Strategic Effects and Incentives in Multi-issue Bargaining Games*

by

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Abstract

The focus of the paper is on issue-by-issue bargaining procedures in which parties are allowed to differ not only in their valuations of the issues but also in their rates of time-preference. We show that the interplay of the forces in the bargaining game is complex and standard assumptions in the literature, such as a common discount factor, can be strong. We investigate the SPE of the game when the order of the issues can be changed and show that parties can have the same preferences over agendas when they both agree over the importance of an issue or when they disagree (if corner solutions are allowed and/or there is a difficult/urgent issue).

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1 Introduction

This paper investigates multi-issue bargaining games in which players attempt to divide each surplus (or cake) as in the standard alternating-offer bargaining model (Rubinstein, 1982). The bargaining is sequential, that is, negotiations over a second surplus can start only after an agreement over the initial division has been reached and the implementation of the agreement is sequential as well, that is, as soon as an agreement is reached it can be implemented.

A similar framework has been investigated by Busch and Horstmann (1997, 1999), Inderst (2000) and In an Serrano (2002, 2003). Their analyses conclude that parties have conflicting preferences over agendas when they are restricted to choose among sequential procedures and, as a result, a simultaneous procedure in which all the issues are discussed at the same time is superior to any sequential procedure. In this paper we show that parties can agree over which one is the best sequential procedure. This consists in discussing the most important issue first, if only interior solutions are allowed, but agreement over agendas can arise even when parties have different evaluations of the issues (and corner solutions are allowed). Moreover, if there is a difficult issue (a definition is in section 4), parties can only agree in postponing such an issue, both in the cases of corner and interior solutions and even when players do not agree over the importance of the issues.

These results are based on the following key assumptions. First, parties are allowed to differ not only in their evaluations of the issues, but also in their rates of time preference. Moreover, after reaching an agreement over an issue there is an interval of time before players attempt to reach an agreement over another issue[1]. In another paper, Flamini (2001), we consider a similar framework, additionally, we allow side payments. In other words, a party can make very large concessions at the initial stage if his overall payoff is non-negative. In this paper, we exclude side-payments and, at most, a party can leave the entire surplus to his opponent. We investigate the interplay of the forces in this game (with interior and corner solutions) and show that parties can have the same preferences over sequential procedure. They can prefer the same agenda not only when there is consensus over the importance of the issues (as in the case of side payments), but also when they disagree over the importance of the issues (and corner solutions are allowed).

These results are not obvious, since there are many different strategic effects that parties need to take into account in choosing their strategies (key features are how different their relative and absolute valuations of the issues are and how different their time preferences are). As a result players may have different incentives: a player may prefer to postpone an issue if unimportant to him but important to his opponent, or he may prefer to postpone an issue important to both parties if relatively more

[1] As Muthoo (1995) points out, in general, not only does this interval exist but it is often larger than the interval of time between a rejection and a new proposal.

important to his opponent. The incentives that player need to take into account may work in opposite directions. However, we show under which conditions parties can have the same preferences over agendas.

The paper is organised as follows. The next section specifies the model. The solution of the model and its properties are included in section 2.1. This section also contains the analysis of the strategic effects that characterised the game. In section 3.1, parties are first allowed to negotiate according to a different order of the issues, then they form their preferences over agenda. In section 3, we show that there is an efficient agenda both when there is consensus over the importance of the issue and when there is not. Section 4 focuses on the case of a difficult/urgent issue.

2 The model

We consider a two-stage bargaining game in which at each stage two players, named i with $i = 1, 2$, attempt to divide a surplus. The bargaining game is sequential in the sense that the second stage can start only once a division on the first surplus is agreed. In each stage, players bargain according to a standard alternating-offer procedure (Rubinstein, 1982). That is, time is discrete, $t = 0, 1, ...$, at $t = 0$, player 1 can make an offer to player 2 who can either accept or reject it. If player 2 rejects it, then he can make a counter-proposal after an interval of time Δ passes. If there is an acceptance then the first bargaining stage ends. We assume that an interval of time τ passes between the end of the first bargaining stage (an acceptance) and the beginning of a new bargaining stage (a new proposal). Moreover, the first proposer at the second stage is the last responder at the initial stage. Player i's rate of time preference is indicated by $r_i > 0$ (and $i = 1, 2$). To take into account that there are intervals of time of different lengths, we define players discount factors as follows: player i's *within-cake* discount factor $\delta_i = \exp(-r_i \Delta)$ applies after a rejection and his *between-cake* discount factor $\alpha_i = \exp(-r_i \tau)$ applies after an acceptance. Moreover, players are allowed to have different valuations of the issues. A positive parameter λ_i indicates the relative importance of cake i to player i (see payoff functions below), with $i = 1, 2$.

The implementation of the agreement is assumed to be sequential, that is, a division is implemented as soon as it is agreed. If an agreement is not reached on the partition of a cake, players get zero payoffs (disagreement) at that stage. Then, if disagreement takes place at the first stage, the second stage cannot take place and players' overall payoffs are zero. In our framework, we consider two agendas, agenda i states that cake i is negotiated first, with $i = 1, 2$. In this section we focus on agenda 1. If, after t rounds, an agreement is reached on the division of the first cake, $(x, 1 - x)$, where x is the share player 1 obtains, and after $n + 1$ periods (a period of length τ and n periods of length Δ) another agreement is reached $(1 - y, y)$, then the payoff player i obtains, v_i, is as follows, with $i = 1, 2$.

$$v_1 = \delta_1^t(\lambda_1 x + \delta_1^n \alpha_1 (1 - y)) \quad (1)$$

$$v_2 = \delta_2^t(1 - x + \delta_2^n \alpha_2 \lambda_2 y) \quad (2)$$

Our technical assumption is as follows. Let $\underline{\lambda}_2 \leq \lambda_2 \leq \overline{\lambda}_2$ where

$$\underline{\lambda}_2 = \frac{\alpha_2(1-\delta_1)(1-\delta_2^2)}{\delta_2(1-\delta_1\delta_2)} \text{ and } \overline{\lambda}_2 = \frac{\delta_2(1-\delta_1\delta_2)}{\alpha_2(1-\delta_1)(1-\delta_2^2)}$$

This assumption allows us to simplify the presentation. This is not a restrictive assumption since in the most interesting case in which some frictions tend to disappear (i.e., $\Delta \rightarrow 0$), these bounds tends to include the entire positive real range (i.e., $\underline{\lambda}_2 \rightarrow 0$ and $\overline{\lambda}_2 \rightarrow \infty$).

2.1 Equilibrium

Let

$$a = \frac{\lambda_1(1-\delta_1\delta_2+\alpha_2\lambda_2(1+\delta_2)(1-\delta_1))}{\delta_2(1-\delta_1^2)}, \; b = \frac{\lambda_1[(1-\delta_2^2)\alpha_2\lambda_2 - \delta_2(1-\delta_1\delta_2)]}{\delta_2(1+\delta_1)(1-\delta_2)}$$

$$f = \frac{\lambda_1[(1-\delta_2^2)\alpha_2\lambda_2\delta_1 - (1-\delta_1\delta_2)]}{(1+\delta_1)(1-\delta_2)}, \; g = \frac{\lambda_1\delta_1(1-\delta_1\delta_2+\alpha_2\lambda_2(1+\delta_2)(1-\delta_1))}{(1-\delta_1^2)}$$

For α_1 that varies between the boundaries[2] $f \leq b \leq g \leq a$, we can define different SPE demands. There are at most three SPE with immediate agreement (when $0 <$

[2]The assumption $\lambda_2 \leq \overline{\lambda}_2$ implies that $b \leq g$. When the frictions within the bargaining stage tends to disappear, $\Delta \rightarrow 0$, $f = b$ and $g = a$.

$b < g < 1$). These are described in the following proposition. Some of these SPE do not exist if some of the boundaries do not belong to the[3] interval (0,1).

Proposition 1 *Let* $\lambda_2 \leq \overline{\lambda}_2$ *with* $i = 1, 2$, *then there is a unique SPE in which the agreement is reached immediately over the partition of every single cake. At the second stage, parties play as in the RBM. At the first stage, the equilibrium demand of player* 1 *(*2*) is* x_1 *(*y_2*, respectively), as defined in the following three cases.*

1) If $0 \leq \alpha_1 \leq b$, *then the equilibrium demands at the first stage are* $x_1 = 1$ *and* $y_2 = \widetilde{y}_2 \in (0, 1)$, *defined below*

$$\widetilde{y}_2 = \frac{(1-\delta_1)[(1-\delta_1\delta_2)\lambda_1 + \alpha_1(1+\delta_1)(1-\delta_2)]}{\lambda_1(1-\delta_1\delta_2)} \tag{3}$$

Then the equilibrium payoffs are as follows:

$$v_1 = \frac{(1-\delta_1\delta_2)\lambda_1 + \alpha_1\delta_1(1-\delta_2)}{1-\delta_1\delta_2} \tag{4}$$

$$v_2 = \lambda_2\alpha_2\frac{1-\delta_1}{1-\delta_1\delta_2} \tag{5}$$

2) If $b \leq \alpha_1 \leq g$, *the equilibrium demands are defined in (6) and (7) below*

$$x_1 = \frac{(1-\delta_2)[(1-\delta_1\delta_2)\lambda_1 + (1-\delta_1)(\alpha_2\lambda_1\lambda_2(1+\delta_2) - \delta_2\alpha_1(1+\delta_1))]}{\lambda_1(1-\delta_1\delta_2)^2} \tag{6}$$

$$y_2 = \frac{(1-\delta_1)[(1-\delta_1\delta_2)\lambda_1 + (1-\delta_2)(\alpha_1(1+\delta_1) - \alpha_2\lambda_1\lambda_2\delta_1(1+\delta_2))]}{\lambda_1(1-\delta_1\delta_2)^2} \tag{7}$$

[3] For instance, if $b < 0$, then the first equilibrium specified in the proposition below does not exist.

and the equilibrium payoffs are as follows:

$$v_1 = \frac{1-\delta_2}{(1-\delta_1\delta_2)^2}[(1-\delta_1\delta_2)(1+\alpha_2\lambda_2)\lambda_1 + (\delta_1-\delta_2)(\alpha_1-\alpha_2\lambda_1\lambda_2)] \quad (8)$$

$$v_2 = \frac{(1-\delta_1)\delta_2}{(1-\delta_1\delta_2)^2\lambda_1}[(1-\delta_1\delta_2)(\lambda_1+\alpha_1) + (\delta_1-\delta_2)(\alpha_1-\alpha_2\lambda_1\lambda_2)] \quad (9)$$

3) If $g \leq \alpha_1 \leq 1$, the equilibrium demands are $y_2 = 1$ and $x_1 = \widetilde{x}_1 \in (0,1)$, where

$$\widetilde{x}_1 = \frac{(1-\delta_2)[1-\delta_1\delta_2+\alpha_2\lambda_2(1-\delta_1)(1+\delta_2)]}{(1-\delta_1\delta_2)} \quad (10)$$

and the equilibrium payoffs are as follows:

$$v_1 = \frac{(1-\delta_2)[\lambda_1(1-\delta_1\delta_2+\alpha_2\lambda_2(1-\delta_1)(1+\delta_2)) + \alpha_1\delta_1]}{1-\delta_1\delta_2} \quad (11)$$

$$v_2 = \frac{\delta_2[1-\delta_1\delta_2+\alpha_2\lambda_2(1-\delta_1)]}{1-\delta_1\delta_2} \quad (12)$$

Proof. The indifference conditions between accepting and rejecting an offer are the following

$$\begin{cases} 1-x_1+\alpha_2\lambda_2\frac{1-\delta_1}{1-\delta_1\delta_2} = \delta_2\left(y_2+\alpha_2\lambda_2\frac{(1-\delta_1)\delta_2}{1-\delta_1\delta_2}\right) \\ (1-y_2)\lambda_1+\alpha_1\frac{1-\delta_2}{1-\delta_1\delta_2} = \delta_1\left(x_1\lambda_1+\alpha_1\frac{(1-\delta_2)\delta_1}{1-\delta_1\delta_2}\right) \end{cases} \quad (13)$$

The solution of the system (13) are the demands x_1 and y_2 defined in (6) and (7) above. It can be shown that $x_1 > 0$ if and only if $\alpha_1 < a$ and $x_1 < 1$ if and only if $\alpha_1 > b$. Similarly, $y_2 > 0$ if and only if $\alpha_1 > f$ and $y_2 < 1$ if and only if $\alpha_1 < g$. It is straightforward to see that $f \leq b$ and $g \leq a$. Then, there is an intersection between the interval $[b, a]$ and $[f, g]$ if and only if $g > b$, that is $\lambda_2 \leq \overline{\lambda}_2$. Under this condition we can distinguish three sections on the α_1 axis. First, when $0 \leq \alpha_1 \leq b$, the equilibrium demands are $x_1 = 1$ and $y_2 = \widetilde{y}_2$, where $\widetilde{y}_2$, defined in (3) is such that player 1 is indifferent between accepting the demand $\widetilde{y}_2$ or rejecting it to demand $x_1 = 1$. Since $\alpha_1 \leq b$ and $\lambda_2 \leq \overline{\lambda}_2$, $\widetilde{y}_2 < 1$. Second, when $b \leq \alpha_1 \leq g$, then the solution of the system (13) is given by the demands in (6) and (7), since these are interior, they are SPE demands. Third, when $g \leq \alpha_1 \leq 1$, the equilibrium demands are $y_2 = 1$ and $x_1 = \widetilde{x}_1$, where $\widetilde{x}_1$, defined in (10) is such that player 2 is indifferent between accepting the demand $\widetilde{x}_1$ or rejecting it to demand $y_2 = 1$. Following standard arguments (see for instance, Osborne and Rubinstein, 1990), it can be shown that these solutions define a unique SPE. ■

The equilibrium specified above has interesting characteristics. First of all, players' demands in equilibrium are complicated functions of the parameters of the model and typical results obtained in the context of bargaining over a single cake (for instance a more patient player obtains a larger utility), may not exist in this game. The following corollary presents some comparative statics. These results are immediate consequences of Proposition 1. A discussion of these results follows.

Corollary 1 *The equilibrium outcome defined in Proposition 1, part 2, is characterised by the following:*

1) if α_1 increases, x_1 decreases, v_1 increases (decreases) if $\delta_1 > \delta_2$ ($\delta_1 > \delta_2$ respectively) and v_2 increases;

2) if α_2 (or λ_2) increases, x_1 increases, v_1 increases and v_2 increases (decreases) *if* $\delta_2 > \delta_1 (\delta_2 < \delta_1$ *respectively);*

3*) if* λ_1 *increases,* x_1 *increases,* v_1 *increases and* v_2 *decreases;*

4) if δ_1 *increases,* v_1 *increases (decreases) if*

$$(1+\alpha_2\lambda_2)\delta_2\lambda_1(1-\delta_1\delta_2)+(\alpha_1-\alpha_2\lambda_1\lambda_2)(1-\delta_2^2+\delta_2(\delta_1-\delta_2))>0(<0 \text{ respect.}) \quad (14)$$

while v_2 *decreases (increases) if*

$$-(1+\alpha_2\lambda_2)\lambda_1(1-\delta_1\delta_2)-2(\delta_1-\delta_2)(\alpha_1-\alpha_2\lambda_1\lambda_2)<0 \ (>0 \text{ respect.}) \quad (15)$$

5) if δ_2 *increases* v_1 *decreases (increases) if*

$$[2(\delta_1-\delta_2)(\alpha_2\lambda_1\lambda_2-\alpha_1)-(1-\delta_1\delta_2)(\lambda_1+\alpha_1)]<0 \ (>0 \text{ respect.}) \quad (16)$$

while v_2 *increases (decreases) if*

$$(\alpha_1+\lambda_1)(1-\delta_1\delta_2)+(\alpha_1-\alpha_2\lambda_1\lambda_2)(\delta_1-\delta_2-\delta_2(1-\delta_1^2))>0 \ (<0 \text{ respect.}) \quad (17)$$

The equilibrium outcome defined in Proposition 1, part 3, is characterised by the following:

6) if α_1 *increases,* v_1 *increases while* $\widetilde{x}_1$ *and* v_2 *remain unchanged;*

7) if α_2 *(or* λ_2*) increases,* $\widetilde{x}_1$*,* v_1 *and* v_2 *increase;*

8*) if* λ_1 *increases,* v_1 *increases, while* v_2 *and* $\widetilde{x}_1$ *remain unchanged;*

9) if δ_1 *increases,* $\widetilde{x}_1$ *and* v_2 *decrease, while* v_1 *increases (decreases) if*

$$\alpha_1-\alpha_2\lambda_1\lambda_2(1-\delta_2^2)>0(<0 \text{ respect.}) \quad (18)$$

10) if δ_2 increases v_2 increases, while v_1 decreases (increases) if

$$\lambda_1[(1-\delta_1\delta_2)^2-\alpha_2\lambda_2(1-\delta_1)(\delta_1(1+\delta_2^2)-2\delta_2)]+\alpha_1\delta_1(1-\delta_1)>0\ (<0\text{ respect.})\quad(19)$$

Corollary 1 does not include a discussion of the first equilibrium defined in proposition 1, since this is strictly related to the outcome of a standard one-cake bargaining game (given that player 1 is able to extract the entire surplus at the initial stage). Corollary 1 shows that the interactions of the discount factors (α_i, δ_j) are an important feature of the interplay of forces in this game. When the between-cake discount factor increases for the first mover (α_1), that is, player 1 discounts less strongly the payoff obtained in the second stage, this is not always good news for such a player (see point 1 of Corollary 1). First of all, player 1 makes a larger concession at the first stage (x_1 decreases) to facilitate the initial agreement. Obviously, player 2's payoff increases when he obtains a larger share of the first division. However, player 1's payoff, v_1, increases only if, in the within-stage negotiations, player 1 is more patient than his opponent $(\delta_1 > \delta_2)$. In other words, player 1's concession at the first bargaining stage is too large if he fears a rejection more than his opponent does. This is an interesting feature of the bargaining game since being more patient is often associated with a higher payoff, but in this game this is not necessarily true, it depends on the link between the within-cake discount factors.

A similar effect on the equilibrium outcome can also be shown when, in the between-stage negotiations, the first responder becomes more patient (α_2 increases, see point 2 of Corollary 1). Given the structure of the game under agenda 1, the parameters α_2 and λ_2 play exactly the same role under this agenda. When we consider the equilibrium demand $\widetilde{x}_1$ defined in (10) the effects are all unambiguous. In particular, an increase the between-cake discount factor α_2 (or the parameter λ_2) has a positive effects on both players' payoffs, regardless of players' rates of time preference. Instead, the effect of an increase in the relative importance of cake 1 to player 1 (λ_1) is unambiguous both in the case of corner solutions and in the case of interior solutions. For the latter, player 1 is better off since he is able to extract a larger share at the first division (point 3 of Corollary 1). In other words, if the first issue becomes relatively more important to player 1, he is able to extract a larger share at this stage. Again for the case of corner solutions (point 8 of Corollary 1), the parameters interact in a less complicated way, in particular $\widetilde{x}_1$ is unchanged but obviously player 1 is better off.

The effects of a change in the within-cake discount factor δ_i on players' payoffs highlight much more complex relationships among the parameters (even for the case of corner solutions). Indeed, these effects depend not only on the relationship between players' time preferences but also on players' valuations of the second bargaining stage. To be more precise, the effect of δ_1 on player 1's payoff is positive when player 1 is more patient than his opponent ($\delta_1 > \delta_2$) and he values the second bargaining stage more than his opponent does, that is, $\alpha_1/\lambda_1 \geq \alpha_2\lambda_2$ (see points 4 of corollary

1). Under these conditions also the other effects of δ_j on v_i (with $i, j = 1, 2$) is as in the standard one-cake bargaining theory, that is, negative for $i \neq j$, and positive otherwise[4]. When these conditions are relaxed, the effects of the δ_i on equilibrium payoffs may be ambiguous (see points 4 and 5 of corollary 1). For the case of corner solutions, we can obtain similar effects as in the standard single-cake bargaining game when player 1 values the second bargaining stage more than his opponent does, that is, $\alpha_1/\lambda_1 \geq \alpha_2\lambda_2$ (see points 9 of corollary 1) either he is more impatient than his opponent (expression (19) is negative if $\delta_1 < \delta_2$) or the interval of time between an acceptance and a new proposal goes to zero (again (19) is negative for $\Delta \to 0$).

A common assumption in the literature is to assume that parties have the same discount factors $\delta_i = \delta$, with $i = 1, 2$, in this case the interplay of the forces in the bargaining process with SPE defined by part 2 of proposition 1 is greatly simplified. As a result player i's payoff does not depend on α_i with $i = 1, 2$. Moreover, player 2's payoff is also independent of his relative valuation of cake 2 (λ_2), while the relative importance of the first cake between players λ_1 still plays a role (as indicated in point 3 of Corollary 1). For the equilibrium outcome defined by the demand $\widetilde{x}_1$ (defined in (10)) the assumption of a common discount factor does not have a great impact on the interplay of the forces, since this is already simplified by the fact that a player

[4]The effect of δ_2 on v_2 is negative, since $\alpha_1 + \lambda_1 > \alpha_1 - \alpha_2\lambda_1\lambda_2$ and $1 - \delta_1\delta_2 > \delta_1 - \delta_2 \geq \delta_1 - \delta_2 - \delta_2(1 - \delta_1^2)$.

asks for the entire surplus at the initial stage.

To investigate further the subtle effects of the parameters on the equilibrium outcome of the bargaining game we focus on the case in which the interval of time between a rejection and a new proposal, Δ, tends zero.

Corollary 2 *Under the conditions specified in Proposition 1, in the limit as* Δ *tends to zero, that is, for* $\lambda_i > 0$ *and for* α_1 *that varies in the intervals with extremes* $0 \leq b_l \leq g_l \leq 1$, *with*

$$b_l \equiv \lim_{\Delta \to 0} b = \frac{\lambda_1(2\alpha_2\lambda_2 r_2 - (r_1 + r_2))}{2r_2}, \; g_l \equiv \lim_{\Delta \to 0} g = \frac{\lambda_1(2\alpha_2\lambda_2 r_1 + r_1 + r_2)}{2r_1}$$

there is a unique SPE in which the agreement is reached immediately over the partition of every single cake. At the second stage, player i *demands the Rubinsteinian share* ($\frac{r_j}{r_1+r_2}$ with $i, j = 1, 2$ and $i \neq j$) *while in the first stage the SPE demands by player 1 and 2 respectively are as follows.*

1) If $0 \leq \alpha_1 \leq b_l$, *then the equilibrium demands at the first stage are* $x_1 = 1$ *and* $\widetilde{y}_2^l \equiv \lim_{\Delta \to 0} \widetilde{y}_2 = 0$.

2) If $b_l \leq \alpha_1 \leq g_l$, *the equilibrium demands are defined in (6) and (7) below*

$$_l x_1 = \frac{r_2[(r_1 + r_2)\lambda_2 + 2r_1(\alpha_2\lambda_1\lambda_2 - \alpha_1)]}{\lambda_1(r_1 + r_2)^2} \quad (20)$$

$$_l y_2 = \frac{r_1[(r_1 + r_2)\lambda_1 + 2r_2(\alpha_1 - \alpha_2\lambda_1\lambda_2)]}{\lambda_1(r_1 + r_2)^2} \quad (21)$$

and the equilibrium payoffs are as follows:

$$
{}_l v_1 = \frac{r_2}{(r_1+r_2)^2}[(r_1+r_2)(1+\alpha_2\lambda_2)\lambda_1 + (r_2-r_1)(\alpha_1-\alpha_2\lambda_1\lambda_2)] \qquad (22)
$$

$$
{}_l v_2 = \frac{r_1}{(r_1+r_2)^2\lambda_1}[(r_1+r_2)(1+\alpha_1\lambda_1) + (r_2-r_1)(\alpha_1-\alpha_2\lambda_1\lambda_2)] \qquad (23)
$$

3) If $g \leq \alpha_1 \leq 1$, the equilibrium demands are $y_2 = 1$ and $x_1 = \widetilde{x}_1^l \equiv \lim_{\Delta\to 0}\widetilde{x}_1 =$ 0.

Corollary 3 *For $r_i\tau$ small with $i = 1,2$, demand ${}_l x_1$ defined in (20) is strictly increasing (decreasing) in τ when $r_1 > r_2\lambda_1\lambda_2$ ($r_1 < r_2\lambda_1\lambda_2$, respectively), payoff ${}_l v_1$ is strictly decreasing (increasing) in τ when $r_1 < r_2(1+2\lambda_1\lambda_2)$ ($r_1 > r_2(1+2\lambda_1\lambda_2)$ respectively) and ${}_l v_2$ is strictly decreasing (increasing) in τ when $r_2 < r_1(2+\lambda_1\lambda_2)$ ($r_2 > r_1(2+\lambda_1\lambda_2)$ respectively).*

Proof. When $r_i\tau$ is small, the between-cake discount factor α_i can be approximated by $1 - r_i\tau$. The results then follow directly from Corollary 2. ■

In Corollary 3 the focus is on the SPE demands defined by interior solutions, since for the other cases the demands are extreme (simply 0 or 1). When the interval of time Δ tends to zero, obviously we can re-establish some of the results of the comparative statics presented in corollary 1 (with reference to $-r_i$ rather δ_i). However, as Corollary 3 shows, additional effects on the equilibrium outcome can be highlighted in the case of small interval of time (for Δ that tends to zero and $r_i\tau$ is small). First of all, if the interval τ increases, that is, the future payoffs are discounted more strongly by both players, and players have the same rate of time preference ($r_1 = r_2$), then, as one would expect, both players are worse off (since the frictions to reach the second bargaining stage increase). More interestingly, the effect on the SPE initial division is more subtle. The share that player 1 obtains at the first stage ${}_Ix_1$ decreases, if the relative importance of cake 1 to player 1, λ_1, is larger than the relative importance of cake 1 to player 2, $1/\lambda_2$ (and vice-versa). In other words, player 2 is able to obtain a more profitable initial agreement when he minds relatively less about the initial issue and despite the fact that player 1 minds more about the initial issue is unable to extract a larger surplus. The intuition is that when player 2's relatively more important issue is discussed second and the payoffs obtained at the second bargaining stage are discounted more strongly, he is able to play in a "tougher" way at the initial stage.

When players have different time preferences, some of the previous effects are reversed. In particular a player could obtain a larger payoff when the frictions (τ) increase. Let player 1 be more patient than player 2 ($r_1 < r_2$) then player 1 obtains a smaller share at the initial division, when the relative importance of cake 2 to player 1 is larger than the relative importance of cake 2 to player 2 (i.e., $1/\lambda_1 > \lambda_2$). Under these circumstances the effect of an increase in the interval of time τ on player 2's payoff is ambiguous. Indeed, there is a trade-off, on one hand, player 2 is worse off since the payoff obtained from future negotiations are discounted more strongly when τ increases; on the other hand, player 2 is better off since he can get a larger share at the initial bargaining stage when $r_1 < r_2\lambda_1\lambda_2$. The overall effects of an increase in τ on player 2's payoff is positive only if player 1 is sufficiently more patient than player 2 ($r_1 < r_2/(2+\lambda_1\lambda_2)$), in other words, the initial concessions are sufficiently large.

3 Changing the Order of the Issues

In this section, the focus is on agenda 2, in which the first issue is represented by cake 2. We then show what incentives parties need to take into account in forming their preferences over agendas. Let

$$\begin{aligned} n &= \frac{\lambda_2(1-\delta_1\delta_2)+\alpha_2(1+\delta_2)(1-\delta_1))}{\lambda_1\lambda_2\delta_2(1-\delta_1^2)},\ o = \frac{(1-\delta_2^2)\alpha_2-\lambda_2\delta_2(1-\delta_1\delta_2)]}{\lambda_1\lambda_2\delta_2(1+\delta_1)(1-\delta_2)} \\ p &= \frac{(1-\delta_2^2)\alpha_2\delta_1-\lambda_2(1-\delta_1\delta_2)]}{\lambda_1\lambda_2(1+\delta_1)(1-\delta_2)},\ q = \frac{\delta_1(\lambda_2(1-\delta_1\delta_2)+\alpha_2(1+\delta_2)(1-\delta_1))}{\lambda_1\lambda_2(1-\delta_1^2)} \end{aligned}$$

As for the analysis of agenda 1, for α_1 that varies between the boundaries $p \leq o \leq q \leq n$, we can define different demands in SPE with immediate agreement. These are at most three.

Proposition 2 *Let $\lambda_2 \succeq \underline{\lambda}_2$ with $i = 1, 2$, then there is an SPE in which the agreement is reached immediately over the partition of every single cake. At the second stage, parties play as in the RBM. At the first stage, the equilibrium demand of player 1 (2) is x_1 (y_2, respectively), as defined in the following cases.*

1) If $0 \leq \alpha_1 \leq o$, the equilibrium demands are $x_1 = 1$ and $y_2 = \widetilde{y}_2 \in (0, 1)$, defined *below:*

$$\widetilde{y}_2 = \frac{(1-\delta_1)[1-\delta_1\delta_2+\alpha_1\lambda_1(1+\delta_1)(1-\delta_2)]}{(1-\delta_1\delta_2)} \tag{24}$$

Then the equilibrium payoffs are as follows:

$$u_1 = \frac{1-\delta_1\delta_2+\alpha_1\delta_1\lambda_1(1-\delta_1\delta_2)}{1-\delta_1\delta_2} \tag{25}$$

$$u_2 = \frac{\alpha_2(1-\delta_1)}{1-\delta_1\delta_2} \tag{26}$$

2) If $o \leq \alpha_1 \leq q$*, the equilibrium demands are defined in (27) and (28) below*

$$x_1 = \frac{(1-\delta_2)[(1-\delta_1\delta_2)\lambda_1+(1-\delta_1)(\alpha_2\lambda_1\lambda_2(1+\delta_2)-\delta_2\alpha_1(1+\delta_1))]}{\lambda_1(1-\delta_1\delta_2)^2} \tag{27}$$

$$y_2 = \frac{(1-\delta_1)[(1-\delta_1\delta_2)\lambda_1+(1-\delta_2)(\alpha_1(1+\delta_1)-\alpha_2\lambda_1\lambda_2\delta_1(1+\delta_2))]}{\lambda_1(1-\delta_1\delta_2)^2} \tag{28}$$

and the equilibrium payoffs are as follows:

$$u_1 = \frac{1-\delta_2}{\lambda_2(1-\delta_1\delta_2)^2}[(\lambda_2+\alpha_2)(1-\delta_1\delta_2)+(\alpha_2-\alpha_1\lambda_1\lambda_2)(\delta_2-\delta_1)] \tag{29}$$

$$u_2 = \frac{\delta_2(1-\delta_1)}{(1-\delta_1\delta_2)^2}[\lambda_2(1+\alpha_1\lambda_1)(1-\delta_1\delta_2)+(\alpha_2-\alpha_1\lambda_1\lambda_2)(\delta_2-\delta_1)] \tag{30}$$

3) If $q \leq \alpha_1 \leq 1$*, the equilibrium demands are* $y_2 = 1$ *and* $x_1 = \widetilde{x}_1 \in (0,1)$*, where*

$$\widetilde{x}_1 = \frac{(1-\delta_2)[(1-\delta_1\delta_2)\lambda_2+\alpha_2(1-\delta_1)(1+\delta_2)]}{(1-\delta_1\delta_2)\lambda_2} \tag{31}$$

and the equilibrium payoffs are as follows:

$$u_1 = \frac{(1-\delta_2)[\lambda_2(1-\delta_1\delta_2)+\alpha_2(1+\delta_2)(1-\delta_1)+\alpha_1\delta_1\lambda_1\lambda_2]}{\lambda_2(1-\delta_1\delta_2)}$$

$$u_2 = \frac{\delta_2[\lambda_2(1-\delta_1\delta_2)+\alpha_2\delta_2(1-\delta_1)]}{1-\delta_1\delta_2}$$

Proof. The proof follows the same reasoning as in Proposition 1.

The interplay of the forces in this game is similar to the ones defined for Agenda 1, with the only difference that now the first issue is represented by cake 2 rather than 1. To show which incentives are the dominant ones we look at the case in which parties can form preferences over agendas.

3.1 The Best Agenda

Whenever the differences in player i's payoffs $v_i - u_i$ (with v_i and u_i defined in Proposition 1 and 2) for $i = 1, 2$, have the same sign, players prefer the same agenda. In the following proposition we show that the best agenda exists both in the case in which there is consensus over the importance of the issues, and in the case in which parties have different preferences over issues.

Proposition 3 *In sequential bargaining procedures, there is consensus over agendas. When players demand interior solutions in both agendas, then the best agenda consists in setting the most important issue first, assuming that a player is characterised by a sufficiently small* α_i*. When at least one player demands an extreme share in equilibrium, then consensus over agenda can arise also when parties have different preferences over issues.*

Proof. The proof consists in analysing the sign of the differences $v_i - u_i$ for any i. Since each equilibrium payoff v_i and u_i can assume at most three values, according to the value that α_i assumes (see Proposition 1 and 2), then we need to consider the following seven cases:

1) In both agendas the SPE outcome is that player 1 demands the entire surplus (that is, $\alpha_i < \min\{b, o\}$). In this case, player 1 prefers agenda 1 (2 respectively) if

$$\frac{(\lambda_1 - 1)(1 - \delta_1\delta_2 - \alpha_1\delta_1(1-\delta_2))}{1+\delta_1\delta_2} > 0(< 0, \text{ respect.}) \tag{32}$$

while player 2 prefers Agenda 1 if

$$\frac{\alpha_2(1-\delta_1)(1-\lambda_2)}{1+\delta_1\delta_2} > 0(< 0, \text{ respect.}) \tag{33}$$

At the limit for $\Delta \to 0$, expression (32) is positive when $\lambda_1 > 1$. Then both players prefer the agenda that sets the most important issue first (e.g., agenda 1 if $\lambda_1 > 1$ and $\lambda_2 < 1$).

2) In both agendas the SPE outcome is that player 1 demands the interior solution of the system of indifference condition (that is, $\max\{b, o\} < \alpha_i < \min\{g, q\}$). In this case, agenda 1 is preferred by player 1 when (34) below is positive and vice-versa agenda 2 is favoured when (34) below is negative.

$$\frac{(1-\delta_2)}{(1-\delta_1\delta_2)^2\lambda_2}[(\lambda_1\lambda_2^2 - 1)(1+\delta_2)\alpha_2(1-\delta_1) + \lambda_2(\lambda_1 - 1)(1-\delta_1\delta_2 + \alpha_1(\delta_2 - \delta_1))] \tag{34}$$

Similarly, agenda 1 (2) is preferred by player 2 when (35) below is positive (negative, respectively):

$$\frac{\delta_2(1-\delta_1)}{(1-\delta_1\delta_2)^2\lambda_1}[(1-\lambda_1^2\lambda_2)(1+\delta_1)\alpha_1(1-\delta_2) + \lambda_1(1-\lambda_2)(1-\delta_1\delta_2 + \alpha_2(\delta_1 - \delta_2))] \tag{35}$$

Without loss of generality, let cake 1 represent the most important issue ($\lambda_1 > 1$ and $\lambda_2 < 1$), then if the between-cake discount factor of one player α_i is sufficiently small, that is, $\alpha_i \leq \underline{\alpha_i} = \Phi_i \Gamma_i$ where

$$\Gamma_i = \frac{(1-\lambda_j)\lambda_i}{(\lambda_i^2 \lambda_j - 1)} \text{ and } \Phi_i = \frac{(1-\delta_i\delta_j + \alpha_j(\delta_i - \delta_j))}{(1+\delta_i)(1-\delta_j)} \tag{36}$$

the Pareto superior agenda consists in discussing the most important issue first (see Flamini 2001 for details).

3) In both agenda the SPE outcome is that player 1 demands $\widetilde{x}_1$ (that is, $\max\{g, q\} < \alpha_i < 1$). Then, at the limit for $\Delta \to 0$, the differences $v_i - u_i$ are as follows

$$v_1 - u_1 = \frac{(\lambda_1 - 1)\alpha_1 r_2}{r_1 + r_2} \tag{37}$$

$$v_2 - u_2 = \frac{(1-\lambda_2)(r_1 + r_2(1-\alpha_2))}{r_1 + r_2} \tag{38}$$

Then, we can conclude that players who are sufficiently patient prefer to put the most important issue first.

4) In agenda 1 player 1 demands the interior solution x_1, but in agenda 2 he obtains the entire surplus (i.e., $b < \alpha_i < o$). Then, at the limit for $\Delta \to 0$, the difference $v_1 - u_1$ is as follows

$$\frac{\lambda_1((1-\alpha_1)r_2(r_1+r_2) + 2r_1r_2\lambda_2\alpha_2) - (r_1+r_2)^2 - \alpha_1 r_2(r_1 - r_2)}{(r_1+r_2)^2} \tag{39}$$

while $v_2 - u_2$ becomes

$$\frac{r_1[\lambda_1\lambda_2\alpha_2(r_1 - r_2) + \lambda_1(r_1+r_2)(1-\alpha_2) + 2\alpha_1 r_2}{(r_1+r_2)^2\lambda_1} \tag{40}$$

In this case, if $r_1 > r_2$, that is, player 1 is more patient than player 2, then player 2 always prefers agenda 1 (even for $\lambda_2 > 1$), while player 1 has the same preferences when λ_1 is sufficiently large, so that expression (39) is positive. In other words, players can agree over agendas even when they do not agree over the importance of the issues.

5) In agenda 1 player 1 demands the entire surplus, while in agenda 2 he obtains the interior solution x_1 (i.e., $o < \alpha_i < b$). Then, at the limit for $\Delta \to 0$, the difference $v_1 - u_1$ is as follows

$$\frac{\lambda_1\lambda_2((r_1+r_2)^2 + \alpha_1 r_2(r_1 - r_2)) - \lambda_2 r_2(r_1+r_2)(1-\alpha_1) - 2\alpha_2 r_1 r_2}{(r_1+r_2)^2\lambda_2} \tag{41}$$

while $v_2 - u_2$ is the following:

$$\frac{-r_1[2\alpha_1 r_2\lambda_1\lambda_2 + \lambda_2(r_1+r_2)(1-\alpha_2) + \alpha_2(r_1-r_2)]}{(r_1+r_2)^2} \tag{42}$$

When $r_1 > r_2$, player 2 always prefers agenda 2, while player 1 has the same preference only if $\lambda_1\lambda_2$ is sufficiently small. That is, player 2's relative valuation of cake 1 is larger then player 1 ($\lambda_1 < 1/\lambda_2$). In this case, it does not matter what is the important issue, only the product $\lambda_1\lambda_2$ is relevant.

6) Player 1 obtains $\widetilde{x}_1$ in agenda 1 and the interior demand x_1 in agenda 2. In this case, at the limit for $\Delta \to 0$, player 1 prefers agenda 1 if

$$r_2\frac{\lambda_1\lambda_2\alpha_1(r_1-r_2) - \lambda_2(r_1+r_2)(1-\alpha_1) - 2\alpha_2 r_1}{(r_1+r_2)^2\lambda_2} > 0 \tag{43}$$

while player 2 prefers agenda 1 if

$$-\frac{2\lambda_1\lambda_2\alpha_1 r_1 r_2 + r_1\lambda_2(r_1+r_2)(1-\alpha_2) - (r_1+r_2)^2 + \alpha_2 r_1(r_1-r_2)}{(r_1+r_2)^2} > 0 \qquad (44)$$

In this case if $r_1 > r_2$ and $\lambda_1\lambda_2$ is sufficiently large players have different preferences over agendas.

7) Player 1 obtains the interior demand x_1 in agenda 1 and $\widetilde{x}_1$ in agenda 2. Then, at the limit for $\Delta \to 0$, player 1 prefers agenda 1 if

$$\frac{r_2[2\lambda_1\lambda_2\alpha_2 r_1 + \lambda_1(r_1+r_2)(1-\alpha_1) + \alpha_1(r_1-r_2)]}{(r_1+r_2)^2} > 0 \qquad (45)$$

while player 2 prefers agenda 1 if

$$\frac{-\lambda_1\lambda_2((r_1+r_2)^2 + \alpha_2 r_1(r_2-r_1)) + \lambda_1 r_1(r_1+r_2)(1-\alpha_2) + 2\alpha_1 r_1 r_2}{(r_1+r_2)^2\lambda_1} > 0 \qquad (46)$$

Then, for instance if $r_1 < r_2$ player 1 always prefers agenda 1, while player 2 has the same preference only if $\lambda_1\lambda_2$ is sufficiently small. ■

Proposition 3 establishes an intuitive result on the efficiency of sequential procedures: when there is an important issue, this should be discussed first. However, this result is not obtained in frameworks similar to ours (such as, Busch and Horstmann, 1997, 1999, Inderst, 2000 In and Serrano, 2002, 2003). In addition, proposition 3 shows that other incentives can be dominant. For instance, regardless of whether an issue is the most important or not, a player may prefer either an agenda where he gets a positive share in the initial agreement (see, e.g., case 4 in proof of Proposition 3) or an agenda where the issue that is valued relatively more strongly by his opponent is postponed (see, e.g., case 5 in proof of Proposition 3). Indeed, an important feature of the bargaining game that affects parties' preferences over agendas is that concessions can be made only at the negotiations on the first issue (before the second has been settled) and these concessions can be large or small depending on the *difference* in the relative importance of an issue, not simply on the value of the importance of an issue.

In some cases, more than one incentive work in the same direction. For instance, if players have opposite preferences over issues (e.g., $\lambda_i = \lambda$ with $i = 1, 2$), then the incentive to put the most important issue first and the incentive to postpone the rival's most important issue coincide. However, when players have the same preferences over issues (e.g., $\lambda_1 > 1$ and $\lambda_2 < 1$), the incentive to put the most important issue first is in contrast with the incentive to postpone the rival's most important issue. Proposition 3 shows that both incentives can be dominant (and under which conditions).

In conclusion, the key elements in defining players' preferences over agendas are not only the value of the importance of an issues (is λ_i larger or smaller than 1?), but also their difference (is $\lambda_1\lambda_2$ larger or smaller than 1?), players' between-cake discount factors and in general their rates of time preferences.

4 Urgent/difficult issue

In this section we assume that one issue is *difficult* in the sense that a rejection of a proposal regarding this issue may lead to the negotiations breaking down. For instance, in a peace process there can be an issue characterised by this feature, similarly, in the bargaining between a buyer and a seller there can be a difficult item. In these cases, what are the driving forces in the bargaining games and as a consequence, how should the agenda be set?

To investigate this case we modify the model described in section 2 in two ways. First, we assume that there is no time lapse between bargaining stages ($\tau = 0$), this is a simplifying assumption (the result below can be re-established when τ is positive). Second, the parameter α now represents the probability of game continuation after a rejection of a proposal regarding the difficult issue, say cake 1. In other words, after a rejection of a proposal regarding the division of cake 1, not only does the discount factor δ_i apply but also the probability of game continuation α, while after a rejection regarding the proposal of cake 2, only the discount factor δ_i applies. This does not imply that cake 1 also represents the most important issue. The importance of an issue still depends on the parameters λ_i with $i = 1, 2$ as in the model described in section 2. When there is a rejection in the bargaining stage related to the division of cake 1, it is as if players are characterised by a smaller discount factor, $\delta_i\alpha$ (rather than δ_i). In other words, cake 1 represents an *urgent* issue in the sense that the bargaining round related to the division of cake 1 is longer than the bargaining round in which players attempt to divide cake 2. Bearing in mind this double interpretation, we derive the players' preferences over agendas among the issue-by-issue procedures in the presence of a difficult/urgent issue.

Moreover, to simplify the presentation, we focus on the case in which players are symmetric and that some frictions tend to disappear ($r_i = r, \lambda_i = \lambda$ for $i = 1, 2$ moreover, $\Delta \to 0$).

Proposition 4 *In agenda 1, for $\Delta \to 0$, the SPE demands are as follows:*

$$_{\iota}\widetilde{x}_1 = \frac{(2+\lambda)(1-\alpha)}{2}, \; _{\iota}y_2 = 1 \tag{47}$$

for $\sqrt{1+\alpha} - 1 < \lambda < \frac{\sqrt{\alpha(1+\alpha)}-\alpha}{\alpha}$;

$$_{\iota}x_1 = \frac{\lambda^2 + 2\lambda - \alpha}{2\lambda(1+\alpha)}, \; _{\iota}y_2 = \frac{1 + 2\lambda - \lambda^2\alpha}{2\lambda(1+\alpha)} \tag{48}$$

for $\frac{\sqrt{\alpha(1+\alpha)}-\alpha}{\alpha} < \lambda < \alpha + \sqrt{\alpha(1+\alpha)}$ *and*

$$_{\iota}x_1 = 1, \; _{\iota}\widetilde{y}_2 = \frac{(1+2\lambda)(1-\alpha)}{2\lambda} \tag{49}$$

for $\lambda > \alpha + \sqrt{\alpha(1+\alpha)}$.

Proof. For $r_i = r, \lambda_i = \lambda$ for $i = 1, 2$, The solution of the indifferent conditions are as follows:

$$x_1 = \frac{(\lambda^2 - \alpha\delta)(1-\alpha\delta^2) + \lambda(1+\delta)(1-\alpha\delta)}{\lambda(1+\delta)(1-\alpha^2\delta^2)} \tag{50}$$

$$y_2 = \frac{\lambda(1+\delta)(1-\alpha\delta) + (1-\alpha\delta\lambda^2)(1-\alpha\delta^2)}{\lambda(1+\delta)(1-\alpha^2\delta^2)} \tag{51}$$

To simplify let's assume that $\lambda < 1$. Then the demand x_1 belongs to (0,1) for $r_3 < \alpha < r_1$, with

$$r_3 = \frac{(1+\lambda)(1+\delta\lambda) - \sqrt{\Delta_{r3}}}{2\delta(\lambda+\delta\lambda+\delta)^2}, \; r_1 = \frac{(1+\lambda)(1+\delta\lambda) - \sqrt{\Delta_{r1}}}{2\delta^2}, \tag{52}$$

$$\Delta_{r1} = 1 - 4\delta^2\lambda + 2\lambda(1-\alpha) + \lambda^2(1+\delta^2) + 2\lambda^3\delta^2(1+\delta) + \delta^2\lambda^4 \tag{53}$$

$$\Delta_{r3} = (1+\delta^2\lambda^2)(1+\lambda^2) + 2\lambda(1-\lambda^2-\lambda(1+\delta^2)+\delta(1+\lambda)) \tag{54}$$

Moreover, r_1 tends to 1 for $\delta \to 1$ while r_3 can be larger than 1 for λ close to 1 and $\delta < 0.5$. When α is smaller than r_3, then the SPE demand is a corner solution $x_1 = 1$. When α is larger than r_1 (however, this case is not interesting for $\delta \to 1$), then we have a corner solution of the system of indifference conditions and $x_1 = 0$. For $\lambda < 1$, the demand y_2 defined in (51) is always positive, however, it will be also larger than 1 unless λ is close to 1 and α is sufficiently large for δ large.

The conditions becomes more transparent for $\delta \to 1$, then the demands (50) and (51) becomes

$$_{\iota}x_1 = \frac{\lambda^2 + 2\lambda - \alpha}{2\lambda(1+\alpha)} \tag{55}$$

$$_{\iota}y_2 = \frac{1 + 2\lambda - \lambda^2\alpha}{2\lambda(1+\alpha)} \tag{56}$$

the demand $_{\iota}x_1$ is in (0,1) for $\sqrt{1+\alpha} - 1 < \lambda < \alpha + \sqrt{\alpha(1+\alpha)}$. Moreover, for $\lambda < \sqrt{1+\alpha} - 1$, then $_{\iota}x_1 < 0$ while for $\lambda > \alpha + \sqrt{\alpha(1+\alpha)}$, then $_{\iota}x_1 > 1$. The demand $_{\iota}y_2$ is in (0,1) for $\frac{\sqrt{\alpha(1+\alpha)}-\alpha}{\alpha} < \lambda < \frac{1+\sqrt{(1+\alpha)}}{\alpha}$. Moreover, for $\lambda < \frac{\sqrt{\alpha(1+\alpha)}-\alpha}{\alpha}$,

${}_{\iota}y_2 > 1$ and for $\lambda > \frac{1+\sqrt{(1+\alpha)}}{\alpha}$, ${}_{\iota}y_2 < 0$. Let $\alpha > 1/3$, then, since

$$0 < \sqrt{1+\alpha} - 1 < \frac{\sqrt{\alpha(1+\alpha)} - \alpha}{\alpha} < \alpha + \sqrt{\alpha(1+\alpha)} < \frac{1+\sqrt{(1+\alpha)}}{\alpha}$$

the SPE demands are as defined in proposition 4, where the demand ${}_{\iota}\widetilde{x}_1$ and ${}_{\iota}\widetilde{y}_2$ by player 1 and 2 respectively are such that the responder is indifferent between accepting or rejecting the proposal so as to demand the entire surplus. ■

Similarly in agenda 2 where cake 2 is shared first the SPE is characterised by the following proposition.

Proposition 5 *In agenda 2, for* $\Delta \to 0$, *the SPE demands are as follows: if* $\lambda < 1$, *player demands the entire surplus, while player 2 demands a share* ${}_{ag2}\widetilde{y}_2 = \frac{\lambda(1-\alpha)}{1+\alpha}$. *If* $\lambda > 1$, *player 2 demand the entire surplus, while player 1 demands the share* ${}_{ag2}\widetilde{x}_1 = \frac{1-\alpha}{(1+\alpha)\lambda}$.

Proof. The solution of the indifferent conditions give demands:

$$ {}_{ag2}x_1 = \frac{(1-\delta\lambda^2)(1-\alpha\delta^2) + \lambda(1-\delta)(1+\alpha\delta)}{\lambda(1+\alpha\delta)(1-\delta^2)} \tag{57}$$

$$ {}_{ag2}y_2 = \frac{\lambda(1-\delta)(1+\alpha\delta) + (\lambda^2-\delta)(1-\alpha\delta^2)}{\lambda(1+\alpha\delta)(1-\delta^2)} \tag{58}$$

These are SPE demands if they are in (0,1). At the limit for $\Delta \to 0$, the demands ${}_{ag2}x_1$ and ${}_{ag2}y_2$ in (57) and (58) tends to $sgn(1-\lambda)\infty$ and $sgn(\lambda-1)\infty$ respectively. This implies that the SPE demands in agenda 2 are as follows: if $\lambda < 1$, player 1 demands $x_1 = 1$ while Player 2 demands a share equal to ${}_{ag2}\widetilde{y}_2$ where ${}_{ag2}\widetilde{y}_2$ is such that player 1 is indifferent between accepting and rejecting this offer so as to demand for the entire surplus:

$$_{ag2}\widetilde{y}_2 = \lim_{\Delta \to 0} \frac{(1-\alpha\delta^2)(1+\lambda) - \delta(1-\alpha)}{1+\alpha\delta} = \frac{\lambda(1-\alpha)}{1+\alpha} \tag{59}$$

On the other hand, if $\lambda > 1$, player 2 demands $y_2 = 1$ while player 1 demands a share equal to $_{ag2}\widetilde{x}_1$ where $_{ag2}\widetilde{x}_1$ is such that player 2 is indifferent between accepting and rejecting this offer so as to demand for the entire surplus:

$$_{ag2}\widetilde{x}_1 = \lim_{\Delta \to 0} \frac{(1-\alpha\delta^2)(1+\lambda) - \delta\lambda(1-\alpha)}{(1+\alpha\delta)\lambda} = \frac{1-\alpha}{(1+\alpha)\lambda} \tag{60}$$

■

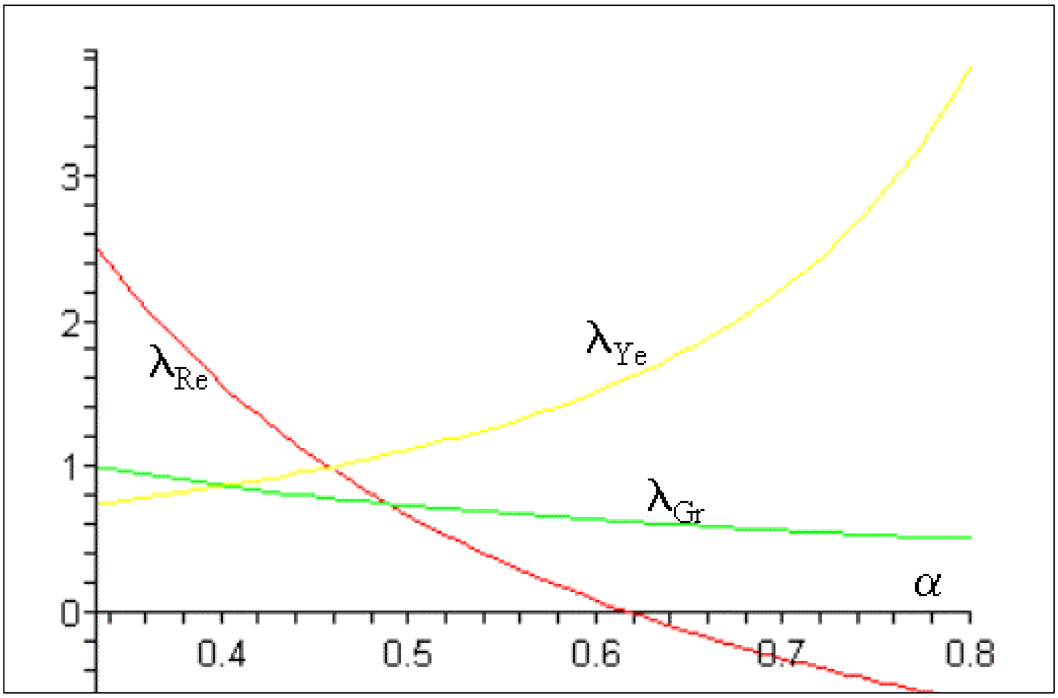
We can now show that when there is a difficult/urgent issue, parties prefer to postpone it and to agree over the easy issue first, even if this is not very important.

Proposition 6 *When there is a difficult/urgent issue, parties can only agree in postponing such an issue regardless of its importance.*

Proof. Let's assume $\alpha > 1/3$, given the SPE demands in Agenda 1, we distinguish 5 cases:

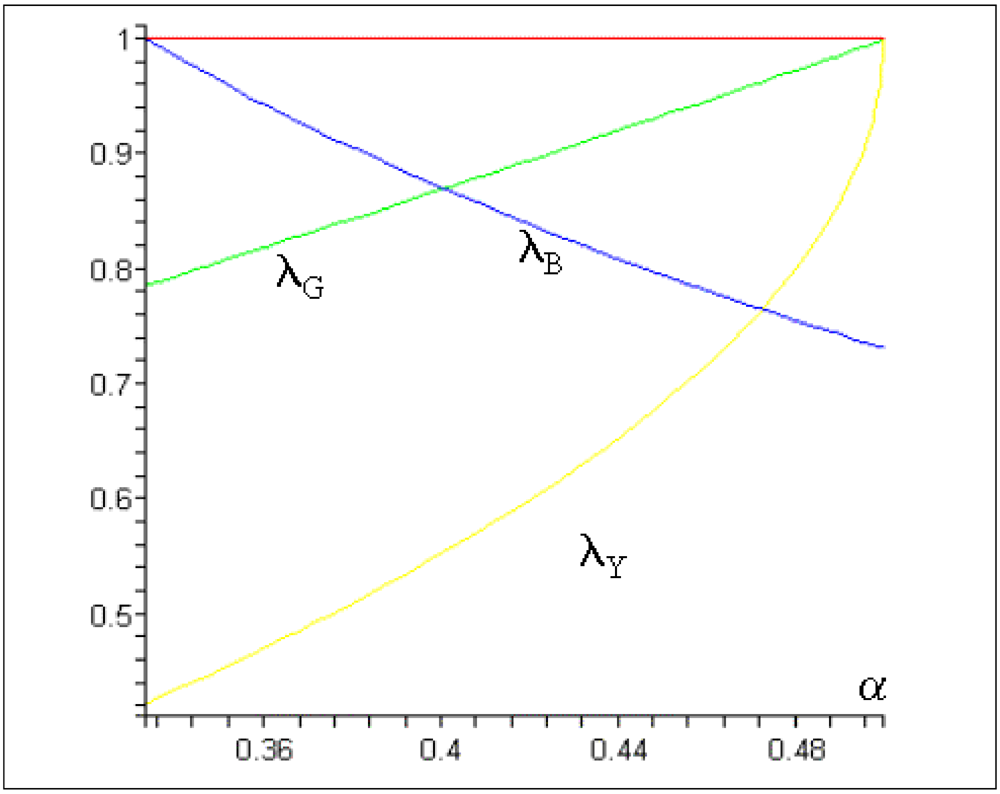
A) Let $0 < \lambda < \lambda_{GR}$, where $\lambda_{Gr} = \frac{\sqrt{\alpha(1+\alpha)}-\alpha}{\alpha}$, in this case in agenda 1 Player 1 demands $\frac{(2+\lambda)(1-\alpha)}{2}$, while in Agenda 2 player 1 obtains the entire surplus. In this case player 1 obtains a larger payoff in agenda 1 if λ is sufficiently large (at the limit $\lambda > \lambda_{Ye}$ with $\lambda_{Ye} = \frac{\alpha(1+a)-1+\sqrt{2-a-2a^2+a^3+a^4}}{1-\alpha^2}$), otherwise, he prefers agenda 2.

Similarly player 2 prefers agenda 1 when λ is sufficiently large (at the limit $\lambda > \lambda_{Re}$ with $\lambda_{Re} = \frac{\alpha(1+a)-1}{\alpha(1+\alpha)}$), otherwise, he prefers agenda 2. Since in this case λ cannot be larger than λ_{GR}, then both players prefer agenda 2 if $0 < \lambda < \min\{\lambda_{Ye}, \lambda_{\text{Re}}, \lambda_{GR}\}$ (see fig. 1) otherwise players have different preferences over agendas.



Agreement arises only for $0 < \lambda < \min\{\lambda_{Ye}, \lambda_{\text{Re}}, \lambda_{GR}\}$

B) Let $\lambda_B < \lambda < 1$, with $\lambda_B = \frac{\sqrt{\alpha(1+\alpha)}-\alpha}{\alpha}$, then Player 1 demands x_1 as in (55) while in Agenda 2 player 1 obtains the entire surplus. Player 1 prefers agenda 1 if $\lambda > \lambda_G$ with $\lambda_G = -(1-a) + \sqrt{2+a^2}$ and agenda 2 otherwise, while Player 2 always prefers agenda 1 unless $\alpha < 0.5$ and λ is in $[\lambda_Y, 1]$ with $\lambda_Y = \frac{(1-a)-\sqrt{1-2a}}{\alpha}$. Then, for $\lambda_B < \lambda < 1$ and $\alpha > 0.5$ players never agree over agenda, while for $\alpha < 0.5$ and $\max\{\lambda_B, \lambda_Y\} < \lambda < \lambda_G$ (see fig. 2) both players prefer agenda 2.



Agreement arises for $\max\{\lambda_B, \lambda_Y\} < \lambda < \lambda_G$

C) Let $1 < \lambda < \alpha + \sqrt{\alpha(1+\alpha)}$, then in agenda 1 player 1 demands x_1 as for case B), while in agenda 2 in equilibrium player 1 obtains ${}_{ag2}\widetilde{x}_1$ as defined in (60). In this case, since $\lambda > 1$, it is straightforward to show that player 1 always prefers agenda 1 while player 2 always prefers agenda 2.

D) Let $\lambda > \alpha + \sqrt{\alpha(1+\alpha)}$, then player obtains the entire surplus in agenda 1 and

${}_{ag2}\widetilde{x}_1$ as defined in (60) in agenda 2. Then, as for the previous case, player 1 prefers agenda 1 while player 2 prefers agenda 2. ■

In conclusion, as in the case of side payments (Flamini, 2001) when there is a difficult/urgent issue parties prefer to postpone it and enjoy an initial agreement rather than to compromise the entire negotiation process by setting the most difficult/urgent issue first.

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