Nelson-Plosser revisited: the ACF approach

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ABSTRACT

We detect a new stylized fact about the common dynamics of macroeconomic and financial aggregates. The rate of decay of the memory of these series is depicted by their Auto-Correlation Functions (ACFs). They all share a common four-parameter functional form that we derive from the dynamics of an RBC model with heterogeneous firms. We find that, not only does our formula fit the data better than the ACFs that arise from autoregressive models, but it also yields the correct shape of the ACF. This can help policymakers understand better the lags with which an economy evolves, and the onset of its turning points. (JEL E32, E52, E63).

Since the seminal paper of Nelson and Plosser (1982), to be referred to henceforth as NP, a growing literature has debated the nature of the dynamics of macroeconomic time series. Naturally, one would like economic theory to inform us on the type of processes which we could expect to encounter. Baseline business cycle models can motivate trend-stationary or difference-stationary processes, but adding more realistic structure to these models generally means that it is hard to explicitly derive the dynamic process for the aggregate variables of interest. However, Abadir and Talmain (2002), to be referred to henceforth as AT, derived the process generating aggregate output in an RBC model with heterogeneous firms, and characterized its Auto-Correlation Function (ACF). ACFs depict the decay of memory with time, as they evaluate the correlation of a series with its past. AT's model implied that the ACF of real GDP per capita should exhibit an initial concave shape, followed by a sharp drop, a prediction which they validated empirically for the UK and the US. They showed that linear Auto-Regressive Integrated Moving-Average (ARIMA) models, as well as their three separate components (including the special case of random walks), all exhibit different types of decays of memory from the one they found. This simple yet accurate shape for GDP invites us here to investigate the shape of the ACFs for all the main macro variables, including all those in the NP dataset and others.

The ACF of AT was the leading term of an expansion of an elaborate integral, and was only suitable as a rough approximation of the broad features of GDP's ACF. Another novel feature here (apart from considering all main macro series in addition to GDP) is that we go beyond the 1-term asymptotic approximation of the ACF of AT, taking into account the remaining terms of the ACF expansion. The resulting functional form typically combines the original shape in AT (plateau plus drop-off) with a cycle. As we shall see, this augmented version of the ACF shape fits closely the ACF of all of the variables studied by NP, and this fit is better than the one produced by AR

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processes, including the special case of the unit root. In addition, it also fits very well the ACFs of variables not considered by NP, some of them known to have notoriously difficult dynamics; e.g. investment, components of the trade and fiscal deficits.

One of the legacies of NP was the unified modelling of the process generating many macroeconomic data. If anything, our paper reinforces this message by offering a parsimonious functional form of only 4 parameters that can model the ACF of most economic aggregates. This empirical regularity is truly impressive, and helps us detect new stylized facts that are common to all macroeconomic series.

Our functional form is rich enough to produce a variety of observed shapes. We find that most of the variables can be classified into only two broad types. The shape of the ACF of most level variables is dominated by the plateau-shape. The ACFs of the rate variables are dominated by an attenuated cycle, the original AT form providing the attenuation. Interestingly, the length of the estimated cycles matches those of the medium run cycles proposed by Comin and Gertler (2006). One feature of the data that comes in strongly when studying ACFs is the presence of a (business) cycle, whether by our method or the more standard ones.

The shape of an ACF is important. An econometric model that does not give rise to the shape of the ACF observed from the data is misspecified, but this might be tolerated if the approximation is good enough.[1] More importantly, the pattern of retention of old information and absorption of new one can be read off an ACF, and this is valuable information that we cannot afford to misread. Getting the ACF shape right means that we will be able to understand the lags with which macroeconomic variables evolve,

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[1]In Section III, we will see the rare illustrations of this adequacy of AR models, with bond yields and the *nominal* money stock. On the other hand, even the real money stock is badly approximated by AR models.

and how quickly situations turn. For example, this can enable us to design monetary policies more effectively.

Section I reviews briefly the relevant literature, and how it relates to the development of our method. Section II presents our estimation procedure. Section III applies it to macro variables and the results are compared to the traditional ones. We show how our estimation method can be augmented to incorporate checks for structural breaks and other deterministic trends. Our earlier results turn out to be robust and accurate. Section IV concludes by considering the implications for the implementation and timing of macroeconomic policy.

I. Development of the literature

According to NP, most macroeconomic time series become stationary after differencing once. Such series are called integrated of order 1, denoted by I(1). The econometric implication of NP's result is that trends are stochastic, rather than deterministic and predictable, and that all shocks to trends are permanent. The economic implication is that the fluctuations of the business cycle can no longer be dissociated from long run growth. Another implication is to invalidate the traditional idea that the conduct of stabilization policy could be separated and had no implication for policies aimed at economic growth.

Many authors, for instance Cochrane (1988) or Rudebusch (1993), have pointed out that a difference-stationary series is not easily distinguishable from a trend-stationary series, when attention is restricted to ARMA models. Some authors, such as Diebold and Senhadji (1996), have shown that longer spans can bring the weight of evidence onto one side, in their instance the trend-stationary side. Others, such as Ireland (2001), have shown that trend-stationary models with a root ρ very close to one ($\rho = 0.9983$) produce

out-of-sample forecasts that are more accurate than the difference-stationary alternative. Note that models with ρ so close to 1 still produce I(0) series, where the memory decays exponentially, whereas $\rho = 1$ leads to I(1) series having infinite (permanent) memory.

To bridge the gap between I(0) and I(1) models, and to deal with borderline cases such as the one described in the previous paragraph, fractionally-integrated models have been introduced and denoted by I(d) where d is not necessarily an integer. A larger d indicates higher persistence (more memory), but for $d < 1$ the effect of shocks to the series eventually decay, unlike when $d = 1$. Such models have been applied to macroeconomic series by Diebold and Rudebusch (1989), Baillie and Bollerslev (1994), Gil-Alaña and Robinson (1997), Chambers (1998), Michelacci and Zaffaroni (2000), and Abadir, Distaso, and Giraitis (2005). The results show that d is generally less than 1. However, fractionally-integrated models imply convex hyperbolic decay rates for the ACFs.[2] They give a good indication of the rate of decay of ACFs in the tails (distant past), but do not tell us what happens in the interim, an information that is of great interest to policymakers and that we shall be modelling in the next sections.

Apart from I(d) models, there is an even larger area of support for the view that macroeconomic series can indeed be stabilized if necessary. It is a nonlinear model that was successfully initiated by Perron (1989), showing that most of these series are best represented as stationary around deterministic trends, with infrequent structural breaks in the trend. This is confirmed by recent evidence in Andreou and Spanos (2003).

Both trend and difference stationary models are linear processes. The two extensions that followed, I(d) and breaks, tackled intermediate memory

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[2]They also do not allow for long cycles, because the peak of the spectrum is at the origin. However, some progress has been made on this aspect by Giraitis, Hidalgo, and Robinson (2001) and Hidalgo (2005).

and nonlinearity, respectively. In the rest of this paper, we present a method that combines a different type of long-memory and nonlinearity in a simple yet accurate way, arising from the economic model of AT.

II. Estimation procedure

There are two traditional ways to look at macroeconomic time series: the time domain and the frequency domain. Here, we introduce estimation in the related ACF domain. Many papers, including NP, have looked at autocorrelations from a descriptive perspective, but reporting them only for a few lags and not estimating their pattern. In this paper, we are going to evaluate whether an extension of the functional form proposed in AT represents the ACF of macroeconomic times series better than traditional AR processes, even when structural breaks are allowed for.

The ACF $\rho_1, \rho_2, \ldots$ of a process $\{z_t\}_{t=1}^{T}$ is defined as the sequence of correlations of the variable with its τ-th lag:

$$(1) \qquad \rho_\tau \equiv \frac{\text{cov}\left(z_t, z_{t-\tau}\right)}{\sqrt{\text{var}\left(z_t\right)\text{var}\left(z_{t-\tau}\right)}},$$

where $\rho_0 \equiv 1$ and $\text{cov}(z_t, z_{t-\tau}) \equiv \text{E}[(z_t - \text{E}z_t)\left(z_{t-\tau} - \text{E}z_{t-\tau}\right)]$. This general definition allows for a time-varying expectation of z_t, but without imposing any parametric structure on the evolution of $\text{E}(z_t)$. This definition was also used in AT.

We begin with the simplest setup of an $\text{AR}(p)$ process, which we will show in footnote 6 to cover the random walk as a special case of the $\text{AR}(1)$. We will also deal with adding deterministic trends and/or breaks at the end of next section. To start, consider a stationary $\text{AR}(p)$ process

$$x_t = a_0 + a_1 x_{t-1} + \cdots + a_p x_{t-p} + \varepsilon_t,$$

where $\{\varepsilon_t\}$ is a sequence of $\text{IID}(0, \sigma^2)$ residuals. The ACF of this process is denoted by ρ_τ^{AR}. The first p values are given by the Yule-Walker equations

$$
(2) \quad
\begin{bmatrix}
1 & \rho_1^{\text{AR}} & \rho_2^{\text{AR}} & \cdots & \rho_{p-1}^{\text{AR}} \\
\rho_1^{\text{AR}} & 1 & \rho_1^{\text{AR}} & \ddots & \vdots \\
\rho_2^{\text{AR}} & \rho_1^{\text{AR}} & 1 & \ddots & \rho_2^{\text{AR}} \\
\vdots & \ddots & \ddots & \ddots & \rho_1^{\text{AR}} \\
\rho_{p-1}^{\text{AR}} & \cdots & \rho_2^{\text{AR}} & \rho_1^{\text{AR}} & 1
\end{bmatrix}
\begin{bmatrix}
a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_p
\end{bmatrix}
=
\begin{bmatrix}
\rho_1^{\text{AR}} \\ \rho_2^{\text{AR}} \\ \rho_3^{\text{AR}} \\ \vdots \\ \rho_p^{\text{AR}}
\end{bmatrix} ;
$$

e.g. see Granger and Newbold (1986). This is a linear system of p equations in the p values $\{\rho_1^{\text{AR}}, \ldots, \rho_p^{\text{AR}}\}$ that can therefore be determined uniquely; e.g. see Abadir and Magnus (2005). Since the system is linear, it is numerically straightforward to evaluate the first p values of the ACF of the AR(p). The remaining values are given by the recursive relation

$$
(3) \quad \rho_\tau^{\text{AR}} = a_1 \rho_{\tau-1}^{\text{AR}} + \cdots + a_p \rho_{\tau-p}^{\text{AR}},
$$

for all $\tau > p$.

The alternative to the AR is an extension of the ACF functional form proposed in AT. The new ACF is

$$
(4) \quad \rho_\tau^{\text{AT}} = \frac{1 - a\left[1 - \cos\left(\omega\tau\right)\right]}{1 + b\tau^c},
$$

where we have the parameters a, b, c, ω. The extension of the AT form into (4) is explained as follows. The model of AT showed that GDP and other variables driven by it follow a new type of long-memory process that reverts to a possibly time-varying mean. The functional form of the ACF in AT is just the denominator of (4), obtained as the leading term of an expansion with alternating signs; see expressions (A.3) of AT and their subsequent derivations. Here, we capture these higher-order oscillating terms with the numerator of (4) in order to approximate the complete ACF formula more accurately. The denominator still controls the decay of memory. When $a = 0$ or $\omega = 0$, we are back to the old form of the ACF.

We want to decide which of the two models best represents the ACF data. We will need to start by selecting the order p of the AR model for each time series under consideration. The AR model has p parameters and our model has 4. Since the two models do not necessarily have the same degrees of freedom, we need to use an information criterion to determine which model fits best in the ACF domain.[3] We use the Schwarz information Criterion (SC), which is known to be consistent. The alternatives are the Akaike criterion and the Hannan–Quinn criterion. The former was shown by Nishii (1988) to be inconsistent. The latter is designed to determine the orders p and q of ARMA(p, q) processes and, since the AT process does not belong to this class, we use the broader Schwarz criterion instead.[4]

Given the empirical ACF of a series, we can estimate by nonlinear least squares the two theoretical ACFs seen earlier. We find that $p \leq 4$ in all the cases at hand.[5] Only a few data points from $\{z_t\}$ contribute to the calculation of the tail end of the empirical ACF. Consequently, the tail of the empirical ACF is typically very erratic and is not a reflection of the true

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[3]One can also estimate the parameters of an AR process from the time domain. By construction, the resulting fit for the ACF would not be as good as fitting directly the ACF, although the resulting estimates would be of comparable magnitudes, as one recognizes from the empirical results in Section III. Our choice to fit ACFs for both models keeps an even field for the comparison.

[4]Our aim in this paper is to compare and rank the fits of the ACFs, not to conduct hypothesis tests on an ACF or its parameters. For this, the reader is referred to Brockwell and Davis (1991) where Bartlett's formula gives the required distribution theory for empirical ACFs, implying their consistency as estimators of ρ_τ. The corresponding distributional results for the ACF parameters follows by the Delta method. This is covered in Caggiano (2006), as are resampling and subsmpling approaches to estimating confidence bands for ACFs.

[5]It was often the case in the ACFs that we fitted for the AT form to have $a \approx 1$, so we could have done away with one more parameter and reduced the penalty in SC to only 3 parameters instead of 4. We preferred not to do so, in order to give AR models their best shot.

ACF. A common practice in time series is to discard a proportion of the end lags of the empirical ACF; see for instance Box and Jenkins (1976). Here, we discard the last 1/4 of these lags and use the rest for fitting the ACFs. This leaves plenty of data points to estimate the ACF parameters that measure the initial slope, curvature, amplitude, and first turning point (hence frequency); which can be inferred from the early part of the ACFs.

III. Estimation results

A. *Comparison of the AT and AR models*

We obtained annual data for all our macro variables from the Bureau of Economic Analysis (BEA), the Bureau of Labor Statistics (BLS), and the Federal Reserve Economic Data (FRED®). In order to minimize the possibility of error due to data manipulation, we do not splice series. Although the lack of splicing means that some of our series start later than the corresponding ones in NP, all of our series end in 2004 which adds 34 years of data over the 1970 end date in NP. With so much additional data, we had enough observations in the high quality datasets provided by the BEA, the BLS, and the FRED to conduct our analysis. This also allowed us to stick to annual data, rather than a higher frequency, so that our conclusions are in no way affected by treatments of seasonality. From these series, we calculated the real counterparts of the variables, and the growth rates of the level variables. We then computed the ACFs of the logarithm of the level variables, and the ACFs of the rate variables. (The programs and data files that we use are appended to the paper in electronic format.)

Table 1 presents the Schwarz criterion for the AT and the best AR model, the order p of the best AR(p), and the R^2 of the two models. In terms of R^2, the fit of the AT model is always superior. When taking into account the number of degrees of freedom through the Schwarz criterion, the AT fit

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is superior in all cases except for the nominal money stock and bond yields, where AR has a slight advantage. Even so, the fit of the two models for these cases is basically the same, and the AR has a better SC only because it has one parameter less than AT; see also footnote 5. However, comparing the fit for the *growth* rate of the money stock, AT clearly dominates AR by SC and R^2. Also, the AR fit for the *real* money stock is nowhere near as good as the AT fit.

This impressive fit for money is particularly striking in Figure 1, where we also report the fiscal components (government expenditure and tax), wages, and prices (CPI and GDP deflator). We see that the fit for prices is also outstanding. From the figure, we see a broad picture emerging whereby the memory of macro variables is of neither of the two types that AR models can produce: exponential speed of decay for I(0) or approximately linear for I(1).[6] For example, the best AR approximation for real money is basically a unit root with the implied linear ACF (clearly not the pattern displayed by the empirical ACF in the graph) and, for real wages, it is a cycle which dampens too fast because the roots of the AR are stationary.

In Figure 2, we see that GDP has dynamics that are much better approximated by AT than AR. This is true for nominal, real, and per capita GDP. The same is true also for employment and industrial production. In the case of nominal industrial production, we can see an unusual pattern of dynamics in the data: cycling that persists for a long time (does not decay fast), but that starts with an early drop in memory that misleads linear models (such as ARMA) into thinking that the memory will continue to decay fast. This type of persistent cyclical behavior is picked up by our ACF, but not by

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[6]The ACF of a unit-root process is $(1 + \tau/t)^{-1/2} \approx 1 - k\tau$ where $k \equiv 1/(2t)$ is a small constant when the process started in the distant past; see AT for details. For a given sample, this ACF can be approximated numerically by the stationary AR's ACF since $\alpha^{\tau} \equiv \exp\left((\log \alpha)\,\tau\right) \approx 1 + (\log \alpha)\tau$ by the exponential expansion when $\log \alpha \approx 0$ (i.e. $\alpha \approx 1$).

the ACF of the autoregressive model which produces cyclical but stationary roots (exponentially-fast decay of memory).

Figure 3 illustrates a series that has given so much difficulty to macro-economic modelers, and which is not in the original NP dataset. Investment, both in nominal and real terms, evolves along the lines suggested here, not as AR models would imply. Notice how closely the ACF of investment resembles the ACF of nominal industrial production seen in Figure 2.

Figure 4 displays common stock prices, a variable that was in the NP dataset. Our ACFs show that there is no stochastic trend of the unit-root type, but rather a long and asymmetric cycle. The memory drops off very rapidly after some point, unlike the prediction of unit-root models. The high autocorrelation at low lags will force a root close to one when AR models are fitted. However, inspections of the ACF indicates that this is not appropriate. Our findings are in line with the results, for individual stocks, that were first noted by De Bondt and Thaler (1985, 1987, 1989).

Figure 5 contains the remainder of the ACFs from Table 1. These include components of the trade deficit, which are so eagerly followed by practitioners because of their impact on policymakers' decisions. Again, our dynamics are much more accurate than the ones arising from ARs.

One final observation can be made. An AR(1) with a positive AR root has a globally-convex ACF, while an AR(2) or AR(3) with complex-conjugate roots has a *locally* concave ACF within each half-cycle (although the ACF decays at an exponential rate, hence "convexly" in the long run). This is why ACF estimation produces few AR(1) models in Table 1.

B. Comparison of the two models after accounting for structural breaks

In this part, we show that our results are not an artifact of the presence of a structural break. We show that, for a dataset in which there are no structural breaks, the information criterion for the AT model is still better

than for the AR model.[7] We now switch to the original NP dataset which has been extensively studied. Perron (1989) did not detect any structural breaks in the period 1946-1970 for velocity, and in the period 1930-1970 for all of his other series. We now apply the previous analysis to these periods.

Table 2 compares the two models. We find that the AT model produces a better information criterion than AR models, for all the variables, even bond yields and the money stock. In the previous dataset (Section IIIA), the two models were hard to tell apart for these two variables. However, we now find that our model still fits very well, even better than before, while the AR fit for these two variables has worsened.

C. Comparison of the two models for data that may contain deterministic trends

It is possible to incorporate deterministic trends in the analysis. If a series is suspected of having a trend, then the data can be detrended and the procedure of Section IIIA repeated. In addition, we can compare the models with and without trend by adjusting the penalty factor of SC when using detrended data. For example, if a simple linear trend is removed, then one more parameter is added to the penalty factor of SC. The intercept is the mean which is always estimated by definition in (1), and so it does not require an additional penalty. The comparison of models with and without trends should be in terms of SC and not R^2, unless R^2 is augmented to incorporate the trend's contribution to the explained sum of squares (normal SC does not depend on this quantity).

Table 3 compares the two models when a linear trend may be present. Variables in rates, such as unemployment rates, are excluded from this table,

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[7]It is also possible to estimate ACFs for series with an identified break, by a similar procedure to the one to be introduced in Section IIIC. This can be done for AR and AT models, but it did not add much to the analysis here, and it was therefore omitted.

since their generating process cannot possibly contain a simple linear trend. The only case where AR has a better SC than AT is for detrended log of real exports, with $-3.26 < -3.17$. However, this is a case where a model with trend is worse than a model without. This is evidenced by comparing the four SCs of real exports in Tables 1 and 3: the best of the four models is the AT without a trend, which has the best SC of -7.22. Incidentally, comparing the SCs of Tables 1 and 3, the only instances where accounting for a linear trend improves the AT fit (in the sense of SC) are the cases of real industrial production and real wages.

IV. Implementation and timing of macroeconomic policy

This paper does not concern itself with welfare, so we cannot study directly optimal economic policy. However, our study is still helpful in the implementation of economic policy because it reveals the dynamics of macroeconomic series. Our model predicts that changes in economic policy take time to work through the system, but not in a gradual way as was previously thought: the result is seeming inertia in the direction taken by the economy, followed by a seemingly sudden turning point. But this pattern is predictable with a good degree of confidence. Our ACFs' patterns have been substantiated by past events and have relevance for current and future debates on the timing and magnitude of macroeconomic policy interventions. They are different from existing models that misinterpret the inertia, projecting it into the future, hence missing these sudden turns.

From the previous section, the shape of the ACF of level variables (such as GDP) indicates that any impulse will decay only very slowly until the end of the ACF's plateau is reached, and that the course of these variables takes a long time to alter. Hence, economic policy should be guided by the long lags over which it operates. For instance, if the size of an economic

intervention is enough to turn around GDP quickly, the momentum imparted to it will lead to a period of overheating. Likewise, an economic policy that imparts, period after period, a stimulus to the economy will eventually build up momentum. Therefore, if a policy intervention is needed to counter the signs of a slowdown, it should:

1. occur as soon as possible to give time to the policy to operate;

2. impart a stimulus sufficient to achieve the objective, taking into account the increments that will keep occurring afterwards due to inertia; and

3. revert to a neutral stance well before the objective is achieved, letting the economy ease onto its intended path.

Interestingly, a number of recent policy oriented papers have advocated policies which react promptly to new information; see Mishkin (1999), Clarida, Galí and Gertler (1999), Bernanke and Gertler (2001). Similarly, recent speeches from Fed governors have started to favour the recommendations that we enumerated earlier; e.g. see Mishkin (2008) on the observed non-linear macroeconomic dynamics and Bernanke (2008) on the sudden turning points in the economy and the need for quick reactions. Since the end of 2007, Fed actions have been more aggressively expansionary to counter the threat of a recession, and our recommendations show that this is the right course of action.

Mishkin (2007), speaking from an empirical perspective, stresses that "what drives many macroeconomic phenomena that are particularly interesting is heterogeneity of economic agents". It is worth recalling that our new ACF's functional form arose from solving explicitly a general equilibrium model with heterogeneous agents.

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Table 1: Comparison of the AT and AR models.

In N-P?	Series	T	n	Schwarz criterion AT model	Schwarz criterion AR model	AR(p)	R^2 AT	R^2 AR
Yes	GDP (nom.)	76	55	-11.93	-8.66	2	99.5%	85.6%
Yes	GDP (real)	76	55	-10.27	-9.33	2	95.5%	86.8%
Yes	GDP per capita (real)	76	55	-11.54	-8.05	2	99.6%	83.2%
Yes	Industrial production (nom.)	76	55	-9.67	-8.39	2	96.2%	84.2%
No	Industrial production (real)	76	55	-3.57	-2.80	2	92.2%	80.4%
Yes	Employment	57	41	-11.06	-10.59	3	95.8%	92.7%
Yes	Unemployment rate	57	41	-3.41	-2.79	1	78.1%	46.6%
Yes	GDP deflator	76	55	-9.23	-6.58	2	99.2%	86.2%
Yes	Consumer prices (CPI)	76	55	-9.07	-6.35	2	99.0%	81.9%
Yes	Wages (nom.)	41	29	-11.86	-11.61	3	90.1%	85.7%
Yes	Wages (real)	41	29	-3.28	-2.82	3	92.0%	85.8%
Yes	Money stock (nom.)	46	33	-11.41	-11.46	3	89.5%	88.8%
No	Money stock (real)	46	33	-6.99	-6.57	1	84.3%	67.0%
Yes	Velocity	46	33	-6.98	-6.54	3	96.0%	93.1%
Yes	Bond yield (nom.)	76	55	-2.86	-2.85	3	87.3%	86.2%
No	Bond yield (real)	76	55	-4.66	-2.72	3	95.8%	68.2%
Yes	Common stock prices (S&P 500) (nom.)	76	55	-7.93	-4.63	2	98.8%	61.3%
No	Investment (nom.)	76	55	-8.90	-7.88	2	95.4%	85.2%
No	Investment (real)	76	55	-6.61	-6.02	2	86.8%	72.6%
No	Exports (nom.)	76	55	-8.02	-6.80	2	95.1%	80.8%
No	Exports (real)	76	55	-7.20	-6.54	2	91.3%	80.5%
No	Imports (nom.)	76	55	-8.62	-8.05	2	95.5%	90.9%
No	Imports (real)	76	55	-6.05	-5.29	2	96.7%	91.7%
No	Government current expenditures (nom.)	76	55	-11.35	-8.34	1	98.6%	63.9%
No	Government current expenditures (real)	76	55	-9.60	-7.80	3	94.5%	64.2%
No	Current tax receipts (nom.)	76	55	-12.14	-9.37	1	98.5%	69.6%
No	Current tax receipts (real)	76	55	-9.55	-6.78	3	97.6%	58.5%
No	Inflation (growth rate of CPI)	75	54	-3.74	-2.77	3	88.2%	66.4%
No	Growth rate of money stock (nom.)	45	32	-3.62	-3.14	1	80.3%	56.0%
No	Growth rate of money stock (real)	45	32	-3.29	-3.20	2	65.1%	52.3%
No	Growth rate of wages (nom.)	40	28	-2.83	-2.44	1	75.8%	48.8%
No	Growth rate of wages (real)	40	28	-3.72	-2.19	2	89.0%	35.4%

Note: T is the sample size, n is the number of ACF lags used for fitting, and p is the number of AR lags selected by SC.

Table 2: Comparison of the AT and AR models, breaks excepted.

Series	T	n	SC AT	SC AR	AR(p)	R^2 AT	R^2 AR
GDP (nominal)	42	29	-7.29	-6.59	2	87.1%	68.6%
GDP (real)	42	29	-7.01	-6.72	3	61.5%	49.9%
GDP per capita (real)	42	29	-5.89	-5.71	3	64.3%	59.6%
Industrial production (nominal)	42	29	-6.99	-6.52	3	56.5%	36.7%
Employment	42	29	-6.27	-6.06	3	67.6%	62.7%
GDP deflator	42	29	-5.66	-4.58	2	89.4%	62.2%
Consumer prices (CPI)	42	29	-5.41	-3.70	2	94.2%	61.4%
Wages (nominal)	42	29	-7.80	-5.84	2	97.5%	78.8%
Wages (real)	42	29	-8.25	-6.39	1	96.1%	67.5%
Money stock (nominal)	42	29	-8.76	-7.21	3	95.8%	79.7%
Velocity	25	17	-5.23	-5.17	2	88.0%	84.2%
Bond yield (nominal)	42	29	-5.54	-4.51	2	99.3%	97.8%
Common stock prices (S&P 500) (nominal)	42	29	-6.17	-5.14	2	99.2%	97.4%

Note: T is the sample size, n is the number of ACF lags used for fitting, and p is the number of AR lags selected by SC.

Table 3: Comparison of the AT and AR models, detrended data.

Series	Schwarz criterion AT model	Schwarz criterion AR model	AR(p)
GDP (nom.)	-4.46	-3.52	2
GDP (real)	-4.08	-1.73	2
GDP per capita (real)	-4.66	-3.43	4
Industrial production (nom.)	-4.77	-2.37	2
Industrial production (real)	-3.66	-2.73	3
Employment	-4.39	-3.64	2
GDP deflator	-3.15	-2.34	2
Consumer prices (CPI)	-3.44	-2.43	2
Wages (nom.)	-6.98	-2.48	2
Wages (real)	-4.37	-3.99	3
Money stock (nom.)	-6.80	-2.84	2
Money stock (real)	-2.68	-2.07	1
Velocity	-3.88	-3.38	2
Common stock prices (S&P 500) (nom.)	-5.18	-3.87	2
Investment (nom.)	-3.79	-2.64	2
Investment (real)	-4.89	-2.12	2
Exports (nom.)	-3.49	-3.37	2
Exports (real)	-3.07	-3.17	1
Imports (nom.)	-3.28	-2.56	2
Imports (real)	-2.79	-2.62	1
Government current expenditures (nom.)	-4.56	-3.17	2
Government current expenditures (real)	-3.86	-1.99	1
Current tax receipts (nom.)	-3.76	-2.93	2
Current tax receipts (real)	-5.69	-4.16	2

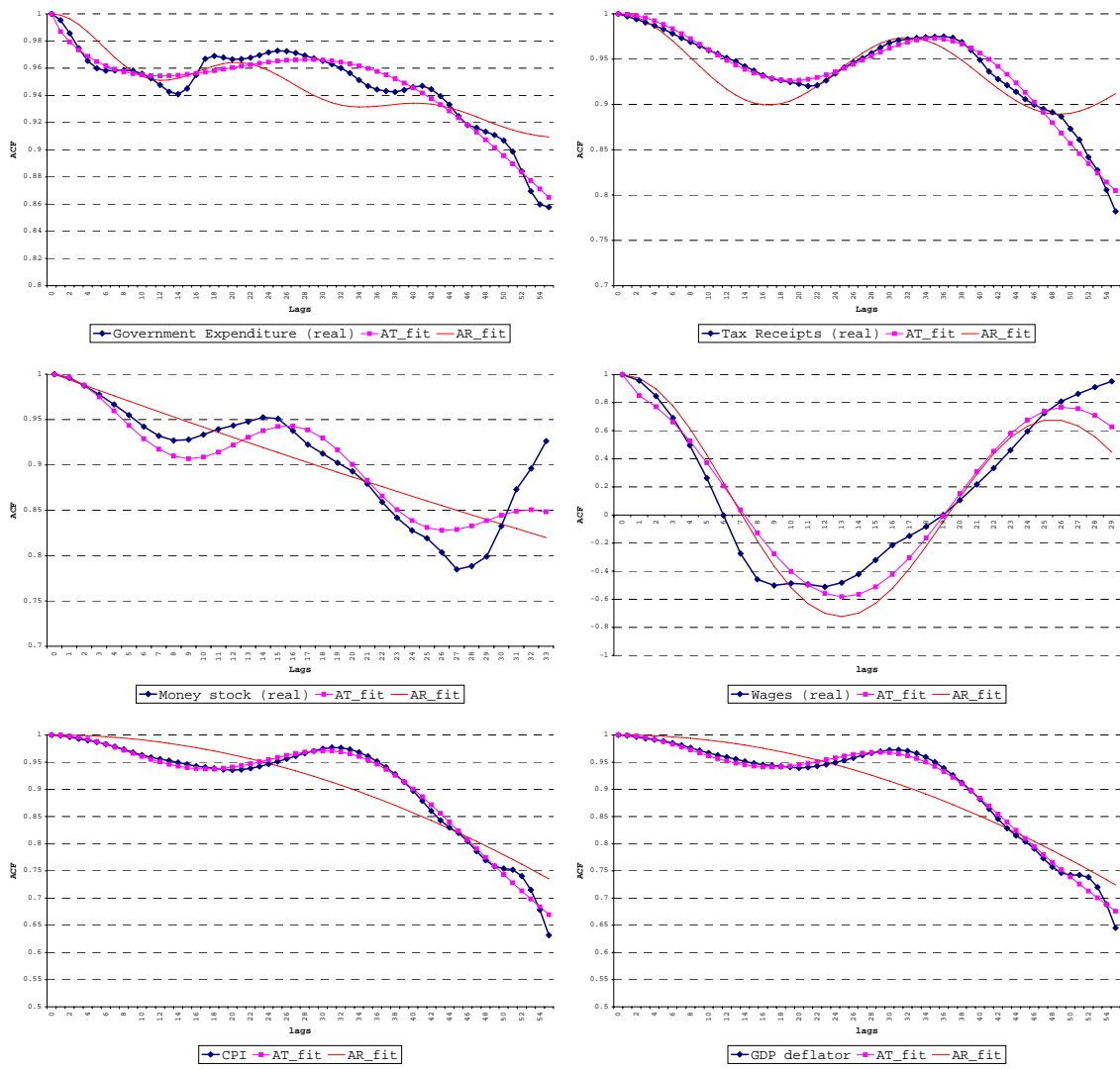Note: p is the number of AR lags selected by SC.

Figure 1. Actual ACFs, and their fits by AT and AR models:
real government expenditure and tax, real money, real wages, CPI, and
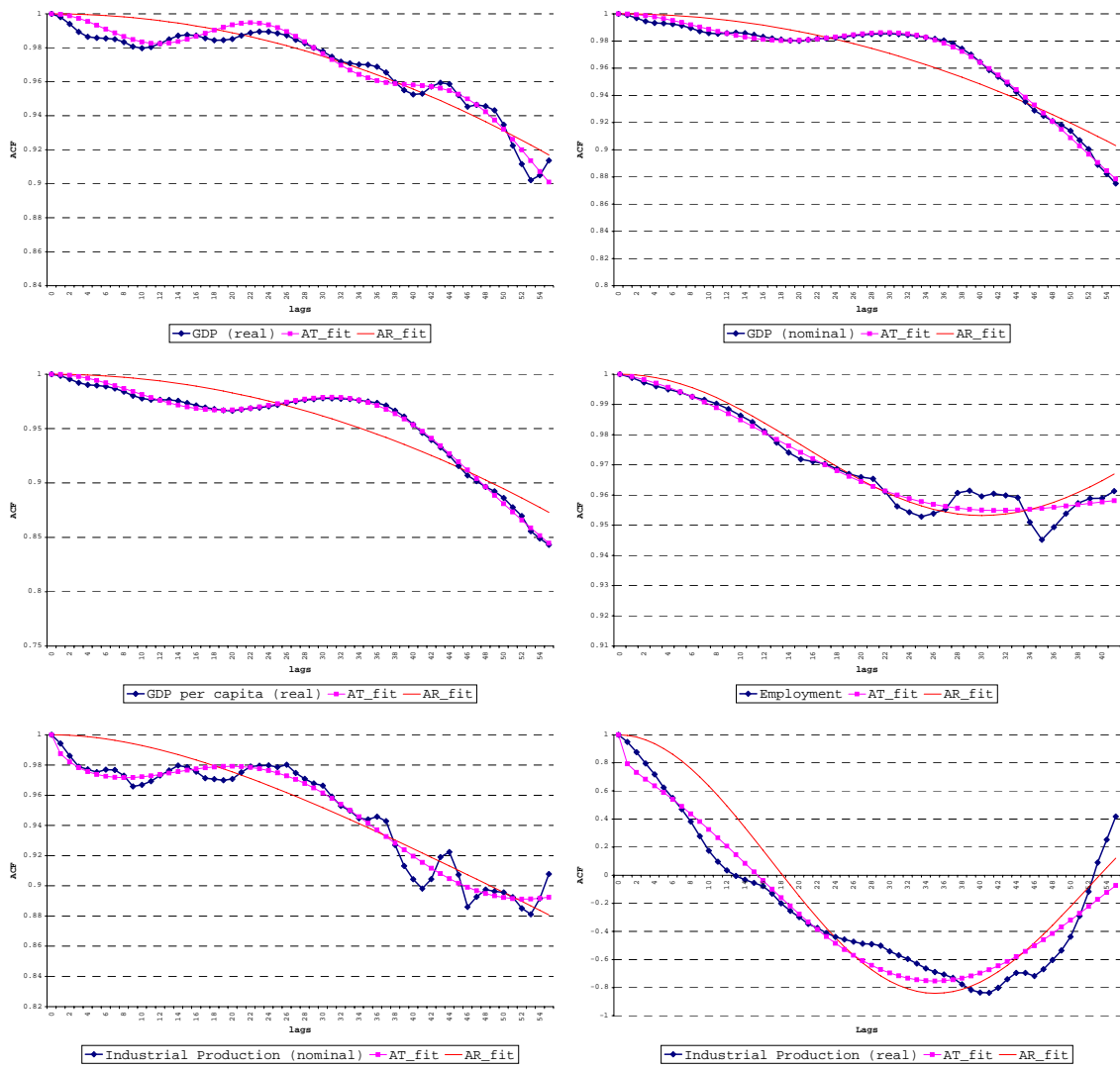GDP deflator.

Figure 2. Actual ACFs, and their fits by AT and AR models: GDP (real, nominal, and real per capita), employment, and industrial production (real and nominal).
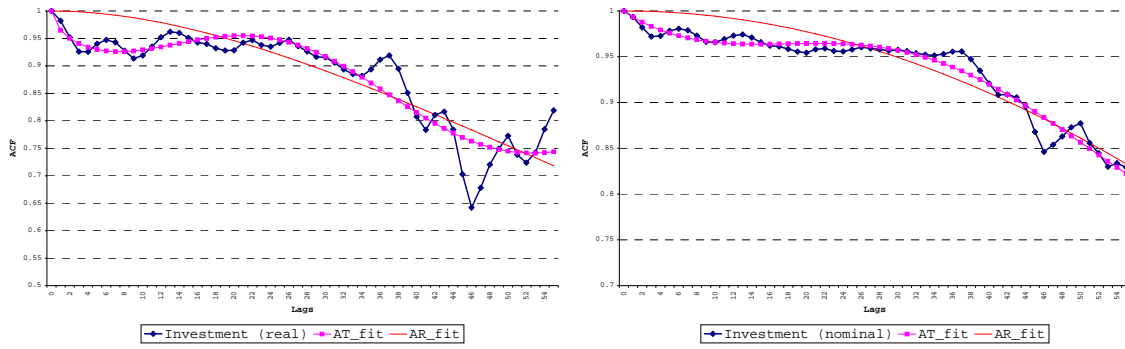
Figure 3. Actual ACFs, and their fits by AT and AR models:
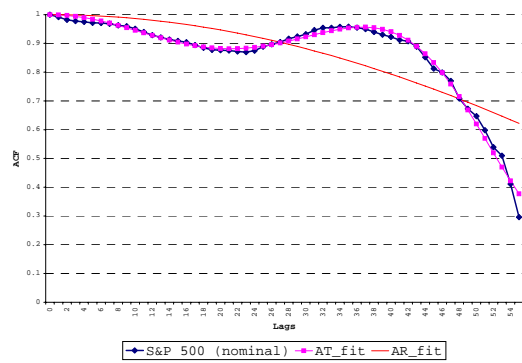investment (real and nominal).

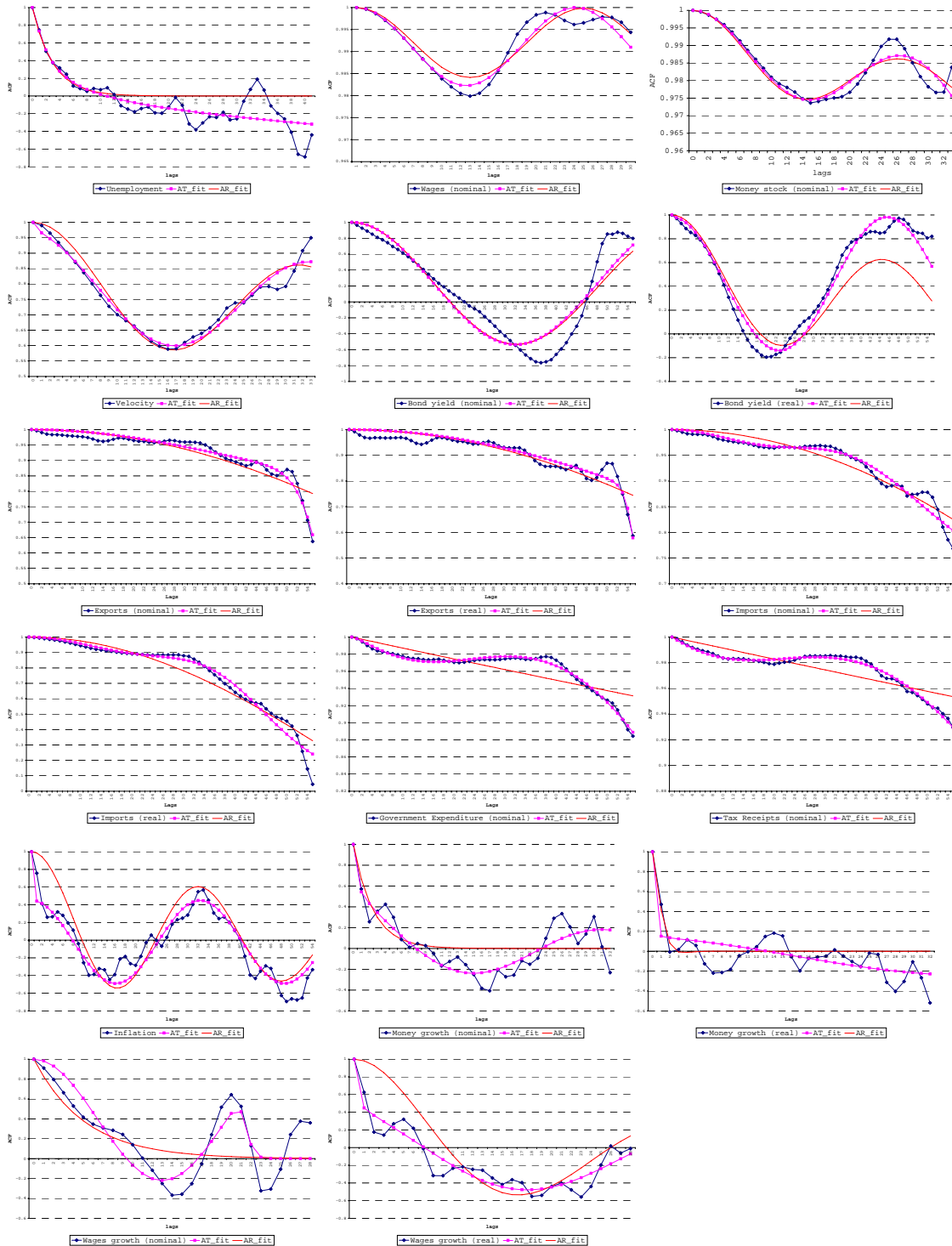Figure 4. Actual ACF and its fit by AT and AR models: S&P 500.

Figure 5. Actual ACFs, and their fits by AT and AR models:
all the other variables.