Reexamining the linkages between inflation and output growth: A bivariate ARFIMA-FIGARCH approach

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Abstract

In this paper, given recent theoretical developments that inflation can exhibit long memory properties due to the output growth process, we propose a new class of bivariate processes to simultaneously investigate the dual long memory properties in the mean and the conditional variance of inflation and output growth series. We estimate the model using monthly UK data and document the presence of dual long memory properties in both series. Then, using the conditional variances generated from our bivariate model, we employ Granger causality tests to scrutinize the linkages between the means and the volatilities of inflation and output growth.

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1 Introduction

One fundamental empirical problem that researchers scrutinize in macroeconomics is the relationship between inflation and real economic activity. Okun (1971) claims that given a stable Philips curve, due to changes in demand management policies, high inflation leads to high inflation variability and that there exists a positive relationship between inflation variability and output variability. In his Nobel lecture, Friedman (1977) argues that a rise in the average rate of inflation and its variability, reducing the efficiency of the pricing system, lead to a lower output.[1] In this context, given the vast empirical literature that demonstrates the presence of a positive relationship between inflation and price variability with its potential implications on output growth and output growth volatility, it is not surprising to see that achieving low and stable inflation constitutes a major macroeconomic policy goal.

Several researchers have empirically investigated the linkages between the rate and variability of inflation and output growth. Okun (1971) finds a high correlation between inflation and its volatility, yet reports a low correlation between inflation and output growth variability. Logue and Sweeney (1981) document a positive relationship between the average inflation rate and the variability of output growth, but inflation rate variability does not affect real growth variability. Katsimbris (1985) finds a positive association between inflation variability and industrial production growth rate for several countries. Fountas, Karanasos and Kim (2001), and Grier, Henry and Shields (2004) report that inflation uncertainty has a significant negative effect on output growth. Caporale and Mckiernan (1996) present evidence of a positive relationship between output growth and output growth uncertainty. Clark (1997), and Iscan

[1] Several researchers, including Vining and Elwertowski (1976), Lach and Tsiddon (1992), Grier and Perry (1998), have found a positive relationship between inflation and price variability.

and Osberg (1998) show that neither average inflation nor inflation volatility significantly affects economic growth.

Several other researchers have approached the question on the relationship between uncertainty and economic activity from a different angle. In particular, to demonstrate the negative impact of monetary instability through its effects on the informational content of prices, Beaudry, Caglayan and Schiantarelli (2001) lay out a simple theoretical model and examine the impact of aggregate price uncertainty on the time-variation in the cross-sectional distribution of capital investment spending at the aggregate and the industry level. They show an increase in macroeconomic uncertainty could lead to a significant reduction in the cross-sectional dispersion of the investment rate indicating significant resource allocation problems. Along the same lines, Baum, Caglayan, Ozkan and Talavera (2006) find support from the data that increased uncertainty distorts firms' cash holding behavior hindering the efficient allocation of firms' resources between capital spending and short-term liquidity needs.

In this paper we have two main objectives. Recent theoretical work, including Morana (2002), suggests that inflation can exhibit long memory properties due to the output process, and that it would be desirable to investigate these two series within a bivariate framework.[2] Hence, our first goal is to develop a new class of bivariate processes—bivariate *Constant Conditional Correlation ARFIMA-FIGARCH* model—to investigate the dual long memory properties present in inflation and output growth series extending Baillie, Han and Kwon (2002) to a bivariate framework using the multivariate FIGARCH structure

[2]Many researchers have looked into time series properties of inflation and output growth series. Researchers including Barsky (1987), Brunner and Hess (1993) argue that inflation contains unit root, others, for example, Baillie, Chung and Tislau (1996), Baum, Barkoulas and Caglayan (1999) find evidence in favor of fractional integration. Baillie, Han and Kwon (2002) demonstrate that not only inflation but also its conditional variance exhibits long memory features. Conrad and Karanasos (2005) also present evidence for dual long memory properties of inflation. Finally, Diebold and Rudebusch (1989) indicate that aggregate output can be represented by an ARFIMA process.

proposed by Teyssiere (1997).[3] This methodology serves us to achieve our second objective by allowing us to construct volatility measures from our bivariate framework in understanding the causal linkages between the means and the volatilities of inflation and output growth that have been investigated by many researchers in the past.

We carry out our empirical investigation using monthly inflation and output growth series for the U.K. over the period between February 1957 and May 2005. We start our empirical analysis estimating the Baillie et al. (2002) model. We then implement the bivariate *ARFIMA-FIGARCH* (dual long memory) model. Having obtained the dual long memory parameters for both series, we exploit the generated conditional variances as proxies for inflation and output growth volatilities to investigate various causal relationships *à* la Granger.

The causality tests reveal the following findings. Similar to the findings in the literature we observe that i) inflation causes higher inflation uncertainty and a reduction in output growth as Friedman (1977) claims; ii) output growth causes an increase in inflation and inflation variability. We also find that output growth volatility leads to an increase in inflation, inflation uncertainty and output growth. These findings are in line with earlier theoretical work.

The rest of the paper is organized as follows. Section 2 discusses the theoretical underpinnings of the univariate and bivariate *ARFIMA-FIGARCH* model and presents the small sample properties of the quasi-maximum-likelihood estimation method we implement from a Monte Carlo exercise. Section 3 documents our findings. Section 4 concludes and gives suggestions for further research.

[3] This methodology is useful due to its capability to jointly capture dual fractional differencing parameters in both the mean and the squared values of the residuals of the series within a bivariate framework.

2 Modeling inflation and output growth

Based on the specific assumptions of the analytical models, one can arrive at a positive, negative, or zero effect of inflation on output growth. Mirroring the diverse analytical findings, empirical research, some of which we refer to in the introduction, fails to reach firm conclusions. However, it is possible to arrive at erroneous or biased conclusions should one did not take the proper time series properties of the underlying variables into account. Furthermore, if inflation exhibits long memory properties due to the output process, it would be adequate to model the two series using a bivariate structure. In what follows below, we propose a new methodology to model the mean and the variance of inflation and output growth series within a bivariate framework to account for the possible linkages between the two while allowing the two series to possess long memory in their means as well as their variances.

2.1 Univariate fractionally integrated model

In this section we outline the dual long memory *ARFIMA-FIGARCH* model proposed by Baillie, Han and Kwon (2002), which we extend to a bivariate framework in the following section. They propose the following model to study the presence of dual long memory properties in the conditional mean and the conditional variance:

$$\begin{aligned} \varphi(L)(1-L)^{d_m}(y_t-\mu) &= \alpha(L)\varepsilon_t, & (1)\\ \varepsilon_t &= \xi_t \sqrt[2]{h_t}, & (2)\\ \lambda(L)(1-L)^{d_v}\varepsilon_t^2 &= \omega+(1-\beta(L))v_t, & (3)\end{aligned}$$

where d_m and d_v capture the presence of long memory behavior of the mean, (m), and the variance, (v), of the series. The skedastic innovation is defined as

$$v_t = \varepsilon_t^2 - h_t \quad (4)$$

and ξ_t is an independent identically distributed (i.i.d.) random process with mean zero and unity variance, $E_t(\varepsilon_t) = 0, Var_t(\varepsilon_t) = h_t$, and $E(\varepsilon_t \varepsilon_s) = 0$ for $s \neq t$. L denotes the lag operator and $\varphi(L) = 1 - \varphi_1 L - \varphi_2 L^2 - ... - \varphi_p L^p$ and $\alpha(L) = 1 + \alpha_1 L + \alpha_2 L^2 + ... + \alpha_q L^q$. Furthermore, we define $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + ... + \lambda_s L^s$, and $\beta(L) = \beta_1 L + \beta_2 L^2 + ... + \beta_r L^r$. For stationarity, all the roots of $\varphi(L)$, $\alpha(L)$, $\lambda(L)$ and $(1 - \beta(L))$ must lie outside the unit circle.

The long memory operator can be conveniently expanded as a hypergeometric function

$$(1-L)^d = F(-d,1,1;L) = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} L^k = \sum_{k=0}^{\infty} \lambda_k L^k, \tag{5}$$

where $\Gamma(\cdot)$ denotes the gamma function with $\Gamma(k+1) = k! = k \times \Gamma(k)$ and $\lambda_k = \frac{k-d-1}{k}\lambda_{k-1}$. When $-0.5 < d < 0.5$, the process is said to be stationary and ergodic. For $0 < d < 0.5$, the autocorrelations of the process decay at a hyperbolic rate and the process is said to exhibit long memory. For $-0.5 < d < 0$, the sum of absolute values of the autocorrelation of the process converges to a constant and it is said to have short memory. For $d \in [0.5, 1)$ the process is mean reverting, even though it is not covariance stationary, as there is no long-run impact of an innovation on future values of the process.

Using equation (4) we can rewrite equation (3) as:

$$(1 - \beta(L)) h_t = \omega + \left[1 - \beta(L) - \lambda(L)(1-L)^{d_v}\right] \varepsilon_t^2. \tag{6}$$

Assuming that equation (6) follows FIGARCH (1,d_v,1), Bollerslev, Mikkelsen and Ole (1996) show that the conditional variance will be positive provided that $w > 0$, $\beta - d_v \leq \frac{1}{3}(2 - d_v)$ and $d_v\left[\lambda - \frac{1}{2}(1 - d_v)\right] \leq \beta(d_v + \lambda - \beta)$. Furthermore, when $d_v = 0$, we obtain the standard $GARCH(r,s)$ model. When $d_v = 1$, we have an *Integrated GARCH* model. Note that if $0 < d_v < 1$, the process captures the long run persistence in the conditional volatility. Consequently, the fractionally integrated GARCH model nests both variance-covariance stationary GARCH and integrated GARCH models, allowing flexibility in the representation of the model. The added benefit of this approach is that the model permits one to study the long run dependence in the conditional variance-covariance structure along with that in the mean of the series.

2.2 A bivariate ARFIMA-FIGARCH model

Here, we propose a new approach to investigate the presence of long memory in the series while estimating the parameters of interest within a bivariate framework. In doing so, we extend the standard univariate *ARFIMA-FIGARCH* framework suggested by Baillie et al. (2002) using the multivariate ARFIMA structure proposed by Teyssiere (1997) assuming a constant correlation coefficient.

Let us define vector $\mathbf{Y_t} = \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix}$ and assume that the conditional mean process of $\mathbf{Y_t}$ can be expressed as a bivariate $ARFIMA(p, d, q)$ model:

$$\mathbf{\Phi}\left(L\right) \begin{bmatrix} (1-L)^{d_{1m}} & 0 \\ 0 & (1-L)^{d_{2m}} \end{bmatrix} \mathbf{Y}_t = \mathbf{\Psi}(L)\boldsymbol{\varepsilon}_t, \tag{7}$$

where $\boldsymbol{\varepsilon}_\mathbf{t} = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$, with $E_t\left(\boldsymbol{\varepsilon}_\mathbf{t}\right) = 0$, $Var_t\left(\boldsymbol{\varepsilon}_t\right) = \mathbf{H_t}$, $\mathbf{\Phi}\left(L\right) = \mathbf{I} - \mathbf{\Phi}_1 L - ... - \mathbf{\Phi}_p L^p$, and $\mathbf{\Psi}\left(L\right) = I + \mathbf{\Psi}_1 L + ... + \mathbf{\Psi}_q L^q$. To achieve stationarity, all the roots of $\mathbf{\Phi}(L)$ and $\mathbf{\Psi}(L)$ are assumed to be outside the unit circle. The coefficients of the matrices for $\mathbf{\Phi}$ and $\mathbf{\Psi}$ of the lag operators are defined as:

$$\mathbf{\Phi_i} = \begin{pmatrix} \varphi_{1,i} & \varphi_{12,i} \\ \varphi_{21,i} & \varphi_{2,i} \end{pmatrix} \qquad \left(i = 1, 2, ...p\right),$$

$$\mathbf{\Psi_j} = \begin{pmatrix} \alpha_{1,j} & \alpha_{12,j} \\ \alpha_{21,j} & \alpha_{2,j} \end{pmatrix} \qquad \left(j = 1, 2, ...q\right).$$

Next, we characterize the bivariate fractionally integrated $FIGARCH(r, d_v, s)$ model with constant conditional correlation for $\mathbf{Y_t}$ as

$$\mathbf{\Theta}\left(L\right)\begin{pmatrix} (1-L)^{d_{1v}} & 0 \\ 0 & (1-L)^{d_{2v}} \end{pmatrix}\boldsymbol{\varepsilon}_t^2 = \mathbf{w} + (\mathbf{I}-\mathbf{B}(L))\mathbf{v}_t \tag{8}$$

where $\boldsymbol{\varepsilon}_t^2 = \begin{pmatrix} \varepsilon_{1t}^2 \\ \varepsilon_{2t}^2 \end{pmatrix}$, $\mathbf{\Theta}\left(L\right) = \mathbf{\Theta}_1 L + ... + \mathbf{\Theta}_s L^s$, $\mathbf{B}(L) = \mathbf{B}_1 L + ... + \mathbf{B}_r L^r$ and all roots of $\mathbf{\Theta}\left(L\right)$ and $\mathbf{B}(L)$ lie outside the unit circle to achieve stationarity. Similar as in the univariate case, we set the skedastic innovation matrix as $\mathbf{v_t} = \boldsymbol{\varepsilon}_t^2 - VECH(\mathbf{H}_t)$, where $\boldsymbol{\varepsilon}_t|\mathbf{H_{t-1}} \sim N\left(\mathbf{0}, \mathbf{H_t}\right)$, and $\mathbf{H_t} = \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix}$. Substituting $\mathbf{v_t}$ into equation (8), we obtain

$$\left(\mathbf{I}-\mathbf{B}\left(L\right)\right)VECH(\mathbf{H}_t) = \boldsymbol{\omega} + \left[\mathbf{I}-\mathbf{B}\left(L\right) - \left(\mathbf{I}-\mathbf{\Theta}\left(L\right)\right)(1-L)^{\mathbf{d}_v}\right]\boldsymbol{\varepsilon}_t^2, \tag{9}$$

and express the elements of this matrix as:

$$\begin{aligned} (1-\beta_{ii}\left(L\right))h_{ii,t} &= \omega_i + \left[1-\beta_{ii}\left(L\right) - \left(1-\lambda_{ii}\left(L\right)\right)(1-L)^{d_{iv}}\right]\varepsilon_{i,t}^2 && (10) \\ h_{12,t} = h_{21,t} &= \rho\sqrt[2]{h_{11,t}}\sqrt[2]{h_{22,t}}, && (11) \end{aligned}$$

where $i = 1, 2$ and ρ denotes the constant conditional correlation coefficient. Equation (10) denotes the presence of long memory in the variance and equation (11) captures the cross correlation between the residuals given, ρ, the constant correlation coefficient.

Here, stationarity is ensured by imposing restrictions on the diagonal elements of the variance–covariance matrice. Hence, positive definiteness in the bivariate diagonal CCC-FIGARCH model is assured when $|\rho| < 1$, $\beta_{ii} - d_{iv} \leq \frac{1}{3}\left(2-d_{iv}\right)$ and $d_{iv}\left[\lambda_{ii} - \frac{1}{2}\left(1-d_{iv}\right)\right] \leq \beta_{ii}\left(d_{iv} + \lambda_{ii} - \beta_{ii}\right)$.

2.2.1 Estimation method and a Monte-Carlo study

Inference in the estimated models is based on quasi-maximum likelihood (QMLE) estimation. Assuming the residuals are conditionally normal, the logarithm of the loglikelihood function can be described as

$$L\left(\theta, \boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, ..., \boldsymbol{\varepsilon}_T\right) = -(T/2)n\ln(2\pi) - (1/2)\sum_{t=1}^{T}\ln|H_t| - (1/2)\sum_{t=1}^{T}\boldsymbol{\varepsilon}_t' H_t^{-1}\boldsymbol{\varepsilon}_t \tag{12}$$

where θ denotes the parameters that will be estimated. The empirical methodology we employ is similar to that in Baillie et al. (1996) where we maximize the loglikelihood function conditional on initial values of the vectors of residuals ε_t^2, $t = 0, -1, -2, ...$, are set to sample unconditional variance. The initial observations y_0, y_1, $y_2, ...$ are set to the sample average.

Earlier researchers have shown that QMLE is consistent and asymptotically normal for specific cases of the ARFIMA and or the FIGARCH models.[4] In particular, simulation evidence for FIGARCH and some complicated ARCH models suggest that QMLE is consistent and asymptotically normal. However, full theoretical treatment is not available, yet. In our Monte Carlo simulation experiment, we conjecture that constants associated with the mean and the conditional variance are known, the limiting distribution of the QMLE is

$$D_T\left(\theta - \theta_0\right) \rightarrow N\left(0, \left[D_T^{-1}\Gamma^{-1}\left(\theta_0\right)\Upsilon\left(\theta_0\right)\Gamma^{-1}\left(\theta_0\right)D_T^{-1}\right]^{-1}\right)$$

where $\Gamma^{-1}\left(\theta_0\right)$ and $\Upsilon\left(\theta_0\right)$ are the Hessian and outer product gradient, evaluated at the true parameter values θ_0, respectively and $diag(D_T) = (T^{(1/2)-d}, T^{(1/2)}, ..., T^{(1/2)})$.

We specifically carry out a detailed Monte Carlo simulation exercise for bivariate $ARFIMA(24, d, 0) - FIGARCH(1, d, 1)$ model to verify the adequacy of the estimation method which we implement in the next section. The model is simulated with the parameter designs of $d_{\pi,m} = d_{y,m} = 0.20$ and $d_{\pi,v} = 0.45$ and $d_{y,v} = 0.30$ where π and y denotes inflation and output growth, respectively. We investigate the above designs for sample size T=500 and T=1000 for 1000 replications in all cases, respectively. Table 1, based on our simulation results, presents average biases, root mean squared errors (RMSE) as well as the standard errors (SD) for our designs. The distributions of the QMLE for $d_{\pi,m}$, $d_{y,m}$, $d_{\pi,v}$, and $d_{y,v}$ are provided in Figures 1a-1b and 2a-2b, respectively.

[4]See Li and McLeod (1986), Dahlhaus(1989), Moehring (1990), Lee and Hansen (1994) and Lumsdaine (1996).

Results obtained from our Monte Carlo experiments are very satisfactory and provide evidence of a very small parameter estimate bias for each parameter. In other words, our simulation exercise suggests that our estimators are close to their true values with high precisions. Note that corresponding results for other parameters also yield similar results with small biases and RMSE. Although we refrain from tabulating those results due to space considerations here, they are available from the authors upon request.

3 Empirical Analysis

3.1 Data

The empirical investigation is carried out on monthly consumer price index (CPI) and industrial production index (IPI) series derived from DATASTREAM database over the period between February 1957 and May 2005 for the United Kingdom. We measure the inflation (π_t) and output growth (y_t) series as the monthly difference in the natural logarithm of the CPI and IPI; $\pi_t = 100\Delta \log (CPI_t)$ and $y_t = 100\Delta \log (IPI_t)$, respectively. Over the sample period, the mean inflation and output growth rates are 0.48% and 0.13%, respectively. Interestingly, the standard deviation of output growth happens to be much larger than that of the inflation series (1.39% *versus* 0.68%) implying that the output growth is more volatile than inflation.

Next we investigate the autocorrelation coefficients for inflation and output growth series. Figures 3 and 4 display the autocorrelations of inflation and output growth up to 100 lags, respectively. Different from the behavior of a standard stationary ($I(0)$) or a non-stationary ($I(1)$) process, we can see from Figure 3 that the autocorrelations of inflation decay at a slow hyperbolic rate. Figure 4 shows that the autocorrelations of output growth also decay slowly and display cyclical dependence. Finally, although we do not present the

KPSS and PP test statistics, they provide evidence that neither of the series can be characterized as non-stationary nor as I(0).[5] Therefore, based on these observations, it seems plausible that both inflation and output growth series exhibit long run dependence; a hypothesis that we can test.

3.2 Estimating the univariate ARFIMA–FIGARCH model

Prior to estimating our bivariate ARFIMA-FIGARCH model for inflation and output growth series, we run a univariate $ARFIMA(p, d_m, 0){-}FIGARCH(1, d_v, 1)$ model for both series separately to gain a sense of the presence of dual long memory in these series. We define the mean equation as

$$\left(1 - \varphi_1 L - ... - \varphi_{24} L^{24}\right)(1 - L)^{d_m} x_t = c + \varepsilon_t \tag{13}$$

to eliminate higher higher order serial correlation and the potential influence of seasonality. Here, x_t depicts the output growth or inflation, and we assume that $d_m \leq 1$ and we assume that $\varepsilon_t \sim N(0, h_t)$. The structure of the conditional variance is captured by a $FIGARCH(1, d_v, 1)$ model

$$(1 - \beta L)\, h_t = \varpi + \left[1 - \beta L - \lambda L(1 - L)^{d_v}\right] \varepsilon_t^2. \tag{14}$$

where $0 \leq d_v \leq 1$, $\varpi > 0$ and $\lambda, \beta < 1$.

Note that although we estimate up to 24 AR lags for the mean equations, Table 2 displays those parameters that are significant at the 10% level and better along with the other parameters of interest. We carry out the estimation using the QMLE method and use a range of starting values to check for the robustness of our findings.

Table 2 reveals that the long memory parameter estimates of mean inflation and mean output growth series are between zero and 0.5 ($d_{\pi,m} = 0.209$ and $d_{y,m} = 0.202$) and they are significant at the 1% level. We have similar observations when we investigate the long memory parameters of the volatilities

[5] These results are available from the authors upon request.

of the series; $d_{\pi,v} = 0.438$ and $d_{y,v} = 0.308$, respectively.[6] Furthermore, given the presence of long memory in the mean equations, the AR(24) specification captures the serial correlation for the inflation and the output growth series. Ljung-Box Q and Q^2 test statistics indicate that there is no significant presence of higher order serial correlation in the standardized and squared standardized residuals of either series.

3.3 Estimating bivariate ARFIMA–CCC–FIGARCH model

Armed with the information above we now estimate a bivariate $ARFIMA(p, d_m, 0)-FIGARCH(1, d_v, 1)$ model for the inflation and output growth series assuming a constant correlation coefficient structure. We write the mean process of the vector $\mathbf{Y}_t = \begin{pmatrix} \pi_t \\ y_t \end{pmatrix}$ as:

$$\left(\mathbf{I} - \mathbf{\Phi}_1 L - \cdots - \mathbf{\Phi}_{24} L^{24}\right) \begin{bmatrix} (1-L)^{d_{\pi,m}} & 0 \\ 0 & (1-L)^{d_{y,m}} \end{bmatrix} \mathbf{Y}_t = \begin{pmatrix} c_{\pi,m} \\ c_{y,m} \end{pmatrix} + \boldsymbol{\varepsilon}_t, \tag{15}$$

where the roots of $\mathbf{\Phi}(L) = \left(\mathbf{I} - \mathbf{\Phi}_1 L - ... - \mathbf{\Phi}_{24} L^{24}\right)$ lie outside the unit circle, the coefficient matrix $\mathbf{\Phi}_p = \begin{pmatrix} \phi_{1,p} & \phi_{12,p} \\ \phi_{21,p} & \phi_{22,p} \end{pmatrix}$ (p = 1, 2,...,24) and the vector of residuals $\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{\pi,t} \\ \varepsilon_{y,t} \end{pmatrix}$ is conditionally normally distributed with $\boldsymbol{\varepsilon}_t | \mathbf{H}_{t-1} \sim N(0, \mathbf{H}_t)$ and

$$\mathbf{H}_t = \begin{pmatrix} h_{\pi,t} & h_{\pi y,t} \\ h_{y\pi,t} & h_{y,t} \end{pmatrix}.$$

The bivariate volatility process, is defined as:

$$\begin{pmatrix} (1-b_1 L) & 0 \\ 0 & (1-b_2 L) \end{pmatrix} \begin{pmatrix} h_{\pi,t} \\ h_{y,t} \end{pmatrix} = \begin{pmatrix} c_{\pi v} \\ c_{yv} \end{pmatrix} + \begin{pmatrix} 1 - b_1 L - (1-a_1 L)(1-L)^{d_{\pi v}} \\ 1 - b_2 L - (1-a_2 L)(1-L)^{d_{yv}} \end{pmatrix} \begin{pmatrix} \varepsilon_{11,t}^2 \\ \varepsilon_{22,t}^2 \end{pmatrix} \tag{16}$$

[6] We carry out a battery of tests to see if data support these findings. The Wald test statistics illustrate the presence for long memory for all volatility processes of inflation and output growth series. Similar finding are obtained when we carry out likelihood ratio tests. All tests statistics are available from the authors upon request.

with constant conditional correlation ρ, where the covariance component is described as $h_{\pi y,t} = \rho\sqrt{h_{\pi,t}}\sqrt{h_{y,t}}$. Similar to our previous estimation approach, we compute the AR coefficients up to 24 lags for the mean inflation and the mean output growth series.

Table 3 depicts coefficients that are significant at the 10% level, or better. Note that the fractional differencing parameters in the mean and the variance of the series obtained from our bivariate model are very similar to those computed using the univariate approach proposed by Baillie et al. (2002). This is possibly due to the observation that the constant conditional correlation coefficient, ρ is not significantly different from zero, implying that the variance-covariance matrix is diagonal.[7] The Ljung-Box Q and Q^2 test statistics suggest that the AR specification we implement adequately captures the data generating mechanism for inflation and output growth series after taking into account the long memory in both series.

3.4 Granger-Causality Tests

Now that we have estimated the model and derived the associated conditional variances of each of the series, we can examine the bidirectional causal relationships between the means and the variances of inflation, π_t, and output growth, y_t, using Granger causality approach. Table 4 reports the size of the coefficients and provides the F statistics for Granger-causality using four, eight and twelve lags to ensure that the results are not driven by the choice of lag length.

Panel 4a reports our results on the causal relationship from inflation and output growth to inflation and output growth uncertainty (variability). We first report in Panel 4a that increased inflation leads to an increase in inflation uncertainty. The relationship is positive and highly significant at all lags sup-

[7]One can in fact let the data determine ρ relaxing the constant correlation coefficient assumption. However, this extension is beyond the purpose of the current paper.

porting the earlier research such as Friedman (1977). The next two columns document the causal link from inflation and output growth to output growth variability. For both cases, we find a positive yet an insignificant causal relationship. The last column of Panel 4a presents evidence on the causal effect from output growth to inflation uncertainty. We document that output growth causes an increase in inflation volatility at eight and twelve lags. Recall that output growth generally leads to an increase in inflation due to the 'Philips curve' effect. In fact, third column of Panel 4c presents evidence to this prediction. Therefore, by the Friedman (1977) hypothesis, an increase in inflation triggers higher inflation volatility.

Panel 4b turns to investigate the causal effects of inflation uncertainty and output growth volatility on inflation and output growth. We first provide evidence that inflation volatility leads to a reduction in inflation and this relationship is highly significant at twelve lags only. In fact Holland (1995) points out that monetary authorities will contract growth rate of money supply when inflation uncertainty increases, which in turn leads to a fall in inflation. Findings supporting the above hypothesis have been reported in the literature, for instance by Grier and Perry (1998). Column three of panel 4b shows that output growth volatility leads to a higher inflation in the UK. This can be explained within the context of a model by Deveraux (1989). Extending the Barro and Gordon (1983) model, Deveraux shows that higher real uncertainty reduces the optimal amount of wage indexation and induces the policy-maker to engineer more inflation surprises to generate favorable real effects. Next, we turn to the causal effects from output growth uncertainty to output. Our analysis reveals a positive relationship. This observation can be justified for instance by Sandmo (1970) who points out that income volatility would lead to a higher saving rate due to precautionary motives, which in turn leads to higher growth according to Solow's growth theory. Models developed by other researchers including Black

(1987), Blackburn and Pelloni (2004) also arrive at a similar prediction. Finally, we look into the relationship from inflation uncertainty to output growth. However, although the causal relationship is consistently negative, it is insignificant.

Theoretical studies generally conclude that an increase in inflation will either reduce output growth or will have no impact.[8] Column two of Panel 4c displays a negative relationship which is significant at eight and 12 lags. The next column presents evidence in support of the traditional 'Phillips curve'; an increase in output growth will lead to an increase in inflation. We then turn to investigate the causal effects of output growth uncertainty on inflation uncertainty. It can be argued that policy makers are interested in minimizing the variability of inflation and output growth and there exists a tradeoff between the two. Column four documents a significant positive causal relationship at all lags. Finally, we look at the causal effects from inflation uncertainty to output growth uncertainty. We find that the relationship is positive but insignificant at all lags.

4 Conclusion

In this paper, we have two main objectives. First, given the recent theoretical work, including Morana (2002), that inflation can exhibit long memory properties due to the output process, we propose a new class of bivariate processes— ARFIMA-FIGARCH model— to investigate the dual long memory properties in the mean and the conditional variance of inflation and output growth series. The model we propose extends Baillie et al. (2002) univariate dual long memory model into a bivariate framework by using the multivariate FIGARCH approach Tyerssiere (1997) proposed. The methodology we propose is important due to its capability to jointly capture dual fractional differencing parameters in both the means and the variances of the series. Second, using the conditional variances generated from our bivariate model, we investigate several causal linkages

[8]See Gillman and Kejak (2005) for a survey.

between the means and the variances of output growth and inflation series that researchers have scrutinized in macroeconomics.

We carry out our empirical analysis for using monthly UK data spanning the period between February 1957 and May 2005. Our findings can be summarized as below. We first show that both inflation and output growth series exhibit long memory in the means and conditional variances. We then carry our causal investigation. We show that an increase in inflation as well as output growth (which in turn causes an increase in inflation) will lead to an increase in inflation variability. Our results also depict that output growth volatility has a positive causal impact on both inflation and output growth. Finally we show that while output growth volatility induces higher inflation volatility inflation leads to a reduction in output growth. These findings are in line with earlier theoretical work.

Given the ultimate objective of the policy makers is to weigh the deviation of mean inflation and output growth from a set target value while keeping an eye on their variances, the proper modelling of these variables is extremely important. We believe that the approach we propose here might be useful for further research in this area.

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Table 1. Simulation results

Panel A_500	True	Bias	RMSE	SD
$d_{\pi,m}$	0.20	-0.0150	0.0942	0.0929
$d_{y,m}$	0.20	-0.0148	0.0668	0.0652
$d_{\pi,v}$	0.45	0.0210	0.2279	0.2271
$d_{y,v}$	0.30	0.0225	0.2233	0.2224
Panel B_1000	**True**	**Bias**	**RMSE**	**SD**
$d_{\pi,m}$	0.20	-0.0075	0.058	0.0578
$d_{y,m}$	0.20	-0.0065	0.0509	0.0505
$d_{\pi,v}$	0.45	0.0140	0.2173	0.2170
$d_{y,v}$	0.30	0.0161	0.2009	0.2004

Table reports the averages of biases and RMSE of the QMLE of the estimates of the fractional differencing parameter.

Table 2. ARFIMA-FIGARCH modelling of inflation and output growth

	Inflation (π_t)	Output growth (y_t)
c_m	0.019	0.050
	(1.136)	(1.263)
d_m	0.209	0.202
	(3.967)***	(4.146)***
φ_1	-0.032	-0.425
	(-0.715)	(-7.375)***
φ_2	-0.031	-0.231
	(-0.821)	(-5.123)***
φ_{12}	0.416	-0.069
	(10.700)***	(-2.089)**
φ_{24}	0.251	-0.052
	(6.894)***	(-1.653)*
c_v	0.0001	0.191
	(0.020)	(2.903)***
d_v	0.438	0.308
	(6.652)***	(3.724)***
α	0.281	0.303
	(8.530)***	(2.532)***
β	0.511	0.017
	(8.396)***	(0.07)
Q(20)	16.253	19.183
	[0.135]	[0.319]
$Q^2(20)$	12.005	7.262
	[0.807]	[0.989]
AIC	1.372	3.119
HQ	1.402	3.148
S	1.346	3.092
$LogL$	-388.095	-894.473

Notes: 1) ***, ** and * denote significance at the 1%, 5% and 10% levels.
2) Q(20) and $(Q)^2(20)$ are the Ljung-Box tests for serial correlation in the standardized and squared standardized residuals, respectively.

Table 3. Bivariate ARFIMA-CCC-FIGARCH modelling

	Inflation (π_t)		Output (y_t)
$c_{\pi,m}$	0.027	$c_{y,m}$	0.071
	(1.753)*		(1.859)*
$d_{\pi,m}$	0.195	$d_{y,m}$	0.205
	(3.826)***		(4.496)***
ϕ_{11_1}	-0.021	ϕ_{22_1}	-0.437
	(-0.510)		(-7.849)***
ϕ_{11_12}	0.417	ϕ_{22_2}	-0.237
	(10.750)***		(-5.399)***
ϕ_{11_24}	0.249	ϕ_{22_12}	-0.067
	(6.842)***		(-2.089)**
ϕ_{12_1}	0.016	ϕ_{21_1}	-0.129
	(1.176)		(-1.918)*
ϕ_{12_12}	0.020	ϕ_{21_12}	-0.053
	(1.444)		(-0.802)
$c_{\pi,v}$	0.00001	$c_{y,v}$	0.185
	(0.00001)		(2.867)***
$d_{\pi,v}$	0.442	$d_{y,v}$	0.328
	(6.727)***		(3.787)***
a_1	0.279	a_2	0.301
	(8.494)***		(3.461)***
b_1	0.513	b_2	0.010
	(8.446)***		(0.289)
rho	0.032		
	(0.772)		
$Q_1\,(20)$	14.580	$Q_2\,(20)$	19.453
	[0.508]		[0.305]
$Q_1^2\,(20)$	13.198	$Q_2^2\,(20)$	10.221
	[0.702]		[0.899]
AIC	4.497	S	4.429
HQ	4.565	$LogL$	-1281.229

Notes: 1) ***, ** and * denote significance at the 1%, 5% and 10% levels.
2) Q(20) and $Q^2(20)$ represent the Ljung-Box Q test and Q^2 tests.

Table 4: Granger-Causality Test

Panel 4a:

Lags	H_0: $\pi_t \rightarrow h_{\pi_t}$	H_0: $\pi_t \rightarrow h_{yt}$	H_0: $y_t \rightarrow h_{yt}$	H_0: $y_t \rightarrow h_{\pi t}$
4	0.2826	0.7655	–0.7325	–0.0269
	(3.529)***	(1.294)	(0.907)	(2.037)*
8	0.2456	0.7416	–0.8154	0.0087
	(2.075)**	(0.698)	(0.990)	(4.465)***
12	0.1649	1.1564	–0.6364	0.0060
	(1.577)*	(0.889)	(1.097)	(3.395)***

Panel 4b:

Lags	H_0: $h_{\pi t} \rightarrow \pi_t$	H_0: $h_{yt} \rightarrow \pi_t$	H_0: $h_{yt} \rightarrow y_t$	H_0: $h_{\pi t} \rightarrow y_t$
4	–0.0937	0.0073	0.0751	–0.2093
	(0.613)	(1.505)	(9.980)***	(1.113)
8	–0.0968	0.0187	0.0600	–0.2617
	(0.968)	(2.080)**	(6.153)***	(0.823)
12	-0.0667	0.0325	0.0497	–0.2592
	(2.760)***	(2.954)***	(5.424)***	(0.772)

Panel 4c:

Lags	H_0: $\pi_t \rightarrow y_t$	H_0: $y_t \rightarrow \pi_t$	H_0: $h_{yt} \rightarrow h_{\pi t}$	H_0: $h_{\pi t} \rightarrow h_{yt}$
4	–0.2496	0.0375	0.0033	0.3106
	(1.101)	(2.623)**	(1.894)*	(0.358)
8	–0.3375	0.0029	0.0272	0.1573
	(1.786)*	(2.097)**	(4.424)***	(0.358)
12	–0.3461	0.1632	0.0324	0.1122
	(2.101)***	(2.253)***	(3.265)***	(0.627)

Notes: 1) ***, ** and * denote significance at the 1%, 5% and 10% levels.
2) Sum of the coefficients of lagged exogenous variables are reported.
3) F-values are reported in the brackets.

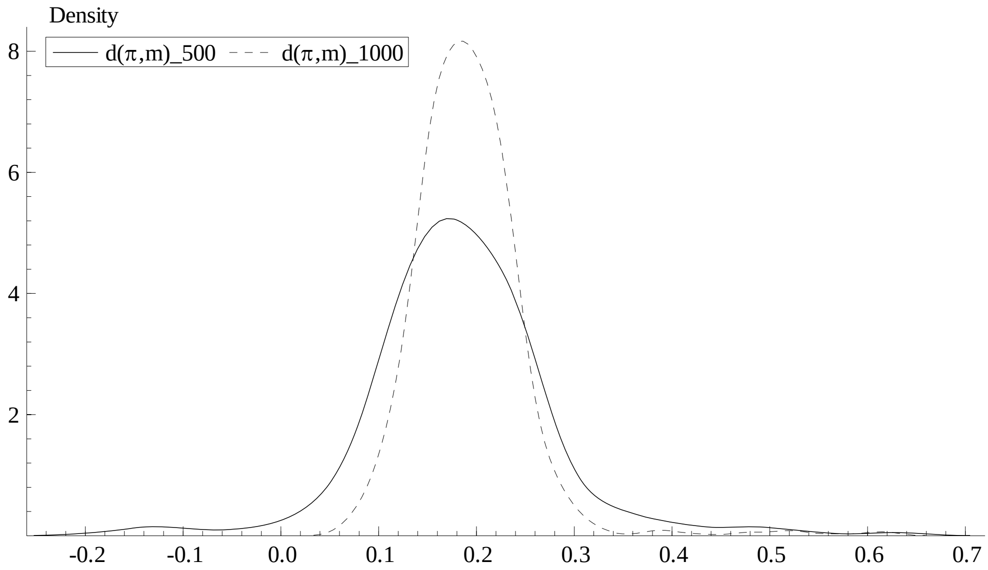

Figure 1a. Monte Carlo Simulation for d(π,m)

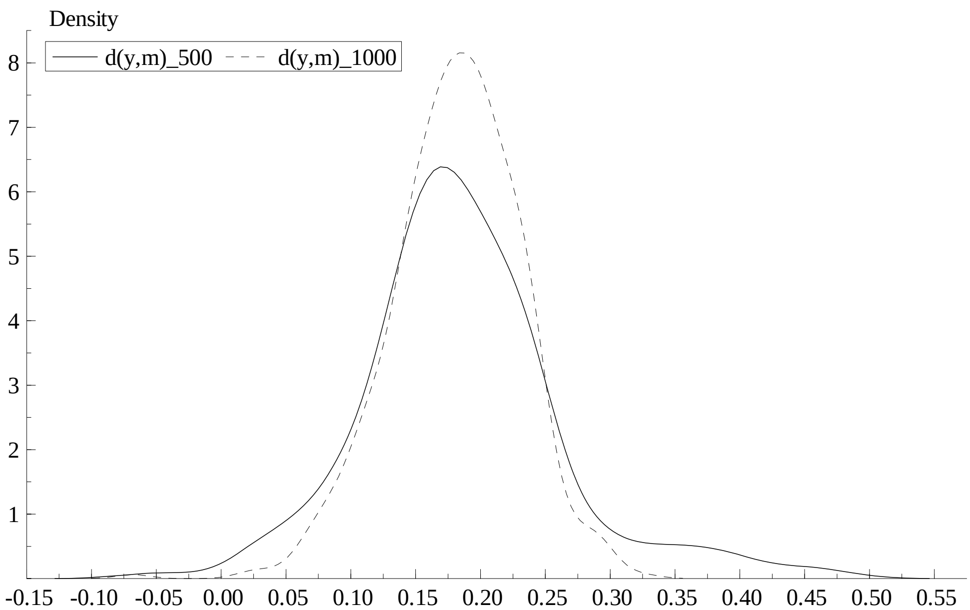

Figure 1b. Monte Carlo Simulation for d(y,m)

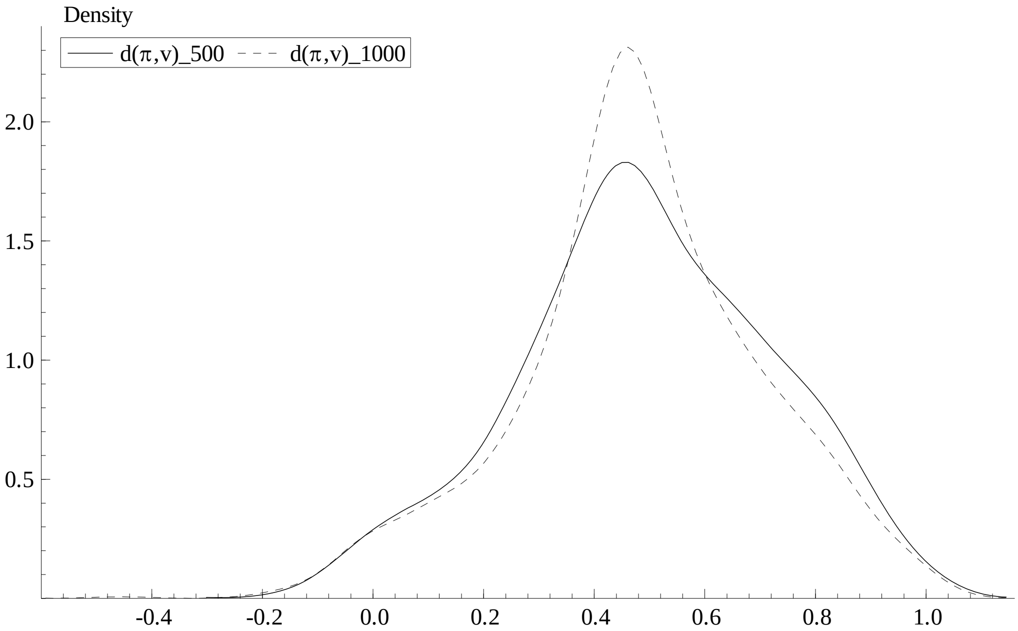

Figure 2a. Monte Carlo Simulation for d(π,v)

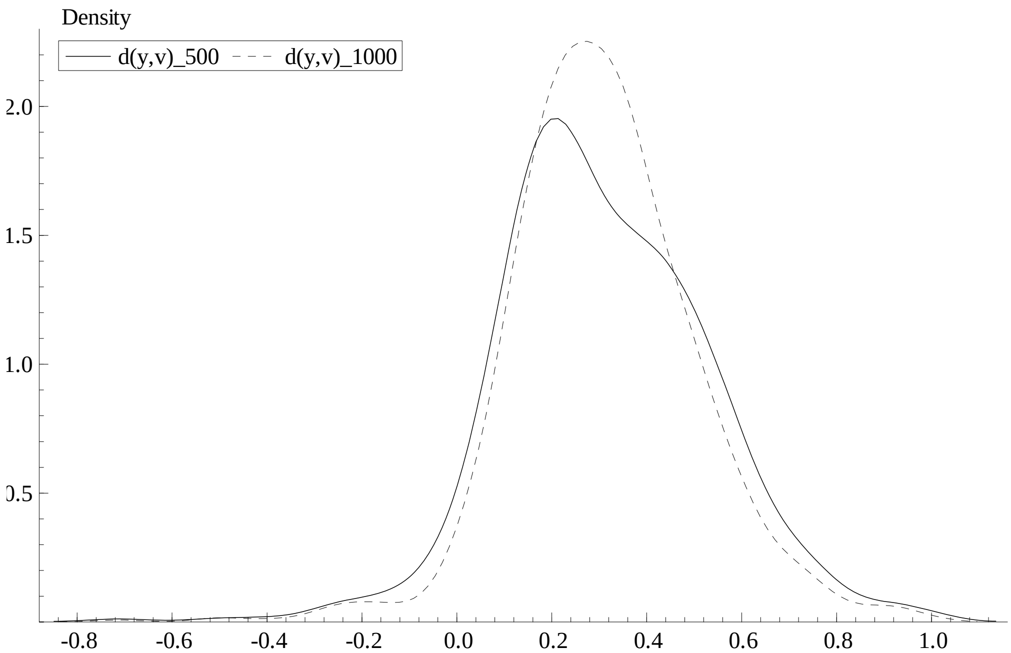

Figure 2b. Monte Carlo Simulation for d(y,v)

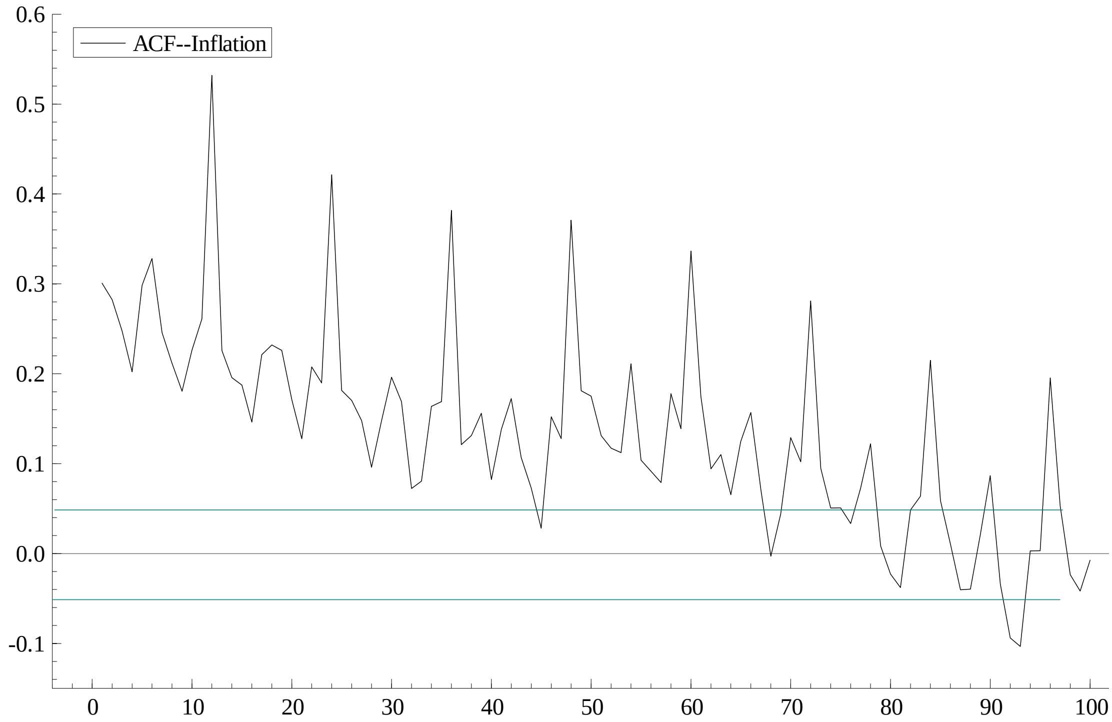

Figure 3: ACF of UK Inflation Series

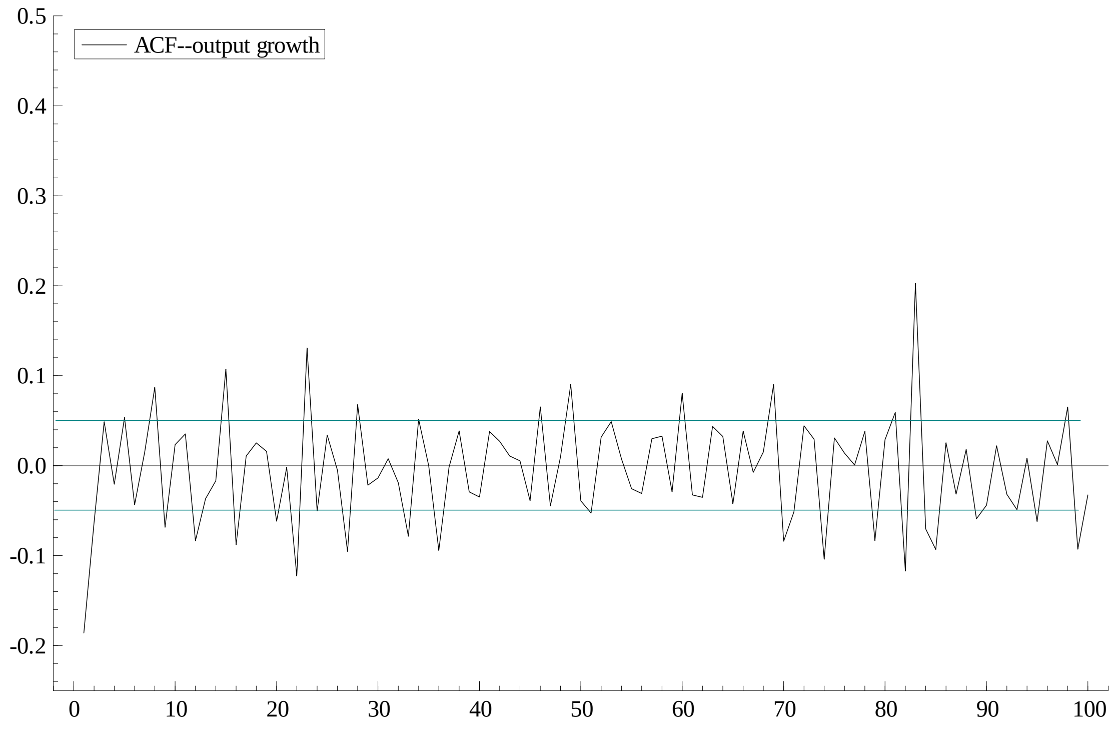

Figure 4: ACF of UK Output Growth Series