($\frac{1-a-r}{1+a+r} < \delta < \frac{2-a-r}{1+a+r}$) *a proposer asks for the largest share which maximises the continuation game,* $a + r$ *(SPE of the third kind). Finally, for* $\delta \leq \frac{1-a-r}{1+a+r}$, *the game is played as the RBM.*

Then, for the same mechanism explained in the previous section, the solution to the RBM is robust to this modification when the discount factor is sufficiently small ($\delta \leq \frac{1-a-r}{1+a+r}$ implies that player 1's demand is larger than any share that allows the continuation game, $a + r \leq \frac{1}{1+\delta}$). Moreover, when player 2 is sufficiently powerful, $a + r < 1/2$, the equilibrium of the third type, in which player 1 extracts the largest share which allows the continuation game, can be sustainable for large δ.

Finally, the equilibrium in which the proposer makes some concessions at the first stage and the second stage is played with certainty (SPE of the first kind) is sustainable for large values of the discount factor δ only if the interval $[a-r, a+r)$ includes the partition $(\frac{1}{2}, \frac{1}{2})$ (that is the lower bound $\frac{2-a-r}{1+a+r}$ is smaller than 1 while the upper bound $\frac{2-a+r}{1+a-r}$ is not smaller than 1). In terms of our example, this case implies that even when player 1 is the powerful party (i.e., $[a-r, a+r] = [1/2, 1]$) and parties are sufficiently patient, the peace process can continue if the powerful party, makes some concessions in the initial stage. The investigation of the peace process continues under a different specification of the continuation probability in section 4.

4 Negotiations with Powerful Players

In this section the focus is on negotiations with one or two powerful players. In the first case, the probability of continuation is a positive monotonic linear function of the share a specific player obtains. Such a player is powerful since he can pursue the continuation of the bargaining game. We then investigate two cases in which the final bargaining stage is ensured by extreme divisions at the initial stage. The first case is characterised by a probability of continuation which is equal to one when a player gets the entire cake and zero otherwise. In the second case, the probability of game continuation is symmetric to the case studied in section 2 (that is, decreases linearly from 1 to $\frac{1}{2}$ if x_1 increases from 0 to 1/2, then increases to 1 for $x_1 = 1$). Then, if a non-extreme partition is implemented, there is a positive probability of game continuation (unless the division is $(\frac{1}{2}, \frac{1}{2})$), which increases if the partition agreed becomes more and more extreme.

The main result is that when there is only one powerful player, if he is sufficiently powerful he is able to extract all the surplus at the first stage, otherwise he leaves a positive share to his rival. However, when both players are powerful (and sufficiently patient) they share two cakes with certainty in equilibrium. In other words, multiple SPE arise (Proposition 11 and Proposition 12). Moreover, in this case neither the solution of the RBM nor the indifference conditions can define an SPE for a large δ.

4.1 One Powerful Player

To fix ideas, we continue the peace process example, in particular if in the initial agreement the powerful party, player 1, does not make any concession, then he will pursue the game continuation. However, according to how drastic the concessions from the powerful player are in the initial agreement, the continuation of the peace process is under risk of breakdown with a positive probability.

To represent this case we assume that the probability of continuation is an increasing function of the share that player 1 obtains in the agreement of the first issue.

In particular, it has the following specification:

$$\alpha(x_1) = b + (1-b)x_1 \tag{12}$$

where[2] $b \in [0,1]$. The intercept b indicates the power of player 1. If b tends to zero, player 1 is very powerful, in the sense that only when he gets the whole cake, the second stage will take place with certainty. On the contrary, if b tends to one, then the negotiation in the first stage is not critical, since whatever is the share player 1 gets, the second stage takes place with certainty. For the case of players with the same valuation of the issues the SPE is specified in the following Proposition.

Lemma 7 ***Proposition 8*** ***Proposition 9*** *For $\lambda = 1$ and δ and b sufficiently large (i.e., $\delta \geq \delta_b = \frac{\sqrt{1+16b-8b^2}-1}{b}$, $b > \frac{1}{3}$), the SPE demands are as follows:*

$$x_1 = \frac{1-\delta^2 b^2 + 1 - b^2 + b + \delta}{(2-b)b(1+\delta^2)+2\delta} \tag{13}$$

$$y_2 = \frac{2b + \delta(1-\delta) - \delta^2(1-b)}{(2-b)b(1+\delta^2)+2\delta} \tag{14}$$

Proof. *Player 1 has incentives to demand as much as possible at the first stage, since this not only increases his first stage utility, but it also increases the probability of game continuation. However, for large δ and b, player 2 will always reject the extreme division (1,0), even though he is a proposer at the second stage with certainty. When*

[2] Similar results can be re-established when player 2 is the powerful player (that is, $\alpha = b(1-x_1)$), although, given the structure of the game, the results cannot be perfectly symmetric.

δ and b are sufficiently large ($\delta \geq \delta_b = \frac{\sqrt{1+16b-8b^2}-1}{b}$, $b > \frac{1}{3}$), the demands in (13) and (14) are the interior solution of the system of indifference conditions. ■

Then, when b tends to 1, the equilibrium demands defined in (13) and (14) are like the ones defined in the literature on agenda (see for instance, Flamini, 2001, Proposition 1, for $\alpha \to 1$). Indeed, in this case the second stage is always played as long as a first agreement is reached. When δ and b are not sufficiently high, the SPE demands are extreme. For instance, when player 1 has the maximum bargaining power ($b = 0$) and players are sufficiently patient, it is straightforward to show the following result.

Claim 10 *When $b = 0$, players' SPE strategies are as follows. Player 1 demands the entire first cake ($x_1 = 1$) and rejects any demand either larger than y_2 defined in (15) or that leads to a payoff smaller than $\frac{\delta(1+2\delta)}{1+\delta}$, player 2 asks for y_2, such as to make player 1 indifferent between accepting and rejecting his offer. That is*

$$y_2 = \frac{2(1-\delta^2)}{2+\delta} \tag{15}$$

Note that y_2 tends to 0 for $\delta \to 1$. In terms of the peace process example, this result has a strong implication: there should be no concession by the powerful player on the initial issue. However, this conclusion is based on the assumption that the initial issue is perceived as important as the second in the negotiation process. If the ratio λ is allowed to differ form one, then the powerful player should make some concessions on the initial issue. For instance, if b is very small ($b = 0$), and players are infinitely patient ($\delta \rightarrow 1$), it can be shown that the indifference conditions establish an equilibrium in which the proposer does not get the whole first cake, if the second cake is sufficiently more important to him ($\lambda < \sqrt{2/3} = 0.8$).

4.2 Any Division Compromises the Second Stage

To conclude this paper we examine extreme cases of powerful players. To simplify, players value the two issues equally ($\lambda = 1$). First, we assume that the probability of continuation is 1 only for the extreme divisions $(0, 1)$, $(1, 0)$ and null otherwise. We then consider the case where it is linear and symmetric to the case considered in section 3.1, that is,

$$\alpha(x_1) = \begin{cases} 1 - 2x_1 \text{ if } x_1 \leq 1/2 \\ 2x_1 - 1 \text{ if } x_1 > \frac{1}{2}. \end{cases} \tag{16}$$

For the former, the most relevant result is that the solution to the RBM cannot be an equilibrium in this game. When players are sufficiently patient only unequal divisions are implemented as shown in the following Proposition.

Proposition 11 *For $\delta \geq 1/2$, at the first stage, only the two extreme partitions $(0, 1)$ and $(1, 0)$ are subgame perfect.*

Proof. The SPE strategies which sustain the partition (0,1) are the following: player 1 asks for 0 and accepts any share either equal to zero or not smaller than $\delta^2/(1 + \delta)$, while player 2 asks for 1 and accepts any share either equal to one or not smaller than $\delta(1+2\delta)/(1+\delta)$. The condition that $\delta \geq 1/2$ ensures that player 2 refuses to concede all the first cake to his opponent $(1/(1+\delta) \leq \delta(1+2\delta)/(1+\delta))$. To check that these strategies are subgame perfect is straightforward and therefore omitted. The SPE strategies sustaining (1,0) are like the ones defined above, but with players 1 and 2 reversed. If there are other SPE, then it must be that the second stage is not reached. Therefore, only the solution of the RBM can be the sustained as an SPE at the first stage. However, this does not hold, since a responder prefers to accept a deviating demand equal to 1, that is to say, $\delta + 1/(1+\delta) > \delta/(1+\delta)$. Therefore, no other SPE can be sustained. ■

For this extreme case the Rubinsteinian result is never sustainable, since for a responder it is never subgame perfect to reject a deviating demand equal to 1. Moreover, the SPE defined above is sustainable also when the continuation probability is almost always non-null, in particular, $\alpha(x_1)$ is as defined in (16) and players become infinitely patient $(\delta \to 1)$.To show this, we first derive the SPE for players sufficiently patient.

Proposition 12 *For $\alpha(x_1)$ as defined in (16) and $\delta \geq 1/2$, there are two SPE characterised by the following strategies: player i asks for the entire first cake and accepts any demand not larger than x, where $x = 2(1-\delta^2)/(3+\delta)$, while player j demands x and accepts any demand in [0,1], for any $i, j = 1, 2$ and $i \neq j$.*

Proof. in Appendix. ■

This implies that when players becomes infinitely patient ($\delta \to 1$) the extreme divisions $(0,1)$ and $(1,0)$ are implemented in equilibrium (as $x = \frac{2(1-\delta^2)}{3+\delta}$ tends to zero). This is the same result established for the extreme form of two powerful players considered before (Proposition 11).

Despite the fact that the case considered in Proposition 12 is symmetric to the one defined in Proposition 1, the technique used is very different. In particular, in the former, the indifference conditions cannot define an SPE, since a proposer can ask for more at the first stage and make the responder better off (as the probability of game continuation is higher). In conclusion, the way in which the first agreement defines the probability of game continuation strongly affects the results of the negotiation process. When both players are powerful, multiple SPE exist in which the beneficial future negotiations take place with certainty.

5 Conclusion

As far as we know, our analysis represents the first attempt to investigate bargaining processes in which the first agreement affects the future negotiations. Our model has highlighted interesting features of bargaining processes in which initial agreements affect the probability of future negotiations. First of all, players can strategically compromise future beneficial bargaining in equilibrium, if they are not both 'powerful'. This result holds for small or large values of players' discount factors, depending on the specification of the probability of continuation. Moreover, the division of one cake à la Rubinstein is an SPE for a sufficiently small discount factor. Finally, multiple SPE can be sustained. Either the Rubinsteinian equilibrium co-exists with equilibria in which the two cakes are shared (for specific values of the discount factor δ) or different two-stage equilibria can co-exist (for a sufficiently high δ).

Our framework can be interpreted at least in two other ways. First, as pointed out in the introduction the probability of game continuation α can play the role of a between-cake discount factor. Then, for instance, in section 3, when the initial agreement is equal, the interval of time between the two bargaining stages is much shorter than in the case of an unequal initial agreement. Another interpretation for our framework is related to the literature on third parties. Suppose that there is a third party who is able to give additional resources to the bargaining players who reached an initial agreement. Then an *egalitarian* third party would give more resources when the first agreement is 'fair' (section 3), while a *partisan* third party would give resources mainly when there is a powerful party.

Future research could consider the cases where the relevance of the second issue can affect the probability of game continuation after the first agreement. Our model offers a framework within which these issues can be investigated further. Future research could also attempt to solve the problem in which players can have a different 'willingness' to bargain after the first agreement has been reached. If this is private information, the resulting game is an interesting signalling problem on the first cake.

Appendix

Proof of Proposition 1. The proof is divided in two parts. First, the indifference conditions are solved, then the SPE demands are defined.

(i) *The indifference conditions.* If each player attempts to make his rival indifferent between accepting and rejecting his proposal, then such a proposal is defined by the solution of the following system:

$$\begin{cases} 1-x_1+\frac{\lambda\left(\left\{\begin{array}{c} 2x_1 \text{ if } x_1\leq 1/2 \\ 2(1-x_1) \text{ if } x_1>1/2 \end{array}\right.\right)}{1+\delta} = \delta\left(y_2+\frac{\delta\lambda\left(\left\{\begin{array}{c} 2y_2 \text{ if } y_2\leq 1/2 \\ 2(1-y_2) \text{ if } y_2>1/2 \end{array}\right.\right)}{1+\delta}\right) \\ \lambda(1-y_2)+\frac{\left\{\begin{array}{c} 2y_2 \text{ if } y_2\leq 1/2 \\ 2(1-y_2) \text{ if } y_2>1/2 \end{array}\right.}{1+\delta} = \delta\left(\lambda x_1+\frac{\delta\left\{\begin{array}{c} 2x_1 \text{ if } x_1\leq 1/2 \\ 2(1-x_1) \text{ if } x_1>1/2 \end{array}\right.}{1+\delta}\right) \end{cases} \tag{17}$$

System (2) includes 4 sets of indifference conditions, according to whether or not the demand of a player is larger or smaller than a half. Then in principle system (2) may lead to 4 solutions: (a) both demands are larger than a half $({}_a x_1 > 1/2, {}_a y_2 > 1/2)$;

(b) both demands are not larger than half (${}_b x_1 \leq 1/2$, ${}_b y_2 \leq 1/2$); (c) player 1's demand is not larger than a half while player 2's is larger than a half (${}_c x_1 \leq 1/2$, ${}_c y_2 > 1/2$); finally (d) player 2's demand is not larger than a half while player 1's is larger than a half (${}_d x_1 > 1/2$, ${}_d y_2 \leq 1/2$).

The solution of system (2) when both the demands are larger than a half (case a) is the following:

$$
\begin{aligned}
{}_a x_1 &= \frac{2\delta^3 - 4\lambda\delta^3 + 3\lambda\delta^2 - 4\lambda\delta - 2\delta + 5\lambda + 2\lambda^2 + 2}{2\lambda^2\delta^3 + 2\delta^3 - 5\lambda\delta^3 + 3\lambda\delta^2 - 3\lambda\delta + 2\lambda^2 + 5\lambda + 2} > 1/2 \qquad (18) \\
{}_a y_2 &= \frac{-4\lambda\delta^3 + 2\lambda^2\delta^3 + 3\lambda\delta^2 - 4\lambda\delta - 2\lambda^2\delta + 2\lambda^2 + 5\lambda + 2}{2\lambda^2\delta^3 + 2\delta^3 - 5\lambda\delta^3 + 3\lambda\delta^2 - 3\lambda\delta + 2\lambda^2 + 5\lambda + 2} > 1/2 \qquad (19)
\end{aligned}
$$

with either $\delta \leq \underline{\delta} = (\sqrt{5} - 1)/2$ or $\delta < \underline{\delta} < 1$ and $\lambda_l < \lambda < \lambda_u$ where

$$
\begin{aligned}
\lambda_u &= \frac{5 + 3\delta^2 + \sqrt{\Delta}}{4(\delta^2 + \delta - 1)} \qquad (20) \\
\lambda_l &= \frac{-(5 + 3\delta^2) + \sqrt{\Delta}}{4(\delta^2 + \delta - 1)} \qquad (21)
\end{aligned}
$$

where $\Delta = 9 + 46\delta^2 + 25\delta^4 + 32\delta^3$.

The constraints on the parameters ensure that the demands are larger than a half. In particular, when δ is sufficiently large ($\delta > \underline{\delta}$), the constraint $\lambda > \lambda_l$ implies that the demand ${}_a y_2$ is larger than a half, while $\lambda < \lambda_u$ ensures that the demand ${}_a x_1$ is larger than a half. For δ small ($\delta \leq \underline{\delta}$) these constraints are not required, as the demands are always larger than a half.

When players have the same valuation of the issues ($\lambda = 1$), the demands in (18) and (19) are equal to $(3 - 2\delta)/(3 - \delta)$, which is always in the interval $[1/2, 1]$.

Moreover, if players are infinitely patient ($\delta \to 1$), these tend to a half.

For the case in which the demands are not larger than a half (case b), there is no solution. In particular, system (2) leads to the following expressions

$$
\begin{aligned}
{}_b x_1 &= \frac{\lambda\delta^2 + 2\lambda^2\delta^2 - \lambda + 2}{5\lambda\delta^3 + 2\lambda^2\delta^3 + 2\delta^3 - 3\lambda\delta^2 + 3\lambda\delta + 2 - 5\lambda + 2\lambda^2} \qquad (22)\\
{}_b y_2 &= \frac{\lambda\delta^2 + 2\delta^2 + 2\lambda^2 - \lambda}{5\lambda\delta^3 + 2\lambda^2\delta^3 + 2\delta^3 - 3\lambda\delta^2 + 3\lambda\delta + 2 - 5\lambda + 2\lambda^2} \qquad (23)
\end{aligned}
$$

However, the requirement that both the demands are simultaneously not larger than 1/2 is never satisfied (for $\delta < 1$).

In case (c), that is player 1 demands for a share not larger than a half (${}_c x_1 \leq 1/2$) and player 2 for share larger than a half (${}_c y_2 > 1/2$), the solution of the indifference conditions is defined below and it holds for $\lambda \leq \lambda_l$ and $\delta \geq \underline{\delta}$.

$$
\begin{aligned}
{}_c x_1 &= \frac{\lambda\delta + 2 + \lambda}{2\delta^2 - 3\lambda\delta^2 - 2\lambda^2\delta^3 + 2\lambda\delta + 2\delta - 2\lambda^2\delta + 2 - 3\lambda - 2\lambda^2} \leq 1/2 \qquad (24)\\
{}_c y_2 &= \frac{2 - 2\lambda^2\delta^2 - 4\lambda\delta^2 + 2\delta + \lambda\delta - 2\lambda^2\delta - 2\lambda^2 - 3\lambda}{2\delta^2 - 3\lambda\delta^2 - 2\lambda^2\delta^3 + 2\lambda\delta + 2\delta - 2\lambda^2\delta + 2 - 3\lambda - 2\lambda^2} > 1/2 \qquad (25)
\end{aligned}
$$

As for the previous case the constraints on the parameters ensure that the demands are consistent with the assumed level of the probability of game continuation. For players infinitely patient ($\delta \to 1$), the demands become as follows:

$$
\begin{aligned}
{}_c x_1 &= \frac{(\lambda + 1)}{3 - 2\lambda - 3\lambda^2} \qquad (26)\\
{}_c y_2 &= \frac{2 - 3\lambda - 3\lambda^2}{3 - 2\lambda - 3\lambda^2} \qquad (27)
\end{aligned}
$$

for $\lambda < \frac{\sqrt{7}-2}{3} = 0.21$. Finally, in case (d), where player 1's demand is larger than a half but player 2's is not (${}_dx_1 > 1/2$ and ${}_dy_2 \leq 1/2$), the demands are:

$$
\begin{aligned}
{}_dx_1 &= -\frac{4\lambda\delta^2 + 2\delta^2 - \lambda\delta - 2\lambda^2\delta + 2\delta + 3\lambda - 2\lambda^2 + 2}{-3\lambda\delta^2 + 2\lambda^2\delta^2 - 2\delta^2 + 2\lambda\delta + 2\lambda^2\delta + 2\delta - 3\lambda + 2\lambda^2 - 2} > 1/2 \quad (28) \\
{}_dy_2 &= \frac{\lambda(\delta + 2\lambda + 1)}{-3\lambda\delta^2 + 2\lambda^2\delta^2 - 2\delta^2 + 2\lambda\delta + 2\lambda^2\delta + 2\delta - 3\lambda + 2\lambda^2 - 2} < 1/2 \quad (29)
\end{aligned}
$$

with $\lambda > \lambda_u$ and $\delta \geq \underline{\delta}$. At the limit for $\delta \to 1$, these are as follows:

$$
\begin{aligned}
{}_dx_1 &= \frac{2\lambda^2 - 3\lambda - 3}{-3 - 2\lambda + 3\lambda^2} \quad (30) \\
{}_dy_2 &= \frac{\lambda(1+\lambda)}{-3 - 2\lambda + 3\lambda^2} \quad (31)
\end{aligned}
$$

for $\lambda > \sqrt{7} + 2 = 4.65$.

(ii) *The SPE demands.* In this second part of the proof, we show that all the demands derived by the indifference conditions are subgame perfect, after a refinement of case (a), which is the case where both players ask for a share larger than 1. To see this, we first find a player's best demand regardless of the responder's behaviour. If player 1 could maximise his payoff, regardless of player 2's response, he would demand a large share at the first stage ($x_1 = 1$) if λ is larger than $2\delta/(1+\delta)$, even though this decreases the possibility of future negotiations. However, he would prefer to maximise the probability of game continuation to reach the second stage ($x_1 = 1/2$) when λ is smaller than $2\delta/(1+\delta)$. On the other hand, player 2 would demand half of the first cake ($y_2 = 1/2$) if $\lambda > (1+\delta)/2\delta$, while he would prefer the whole first cake ($y_2 = 1$) if $\lambda < (1+\delta)/2\delta$. To sum up, for $\lambda < 2\delta/(1+\delta)$, player 1's best partition would be $(\frac{1}{2}, \frac{1}{2})$, while player 2 would demand $y_2 = 1$. For $2\delta/(1+\delta) < \lambda < (1+\delta)/2\delta$, both players would demand the whole first cake, while for $\lambda > (1+\delta)/2\delta$, player 2 would prefer the partition $(\frac{1}{2}, \frac{1}{2})$, and 1 would demand the entire first cake.

By taking into account the consideration that a player is willing to obtain only half of the first cake, we can define the SPE demands. In particular, we focus on case (a) above, where the indifference conditions define shares larger than a half. For all the other cases the indifference conditions define the SPE demands.

First we consider the case in which player 2 attempts to obtain only half of the first cake. This holds when player 2 strongly prefers the second cake ($\lambda > (1+\delta)/2\delta$). In this case, when it is player 2's turn to propose, player 2 will asks for half of the first cake (instead of ${}_a y_2$, in (19), larger than a half). This implies that player 1 can ask for a share, ${}_e x_1$, such that player 2 is indifferent between accepting this offer or rejecting it so as to propose a half. This demand is

$$ {}_e x_1 = \frac{2+\delta+4\lambda-\delta^2(1+2\lambda)}{2(1+\delta+2\lambda)} \tag{32} $$

which is always in the interval [1/2, 1]. At the limit as δ goes to 1, ${}_e x_1$ tends to a half. Player 2's demand, ${}_e y_2 = 1/2$, is accepted by player 1, if what player 1 obtains in the case of acceptance is not smaller than he would get in the case of rejection (that is to say $\frac{\lambda}{2} + \frac{1}{1+\delta} > \delta(\lambda_e x_1 + 2(1 - {}_e x_1)\frac{\delta}{1+\delta})$. This implies that the ratio representing the relative importance of the issues, λ, has to be sufficiently small ($\lambda < \lambda_u$ where λ_u is defined in (20)). To conclude, infinitely patient players ask for a share equal to a half if $1 < \lambda < \lambda_u$.

We finally consider the case in which player 1 prefers a share equal to a half rather than something larger than this (i.e., ${}_a x_1$) at the first stage. This holds when player 1 strongly prefers cake 2 to 1 ($\lambda < 2\delta/(1+\delta)$). In this case, player 2 can ask for a share ${}_f y_2$ such that player 1 is indifferent between accepting this offer and rejecting it so as to propose the division $(\frac{1}{2}, \frac{1}{2})$. This implies that

$$ {}_f y_2 = \frac{4 + 2\lambda + \delta\lambda - \delta^2(\lambda - 2)}{2(\lambda(1+\delta) + 2)} \tag{33} $$

which is always in $[1/2, 1]$. Moreover, for infinitely patient players, it tends to a half. Player 2 accepts the demand ${}_f x_1 = 1/2$ rather than rejecting it to propose ${}_f y_2$, if $\lambda > \lambda_l$.

To sum up, the indifference conditions define the SPE demands except for the cases in which $\lambda_l < \lambda < 2\delta/(1+\delta)$, where the demands are ${}_f x_1 = 1/2$ and ${}_f y_2$ as defined in (33), and $(1+\delta)/2\delta < \lambda < \lambda_u$, where the demands are ${}_e x_1$, as in (32) and ${}_e y_2 = 1/2$ (see Figure 1 for a representation of the equilibria). Then, at the first stage the SPE responses are to accept these demands, or any demand that implies an equilibrium payoff at least equal to the one obtained by rejecting and demanding the shares defined above. It is straightforward to prove that there are no profitable deviations to the strategies stated above ■

Proof of Proposition 5. The proof consists of three parts in each of which the subgame perfection of the three strategies stated in the proposition is shown. The proof of the uniqueness of the three SPE strategies is omitted since it follows the usual arguments.

The SPE demands described in the first part of the proposition are found by imposing that a responder is indifferent between accepting and rejecting his rival's proposal. The demands belong to the interval which allows the continuation game if and only if δ is sufficiently large ($\delta \geq \delta_1$). It is then straightforward to show that there are no profitable deviations to the strategies described in the first part.

For the second part, the Rubinsteinian division at the first stage is an SPE only if a single cake is divided. To see this, suppose it is not true, then if a proposer asks for a larger share on the first stage ($\frac{1}{1+\delta}+\varepsilon$), which still allows the continuation game, such a deviation would be accepted by the responder (since $\frac{\delta}{1+\delta} - \varepsilon + \frac{1}{1+\delta} > \delta(\frac{1}{1+\delta} + \frac{\delta}{1+\delta})$. Moreover, when $\frac{1}{1+\delta} = \frac{1}{2} + r$, the second cake is divided. In this case, $\delta_r = \frac{1-2r}{1+2r}$ and this belongs to the range $\delta \geq \delta_1$ (first part of the proof). Next, we show that when $x_1 = \frac{1}{1+\delta}$ does not belong to the interval $[\frac{1}{2} - r, \frac{1}{2} + r]$ (i.e., $\frac{1}{1+\delta} > \frac{1}{2} + r$ $\Leftrightarrow \delta < \delta_r$) then the RBM is robust. We check only the most crucial deviation, that is a proposer demands a share that allows the continuation game, the other deviations can be proven in a more straightforward way. Suppose that the proposer asks a share smaller than x1. According to the SPE strategies the responder must accept this proposal. If the deviating proposal, $x_1 - \varepsilon$, allows the division of two cakes, then the deviation ε cannot be smaller than $\frac{1-\delta}{2(1+\delta)} - r$. The proposer's payoff is not larger than the payoff he could obtain by demanding x_1 if and only if $\varepsilon \geq \frac{\delta}{1+\delta}$. In other words, there are profitable deviations whenever $\varepsilon < \frac{\delta}{1+\delta}$. However, these deviations cannot take place if the right hand side of this inequality is not larger than $\frac{1-\delta}{2(1+\delta)} - r$, i.e., $\delta \leq \delta_2$. Finally, since for $\delta \leq \delta_2$, the Rubinsteinian payoff is higher than the maximum payoff an initial proposer can obtain by the division of two cakes $(\frac{1}{2} + \frac{\delta}{1+\delta} + r)$, then the proof of the uniqueness of the SPE follows standard arguments.

For part three of the proposition the proof follows a similar but more complicated logic than the one in Proposition 3. Let's consider the responder's behaviour first. Suppose that player 1 asked for $\frac{1}{2} + r + \varepsilon$ with $\varepsilon > 0$. Then player 2 rejects if and only if $1 - (r + \frac{1}{2} + \varepsilon) \leq \delta(r + \frac{1}{2} + \frac{\delta}{1+\delta})$. That is to say, $\varepsilon \geq \underline{\varepsilon} = \frac{1-3\delta^2}{2(1+\delta)} - r(1+\delta)$. This is always satisfied for $\delta \geq \delta_3 = \frac{-2r+\sqrt{3-4r}}{3+2r}$ with $\delta_2 < \delta_3 < \delta_1$. When the discount factor belongs to the interval $[\delta_2, \delta_3)$, player 2 accepts a deviating offer. However, this is unprofitable to his rival if $r + \frac{1}{2} + \varepsilon < r + \frac{1}{2} + \frac{\delta}{1+\delta}$, that is, if $\varepsilon < \frac{\delta}{1+\delta}$. Then the acceptance of a deviating demand $r + \frac{1}{2} + \varepsilon$ is always unprofitable to player 1 if $\underline{\varepsilon} < \frac{\delta}{1+\delta}$, which implies $\delta \geq \delta_2$. As before, whenever there is a rejection, player 1 is worse off since $\delta(\frac{1}{2} + \frac{1}{1+\delta})) \leq \frac{1}{2} + \frac{\delta}{1+\delta}$.

On the other hand, when player 1 asks for $r + \frac{1}{2} - \varepsilon$, we distinguish two cases. In the first one, the deviating demand still allows the division of two cakes $(r(1+\delta)-(1-\delta)3/2 \leq 2r)$, then player 2 accepts this proposal if $1-(r+\frac{1}{2}-\varepsilon)+\frac{1}{1+\delta} > \delta(\frac{1}{2}+r+\frac{\delta}{1+\delta}))$. That is to say, $r(1+\delta)-(1-\delta)3/2 < \varepsilon$ (which is consistent with $\varepsilon \leq 2r$). This implies that for any $0 < \varepsilon \leq 2r$ there is an acceptance if $r(1+\delta) - (1-\delta)3/2 \leq 0 \Leftrightarrow \delta \leq \delta_1$. In the second case, player 1's deviating demand does not allow the continuation game ($\varepsilon > 2r$). Then, player 2 rejects it if $1 - (\frac{1}{2} + r + \varepsilon) \leq \delta(\frac{1}{2} + r + \frac{\delta}{1+\delta})$, which implies $\varepsilon \leq \frac{(3+2r)\delta^2 + 4r\delta - (1+2r)}{2(1+\delta)} = \widetilde{\varepsilon}$. This condition is consistent with $\varepsilon > 2r$, if and only if $\delta > \delta_4 = \sqrt{\frac{1+2r}{3+2r}}$ with $\delta_4 < \delta_1$. When there is no consistency, $\delta_2 \leq \delta \leq \delta_4$, player 2 always accepts a deviating offer in which player 1 gets $r + \frac{1}{2} - \varepsilon$ (with $\varepsilon > 2r$) and player 1 is clearly worse off. When there is consistency, $\delta_4 < \delta \leq \delta_1$, player 2 rejects the deviating demand $r + \frac{1}{2} - \varepsilon$ with $2r < \varepsilon \leq \widetilde{\varepsilon}$, while he accepts it, if $\varepsilon > \widetilde{\varepsilon}$. In both cases, player 1 is worse off, therefore he is better off in proposing $r + \frac{1}{2}$. ■

Proof of Proposition 12. The demand x specified in Proposition 12 is smaller than a half for players who are sufficiently patient ($\delta > 1/2$). Moreover, it is such that the responder is indifferent between accepting the demand x and rejecting it to propose 1 (i.e., $1 - x + \frac{1-2x}{1+\delta} = \delta(1 + \frac{\delta}{1+\delta})$. Finally, player j accepts zero rather than reject and ask for x (that is, $\frac{1}{1+\delta} > \delta(x + \frac{(1-2x)\delta}{1+\delta})$.

To check that the strategies defined above are subgame perfect, we consider the case in which a player, say j, deviates and asks for a share larger than x, $(x+\varepsilon)$. We omit the other possible deviations that players can implement, because in these cases it is simpler to show that these are not profitable. Suppose that player j asks for $x+\varepsilon$. The deviating demand can be either not larger than a half ($x + \varepsilon \leq \frac{1}{2}$) or larger than a half ($x + \varepsilon > \frac{1}{2}$). In the former ($x + \varepsilon \leq \frac{1}{2}$), player i is better off in rejecting this demand since he not only gets a smaller share at the fist stage, but the probability of game continuation is also smaller, since $x+\varepsilon \leq \frac{1}{2}$. To prove this formally, we need to show that his payoff in the case of acceptance $(1-(x+\varepsilon)+\frac{1-2(x+\varepsilon)}{1+\delta})$ is smaller than his payoff in the case of rejection $(\delta(1+\frac{\delta}{1+\delta})$ which is also equal to $1-x+\frac{(1-2x)}{1+\delta})$. This always holds since ε is a positive number. In this case, player j is worse off, as his payoff in the next period, $\delta(1+\frac{1}{1+\delta})$, is smaller than what he could have obtained by demanding x (i.e., $x+\frac{(1-2x)\delta}{1+\delta}=\frac{2+\delta-\delta^2+2\delta^3}{(1+\delta)(3+\delta)}$).

In the second case, which is characterised by a deviating demand $x+\varepsilon$ in $[\frac{1}{2},1]$, player i accepts this demand, if the smaller share he obtains at the first stage is outweighed by a larger expected utility at the second stage. However, we can show that this never holds. Player i is better off in rejecting the deviating demand, in other words, $(1-(x+\varepsilon)+\frac{1-2(x+\varepsilon)}{1+\delta} \leq \delta(1+\frac{\delta}{1+\delta})$. This inequality can be written as $x+\varepsilon \leq \frac{2\delta^2}{1+\delta}$. But this always holds since the right hand side is larger than 1 (for $\delta > \frac{1}{2}$), while $x+\varepsilon$ is always not larger than one. Then the strategies defined in the proposition are subgame perfect.

Note that in this game, the indifference conditions between accepting and rejecting an offer cannot define SPE demands. These are $y=\frac{\delta}{3\delta-1} \geq \frac{1}{2}$ for $\delta \geq \frac{1}{2}$. However, it can be shown that a proposer is better off in deviating from the demand defined by the indifference conditions. In particular, a proposer can ask for the entire first cake and this is acceptable. That is the responder obtains more by accepting rather rejecting this offer, since with probability one he will be a proposer at the second stage (i.e., $\frac{1}{1+\delta} > \delta(y + \frac{2y-1}{1+\delta})$. In conclusion, the (symmetric) demands defined by the indifference conditions cannot be subgame perfect, since the probability of game continuation increases when the agreement becomes closer to an extreme partition. ■

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