Sticky Prices, Limited Participation, or Both?*

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Abstract

This paper investigates the micro mechanisms by which monetary policy affects and is transmitted through the U.S economy, by developing a unified, dynamic, stochastic, general equilibrium model that nests two classes of models. The first sticky prices and the second limited participation. Limited participation is incorporated by assuming that households' are faced with quadratic portfolio adjustment costs. Monetary policy is characterized by a generalized Taylor rule with interest rate smoothing. The model is calibrated and investigates whether the unified model performs better in replicating empirical stylized facts, than the models that have only sticky price or limited participation. The unified model replicates the second moments of the data better than the other two types of models. It also improves on the ability of the sticky price model to deliver the hump-shaped response of output and inflation. Moreover, it also delivers on the ability of the limited participation model to replicate the fall in profits and wages, after a contractionary monetary policy.

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1. Introduction

Following is the consensus empirical evidence of the effect of an unanticipated contractionary monetary shock in the U.S.: There is a delayed, time-reverting, hump-shaped response of output, with the peak effect occurring after approximately 6 quarters; a hump-shaped response of inflation, with a peak response after approximately 8 quarters; a decrease in profits and real wages[1]; and an immediate decrease in the growth rate of money. With respect to second moments, U.S. data after 1979:3 document a positive correlation between output and lags of output, money growth, inflation, and the interest rate. All the above correlations are negative before 1979:2.

It is an important and still open research question, what are the micro mechanisms by which monetary policy affects, and is transmitted through the economy. During the past few years, two competing classes of models have been used to analyze the monetary transmission mechanism, namely sticky price and limited participation models. The former class is based on the Keynesian idea of nominal price rigidity. The latter emphasizes the role of barriers to flows of funds across financial and goods markets, in other words the inability of agents to costlessly engage in financial transactions in all periods. However until this paper they were not both incorporated in a single model capable of investigating their relative importance and whether in conjunction they can explain more of the empirical regularities.

Previous work by Christiano et. al. (1997), and King and Watson (1996), compared the performance of sticky-price and limited participation models, and tended to view these two classes of models as competing. King and Watson base their comparison on second moments, and find that the sticky price model generates less business cycle variability than in the data, specifically, too little variation in the real interest rate. In the limited participation model, variability is almost entirely due to real rather than monetary shocks and too much variability is present in the real interest rate. In terms of covariability, only the sticky price model reproduces the leading indicator behavior of prices, and only the limited participation model generates a negative relationship between money and future values of output. Both models generate positive covariance of prices and lagged output. Christiano et. al. find that although the sticky-price model fails to reproduce the decrease in profits after a contractionary monetary policy, the limited participation model is able to do so, only with high labor supply elasticity and high average mark-up. Lastly, Christiano et. al. argue that in order to be able

[1] For example, see Christiano et. al. (1997).

to reproduce important empirical characteristics of the data, one should incorporate in those models a source of real rigidities in the labor market.

Therefore, the natural research strategy after King and Watson (1997) and Christiano, Eichenbaum and Evans (1997) is to combine both frictions in a unified model. Dynamic, stochastic, general equilibrium (DSGE) models that allow for only one type of friction cannot explain very well how the economy responds to an unanticipated monetary policy shock. Hendry and Zhang (2001) and Keen (2001) investigate the gains from incorporating both frictions. Hendry and Zhang (2001) develop a DSGE model for Canada, in which limited participation is characterized with a time cost of portfolio adjustment, in the utility function. Sticky prices are introduced with a price-adjustment cost in the intermediate goods market and a wage adjustment cost in the household's decision problem. Henry and Zhang show that portfolio adjustment costs induce persistent deviations of inflation and output from the steady state. On the other hand, price-adjustment costs are less effective, having only a minimal effect on inflation and output. Keen (2001) develops a DSGE model similar to Dow (1995). Limited-participation is introduced as a time cost of portfolio adjustment. Sticky prices are introduced as in Levin (1991) wage contract rules. The model can match the empirical responses of real and nominal variables to an expansionary monetary shock (the rise in output, the larger relative increase in investment than consumption, the gradual increase in the price level and the decline in the nominal interest rate) without imposing any implausible assumptions (e.g. large capital adjustment costs). Although both models are able to get better results by incorporating both frictions, it should be stated that none of them is able to reproduce the hump-shaped response of output to a monetary shock.

This paper brings these two lines of research together. It develops a unified, coherent, dynamic, stochastic, general, equilibrium model that nests these two structural specifications, and investigates what additional insights into understanding the business cycle can be gained, by allowing both frictions to coexist. In particular, it asks whether the unified (U) model performs better than the models that have only one friction, the sticky-price-only model (SP) and the limited-participation-only model (LP). Sticky prices are incorporated by assuming that monopolistically competitive firms face a quadratic cost of nominal price adjustment, following Rotemberg (1982). Limited participation is incorporated by assuming that households face a quadratic cost of portfolio adjustment, following Cooley and Quadrini (1999). The unified model is shown to perform better than either the sticky-price-only or the limited-participation-only versions of the model on empirical grounds. It does so both in terms of matching second moments of the data and the shape of empirical impulse responses. Only the combined model can deliver the hump-shaped response of output and inflation and the decrease in profits and wages after a contractionary monetary policy, something that previous models that incorporate both frictions are unable to do so.

The model developed in this paper, differs from those in the literature in that it allows for endogenous interest rate setting and considers a different specification of the portfolio adjustment cost. In order to describe the monetary policy of the Federal Reserve, an interest rate rule that generalizes Taylor's (1993) specification is used, allowing the interest rate to respond to its lags as well as to output and inflation. Concerning the portfolio adjustment cost, a quadratic adjustment cost is introduced in the utility function of the representative household rather than in its budget constraint like Cooley and Quadrini (1999). The representative agent faces utility costs of portfolio adjustments, generated by factors such as the time required to obtain information about new opportunities in financial markets or to contact the stock broker to arrange transactions. Both of these additional features turn out to be important in helping the model explain key features of the data.

The model is used to analyze the effect of four individual types of shocks: policy, technology, preference, and money demand shocks. To gain intuition on the functioning of the model, impulse responses generated by its three possible versions are compared: the sticky-price-only, the limited-participation-only and the unified frameworks. Only the unified version of the model embodies strong internal propagation mechanisms in order to deliver the shape of empirical impulse responses: the hump-shaped response of output and inflation and the decrease in wages and profits after a monetary policy shock. Christiano, Eichenbaum and Evans (2001) show that a model with habit persistence in preferences for consumption, adjustment costs in investment and variable utilization accounts for the delayed, hump-shaped response of output and inflation after a monetary policy shock. The model developed in this paper shows that time-separable preferences can deliver this pattern with the proper specification of the portfolio adjustment. The introduction of the portfolio adjustment cost introduces consumption rigidities. Since in this setup, portfolio adjustment costs is introduced in the utility function, the model mimics habit persistence in preferences for consumption. In addition, the unified model can generate the 0.4-0.6 percent deviation of output from the steady state after a 1 percent policy shock, whereas the sticky-price-only model produces an excessively large output response. This is a common limitation of sticky price models. The limited-participation-only model produces a decrease in profits and real wages after a contractionary monetary policy shock, a result that is also inherited in the unified framework.

Although impulse response analysis provides an intuitive way to analyze the monetary transmission mechanism, empirical studies have shown that the unsystematic portion of policy-instrument variability is quantitatively small in relation to the variability of the systematic component. Therefore, it is more informative to analyze both the systematic and unsystematic component of monetary policy by studying second moments. Accordingly, model-generated vector autocorrelations are compared with those of U.S. data. The most important finding of this paper is the ability of the unified framework to replicate most of the second moments in the data.

The remainder of this paper is organized as follows. Section 2, below, sets up the model. Section 3 describes the data and the parameterization of the model. Section 4 discusses the results and the importance of the degree of the portfolio adjustment cost. Section 5 summarizes and concludes.

2. The Model

2.1. Overview

The DSGE model combines elements of existing sticky price and limited participation specifications. Sticky prices are incorporated, following Rotemberg (1982), by assuming that monopolistically competitive firms face a quadratic cost of nominal price adjustment. Limited participation is incorporated, following Cooley and Quadrini (1999), by assuming that households face a quadratic cost of portfolio adjustment. These costs of price and portfolio adjustments permit the monetary authority to influence the behavior of real variables in the short run.

There are five types of agents in the economy: a representative household, a representative finished goods-producing firm, a representative bank, a continuum of intermediate goods-producing firms indexed by $i \in [0, 1]$, and a monetary authority. Time periods are indexed by $t = 0, 1, 2, ...$ The behavior of each agent is described in the subsections below.

2.2. The Representative Household

The representative household enters period t with M_{t-1} units of money and K_t units of capital. Immediately following the realization of the period-t shocks, the household must decide how to divide its funds into an amount D_t to be deposited in the representative bank and an amount $M_{t-1} - D_t$ to be used to facilitate goods purchases.

When choosing D_t, the household faces a quadratic portfolio adjustment cost, measured in terms of units of utility, and given by

$$\tau_t = \frac{\phi_d}{2}\left(\frac{D_t}{\mu D_{t-1}} - 1\right)^2, \tag{1}$$

where $\phi_d \geq 0$ governs the magnitude of the adjustment cost and where, as noted below, $\mu \geq 1$ denotes the gross steady-state rate of money growth. In the steady state, since D_t grows at the money growth rate, the adjustment cost is equal to zero. Differently from Cooley and Quadrini (1999), the quadratic cost of adjustment in this specification is introduced in the utility function of the household rather than in its budget constraint, since it is assumed that the agent faces utility costs of portfolio adjustments. These costs can be generated by factors such as the time required to obtain information about new opportunities in financial markets or to contact the stock broker to arrange transactions.

During period t, the household supplies $h_t(i)$ units of labor at the nominal wage W_t and $K_t(i)$ units of capital at the nominal rental rate Q_t to each intermediate goods-producing firm $i \in [0,1]$. The household's choices must satisfy

$$h_t = \int_0^1 h_t(i)di,$$

where h_t denotes total hours worked, and

$$K_t = \int_0^1 K_t(i)di$$

for all $t = 0, 1, 2, ...$

During period t, the household purchases output from the representative finished goods-producing firm at the nominal price P_t, and then divides its purchases into an amount C_t to be consumed and an amount I_t to be invested. Since it is assumed that the household receives its wages before making its goods purchases, it faces the cash-in-advance constraint

$$\frac{M_{t-1} - D_t + W_t h_t}{P_t} \geq v_t(C_t + I_t) \tag{2}$$

for all $t = 0, 1, 2, ...$ In (2), v_t is a random term that measures the amount of money the household must carry to facilitate its purchases of goods. It is basically a shock to the inverse of the quarterly rate of the income velocity of M2, and it is assumed to follow the autoregressive process

$$\ln(v_t) = (1-\rho_v)\ln(v) + \rho_v \ln(v_{t-1}) + \varepsilon_{vt}, \tag{3}$$

where $v > 0$, $1 > \rho_v > 0$, and the serially uncorrelated innovation ε_{vt} is normally distributed with mean zero and standard deviation σ_v.

By investing I_t units of the finished good during each period t, the household increases the capital stock over time according to

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\phi_k}{2}\left(\frac{K_{t+1}}{gK_t} - 1\right)^2 K_t, \tag{4}$$

where $1 > \delta > 0$ is the depreciation rate, the parameter $\phi_k \geq 0$ governs the magnitude of capital adjustment costs, and g is the gross steady-state growth rate of the capital stock.

At the end of period t, the household receives its rental payments Q_tK_t along with principal plus interest $r_t^d D_t$ from the bank; hence, r_t^d measures the gross interest rate on deposits. The household also receives nominal profits B_t from the representative bank and $F_t(i)$ from each intermediate goods-producing firm $i \in [0,1]$, for a total of $B_t + F_t$ in nominal profits, where

$$F_t = \int_0^1 F_t(i)di.$$

The household then carries M_t units of money into period $t+1$; it faces the budget constraint

$$\frac{M_{t-1} + (r_t^d - 1)D_t + W_t h_t + Q_t K_t + B_t + F_t}{P_t} \geq C_t + I_t + \frac{M_t}{P_t} \tag{5}$$

during each period $t = 0, 1, 2, ...$

The household, therefore, chooses C_t, h_t, τ_t, D_t, M_t, I_t, and K_{t+1} for all $t = 0, 1, 2, ...$ to maximize its expected utility function, given by

$$E_0 \sum_{t=0}^{\infty} \beta^t a_t [\ln(C_t) - \gamma(h_t + \tau_t)], \tag{6}$$

subject to the constraints imposed by (1), (2), (4), and (5) for all $t = 0, 1, 2, ...$ In the utility function, $1 > \beta > 0$, and $\gamma > 0$. The preference shock a_t follows the autoregressive process

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \tag{7}$$

where $1 > \rho_a > 0$, and the serially uncorrelated innovation ε_{at} is normally distributed with mean zero and standard deviation σ_a. It resembles a shock to the IS curve in more traditional Keynesian analysis.

Let Λ_{1t} denote the Lagrange multiplier on the budget constraint (5) and let Λ_{2t} denote the Lagrange multiplier on the cash-in-advance constraint (2). Then the household's first-order conditions include (1), (2), (4), and (5) with equality, along with

$$a_t = (\Lambda_{1t} + v_t \Lambda_{2t}) C_t, \tag{8}$$

$$\gamma a_t = (\Lambda_{1t} + \Lambda_{2t})(W_t/P_t), \tag{9}$$

$$\frac{\Lambda_{1t}}{P_t} = \beta E_t \left(\frac{\Lambda_{1t+1} + \Lambda_{2t+1}}{P_{t+1}} \right), \tag{10}$$

$$\begin{aligned} & \gamma \phi_d a_t \left(\frac{D_t}{\mu D_{t-1}} - 1 \right) \frac{D_t}{\mu D_{t-1}} \\ = \quad & \frac{[\Lambda_{1t}(r_t^d - 1) - \Lambda_{2t}] D_t}{P_t} + \beta \gamma \phi_d E_t \left[a_{t+1} \left(\frac{D_{t+1}}{\mu D_t} - 1 \right) \frac{D_{t+1}}{\mu D_t} \right], \end{aligned} \tag{11}$$

and

$$\begin{aligned} & (\Lambda_{1t} + v_t \Lambda_{2t}) \left[1 + \frac{\phi_k}{g} \left(\frac{K_{t+1}}{g K_t} - 1 \right) \right] \\ = \quad & \beta E_t \left[\Lambda_{1t+1}(Q_{t+1}/P_{t+1}) + (\Lambda_{1t+1} + v_{t+1} \Lambda_{2t+1})(1 - \delta) \right] \\ & + \beta \phi_k E_t \left\{ (\Lambda_{1t+1} + v_{t+1} \Lambda_{2t+1}) \left[\left(\frac{K_{t+2}}{g K_{t+1}} - 1 \right) \left(\frac{K_{t+2}}{g K_{t+1}} \right) - \frac{1}{2} \left(\frac{K_{t+2}}{g K_{t+1}} - 1 \right)^2 \right] \right\} \end{aligned} \tag{12}$$

for all $t = 0, 1, 2, \ldots.$

2.3. The Representative Finished Goods-Producing Firm

The representative finished goods-producing firm uses $Y_t(i)$ units of each intermediate good $i \in [0, 1]$, to produce Y_t units of the finished good according to the constant returns to scale technology described by

$$\left[\int_0^1 Y_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \geq Y_t, \tag{13}$$

with $\theta > 1$. Intermediate good i sells at the nominal price $P_t(i)$, while the finished good sells at the nominal price P_t. Given these prices, the finished goods-producing firm chooses Y_t and $Y_t(i)$ for all $i \in [0, 1]$ to maximize its profits,

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di, \tag{14}$$

for each $t = 0, 1, 2, \ldots.$

The first-order conditions for this problem imply that

$$Y_t(i) = [P_t(i)/P_t]^{-\theta} Y_t \tag{15}$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, ...$, which reveals that $-\theta$ measures the price elasticity of demand for intermediate good i. Competition in the market for the finished good requires that the representative firm earn zero profits in equilibrium. This zero-profit condition determines P_t as

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{1/(1-\theta)} \tag{16}$$

for all $t = 0, 1, 2, ...$

2.4. The Representative Bank

At the beginning of period t, the representative bank accepts deposits D_t from the representative household. It also receives a lump-sum nominal transfer X_t from the monetary authority. Thus, it can lend $L_t(i)$ to each intermediate goods-producing firm $i \in [0, 1]$, subject to the constraint

$$D_t + X_t \geq L_t, \tag{17}$$

where

$$L_t = \int_0^1 L_t(i) di.$$

At the end of period t, the bank collects $r_t L_t(i)$ in principal and interest from each intermediate goods-producing firm $i \in [0, 1]$; hence, r_t denotes the gross nominal interest rate on loans. Since the bank owes $r_t^d D_t$ to its depositors, its profits are given by

$$B_t = r_t L_t + D_t + X_t - L_t - r_t^d D_t. \tag{18}$$

Competition among banks for loans and deposits guarantees that

$$r_t = r_t^d \tag{19}$$

for all $t = 0, 1, 2, ...$ So long as the net nominal interest rate $r_t - 1$ is positive, the bank will lend all of its funds and (17) will hold with equality.

2.5. The Representative Intermediate Goods-Producing Firm

The representative intermediate goods-producing firm hires $h_t(i)$ units of labor and $K_t(i)$ units of capital from the representative household during period t in order to produce $Y_t(i)$ units of intermediate good i according to the constant returns to scale technology described by

$$K_t(i)^{\alpha}[g^t z_t h_t(i)]^{1-\alpha} \geq Y_t(i), \tag{20}$$

where $1 > \alpha > 0$ and where $g \geq 1$ denotes the gross rate of labor-augmenting technological progress. The aggregate technology shock z_t follows the autoregressive process

$$\ln(z_t) = (1 - \rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt}, \tag{21}$$

where $z > 0$, $1 > \rho_z > 0$, and the serially uncorrelated innovation ε_{zt} is normally distributed with mean zero and standard deviation σ_z.

The firm rents capital on credit, but must pay its wage bill with funds $L_t(i)$ borrowed from the representative bank. Therefore it faces the finance constraint

$$L_t(i) \geq W_t h_t(i) \tag{22}$$

for all $t = 0, 1, 2, ...$ Since these funds are borrowed at the gross rate r_t, the firm must repay principal plus interest $r_t L_t(i)$ at the end of the period.

Since intermediate goods substitute imperfectly for one another as inputs in the production of the finished good, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market. Thus, during each period t, the representative intermediate goods-producing firm sets a nominal price $P_t(i)$ for its output, subject to the requirement that it satisfy the representative finished goods-producing firm's demand, taking P_t and Y_t as given.

In addition, the intermediate goods-producing firm faces a quadratic cost of adjusting its nominal price, measured in terms of the finished good and given by

$$\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 Y_t, \tag{23}$$

where $\phi_p \geq 0$ governs the magnitude of the adjustment cost and $\pi \geq 1$ denotes the gross steady-state rate of inflation and is measured in terms of the finished good and increases proportionally with the size of Y_t of the overall economy. Following Rotemberg (1982), this cost captures the negative effect of price changes, which increase in magnitude with the size of the price change and the total output of the economy Y_t.

These costs of price adjustment make the intermediate goods-producing firm's problem dynamic. It chooses $h_t(i)$, $K_t(i)$, $Y_t(i)$, $L_t(i)$, and $P_t(i)$ for all $t = 0, 1, 2, ...$ to maximize its total market value, equal to

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_{1t}[F_t(i)/P_t], \tag{24}$$

subject to the constraints imposed by its production possibilities, by the finance constraint (22), and by the demand curve

$$Y_t(i) = [P_t(i)/P_t]^{-\theta} Y_t$$

for all $t = 0, 1, 2, ...$

In (24), Λ_{1t} is the Lagrange multiplier on the budget constraint (5) from the representative household's problem, so that $\beta^t \Lambda_{1t}/P_t$ represents the marginal utility of an additional dollar of profits during period t for the representative household and

$$\begin{aligned} F_t(i) &= P_t(i)Y_t(i) + [L_t(i) - W_t h_t(i)] - Q_t K_t(i) \\ &\quad - r_t L_t(i) - \frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 P_t Y_t. \end{aligned}$$

measures the firm's nominal profits during period t.

When the net nominal interest rate $r_t - 1$ is positive, the finance constraint (22) holds with equality. In this case, the firm's problem simplifies to one of choosing $h_t(i)$, $K_t(i)$, and $P_t(i)$ to maximize its total market value, where

$$\frac{F_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t}\right]^{1-\theta} Y_t - \frac{Q_t K_t(i) + r_t W_t h_t(i)}{P_t} - \frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 Y_t, \tag{25}$$

subject to the single constraint

$$K_t(i)^\alpha [g^t z_t h_t(i)]^{1-\alpha} \geq [P_t(i)/P_t]^{-\theta} Y_t \tag{26}$$

for all $t = 0, 1, 2, ...$ The first-order conditions for this problem are (26) with equality,

$$\Lambda_{1t} r_t (W_t/P_t) h_t(i) = (1-\alpha)\Xi_t K_t(i)^\alpha [g^t z_t h_t(i)]^{1-\alpha}, \tag{27}$$

$$\Lambda_{1t}(Q_t/P_t)K_t(i) = \alpha \Xi_t K_t(i)^\alpha [g^t z_t h_t(i)]^{1-\alpha}, \tag{28}$$

and

$$\begin{aligned} 0 &= (1-\theta)\Lambda_{1t}\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\left(\frac{Y_t}{P_t}\right) + \theta \Xi_t \left[\frac{P_t(i)}{P_t}\right]^{-\theta-1}\left(\frac{Y_t}{P_t}\right) \\ &\quad - \phi_p \Lambda_{1t}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]\left[\frac{Y_t}{\pi P_{t-1}(i)}\right] \\ &\quad + \beta \phi_p E_t \left\{\Lambda_{1t+1}\left[\frac{P_{t+1}(i)}{\pi P_t(i)} - 1\right]\left[\frac{P_{t+1}(i) Y_{t+1}}{\pi P_t(i)^2}\right]\right\} \end{aligned} \tag{29}$$

for all $t = 0, 1, 2, ...$, where Ξ_t is the Lagrange multiplier on (26). The log-linearized version of (29), yields the New Keynesian forward looking Phillips Curve.

2.6. The Monetary Authority

The monetary authority conducts monetary policy by gradually adjusting the short-term nominal interest rate r_t in response to deviations of detrended inflation $\pi_t = P_t/P_{t-1}$ and output $y_t = Y_t/g^t$ and from their steady-state values π and y, according to the policy rule

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + \rho_\pi \ln(\pi_t/\pi) + \rho_y \ln(y_t/y) + \varepsilon_{rt}, \tag{30}$$

where r, π, and y are the steady-state values of r_t, π_t and y_t, respectively. In (30), the parameters ρ_r, ρ_π, and ρ_y are positive. The parameter ρ_r captures the degree of interest rate changes smoothing, and ρ_π and ρ_y the degree of the interest rate reactions to inflation and output deviations from their steady state values, respectively. The case in which $\rho_\pi/(1 - \rho_r)$ is greater than one is consistent with the Fed's policy to stabilize inflation. The same stands for output if $\rho_y/(1 - \rho_r)$ is greater than zero. The serially uncorrelated innovation ε_{rt} is normally distributed with mean zero and standard deviation σ_r.

The above policy rule resembles the one put forth by Taylor (1993) to describe Federal Reserve's behavior from 1987 through 1992, but it generalizes Taylor's specification by allowing policy to respond to the lagged interest rate as well as output and inflation. As discussed in Clarida, Gali and Gertler (1997), the Federal Reserve has a tendency to smooth changes in interest rates. Therefore, each period it adjusts the Funds rate to a linear combination of its past values.

2.7. Symmetric Equilibrium

In a symmetric equilibrium, all intermediate goods-producing firms make identical decisions, so that $h_t(i) = h_t$, $K_t(i) = K_t$, $F_t(i) = F_t$, $Y_t(i) = Y_t$, $P_t(i) = P_t$, and $L_t(i) = L_t$ for all $i \in [0, 1]$ and $t = 0, 1, 2, ...$ In addition, the market-clearing condition $M_t = M_{t-1} + X_t$ must hold for all $t = 0, 1, 2, ...$ These equilibrium conditions, together with the first-order conditions for the representative agents' problems, the laws of motion for the aggregate shocks, and the policy rule, form a system of difference equations describing the model's equilibrium. In the absence of shocks, the economy converges to a steady state. The system is log-linearized around its steady state, and Klein's method (2000) is applied to obtain a solution of the form

$$\mathrm{f}_t = Us_t \tag{31}$$

and

$$s_t = \Pi s_{t-1} + W\varepsilon_t \tag{32}$$

for all $t = 0, 1, 2, ...$

In (31) and (32), f_t is the vector of the model's flow variables which includes output $y_t = Y_t/g^t$, inflation π_t, the current values of the bank deposits $d_t = D_t/M_t$, money growth μ_t, consumption $c_t = C_t/g^t$, investments $i_t = I_t/g^t$, the multipliers $\lambda_{1t} = g^t\Lambda_{1t}$, $\lambda_{2t} = g^t\Lambda_{2t}$, and $\xi_t = g^t\Xi_t$, the real factor prices $w_t = (W_t/P_t)/g^t$, and $q_t = Q_t/P_t$, banks profits $b_t = B_t/M_t$, bank loans $l_t = L_t/M_t$, hours worked h_t, and real profits $f_t = (F_t/P_t)/g^t$. s_t is the vector that includes the model's endogenous state variables and the model's four shocks. The model's endogenous state variables are the lagged values of real balances $m_{t-1} = (M_{t-1}/P_{t-1})/g^{t-1}$, the lagged values of the bank deposits $d_{t-1} = D_{t-1}/M_{t-1}$, the lagged interest rate r_{t-1}, and the current values of the capital stock k_t. The four shocks in the model are the money demand shock v_t, the preference shock a_t, the technology shock z_t and the policy shock ε_{rt}. The vector ε_t includes the four innovations ε_{vt}, ε_{at}, ε_{zt}, and ε_{rt} and is assumed to be normally distributed with zero mean and covariance matrix

$$V = E\varepsilon_t\varepsilon_t' = \begin{bmatrix} \sigma_v^2 & 0 & 0 & 0 \\ 0 & \sigma_a^2 & 0 & 0 \\ 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & \sigma_r^2 \end{bmatrix}$$

The parameters that describe private agents' tastes, technologies and the policy rule determine the elements of the matrices Π, W, and U.

3. Data, Parameterization

3.1. Data

In the data, output is measured by real GDP in chained 1996 dollars, while money growth by changes in the M2 money stock. Inflation is measured by changes in the GDP implicit price deflator, and the interest rate by the three-month Treasury bill rate. All series, except for the interest rate, are seasonally adjusted. In addition, the series for output is expressed in per-capita terms by dividing by the civilian noninstitutional population age 16 and above.

Distinct upward trends appear in the resulting data for per capita output, consumption and investments. In the model though it is assumed that these variables fluctuate around a deterministic trend. Therefore, a linear trend is removed from the logarithm of each one. In addition, in the model the variables fluctuate about their steady-state values in response to shocks, therefore the mean is removed from the logarithm of all data as well.

The data are quarterly and run from 1959:1 through 2001:1, and are divided into two subsamples, the first covering the period 1959:1 through 1979:2, and the second covering the period from 1979:3 through 2001:1. The breakpoint corresponds to the widely believed change in the US monetary policy, that occurred at the beginning of Paul Volker's tenure as Chairman of the Federal Reserve System. Prior the 1980s, monetary policy in the US was highly accommodative. After a decrease in the real interest rates while anticipated inflation rose, the Federal Reserve Bank used to increase the nominal interest rate but usually less than the increase in expected inflation. After 1980s though, monetary policy is believed to have been more stabilizing and systematic than before. The Federal Reserve Bank systematically raised real as well as nominal interest rates in response to higher expected inflation, adopting a proactive behavior[2].

3.2. Parameterizing the Model

As mentioned above, distinct upward trends appear in the series of output, real balances, consumption, investment, capital stock, real wages and real profits of the intermediate goods producing firm. The model accounts for these trends in the data by including a deterministic term in the production function, that captures the effects of labor-augmenting technological progress for each intermediate good. The model also implies that the variables listed above grow at the same growth rate g along a balanced growth path. The log of the growth rate is estimated with a regression of the log of real per capita GDP, $\ln(Y_t)$, on a constant and a trend. Thus, g is set equal to 1.0047 and 1.005, pre and post 1979:2, respectively. In the model, this yields steady-state growth in real per-capita output of about 1.9 and 2 percent, pre and post 1979:2, respectively

Regressions of the growth rate of the GDP implicit price deflator, $\ln(P_t/P_{t-1})$, on

[2] As Clarida, Gali and Gertler (2000), and Boivin and Giannoni (2002) show, unexpected exogenous interest rate shocks has a reduced effect after 1980s.

a constant provides estimates of $\ln(\pi)$. Steady-state inflation is estimated to be 1.0107 and 1.0085 pre and post 1979:2, respectively, and per quarter. In the model, this values of π implies that the steady-state annual rate of inflatin is about 4.3 percent and 3.4, pre and post 1979:2, respectively. This is an indication that pre 1980s the U.S. economy experienced higher inflation.

The steady-state equilibrium condition of the cash-in-advance constraint (2), implies that the log of the money demand shock is equal to the log of real balances minus the log of consumption and investment. Therefore, the calibrated steady-state value of the money demand shock, v, is equal to the average of the exponent of the log of the money demand shock, which is equal to 3.0917 and 2.6218, pre and post 1979:2 respectively. A regression of the calculated demeaned log of money demand shock on its lag provides the estimates of the serial correlation and the standard deviation. Thus, serial correlations are 0.9989 and 0.9976, and standard deviations are 0.0408 and 0.0979, pre and post 1979:2 respectively.

The calibrated values for the preference and technology shocks are taken from Ireland (1999), who estimates with maximum likelihood a sticky-price model that incorporates similar specifications. The model assumes that the steady-state preference shock a is equal to 1 for both periods. Its serial correlation, ρ_a, set equal to 0.94 and 0.89, pre and post 1979:2 respectively, with a standard deviation equal to 0.03 for both periods. The steady-state value of the technology shock z is set equal to 4000 and 4500, with serial correlation ρ_z equal to 0.92 and 0.96, and standard deviation equal to 0.015 and 0.008 pre and post 1979:2 respectively.

Some structural parameters are set equal to values commonly used in the literature. The weight on hours worked in the representative household's utility function γ is set equal to 1.5793, which implies that the household spends about one third of its time working in the model's steady state. The quarterly depreciation rate, δ, is set equal to 0.025, and capital's share in production α is set equal to 0.36. As Kim (1998) and King and Watson (1996) suggest, large capital adjustment costs are needed in order for sticky-price models to generate sensible responses of output to monetary shocks, therefore, the degree of capital adjustment cost, ϕ_k, is set equal to 40. The parameter that measures the degree of market power possessed by the representative goods-producing firm, θ, is set equal to 6. This value implies a steady-state markup of price over marginal cost equal to Rotemberg and Woodfords's (1992) benchmark of 20%. Lastly, the discount factor β is set equal to 0.995.

To characterize the Federal Reserve's monetary policy rule, calibrated values are taken to be close to values from Clarida, Gali and Gertler (2000) and Christiano and Gust (1999). The former paper estimates interest rate reaction functions for the U.S. economy, pre and post Volker, while the latter discusses what implications the interest rate rules have on the model's uniqueness, explosiveness and indeterminacy. As discussed by Christiano and Gust (1999), in a model that incorporates limited participation in the financial markets and price flexibility, the interest rate response to inflation ρ_π should be high, while the interest rate response to output ρ_y should be low, results that conflict with the empirical analysis by Christiano, Gali and Gertler (2000). In a sticky-price-only model, there is not a lot of intertemporal substitution in consumption since prices are rigid. Therefore, an increase in expected inflation lowers the real interest rate through the Fischer equation, which causes an increase in investments, output and actual inflation. In this case, if the central bank adopts a tight monetary policy with a high interest rate response to output and inflation, the causality from expected inflation to actual inflation stops, so that high expected inflation is not self-fulfilling. If prices are flexible though, but frictions in financial markets are present, higher anticipated inflation increases consumption today versus future consumption and decreases today's deposits to the financial sector. This causes the nominal interest rate to rise. If the interest rate response to inflation, ρ_π, in the interest rate rule is low, the central bank must inject liquidity into financial markets to prevent a large increase in the interest rate, which will cause an increase in inflation. Therefore this increases the probability that high inflation expectations can be self-fulfilling, and creates indeterminacy or explosiveness. Following the same logic, a high interest rate response to output ρ_y, also results in self-fulfilling inflation expectations, since the interest rate from lower deposits in financial markets results in a lower output and that is going to offset the increase in the interest rate caused by a high ρ_π.

Not surprisingly, the widely known interest rate rule popularized by Taylor (1993), where there is no interest rate smoothing in the policy rule (30), meaning that ρ_r is set equal to zero, and where ρ_π and ρ_y are set equal to 1.5 and to 0.5 respectively, produces an indeterminate equilibrium. Therefore, the degree of interest rate smoothing ρ_r is set equal to 0.75 post Volker and 0.7 pre Volker, and the interest rate response to inflation is set equal to 0.45 and 0.5 pre and post Volker respectively. This values are justified by the estimated interest rate rules by Clarida, Gali and Gertler (2000). The interest rate response to output ρ_y is set equal to 0.01 in both subsamples, in order to have determinacy in the model. The σ_r is set equal to 0.007 pre-1979 and 0.004 post-1979. The different policy rules for the two sub-samples justifies the change in the way that monetary policy was conducted after the 1980s.

The model in this paper incorporates both sticky prices and limited participation. Therefore three versions of the model can be analyzed and compared. In the sticky-price-only (SP) version of the model, ϕ_p is greater than zero, while the portfolio adjustment cost ϕ_d is zero. In the limited-participation-only (LP) version of the model ϕ_d is greater than zero, while prices are flexible (ϕ_p is set to zero). And the last one, the unified-version of the model (U), incorporates both adjustment costs, therefore both ϕ_p and ϕ_d are greater than zero. In the real business cycle version of the model, the economy is frictionless in the sense that there are no portfolio or price adjustment costs, meaning both ϕ_d and ϕ_p are equal to zero.

In this model, $(\theta - 1)/\phi_p$ represents the fraction of the discrepancies between the target prices P^*_{t+j} (the nominal price that would prevail in the absence of price adjustment costs) and the actual prices P_{t+j}, that are eliminated per quarter after after a change in the price level at date t[3]. This means that when the degree of market power possessed by the representative goods-producing firm, θ, increases or when the degree of price adjustment costs ϕ_p is lower, price adjustments become more rapid. The value of ϕ_p in the SP and U model is set equal to 70, a value close to the estimated one from Ireland (1999). Since θ is set equal to 6, around 7.1% of the discrepancies are eliminated every quarter.

The value of ϕ_d in the LP and U models is set equal to 0.8 in the case that the model produces impulse responses that match those in the data. It is set equal to 5 for the model to match second moments of the data. The former is called low ϕ_d^L and the latter high ϕ_d^H portfolio adjustment cost.

4. Results

4.1. Low Portfolio Adjustment Costs

Although four shocks are incorporated in the model (policy, technology, preference and money demand shocks), only two of those are going to be analyzed, the policy and the technology shocks. Figures 1 and 2 display impulse responses of output, money growth, inflation, nominal interest rate, real wages and real profits for the SP, LP and U versions of the model, after those two shocks. All impulse responses shown are for the period

[3] See Ireland (2000). Calvo rationalizes this specification with the assumption that only a fraction of firms adjust their prices each period.

after 1980s[4].

Empirically, after an unanticipated contractionary monetary policy, a delayed hump-shaped response of output, with the peak effect occurring after 6 quarters is observed. Christiano, Eichenbaum and Evans (2001) show that a model with habit persistence in preferences for consumption, adjustment costs in investment and variable utilization accounts for the delayed, delivers the hump-shaped response of output and inflation after a monetary policy shock. They conclude that a model with standard, time-separable preferences cannot be consistent with this pattern.

The model developed in this paper shows that time-separable preferences can deliver this pattern with the proper specification of the portfolio adjustment cost. The introduction of the quadratic portfolio adjustment cost in the utility function, brings in the model deposit rigidity. The cash-in-advance constraint links consumption and deposits, and therefore this rigidity is transmitted to consumption. So, this model mimics habit persistence in preferences for consumption.

The strong internal propagation mechanisms that this setup embodies, is able to deliver the shape of empirical impulse responses, only if it includes both sticky prices, and limited participation. The most interesting finding concerning impulse responses is the fact that when portfolio adjustment costs are low, the response of output is hump-shaped, in the unified model. In addition, the unified model can generate the 0.4-0.6 percent deviation of output from the steady state after a 1 percent policy shock, whereas the sticky-price-only model produces an excessively large output response, a common finding in pure sticky price models. Therefore the portfolio adjustment cost is a necessary but not a sufficient condition for the hump-shaped response of output and inflation to the policy shock.

Another feature of the data that got a lot of attention by Christiano, Eichenbaum and Evans (1997) is the negative response of wages and profits to a contractionary monetary policy. Although sticky price models cannot generate these features, Christiano, Eichenbaum and Evans (1997), finds that a limited participation model does. This is true also in the framework developed in this paper. The LP model produces this decrease in profits and wages, although the SP model does not. The unified model succeeds in this aspect as well. Therefore this is a result that is also inherited in the unified framework.

[4] Similar results can be obtained for the period before 1980s.

4.2. High Portfolio Adjustment Costs

Although impulse response analysis provides a good way to analyze the functioning of the model and the effect of monetary shocks to the economy, McCallum (1999) argues that exogenous shocks, the unsystematic portion, account for a very small fraction of monetary policy-instrument variability. He continues arguing that this variability is quantitatively small in relation to the variability of the systematic component and suggests that more emphasis should be given to the systematic portion of policy behavior. Therefore, it is more informative to analyze both the systematic and unsystematic component of monetary policy by studying second moments.

Instead of data impulse responses, data vector autocorrelation functions are estimated. A VAR system with four lags is used to estimate the vector autocorrelation functions. Since this analysis doesn't focus on the identification of the unsystematic component of monetary policy, the VAR system doesn't have to be shock identified. The purpose of shock identified VAR models is to identify the unsystematic component of monetary policy, not to generate policy-invariant equation systems, that governs the effect of systematic or anticipated policy actions.

In an attempt to match second moments for longer lags as well, the degree of portfolio adjustment costs should be increased[5]. Although the parameterization with low degree of portfolio adjustment costs is able to reasonably replicate impulse responses and the correlations for short lags, it cannot replicate second moments for longer lags. Figures 3 and 4 display the model-generated correlation functions, with high portfolio adjustment costs equal to 5, together with the U.S. data correlation functions for output, money growth, inflation and interest rates, for both periods, pre and post 1980s, respectively. In addition, table 2, displays a summary of the correlations and standard deviations. As a general result, only the U model can matche very well the second moments of the data, especially for the period pre 1980s.

Concerning the behavior of output after 1979:2, during the business cycle, all models produce higher output volatility than observed in the data. This is a common characteristic of SP models, as discussed in Ellison and Scott (2000).

The correlation of interest rate and lagged output is positive in the data post 1980s, indicating that the interest rate is a positive lagging indicator of output. This is ex-

[5]It has been investigated that the parameterization with low degree of portfolio adjustment costs is able to reasonably replicate impulse responses and correlations for only short lags. Figures not shown in the paper. Available by request.

plained by the behavior of the Federal Reserve, which is more likely to implement tight monetary policy to avoid inflationary pressures in periods of growth. Only the U model is able to capture this stylized fact post 1980s. The SP and LP versions of the model together with the SP and LP models in the King and Watson (1996) show that the interest rate is negatively lagging indicator of output. Before 1979:2 the scenario is not the same as the data produces a negative correlation of interest rate and lagged output. This fact justifies the argument that the U.S. economy was working in a completely different regime prior to the 1980s. All models are able to capture this stylized fact in the data.

Prior to 1979:2 the correlation between different leads and lags of inflation and output is negative in the data and this is true for all models considered for that period. Concerning the period after 1980s, that correlation is positive in the data most of the times, and only the U model is able to capture this stylized fact.

All models produce the negative correlations of money growth and lagged output and of lagged money growth and output before 1979:2. On the other hand, those same correlations are positive after 1979:2. Only the U model gives a positive correlation of output and lagged money growth for a short amount of time.

Empirically, it is observed that after an unanticipated contractionary monetary policy, there is a delayed hump-shaped response in inflation, with a peak response after about 8 quarters. Fuhrer and Moore (1995) argues that in order for a DSGE model to replicate this sluggish response of the inflation rate to the shocks hitting the economy, inflation rigidities should be included in the model. In my specification, the unified model is able to capture this sluggishness in inflation without incorporating inflation rigidities. With high portfolio adjustment costs, inflation response to a contractionary monetary policy is hump-shaped, with the peak occurring after 4 quarters. This is consistent with Ireland (2001) that shows that after estimating a DSGE model with price and inflation rigidities, the US data prefer a model with adjustment costs applying to the price level but not to the inflation rate.

5. Conclusions

This paper brings together two classes of models that during the past few years, have been used to study monetary aspects of the business cycle: the sticky price models, which draw on traditional Keynesian ideas of nominal price rigidity, and the limited participation models, which emphasize instead the role of barriers to flows of funds across financial and goods markets. It develops a unified, coherent dynamic, stochastic, general, equilibrium (DSGE) model that nests these two structural specifications, and investigates what extra insights can be obtained by allowing both frictions to coexist. Sticky prices are incorporated by assuming that monopolistically competitive firms face a quadratic cost of nominal price adjustment, following Rotemberg (1982). Limited participation is incorporated by assuming that households face a quadratic cost of portfolio adjustment, following Cooley and Quadrini (1999). In addition the model allows for endogenous interest rate setting, an interest rate rule that generalizes Taylor's (1993) specification, allowing the interest rate to respond to its lags as well as to output and inflation.

The unified model performs better than either the sticky-price-only and limited-participation-only versions of the model on empirical grounds, both in terms of matching second moments of the data and the shape of empirical impulse responses.

Only the unified version of the model with low portfolio adjustment costs embodies strong internal propagation mechanisms in order to deliver the shape of empirical impulse responses: the hump-shaped response of output and inflation and the decrease in wages and profits after a monetary policy shock. Time-separable preferences can deliver this pattern with the introduction of the portfolio adjustment cost in the utility function, and the model is able to mimic habit persistence in preferences for consumption. This means that the portfolio adjustment cost is a necessary but not a sufficient condition for the hump-shaped response of output. In addition, the unified model can generate the 0.4-0.6 percent deviation of output from the steady state after a 1 percent policy shock, whereas the sticky-price-only model produces an excessively large output response. Also, the limited-participation-only model produces a decrease in profits and real wages after a contractionary monetary policy shock, a result that is also inherited in the unified framework. Moreover, it produces a hump-shaped response of inflation to a monetary contraction with a higher degree of portfolio adjustments.

One of the most important finding of this paper is the ability of the unified framework with high portfolio adjustment cost to replicate most of the second moments in the data, both in the short run and the long run: the positive correlations of output with lagged money growth, lagged inflation and lagged interest rate, lagged output with money growth, inflation and interest rate for the period after 1979:2, and the negative ones for periods before 1979.

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Table 1: Calibration Values

Parameter	Full Sample			Parameter	Pre-1980	Post-1980
β	0.995			g	1.0047	1.005
γ	1.5793			π	1.0107	1.0085
δ	0.025					
α	0.36			v	3.0917	2.6218
θ	6			ρ_v	0.998	0.997
ϕ_k	40			σ_v	0.0408	0.0979
	SP	LM	U	ρ_a	0.94	0.89
ϕ_p	70	0	70	σ_a	0.03	0.03
ϕ_d^L	0	0.8	0.8			
ϕ_d^H	0	5	5	z	4000	4500
				ρ_z	0.92	0.96
				σ_z	0.015	0.008
				ρ_r	0.7	0.75
				ρ_π	0.45	0.5
				ρ_y	0.01	0.01
				σ_r	0.004	0.007

Table 2: Summary of Cross-Correlations with high portfolio adjustment costs ($\text{Cor}x_t, y_{t+k}$), where y_t is output and x_t is the series in column one)								
	1959:1 - 1979:2				1979:3 - 2001:1			
	σ	k			σ	k		
		-4	0	4		-4	0	4
Output								
Data	0.058	0.889	1	0.889	0.021	0.652	1	0.652
SP	0.180	0.945	1	0.945	0.162	0.916	1	0.916
LP	0.185	0.931	1	0.931	0.169	0.785	1	0.785
U	0.181	0.909	1	0.909	0.190	0.686	1	0.686
Money Gr.								
Data	0.009	-0.585	-0.592	-0.389	0.008	0.369	0.297	0.406
SP	0.049	-0.226	-0.165	-0.200	0.102	-0.034	-0.009	-0.049
LP	0.021	-0.720	-0.670	-0.624	0.020	-0.523	-0.388	-0.268
U	0.040	-0.226	-0.152	-0.379	0.090	0.224	0.559	0.079
Inflation								
Data	0.008	-0.271	-0.459	-0.639	0.004	0.531	0.407	0.083
SP	0.015	-0.832	-0.858	-0.879	0.009	-0.647	-0.543	-0.677
LP	0.071	-0.180	-0.342	-0.229	0.159	-0.052	-0.386	-0.139
U	0.057	-0.130	-0.032	-0.279	0.127	0.204	0.555	0.065
Int. Rate								
Data	0.005	0.015	-0.116	-0.419	0.006	0.511	0.518	0.126
SP	0.016	-0.844	-0.923	-0.861	0.011	-0.622	-0.795	-0.643
LP	0.047	-0.358	-0.550	-0.412	0.109	-0.228	-0.620	-0.273
U	0.070	-0.089	-0.147	-0.334	0.195	0.397	0.369	0.031
Wages								
SP	0.075	0.845	0.828	0.749	0.066	0.781	0.807	0.699
LP	0.086	0.826	0.879	0.793	0.094	0.601	0.827	0.557
U	0.102	0.635	0.733	0.491	0.166	0.403	0.810	0.334
Profits								
SP	0.221	0.778	0.774	0.831	0.242	0.642	0.516	0.670
LP	0.185	0.931	1.000	0.931	0.169	0.785	1.000	0.785
U	0.697	0.099	0.028	0.302	1.718	-0.232	-0.483	-0.017

Figure 1: Percent Deviation from the Steady State to a One Percent Policy Shock
Period 1979:3 - 2001:1

Sticky Price Model	**Limited Participation Model**	**Unified Model**
SP [Φp=70, Φd=0]	LP08 [Φp=0, Φd=0.8]	U08 [Φp=70, Φd=0.8]

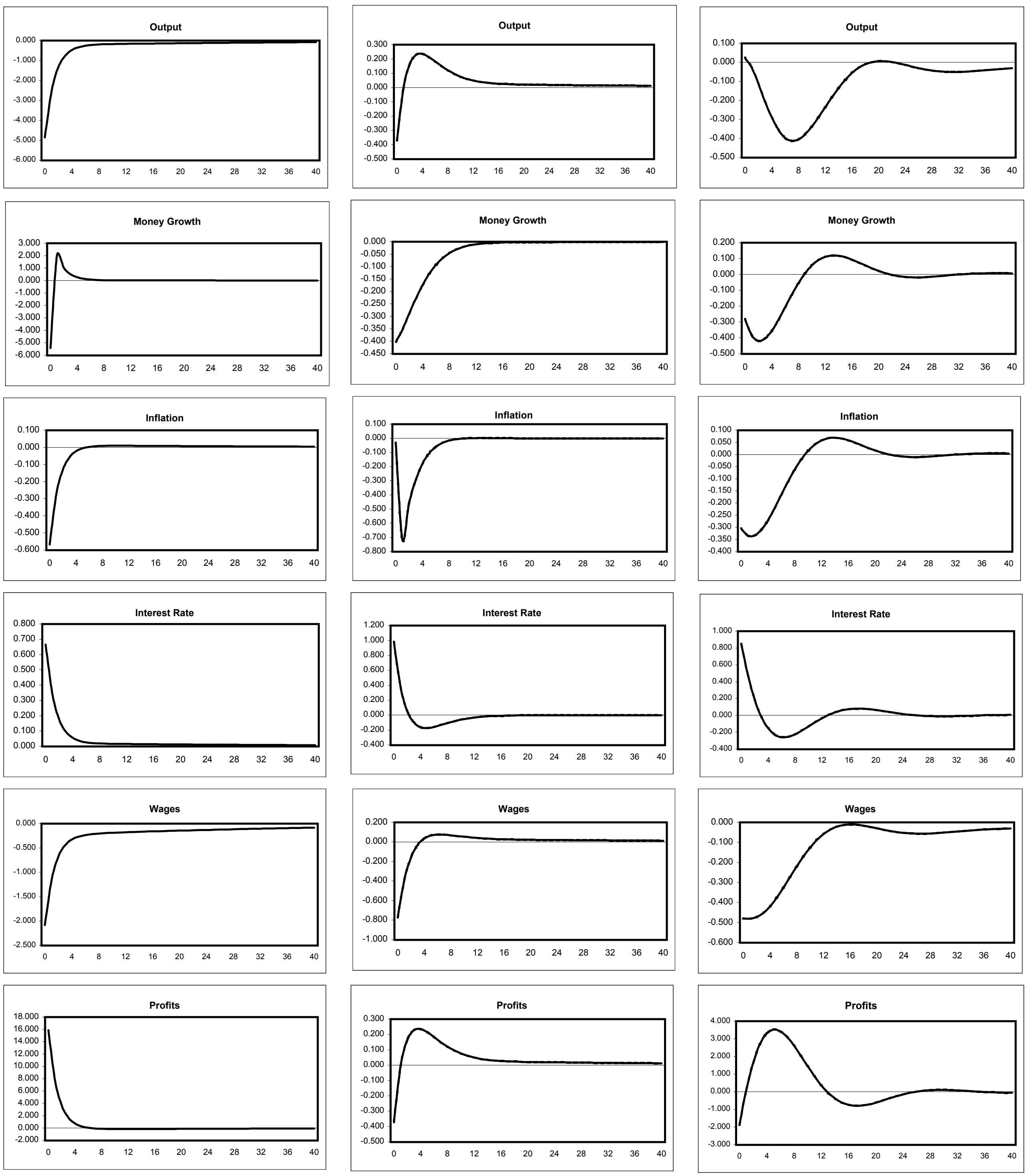


Figure 2: Percent Deviation from the Steady State to a One Percent Technology Shock
Period 1979:3 - 2001:1

Sticky Price Model	**Limited Participation Model**	**Unified Model**
SP [Φp=70, Φd=0]	LP08 [Φp=0, Φd=0.8]	U08 [Φp=70, Φd=0.8]

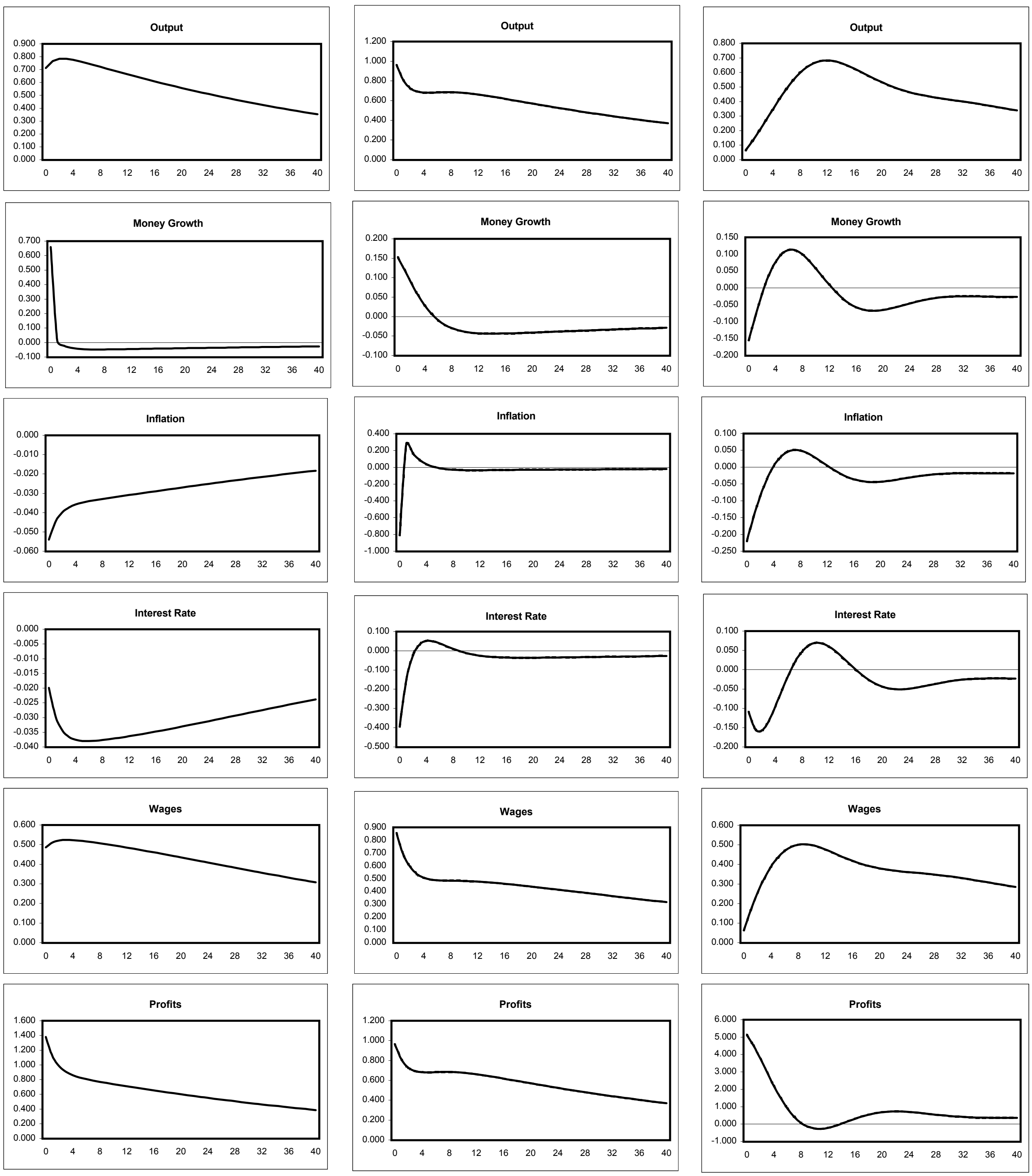


Figure 3: Vector Autocorrelation Functions for the period 1959:1 - 1979:2

Grey Line - Data
Black Line - Unified Model - U5[Φp=70, Φd=5]
Black Dotted Line - Sticky Price Model - SP[Φp=70, Φd=0]
Grey Dotted Line - Limited Participation Model - LP5[Φp=0, Φd=5]

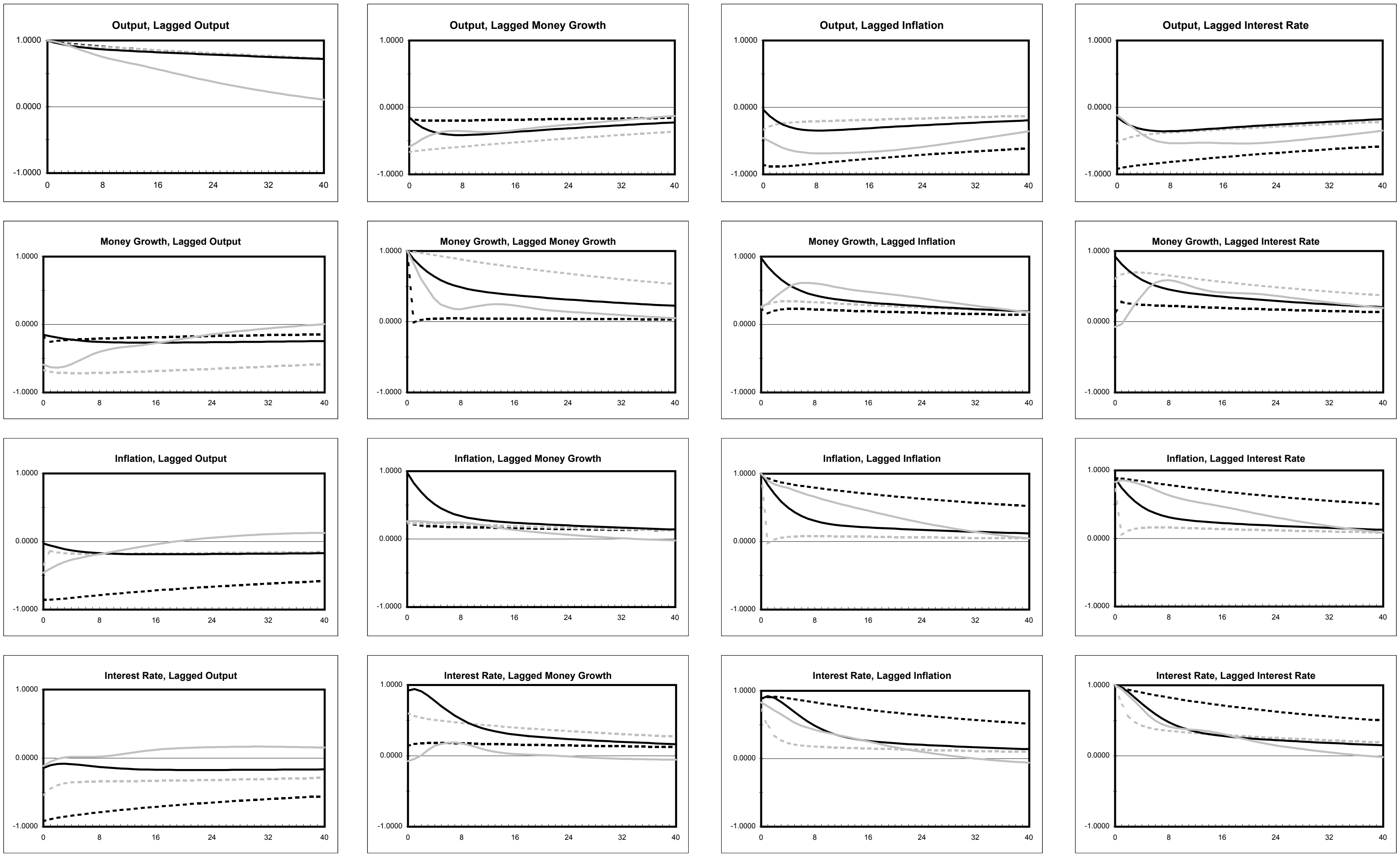
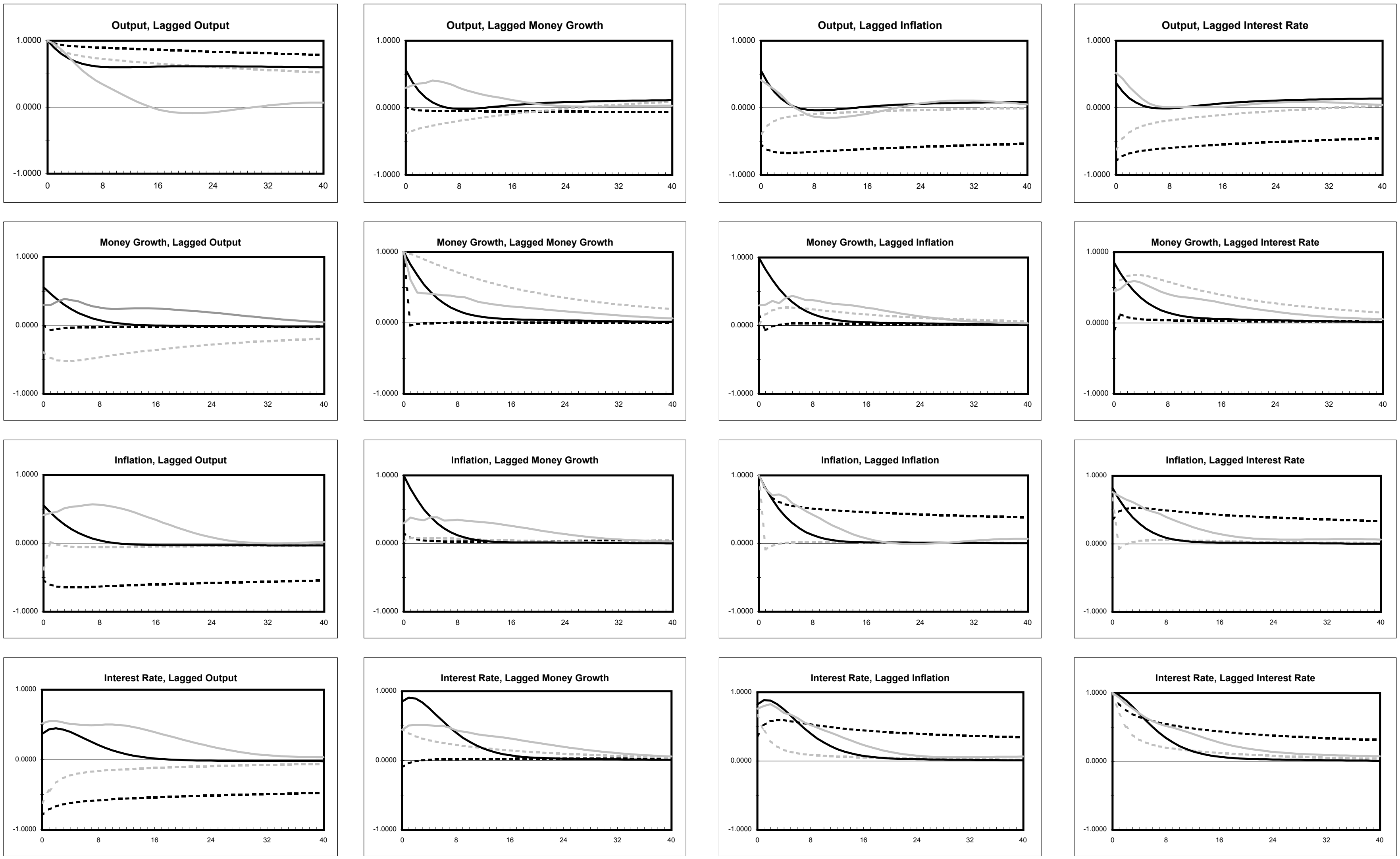


Figure 4: Vector Autocorrelation Functions for the period 1979:3 - 2001:1

Grey Line - Data
Black Line - Unified Model - U5[Φp=70, Φd=5]
Black Dotted Line - Sticky Price Model - SP[Φp=70, Φd=0]
Grey Dotted Line - Limited Participation Model - LP5[Φp=0, Φd=5]



Technical Appendix of
"Sticky Prices, Limited Participation, or Both?"

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March 5, 2004

1. Characterizing the Equilibrium

1.1. Symmetric Equilibrium

In a symmetric equilibrium, $h_t(i) = h_t$, $K_t(i) = K_t$, $F_t(i) = F_t$, $Y_t(i) = Y_t$, $P_t(i) = P_t$, and $L_t(i) = L_t$ for all $i \in [0, 1]$ and $t = 0, 1, 2, \ldots$. In addition, the market-clearing condition

$$M_t = M_{t-1} + X_t \tag{23}$$

must hold for all $t = 0, 1, 2, \ldots$. It is useful to note that these equilibrium conditions, together with (12)-(14), (16), and (17), can be used to rewrite the household's budget constraint (5) as the aggregate resource constraint

$$Y_t = C_t + I_t + \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t, \tag{5}$$

which must also hold for all $t = 0, 1, 2, \ldots$.

Collecting and simplifying (1)-(23) yields

$$\tau_t = \frac{\phi_d}{2}\left(\frac{D_t}{\mu D_{t-1}} - 1\right)^2, \tag{1}$$

$$\frac{M_t}{P_t} = v_t(C_t + I_t), \tag{2}$$

$$\ln(v_t) = (1 - \rho_v)\ln(v) + \rho_v \ln(v_{t-1}) + \varepsilon_{vt}, \tag{3}$$

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{\phi_k}{2}\left(\frac{K_{t+1}}{gK_t} - 1\right)^2 K_t, \tag{4}$$

$$Y_t = C_t + I_t + \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t, \tag{5}$$

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \tag{6}$$

$$a_t = (\Lambda_{1t} + v_t \Lambda_{2t}) C_t, \tag{7}$$

$$\gamma a_t = (\Lambda_{1t} + \Lambda_{2t})(W_t/P_t), \tag{8}$$

$$\frac{\Lambda_{1t}}{P_t} = \beta E_t \left(\frac{\Lambda_{1t+1} + \Lambda_{2t+1}}{P_{t+1}}\right), \tag{9}$$

$$\begin{aligned} & \gamma\phi_d a_t \left(\frac{D_t}{\mu D_{t-1}} - 1\right)\frac{D_t}{\mu D_{t-1}} \\ = \; & \frac{[\Lambda_{1t}(r_t - 1) - \Lambda_{2t}]D_t}{P_t} + \beta\gamma\phi_d E_t \left[a_{t+1}\left(\frac{D_{t+1}}{\mu D_t} - 1\right)\frac{D_{t+1}}{\mu D_t}\right], \end{aligned} \tag{10}$$

$$\begin{aligned} & (\Lambda_{1t} + v_t \Lambda_{2t})\left[1 + \frac{\phi_k}{g}\left(\frac{K_{t+1}}{gK_t} - 1\right)\right] \\ = \; & \beta E_t \left[\Lambda_{1t+1}(Q_{t+1}/P_{t+1}) + (\Lambda_{1t+1} + v_{t+1}\Lambda_{2t+1})(1-\delta)\right] \\ & + \beta\phi_k E_t \left\{(\Lambda_{1t+1} + v_{t+1}\Lambda_{2t+1})\left[\left(\frac{K_{t+2}}{gK_{t+1}} - 1\right)\left(\frac{K_{t+2}}{gK_{t+1}}\right) - \frac{1}{2}\left(\frac{K_{t+2}}{gK_{t+1}} - 1\right)^2\right]\right\}, \end{aligned} \tag{11}$$

$$D_t + X_t = L_t, \tag{12}$$

$$B_t = r_t X_t, \tag{13}$$

$$r_t = r_t^d, \tag{14}$$

$$\ln(z_t) = (1 - \rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt}, \tag{15}$$

$$L_t = W_t h_t, \tag{16}$$

$$\frac{F_t}{P_t} = Y_t - \frac{Q_t K_t + r_t W_t h_t}{P_t} - \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t, \tag{17}$$

$$Y_t = K_t^{\alpha}(g^t z_t h_t)^{1-\alpha}, \tag{18}$$

$$\Lambda_{1t} r_t (W_t/P_t) h_t = (1-\alpha)\Xi_t Y_t, \tag{19}$$

$$\Lambda_{1t}(Q_t/P_t)K_t = \alpha \Xi_t Y_t, \tag{20}$$

$$0 = (1-\theta)\Lambda_{1t} + \theta\Xi_t - \phi_p \Lambda_{1t}\left(\frac{\pi_t}{\pi} - 1\right)\left(\frac{\pi_t}{\pi}\right) \quad (21)$$
$$+\beta\phi_p E_t\left[\Lambda_{1t+1}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\left(\frac{\pi_{t+1}}{\pi}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\right],$$

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + \rho_y \ln(y_t/y) + \rho_\pi \ln(\pi_t/\pi) + \varepsilon_{rt}, \quad (22)$$

and

$$M_t = M_{t-1} + X_t. \quad (23)$$

Together with the definitions $y_t = Y_t/g^t$ and $\pi_t = P_t/P_{t-1}$, these 23 equations determine the behavior of the 23 variables τ_t, D_t, M_t, P_t, v_t, C_t, I_t, K_t, Y_t, a_t, Λ_{1t}, Λ_{2t}, W_t, r_t, Q_t, X_t, B_t, r_t^d, z_t, L_t, h_t, F_t, and Ξ_t.

These equilibrium conditions, the first-order conditions for the representative agents' problems, the laws of motion for the aggregate shocks, and the policy rule (22), form a system of difference equations describing the model's symmetric equilibrium, that implies that in the absence of shocks, the economy converges to a steady state.

1.2. Transformed System

As a first step in solving the model, define the transformed variables $d_t = D_t/M_t$, $m_t = (M_t/P_t)/g^t$, $\mu_t = M_t/M_{t-1}$, $c_t = C_t/g^t$, $i_t = I_t/g^t$, $k_t = K_t/g^t$, $y_t = Y_t/g^t$, $\lambda_{1t} = g^t\Lambda_{1t}$, $\lambda_{2t} = g^t\Lambda_{2t}$, $w_t = (W_t/P_t)/g^t$, $q_t = Q_t/P_t$, $x_t = X_t/M_{t-1}$, $b_t = B_t/M_t$, $l_t = L_t/M_t$, $f_t = (F_t/P_t)/g^t$, and $\xi_t = g^t\Xi_t$. Use (14) to eliminate r_t^d from the system, and rewrite (1)-(13) and (15)–(23) as

$$\tau_t = \frac{\phi_d}{2}\left(\frac{\mu_t d_t}{\mu d_{t-1}} - 1\right)^2, \quad (1)$$

$$m_t = v_t(c_t + i_t), \quad (2)$$

$$\ln(v_t) = (1-\rho_v)\ln(v) + \rho_v \ln(v_{t-1}) + \varepsilon_{vt}, \quad (3)$$

$$gk_{t+1} = (1-\delta)k_t + i_t - \frac{\phi_k}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 k_t, \quad (4)$$

$$y_t = c_t + i_t + \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 y_t, \quad (5)$$

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{at}, \quad (6)$$

$$a_t = (\lambda_{1t} + v_t\lambda_{2t})c_t, \quad (7)$$

$$\gamma a_t = (\lambda_{1t} + \lambda_{2t}) w_t, \tag{8}$$

$$g\lambda_{1t} = \beta E_t \left(\frac{\lambda_{1t+1} + \lambda_{2t+1}}{\pi_{t+1}} \right), \tag{9}$$

$$\begin{aligned} & \gamma \phi_d a_t \left(\frac{\mu_t d_t}{\mu d_{t-1}} - 1 \right) \frac{\mu_t d_t}{\mu d_{t-1}} \\ = \; & [\lambda_{1t}(r_t - 1) - \lambda_{2t}] d_t m_t + \beta \gamma \phi_d E_t \left[a_{t+1} \left(\frac{\mu_{t+1} d_{t+1}}{\mu d_t} - 1 \right) \frac{\mu_{t+1} d_{t+1}}{\mu d_t} \right], \end{aligned} \tag{10}$$

$$\begin{aligned} & g(\lambda_{1t} + v_t \lambda_{2t}) \left[1 + \frac{\phi_k}{g} \left(\frac{k_{t+1}}{k_t} - 1 \right) \right] \\ = \; & \beta E_t \left[\lambda_{1t+1} q_{t+1} + (\lambda_{1t+1} + v_{t+1} \lambda_{2t+1})(1 - \delta) \right] \\ & + \beta \phi_k E_t \left\{ (\lambda_{1t+1} + v_{t+1} \lambda_{2t+1}) \left[\left(\frac{k_{t+2}}{k_{t+1}} - 1 \right) \left(\frac{k_{t+2}}{k_{t+1}} \right) - \frac{1}{2} \left(\frac{k_{t+2}}{k_{t+1}} - 1 \right)^2 \right] \right\}, \end{aligned} \tag{11}$$

$$d_t + x_t / \mu_t = l_t, \tag{12}$$

$$b_t \mu_t = r_t x_t, \tag{13}$$

$$\ln(z_t) = (1 - \rho_z) \ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{zt}, \tag{15}$$

$$m_t l_t = w_t h_t, \tag{16}$$

$$f_t = y_t - q_t k_t - r_t w_t h_t - \frac{\phi_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 y_t, \tag{17}$$

$$y_t = k_t^{\alpha} (z_t h_t)^{1-\alpha}, \tag{18}$$

$$\lambda_{1t} r_t w_t h_t = (1 - \alpha) \xi_t y_t, \tag{19}$$

$$\lambda_{1t} q_t k_t = \alpha \xi_t y_t, \tag{20}$$

$$\begin{aligned} 0 \; = \; & (1 - \theta) \lambda_{1t} + \theta \xi_t - \phi_p \lambda_{1t} \left(\frac{\pi_t}{\pi} - 1 \right) \left(\frac{\pi_t}{\pi} \right) \\ & + \beta \phi_p E_t \left[\lambda_{1t+1} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \left(\frac{\pi_{t+1}}{\pi} \right) \left(\frac{y_{t+1}}{y_t} \right) \right], \end{aligned} \tag{21}$$

$$\ln(r_t / r) = \rho_r \ln(r_{t-1} / r) + \rho_y \ln(y_t / y) + \rho_\pi \ln(\pi_t / \pi) + \varepsilon_{rt}, \tag{22}$$

and

$$\mu_t = 1 + x_t. \tag{23}$$

Note also that the definitions of π_t, m_t, and μ_t imply

$$g m_t \pi_t = \mu_t m_{t-1}. \tag{24}$$

These 23 equations determine the behavior of the 23 stationary variables y_t, π_t, τ_t, d_t, m_t, μ_t, v_t, c_t, i_t, k_t, a_t, λ_{1t}, λ_{2t}, w_t, r_t, q_t, x_t, b_t, z_t, l_t, h_t, f_t, and ξ_t.

1.3. Steady State

In the absence of shocks, the economy converges to a steady state, in which each of the stationary variables is constant. Let π be chosen by policy. Equations (3), (6), and (15) determine v, $a = 1$, and z. Equations (1), (23), and (24) determine

$$\tau = 0,$$

$$\mu = g\pi,$$

and

$$x = \mu - 1.$$

Equations (9), (10), (11), (13), and (21) determine

$$r = \mu/\beta,$$

$$b = rx/\mu,$$

$$\lambda_2 = (r-1)\lambda_1,$$

$$q = (g/\beta - 1 + \delta)[1 + v(r-1)],$$

and

$$\xi = \left(\frac{\theta - 1}{\theta}\right)\lambda_1.$$

Equations (7) and (8) determine

$$c = \frac{1}{\lambda_1 + v\lambda_2}$$

and

$$w = \frac{\gamma}{\lambda_1 + \lambda_2}.$$

Equations (2), (4), (5), (17), (19), and (20) determine

$$k = \frac{c}{(\lambda_1 q/\alpha\xi) - g + 1 - \delta},$$

$$i = (g - 1 + \delta)k,$$

$$m = v(c + i),$$

$$y = \frac{\lambda_1 qk}{\alpha\xi},$$

$$h = \frac{(1-\alpha)\xi y}{\lambda_1 rw},$$

and

$$f = y - qk - rwh.$$

Equations (12) and (16) determine

$$l = wh/m$$

and

$$d = l - x/\mu.$$

Finally, (18) determines

$$\lambda_1 = \left[\frac{\gamma}{(1-\alpha)z}\right]\left(\frac{\theta}{\theta-1}\right)^{1/(1-\alpha)}\left(\frac{q}{\alpha}\right)^{\alpha/(1-\alpha)}.$$

1.4. The Linearized System

Equations (1)-(13) and (15)-(24), can be log-linearized to describe the behavior of the 23 stationary variables as they fluctuate about their steady-state values in response to shocks. Let $\hat{y}_t = \ln(y_t/y)$, $\hat{\pi}_t = \ln(\pi_t/\pi)$, $\hat{\tau}_t = \ln(\tau_t/t)$, $\hat{d}_t = \ln(d_t/d)$, $\hat{m}_t = \ln(m_t/m)$, $\hat{\mu}_t = \ln(\mu_t/\mu)$, $\hat{v}_t = \ln(v_t/v)$, $\hat{c}_t = \ln(c_t/c)$, $\hat{\imath}_t = \ln(i_t/i)$, $\hat{k}_t = \ln(k_t/k)$, $\hat{a}_t = \ln(a_t/a)$, $\hat{\lambda}_{1t} = \ln(\lambda_{1t}/\lambda_1)$, $\hat{\lambda}_{2t} = \ln(\lambda_{2t}/\lambda_2)$, $\hat{w}_t = \ln(w_t/w)$, $\hat{r}_t = \ln(r_t/r)$, $\hat{q}_t = \ln(q_t/q)$, $\hat{x}_t = \ln(x_t/x)$, $\hat{b}_t = \ln(b_t/b)$, $\hat{z}_t = \ln(z_t/z)$, $\hat{l}_t = \ln(l_t/l)$, $\hat{h}_t = \ln(h_t/h)$, $\hat{f}_t = \ln(f_t/f)$, and $\hat{\xi}_t = \ln(\xi_t/\xi)$. Then a log-linear approximation of (1) implies that $\hat{\tau}_t = 0$, while log-linear approximations to (2)-(13) and (15)–(24) yield

$$0 = -m\hat{m}_t + m\hat{v}_t + vc\hat{c}_t + vi\hat{\imath}_t, \tag{2}$$

$$\hat{v}_t = \rho_v \hat{v}_{t-1} + \varepsilon_{vt}, \tag{3}$$

$$0 = -gk\hat{k}_{t+1} + (1-\delta)k\hat{k}_t + i\hat{\imath}_t, \tag{4}$$

$$0 = -y\hat{y}_t + c\hat{c}_t + i\hat{\imath}_t, \tag{5}$$

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{at}, \tag{6}$$

$$0 = -\hat{a}_t + \lambda_1 c\hat{\lambda}_{1t} + v\lambda_2 c\hat{\lambda}_{2t} + v\lambda_2 c\hat{v}_t + \hat{c}_t, \tag{7}$$

$$0 = -\gamma\hat{a}_t + \lambda_1 w\hat{\lambda}_{1t} + \lambda_2 w\hat{\lambda}_{2t} + \gamma\hat{w}_t, \tag{8}$$

$$0 = -r\hat{\lambda}_{1t} + E_t\hat{\lambda}_{1t+1} + (r-1)E_t\hat{\lambda}_{2t+1} - rE_t\hat{\pi}_{t+1}, \tag{9}$$

$$\begin{aligned} 0 = &\left[\lambda_1(r-1)dm\right]\hat{\lambda}_{1t} - \lambda_2 dm\hat{\lambda}_{2t} + (\lambda_1 rdm)\hat{r}_t \\ &-(\gamma\phi_d)\hat{\mu}_t + \gamma\phi_d\hat{d}_{t-1} - (1+\beta)\gamma\phi_d\hat{d}_t \\ &+\beta\gamma\phi_d E_t\hat{\mu}_{t+1} + \beta\gamma\phi_d E_t\hat{d}_{t+1}, \end{aligned} \tag{10}$$

$$\begin{aligned} 0 \;=\; & -g\lambda_1\hat{\lambda}_{1t} - gv\lambda_2\hat{v}_t - gv\lambda_2\hat{\lambda}_{2t} + \phi_k\left(\lambda_1 + v\lambda_2\right)\widehat{\kappa}_t \\ &+[\beta\lambda_1(1+q-\delta)]E_t\hat{\lambda}_{1t+1} + \left[\beta(1-\delta)v\lambda_2\right]E_t\hat{\lambda}_{2t+1} \\ &+\beta\lambda_1 qE_t\hat{q}_{t+1} + \beta v\lambda_2(1-\delta)E_t\hat{v}_{t+1} \\ &-(1+\beta)\phi_k\left(\lambda_1 + v\lambda_2\right)E_t\hat{k}_{t+1} + \beta\phi_k\left(\lambda_1 + v\lambda_2\right)E_t\hat{k}_{t+2}, \end{aligned} \tag{11}$$

$$0 = -l\hat{l}_t + d\hat{d}_t + \left(\frac{x}{\mu}\right)\hat{x}_t - \left(\frac{x}{\mu}\right)\hat{\mu}_t, \tag{12}$$

$$0 = -\hat{b}_t - \hat{\mu}_t + \hat{r}_t + \hat{x}_t, \tag{13}$$

$$\hat{z}_t = \rho_z\hat{z}_{t-1} + \varepsilon_{zt}, \tag{15}$$

$$0 = \hat{m}_t + \hat{l}_t - \hat{w}_t - \hat{h}_t, \tag{16}$$

$$0 = -f\hat{f}_t + y\hat{y}_t - qk\hat{q}_t - qk\hat{k}_t - rwh\hat{r}_t - rwh\hat{w}_t - rwh\hat{h}_t, \tag{17}$$

$$0 = -\hat{y}_t + \alpha\hat{k}_t + (1-\alpha)\,\hat{z}_t + (1-\alpha)\,\hat{h}_t, \tag{18}$$

$$0 = -\hat{\lambda}_{1t} - \hat{r}_t - \hat{w}_t - \hat{h}_t + \hat{\xi}_t + \hat{y}_t, \tag{19}$$

$$0 = -\hat{\lambda}_{1t} - \hat{q}_t - \hat{k}_t + \hat{\xi}_t + \hat{y}_t, \tag{20}$$

$$0 = -\phi_p \hat{\pi}_t + (\theta - 1)\,\hat{\xi}_t - (\theta - 1)\hat{\lambda}_{1t} + \beta\phi_p E_t\left(\hat{\pi}_{t+1}\right), \tag{21}$$

$$0 = -\hat{r}_t + \rho_r \hat{r}_{t-1} + \rho_y \hat{y}_t + \rho_\pi \hat{\pi}_t + \varepsilon_{rt}, \tag{22}$$

$$0 = -\mu\hat{\mu}_t + x\hat{x}_t \tag{23}$$

and

$$0 = -\hat{m}_t - \hat{\pi}_t + \hat{\mu}_t + \hat{m}_{t-1}. \tag{24}$$

In solving the model, it is convenient to use (4) to rewrite (11) as

$$\begin{aligned} 0 \;\; = \;\; & -g\lambda_1\hat{\lambda}_{1t} - gv\lambda_2\hat{v}_t - gv\lambda_2\hat{\lambda}_{2t} + \phi_k\left(\lambda_1 + v\lambda_2\right)\widehat{\kappa}_t \\ & +[\beta\lambda_1(1+q-\delta)]E_t\hat{\lambda}_{1t+1} + [\beta(1-\delta)v\lambda_2]\,E_t\hat{\lambda}_{2t+1} + \beta\lambda_1 q E_t\hat{q}_{t+1} \\ & +\beta v\lambda_2(1-\delta)E_t\hat{v}_{t+1} + \left[\frac{\beta i\phi_k}{gk}\left(\lambda_1 + v\lambda_2\right)\right]E_t\hat{\imath}_{t+1} \\ & + \left[\frac{\beta(1-\delta)}{g} - 1 - \beta\right]\phi_k\left(\lambda_1 + v\lambda_2\right)E_t\hat{k}_{t+1}. \end{aligned} \tag{11}$$

Now the 22 equations, (2)-(13), and (15)-(24) describe the behavior of the 22 variables $\hat{y}_t$, $\hat{\pi}_t$, $\hat{d}_t$, $\hat{m}_t$, $\hat{\mu}_t$, $\hat{v}_t$, $\hat{c}_t$, $\hat{\imath}_t$, $\hat{k}_t$, $\hat{a}_t$, $\hat{\lambda}_{1t}$, $\hat{\lambda}_{2t}$, $\hat{w}_t$, $\hat{r}_t$, $\hat{q}_t$, $\hat{x}_t$, $\hat{b}_t$, $\hat{z}_t$, $\hat{l}_t$, $\hat{h}_t$, $\hat{f}_t$, and $\hat{\xi}_t$.

1.5. The Linear System in Matrix Form

The vector f_t^0 keeps track of the model's flow variables which include output $y_t = Y_t/g^t$, inflation π_t, the current values of the bank deposits $d_t = D_t/M_t$, money growth μ_t, consumption $c_t = C_t/g^t$, investments $i_t = I_t/g^t$, the multipliers $\lambda_{1t} = g^t\Lambda_{1t}$, $\lambda_{2t} = g^t\Lambda_{2t}$, and $\xi_t = g^t\Xi_t$, the real factor prices $w_t = (W_t/P_t)/g^t$, and $q_t = Q_t/P_t$, banks profits $b_t = B_t/M_t$, bank loans $l_t = L_t/M_t$, hours worked h_t, real profits $f_t = (F_t/P_t)/g^t$.

$$\mathrm{f}_t^0 = \begin{bmatrix} \hat{y}_t & \hat{\pi}_t & \hat{d}_t & \hat{\mu}_t & \hat{c}_t & \hat{\imath}_t & \hat{\lambda}_{1t} & \hat{\lambda}_{2t} & \hat{w}_t & \hat{q}_t & \hat{x}_t & \hat{b}_t & \hat{l}_t & \hat{h}_t & \hat{f}_t & \hat{\xi}_t \end{bmatrix}',$$

The vector s_t^0 keeps track of the model's endogenous state variables which include the lagged values of real balances $m_{t-1} = (M_{t-1}/P_{t-1})/g^{t-1}$ (because prices are sticky), the lagged values of the bank deposits $d_{t-1} = D_{t-1}/M_{t-1}$, the lagged interest rate r_{t-1}, the current values of the capital stock, the money demand shock, the preference shock, the technology shock and the policy shock.

$$s_t^0 = \left[\hat{d}_{t-1} \quad \hat{k}_t \quad \hat{m}_{t-1} \quad \hat{r}_{t-1}\right]',$$

The vector z_t^0 keeps track of the model's four shocks, the money demand shock $\hat{v}_t$, the preference shock $\hat{a}_t$, the technology shock $\hat{z}_t$, and the policy shock ε_{rt}.

$$\mathrm{z}_t^0 = [\hat{v}_t \quad \hat{a}_t \quad \hat{z}_t \quad \varepsilon_{rt}]'.$$

Then (2), (4), (5), (7)-(13), (16)-(24) can be written as

$$AE_t\mathrm{y}_{t+1}^0 = B\mathrm{y}_t^0 + CE_t\mathrm{z}_{t+1}^0 + D\mathrm{z}_t^0 \tag{25}$$

where

$$\mathrm{y}_t^0 = \begin{bmatrix} \mathrm{f}_t^0 \\ s_t^0 \end{bmatrix}$$

1and A is a 20x20, B is a 20x20, C is a 20x4 and D is a 20x4.

Equation (2) implies

$a_{119} = -m$

$b_{15} = vc$

$b_{16} = vi$

$d_{11} = m$

Equation (4) implies

$a_{218} = -gk$

$b_{26} = i$

$b_{218} = (1-\delta)k$

Equation (5) implies

$b_{31} = -y$

$b_{35} = c$

$b_{36} = i$

Equation (7) implies

$b_{45} = 1$

$b_{47} = \lambda_1 c$

$b_{48} = v\lambda_2 c$

$d_{41} = v\lambda_2 c$

$d_{42} = -1$

Equation (8) implies

$b_{57} = \lambda_1 w$

$b_{58} = \lambda_2 w$

$b_{59} = \gamma$

$d_{52} = -\gamma$

Equation (9) implies

$a_{62} = -r$

$a_{67} = 1$

$a_{68} = r - 1$

$b_{67} = -r$

Equation (10) implies

$a_{73} = \beta\gamma\phi_d$

$a_{74} = \beta\gamma\phi_d$

$a_{77} = -(1 + \beta)\gamma\phi_d$

$a_{720} = \lambda_1 rdm$

$b_{74} = -\gamma\phi_d$

$b_{77} = \lambda_1(r-1)dm$

$b_{78} = -\lambda_2 dm$

$b_{717} = \gamma\phi_d$

Equation (11) implies

$a_{86} = (\beta i\phi_k/gk)\,(\lambda_1 + v\lambda_2)$

$a_{87} = \beta\lambda_1(1+q-\delta)$

$a_{88} = \beta(1-\delta)v\lambda_2$

$a_{810} = \beta\lambda_1 q$

$a_{818} = [(\beta(1-\delta)/g) - 1 - \beta]\,\phi_k\,(\lambda_1 + v\lambda_2)$

$b_{87} = -g\lambda_1$

$b_{88} = -gv\lambda_2$

$b_{818} = \phi_k\,(\lambda_1 + v\lambda_2)$

$c_{81} = \beta v\lambda_2(1-\delta)$

$d_{81} = -gv\lambda_2$

Equation (12) implies

$b_{93} = d$

$b_{94} = -x/\mu$

$b_{911} = x/\mu$

$b_{913} = -l$

Equation (13) implies

$a_{1020} = 1$

$b_{104} = -1$

$b_{1011} = 1$

$b_{1012} = -1$

Equation (16) implies

$a_{1119} = -1$

$b_{119} = 1$

$b_{1113} = 1$

$b_{1114} = -1$

Equation (17) implies

$a_{1220} = -rwh$

$b_{121} = y$

$b_{129} = -rwh$

$b_{1210} = -qk$

$b_{1214} = -rwh$

$b_{1215} = -f$

$b_{1218} = -qk$

Equation (18) implies

$b_{131} = -1$

$b_{1314} = 1 - \alpha$

$b_{1318} = \alpha$

$d_{133} = 1 - \alpha$

Equation (19) implies

$a_{1420} = -1$

$b_{141} = 1$

$b_{147} = -1$

$b_{149} = -1$

$b_{1414} = -1$

$b_{1416} = 1$

Equation (20) implies

$b_{151} = 1$

$b_{157} = -1$

$b_{1510} = -1$

$b_{1516} = 1$

$b_{1518} = -1$

Equation (21) implies

$a_{162} = \beta\phi_p$

$b_{162} = -\phi_p$

$b_{167} = -(\theta - 1)$

$b_{1616} = \theta - 1$

Equation (22) implies

$a_{1720} = -1$

$b_{171} = \rho_y$

$b_{172} = \rho_\pi$

$b_{1720} = \rho_r$

$d_{174} = 1$

Equation (23) implies

$b_{184} = -\mu$

$b_{1811} = x$

Equation (24) implies

$a_{1919} = -1$

$b_{192} = -1$

$b_{194} = 1$

$b_{1919} = 1$

The presence of d_t in y^0_{t+1} and y^0_t implies

$a_{203} = 1$

$b_{2017} = -1$

Finally (3), (6), and (15)

$$\mathrm{z}^0_{t+1} = P\mathrm{z}^0_t + \varepsilon_{t+1}, \tag{26}$$

where

$$P = \begin{bmatrix} \rho_v & 0 & 0 & 0 \\ 0 & \rho_a & 0 & 0 \\ 0 & 0 & \rho_z & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\varepsilon_t = [\varepsilon_{vt} \quad \varepsilon_{at} \quad \varepsilon_{zt} \quad \varepsilon_{rt}]'.$$

2. Solving the Model

The method of Klein can then be applied to the system of equations (25), (26), in order to obtain a solution of the form

$$\mathrm{f}_t = U s_t \tag{27}$$

and

$$s_t = \Pi s_{t-1} + W \varepsilon_t \tag{28}$$

where U is a 16x8, Π is an 8x8, and W is a 8x4 matrices.

The vector f_t keeps track of the model's flow variables

$$\mathrm{f}_t = \begin{bmatrix} \hat{y}_t & \hat{\pi}_t & \hat{d}_t & \hat{\mu}_t & \hat{c}_t & \hat{\imath}_t & \hat{\lambda}_{1t} & \hat{\lambda}_{2t} & \hat{w}_t & \hat{q}_t & \hat{x}_t & \hat{b}_t & \hat{l}_t & \hat{h}_t & \hat{f}_t & \hat{\xi}_t \end{bmatrix}',$$

the vector s_t keeps track of the model's endogenous state variables and the model's four shocks

$$s_t = \begin{bmatrix} \hat{d}_{t-1} & \hat{k}_t & \hat{m}_{t-1} & \hat{r}_{t-1} & \hat{v}_t & \hat{a}_t & \hat{z}_t & \varepsilon_{rt} \end{bmatrix}',$$

$$W = \begin{bmatrix} 0_{(4x4)} \\ I_{(4x4)} \end{bmatrix}.$$

The vector ε_t keeps track of the four innovations ε_{vt}, ε_{at}, ε_{zt}, and ε_{rt} and it is assumed to be normally distributed with zero mean and covariance matrix V.

$$\varepsilon_t = [\varepsilon_{vt} \;\; \varepsilon_{at} \;\; \varepsilon_{zt} \;\; \varepsilon_{rt}]',$$

$$V = E\varepsilon_t\varepsilon_t' = \begin{bmatrix} \sigma_v^2 & 0 & 0 & 0 \\ 0 & \sigma_a^2 & 0 & 0 \\ 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & \sigma_r^2 \end{bmatrix}.$$

The model's 22 parameters describing private agents' tastes, technologies and policy rule, that determine the elements of the matrices U, Π, W, and V are

$\phi_d \geq 0$	$1 > \beta > 0$	$\sigma_z > 0$
$\mu \geq 1$	$\gamma > 0$	$\phi_p \geq 0$
$v > 0$	$1 > \rho_a \geq 0$	$\rho_r > 0$
$1 > \rho_v \geq 0$	$\sigma_a > 0$	$\rho_y > 0$
$\sigma_v > 0$	$\theta > 1$	$\rho_\pi > 0$
$1 > \delta > 0$	$1 > \alpha > 0$	$\sigma_r > 0$
$\phi_k \geq 0$	$z > 0$	
$g \geq 1$	$1 > \rho_z \geq 0$	

3. Vector Autocorrelation Functions

3.1. Model Vector Autocorrelation Functions

Equations (27) and (28) can give rise to a model of the form

$$\mathrm{d}_t = Cs_t \tag{29}$$

and

$$s_t = \Pi s_{t-1} + W\varepsilon_t \tag{28}$$

where C is formed from the rows of U and Π as

$$C = \begin{bmatrix} U_1 \\ U_4 \\ U_2 \\ \Pi_4 \\ U_9 \\ U_{15} \end{bmatrix}$$

Therefore

$$\mathrm{d}_t = \begin{bmatrix} \hat{y}_t & \hat{\mu}_t & \hat{\pi}_t & \hat{r}_{t-1} & \hat{w}_t & \hat{f}_t \end{bmatrix}'.$$

Equation (28) implies that

$$s_t = \Pi^k_{t-k} s_t + \sum_{j=0}^{k-1} \Pi^j W \varepsilon_{t-j}. \tag{30}$$

Therefore

$$E[s_t s'_{t-k}] = \Pi^k E[s_{t-k} s'_{t-k}] = \Pi^k \Sigma^s \tag{31}$$

where

$$vec(\Sigma^s) = [I_{(64x64)} - \Pi \otimes \Pi]^{-1} vec(WVW'). \tag{32}$$

Using equation (28), the autocovariance matrix can be computed as

$$\Gamma_k^{\mathrm{d}} = E[\mathrm{d}_t \mathrm{d}'_{t-k}] = UE[s_t s'_{t-k}]U' = U\Pi^k \Sigma^s U' \tag{33}$$

Thus, the autocorrelation can be computed as

$$\varrho_k^{\mathrm{d}}(i,j) = \frac{\Gamma_k^{\mathrm{d}}(i,j)}{\sqrt{\Gamma_0^{\mathrm{d}}(i,i)}\sqrt{\Gamma_0^{\mathrm{d}}(j,j)}} \tag{34}$$

3.2. Data Vector Autocorrelation Functions

Consider using the data for output $\hat{y}_t$, money growth $\hat{m}_t$, inflation $\hat{\pi}_t$, and interest rate $\hat{r}_t$, that are obtained from the data collected, the real GDP in chained 1996 dollars Y_t, the M2 money stock M_t, the GDP implicit price deflator P_t, the three-month Treasury bill rate r_t, and the civilian noninstitutional population, age 16 and over n_t,

Therefore

$$\mathrm{d}_t = \begin{bmatrix} \hat{y}_t \\ \hat{m}_t \\ \hat{\pi}_t \\ \hat{r}_t \end{bmatrix} = \begin{bmatrix} \ln\left(\frac{Y_t}{4n_t}\right) - t\ln(g) - \ln(y) \\ \ln(M_t) - \ln(M_{t-1}) - \ln(m) \\ \ln(P_t) - \ln(P_{t-1}) - \ln(\pi) \\ \ln\left(\frac{1}{1-(r_t/400)}\right) - \ln(r) \end{bmatrix}.$$

and

$$\ln(y) = average\left[\ln\left(\frac{Y_t}{4n_t}\right) - t\ln(g)\right],$$

$$\ln(m) = average[\ln(M_t) - \ln(M_{t-1})],$$

$$\ln(\pi) = average[\ln(P_t) - \ln(P_{t-1})],$$

$$\ln(r) = average\left[\ln\left(\frac{1}{1-(r_t/400)}\right) - \ln(r)\right].$$

The autoregression with four lags

$$\mathrm{d}_t = a_1\mathrm{d}_{t-1} + a_2\mathrm{d}_{t-2} + a_3\mathrm{d}_{t-3} + a_4\mathrm{d}_{t-4} + B\varepsilon_t, \tag{35}$$

can be estimated, where more than one lag of d_t can be accommodated by writing the system in companion form.

Thus,

$$\begin{bmatrix} \mathrm{d}_t \\ \mathrm{d}_{t-1} \\ \mathrm{d}_{t-2} \\ \mathrm{d}_{t-3} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ I_{(4x4)} & 0_{(4x4)} & 0_{(4x4)} & 0_{(4x4)} \\ 0_{(4x4)} & I_{(4x4)} & 0_{(4x4)} & 0_{(4x4)} \\ 0_{(4x4)} & 0_{(4x4)} & I_{(4x4)} & 0_{(4x4)} \end{bmatrix} \begin{bmatrix} \mathrm{d}_{t-1} \\ \mathrm{d}_{t-2} \\ \mathrm{d}_{t-3} \\ \mathrm{d}_{t-4} \end{bmatrix} + \begin{bmatrix} I_{(4x4)} \\ 0_{(4x4)} \\ 0_{(4x4)} \\ 0_{(4x4)} \end{bmatrix} \varepsilon_t,$$

or

$$\mathrm{D}_t = A\mathrm{D}_{t-1} + B\varepsilon_t \tag{36}$$

Then it can be written as

$$\mathrm{D}_t = A^k\mathrm{D}_{t-k} + \sum_{j=0}^{k-1} A^j B\varepsilon_{t-j}, \tag{37}$$

Therefore, the autocovariance function can be calculated as

$$\Gamma_k^{\mathrm{D}} = E[\mathrm{D}_t\mathrm{D}'_{t-k}] = A^k E[\mathrm{D}_{t-k}\mathrm{D}'_{t-k}] = A^k\Sigma^{\mathrm{D}} \tag{38}$$

where

$$vec(\Sigma^{\mathrm{D}}) = [I_{(256x256)} - A \otimes A]^{-1} vec(BVB'), \tag{39}$$

and V is the 4x4 covariance matrix of the autoregression estimation.

Thus the autocorrelations can be computed as

$$\varrho_k^{\mathrm{D}}(i,j) = \frac{\Gamma_k^{\mathrm{D}}(i,j)}{\sqrt{\Gamma_0^{\mathrm{D}}(i,i)}\sqrt{\Gamma_0^{\mathrm{D}}(j,j)}} \tag{40}$$