Endogenous vs. Semi-endogenous Growth in a Two-R&D-Sector Model*

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Abstract

This paper contributes to the endogenous versus semi-endogenous growth debate by establishing that the latter emerges as a *general* case, whereas the former becomes a *special* case in a two-R&D-sector growth model. It turns out that endogenous growth requires two knife-edge conditions of parameters. This nding (i) stands against recent two-R&D-sector models which show that long-run growth can be endogenous and (ii) resurrects the stark policy conclusion of semi-endogenous growth. The driving force of our result is knowledge spillovers *between* two R&D activities, which are largely neglected in the existing studies.

Key Words: scale effects, endogenous and semi-endogenous growth, two-R&D-sector model, quality and variety innovation.

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In the endogenous growth literature, one of the major issues is whether long-run growth driven by R&D is endogenous or semi-endogenous. According to Jones (1995a), semi-endogenous growth means that (i) technological change itself is endogenous, but (ii) long-run growth is pinned down by an exogenous population growth. A key implication of (ii) is that the long-run growth is independent of public policy, e.g. R&D subsidies. This striking result is established in one-R&D-sector growth models (see below for a brief literature review). This finding has been recently challenged in several studies which use sophisticated two-R&D-sector models. Their central message is that semi-endogenous growth is limited to one-R&D-sector models, and its associated policy implications have little relevance to a real world in which there are diverse types of research activities.

The present paper contributes to this debate by establishing the generality of semi-endogenous growth even in the two-R&D-sector framework. More specifically, we demonstrate that long-run growth becomes semi-endogenous under very mild conditions. In contrast, endogenous growth requires two "knife-edge" conditions of parameters. Long-run growth can be endogenous if, and only if, such a double coincidence occurs. This finding clearly stands against the key results of the recent two-R&D-sector models and resurrects the stark policy conclusion of semi-endogenous growth.

The endogenous versus semi-endogenous growth debate originates in scale effects (growth is positively related to the size of the economy) predicted by earlier R&D-based growth models (Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990)). This prediction was rejected by Jones (1995a) in his influential empirical work.[1] As a data-consistent alternative, Jones (1995a) proposed the one-R&D-sector model which

[1] He showed that TFP growth of some OECD countries exhibits no persistent rise over the last decades, whereas the number of scientists and engineers dramatically increased. Apart from this study, a cross-sectional analysis of Backus, et al. (1992) provide no support for the prediction at the aggregate level. In contrast, Kremer (1993) shows that scale effects exist in the very long history of the world.

exhibits semi-endogenous growth. Essentially the same theoretical result is also obtained in one-R&D-sector models of Kortum (1997) and Segerstrom (1998a).

In order to counter this argument, several studies proposed two-R&D-sector growth models. As pioneered by Young (1998), these studies (Aghion and Howitt (1998, Ch.12), Dinopoulos and Thompson (1998), Howitt (1997), Peretto (1998), and Peretto and Smulders (1998)) model technological advance in the dual form of variety innovation of new products and their quality (or productivity) improvement. They establish that growth of per capita income is endogenous (i.e. is affected by public policy) and independent of the size of the economy. The key mechanism is that an exogenous population growth pins down the growth of variety goods but not the intensity of quality innovation which, as a result, determines the endogenous rate of technological progress in the long run.

However, these two-R&D-sector models assume no or at best very limited knowledge spillovers *between* quality and variety R&D. If these research activities are interpreted as basic and applied (or scienti c and technological) research respectively, this assumption implies that there is no positive externality between them. This assumption is not only restrictive[2] but the central feature that drives their main results.[3] The present paper relaxes this assumption, introducing inter-R&D knowledge spillovers. This is the key mechanism that leads to the main result that semi-endogenous growth emerges as a *general* case, whereas endogenous growth becomes a *special* case in the two-R&D-sector framework.

Section 1 describes the model, relegating some mathematical details to Appendix. This will enable us to concentrate on the main argument of the paper. Section 2 establishes that semi-endogenous growth arises as a general case, and Section 3 identi es two knife-edge conditions required for endogenous growth. Section 4 concludes.

[2] See Mans eld (1998).

[3] See Jones (1998a) for an illuminating survey.

1 Description of the Model

We essentially merge Grossman and Helpman s (1991, Ch. 3 & 4) two standard growth models based on quality and variety innovation, introducing a positive population growth.

1.1 Consumers and Final Output Sector

Without loss of generality, we assume that the entire population of the economy constitutes one large household. Each member of the household supplies one unit of labour service (a numeraire) at every point in time. The size of the household is given by $L_t = e^{\lambda t}$, $\lambda > 0$. The household derives its income from wages of its members and nancial assets it owns. The household intertemporally maximises the sum of the instantaneous (logarithmic) utility of its members.

Under perfectly competitive environment, homogeneous consumption goods are produced with intermediate goods which are differentiated in variety and quality. These goods do not exist in the economy until they are invented through R&D. The aggregate production function takes the form of

$$Y_t = \left[\int_0^{N_t} \left(\sum_{n_i=0}^{\infty} q_{nit} x_{nit} \right)^{\varepsilon/(1+\varepsilon)} di \right]^{(1+\varepsilon)/\varepsilon}, \qquad \infty > \varepsilon > 0, \tag{1}$$

$$q_{nit} = \gamma^{n_i} Q_\tau^{1/\varepsilon}, \qquad \gamma > 1, \qquad n_i = 0, 1, 2, ... \tag{2}$$

N_t denotes the variety of intermediate goods and rises due to variety innovation (e.g. the invention of the laser). x_{nit} denotes the quantity of inputs in the ith variety after its quality has been improved n_i times. q_{nit} is the quality level of x_{nit} and rises through quality innovation (e.g. surgical applications of the laser). This quality index consists of two parts. First, $Q_\tau^{1/\varepsilon}$ denotes the initial quality level of the ith variety when it was invented at time $\tau \leq t$. We assume that Q_τ is determined by the quality level in the whole economy at τ such that $Q_\tau = (1/N_\tau) \int_0^{N_\tau} q_{ni'\tau}^{\varepsilon} di'$. Second, γ^{n_i} represents a quality improvement achieved as a result of n_i innovations following the invention of the variety.

Note that different quality products in the ith variety enter as perfect substitutes in (1). Thus, following the literature, nal good producers are assumed to use inputs of the lowest quality-adjusted price in each industry (which are the highest quality in equilibrium due to Bertrand competition).

1.2 Intermediate Goods Sector

In each instant, some researchers engage in variety R&D, and others conduct quality R&D. If they succeed in variety R&D at time τ, an entirely new product, say the ith variety with quality $Q_\tau^{1/\varepsilon}$, is introduced into the economy and a patent is granted, which excludes others from producing the new product. The innovator becomes a local monopolist. He faces the factor demand function derived from (1) and uses technology which produces one unit of x_{nit} with one unit of labour services. Solving the pro t maximisation problem generates ow pro ts that the innovator earns over time. But once the new variety is available on the market, its quality can be improved by outside rms. Following the literature, we assume that incumbent rms do not conduct quality R&D.

If the new variety is improved upon once, its quality index becomes $\gamma Q_\tau^{1/\varepsilon}$. The higher quality good captures the whole market share, making the original product of quality $Q_\tau^{1/\varepsilon}$ obsolete. However, such improvement infringes on the patent of the variety innovator. At this stage, the variety and quality innovators reach a license agreement in which the latter pays a xed fraction $1 > \kappa > 0$ of his pro ts as royalty to the former. κ is interpreted as the scope of the *variety* patent set by the government.[4]

[4] Alternatively, κ may measure a relative bargaining power of variety and quality innovators on legal

Of course, the quality of the product can be improved further to $\gamma^{n_i} Q_{\tau_i}^{1/\varepsilon}$, $n_i \geq 2$.... Whenever further quality innovation occurs, lower quality goods are driven out of the market, so that all varieties of goods are produced by patent holders of the highest quality good in the industry.[5] However, the latest quality innovator is still required to pay royalty to the original innovator of the variety. This license agreement, which remains forever irrespective of the level of quality, implies the following profit share

$$\pi_{nit}^{v} = \begin{cases} \pi_{nit} \ \text{ for } n_i = 0, \\ \kappa \pi_{nit} \ \text{ for } n_i \geq 1, \end{cases} \qquad \pi_{nit}^{q} = \begin{cases} 0 \text{ for } n_i = 0, \\ (1-\kappa)\, \pi_{nit} \ \text{ for } n_i \geq 1, \end{cases} \tag{3}$$

π_{nit}^{v} and π_{nit}^{q} denote profits earned by variety and quality innovators, and π_{nit} is the *total* profit flow arising from selling the ith variety at time t.

Let V_t^v and V_{nit}^q denote the expected discounted flow profits π_{nit}^{v} and π_{nit}^{q}, respectively. Namely, V_t^v and V_{nit}^q give the stockmarket value of variety and quality innovation. V_t^v takes into account the risk facing the variety innovator that profits discretely fall when his product is improved upon for the first time. Similarly, V_{nit}^q incorporates the risk facing the quality innovator that his product becomes obsolete once a higher quality product is invented in the industry.

1.3 R&D and Knowledge Stock

First let us consider quality R&D. The nth quality innovation in industry i occurs with a Poisson arrival rate of

$$\xi_{nit} = \frac{R_{it}^q K_t^q}{a^q q_{nit}^{\varepsilon}}, \qquad K_t^q = N_t^{\phi^q} Q_t^{\delta^q}, \qquad \phi^q, \delta^q > 0. \tag{4}$$

enforcement of patent protection that affects royalty payment (see Green and Scotchmer (1995)).

[5] Following the literature, quality innovation is assumed not to infringe on the patent of previous quality innovators.

$a^q > 0$ is a constant, R^q_{it} denotes workers used, and K^q_t represents the *positive* externality of the knowledge stock useful to quality R&D. N_t represents knowledge created through variety innovation, and $Q_t = (1/N_t)\int_0^{N_t} q^{\varepsilon}_{ni't}di'$ captures knowledge generated by quality innovation. Parameters ϕ^q and δ^q are weights attached to each type of technological knowledge. q^{ε}_{nit} $(n_i \geq 1)$ in the denominator represents the *negative* externality of the past innovation, which raised the quality of the ith variety to q^{ε}_{nit}. This means that quality R&D becomes progressively more difficult, as it aims at successively higher quality goods.[6]

Turning to variety innovation, it occurs according to

$$\dot{N}_t = \frac{R^v_t K^v_t}{a^v Q_t}, \qquad K^v_t = N^{\phi^v}_t Q^{\delta^v}_t, \qquad \phi^v, \delta^v > 0 \tag{5}$$

where $a^v > 0$ is a constant, R^v_t denotes workers employed, and K^v_t is the *positive* externality of the knowledge stock useful to variety R&D. Like q^{ε}_{nit} in (4), Q_t in the rst equality of (5) captures the *negative* externality of the past successful innovations. Recall that the latest variety introduced at t has the initial quality, which is a function of Q_t. As technology advances or Q_t rises, variety innovation becomes more and more difficult.[7]

If a rm succeeds in variety R&D, it attains the stockmarket value V^v_t. Similarly, a rm which generates the nth quality innovation in industry i, achieves the stockmarket value V^q_{nit}. Therefore, the number of R&D workers is determined by $\max_{R^v_t} V^v_t \frac{R^v_t K^v_t}{a^v Q_t} - (1-s^v)R^v_t$ for variety R&D and $\max_{R^q_{it}} V^q_{nit}\frac{R^q_{it}K^q_t}{a^q q_{nit}} - (1-s^q)R^q_{it}$ for quality R&D, where s^v and s^q denote the rate of R&D subsidy nanced by lump-sum transfers.

R&D activities are nanced through the stockmarket where consumers savings are

[6] As an example of the negative externality, consider the production of silicon chips, which are created by printing circuit patterns on wafers of silicon. As more and more transistors are condensed in a single chip (now well in excess of ten million), the creation of the next-generation chip becomes more and more difficult, and the conventional method is said to hit a wall , at which circuit patterns begin to blur. This type of R&D difficulty arises because of the past successful R&D efforts.

[7] An alternative speci cation of (5) may be $\dot{N}_t = \frac{R^v_t K^v_t}{a^v}$ and $K^v_t = N^{\phi^v}_t Q^{\delta^v - 1}_t$, where K^v_t incorporates the positive and negative externalities. This modi cation does not change anything in the following analysis.

invested in equities of innovative firms. Successful firms distribute profits as dividend to investors. The household may lose in some investment, as R&D is uncertain.[8] But such risks are fully diversifiable since there are many research firms.

1.4 Equilibrium and Long-run Growth

We are interested in equilibrium where both variety and quality R&D are conducted simultaneously. Such equilibrium requires two things; (i) research firms are indifferent between two types of R&D, and (ii) workers, used for producing intermediate goods and R&D, are fully employed.

Appendix establishes the first useful result:

$$\xi_{nit} = \xi_t \qquad \forall\, i, n_i, \tag{6}$$

i.e. the risk facing quality innovator of his product becoming obsolete is the same for all varieties, irrespective of quality level. (6) must hold if all existing variety goods are subject to quality improvement.

Using the fact that only the state-of-the-art products are in use in all industries, the aggregate production function (1) can be reduced to $y_t = \frac{\varepsilon}{1+\varepsilon} c_t \left(N_t Q_t\right)^{\frac{1}{\varepsilon}}$ where y_t is per capita output and c_t is consumption expenditure per person. In the long run when $\dot{c}_t = 0$, per capita output grows at the rate

$$g_y = \frac{1}{\varepsilon}\left(g_N + g_Q\right) \tag{7}$$

where g_k is the growth rate of a variable k. Furthermore, Appendix shows

$$g_{Qt} = \left(\gamma^{\varepsilon} - 1\right) \xi_t. \tag{8}$$

[8] (5) assumes that variety innovation is deterministic. However, Li (1998a) shows that it is derived from a stochastic process of discrete variety innovation.

In existing two-R&D-sector models, growth of variety is proportional to population growth, i.e. $g_N = \lambda$ (λ can be zero) and only g_Q is determined endogenously in balanced growth equilibrium.[9] The following sections will show that both g_N and g_Q are pinned down by λ in general and they are endogenously determined under very restrictive conditions.

2 Semi-endogenous Growth

To establish the generality of semi-endogenous growth, we concentrate on the balanced growth path. This path is characterised by (i) a constant share of workers allocated to manufacturing and two types of R&D, and (ii) constant g_N and g_Q. We focus on requirement (ii), since doing so is sufficient to establish our desirable result.

Note that g_N and g_Q can be expressed as

$$g_{Nt} = \frac{R_t^v \chi_t^v}{L_t a^v}, \qquad \frac{g_{Qt}}{\gamma^\varepsilon - 1} = \xi_t = \frac{R_t^q \chi_t^q}{L_t \gamma^\varepsilon a^q} \tag{9}$$

where $\chi_t^s = L_t / N_t^{1-\phi^s} Q_t^{1-\delta^s}$, $s = q, v$ and R_t^q is the number of workers in quality R&D in the whole economy. The first equation results from rearranging (5). The second equation is derived by summing (4) over i.[10] Since R_t^q / L_t and R_t^v / L_t are constant along the balanced growth path, so is χ_t^s, $s = q, v$. Furthermore, constant χ_t^s, $s = q, v$ means that L_t and $N_t^{1-\phi^s} Q_t^{1-\delta^s}$, $s = v, q$, grow at the same rate:

$$\lambda = (1 - \phi^v)\, g_N + (1 - \delta^v)\, g_Q, \qquad \lambda = (1 - \phi^q)\, g_N + (1 - \delta^q)\, g_Q. \tag{10}$$

For $\phi^v \neq \phi^q$ or/and $\delta^v \neq \delta^q$, these equations are linearly independent and pin down g_N and g_Q. This is the general case, which this section analyses. The special case of $\phi^v = \phi^q$

[9] Segerstrom (1998b) is an exception.

[10] This is derived as follows. (4) and (6) imply $R_{it}^q = q_{nit}^\varepsilon \xi_t a^q / K_t^q$ where $q_{nit}^\varepsilon = \gamma^{\varepsilon n_i} Q_\tau$ for $n_i \geq 1$. Summing it over i gives $R_t^q = \int_0^{N_t} R_{it}^q di = \int_0^{N_t} q_{nit}^\varepsilon di a^q \xi_t / K_t^q = N_t Q_t \gamma^\varepsilon a^q \xi_t / K_t^q = \gamma^\varepsilon a^q \xi_t L_t / \chi_t^q$ where the third equality uses $Q_t = (1/N_t) \int_0^{N_t} q_{nit}^\varepsilon di$ where $q_{nit}^\varepsilon = \gamma^{\varepsilon n_i} Q_\tau$ for $n_i \geq 0$.

and $\delta^v = \delta^q$ will be considered in the following section.

Solving (10) and defining $D = (1-\phi^q)(1-\delta^v) - (1-\phi^v)(1-\delta^q) \neq 0$, we obtain

$$g_N = \frac{\delta^q - \delta^v}{D}\lambda, \qquad g_Q = \frac{\phi^v - \phi^q}{D}\lambda. \tag{11}$$

Clearly, an interior solution $g_N, g_Q > 0$ requires that both numerators and D must have the same signs. There are two cases:

$$\text{(i) } \phi^v > \phi^q \text{ and } \delta^v < \delta^q, \qquad \text{(ii) } \phi^v < \phi^q \text{ and } \delta^v > \delta^q. \tag{12}$$

A unique interior solution is obtained if either case holds true. In Case (i), for each variety and quality R&D, knowledge created through its own R&D is more important than knowledge generated by the other type of R&D. Case (ii) is the reverse situation.

Corner solutions are essentially equivalent to one-R&D-sector models. For example, consider the case of $1 > \phi^s, \delta^s > 0$, $s = q, v$. Diagrammatical analysis easily verifies that $g_N = \lambda/(1-\phi^v)$ and $g_Q = 0$ arises for $\phi^v \leq \phi^q$ and $\delta^v < \delta^q$. In this case, growth is driven by variety innovation only and is equivalent to the one-R&D-sector model of Jones (1995a). Alternatively, $g_N = 0$ and $g_Q = \lambda/(1-\delta^q)$ occurs for $\phi^v > \phi^q$ and $\delta^v \geq \delta^q$. This is essentially equivalent to Segerstrom (1998a) where only quality R&D is conducted.[11]

Substituting (11) into (7) gives

$$g_y = \frac{\phi^v - \phi^q + \delta^q - \delta^v}{\varepsilon D}\lambda. \tag{13}$$

This states that the growth rate of output per capita depends only the growth rate of labour force λ and parameters ϕ^s and δ^s, $s = q, v$, which determine the degree of knowledge spillover in the R&D sectors. Importantly, these parameters are typically assumed to be exogenous.[12]Since (13) is obtained by differentiating $\ln \chi_t^s$, $s = q, v$, and setting them

[11] This case is in fact identical to the model of Li (1998b) which generalises Segerstrom (1998a).

[12] Jones (1998b) endogenises the rate of population growth to generate endogenous growth in an R&D-based model.

zero, g_y is clearly independent of consumers preferences and government policy, such as investment tax credits and R&D subsidies.

In this class of the model, what is endogenous in the long run is the share of workers used in the R&D sectors. Its expression is not reported here, since it is not informative. But it can be veri ed that that share is increasing in R&D subsidies (s^q and s^v).

Note that (12) remains valid even for $\phi^s > 1$ and $\delta^s > 1$, $s = q, v$. For equations (10) to make sense, however, only either ϕ^v or δ^v (ϕ^q or δ^q) in the rst (second) equation is allowed to be larger than one as long as the other parameter is strictly smaller than one, since, if not, the right-hand side becomes negative. For example, $\phi^s > 1 > \delta^s$, $s = q, v$, meets this condition. This fact leads to the following novel result.

In the one-R&D-sector (variety) model of Jones (1995a), semi-endogenous growth requires that an increase in variety knowledge $\dot{N}_t$ is less than linear in the stock of variety knowledge N_t, i.e. $\phi^v < 1$ in equation (5). For $\phi^v \geq 1$, output growth per capita would explode in Jones s model. This unrealistic feature is eliminated in our two-R&D-sector model. Our model exhibits a *constant* semi-endogenous growth even for $\phi^v \geq 1$ as long as $\delta^v < 1$. This highlights the fact that conditions for semi-endogenous growth markedly differ between one-R&D-sector and two-R&D-sector models.

3 Endogenous Growth

This section examines the special case of $\phi \equiv \phi^v = \phi^q$ *and* $\delta \equiv \delta^v = \delta^q$. We rst assume $\phi \neq \delta$. An important consequence of these parameter restrictions is that two equations in (10) are now identical, i.e.

$$\lambda = (1 - \phi)\, g_N + (1 - \delta)\, g_Q. \qquad (BG)$$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	ρ	s^q	s^v	$s^q = s^v$	κ	a^q	a^v	ϕ	δ	γ^ε	λ
g_Q	$+$	$+$	$-$	0	$-$	$-$	$+$	$+$	$+$	$\pm$	$+$
g_N	$-$	$-$	$+$	0	$+$	$+$	$-$	$\pm$	$\pm$	$\pm$	$+$
g_y	$-$	$-$	$+$	0	$+$	$+$	$-$	$\pm$	$\pm$	$\pm$	$+$

Table 1: Comparative Statics for $\phi > \delta$

This equation de nes the combination of g_N and g_Q which must hold along the *b*alanced *g*rowth path. But this condition alone cannot determine g_N and g_Q. This fact sets the stage for endogenous growth.

We are interested in equilibrium in which quality and variety R&D are both conducted. In other words, entrepreneurs must be indifferent between two types of R&D. Appendix demonstrates that such *r*esearch *a*rbitrage is met by the following condition:

$$g_Q = \frac{(\gamma^\varepsilon - 1)(1 - \kappa - a)(\rho + \phi g_N)}{(\gamma^\varepsilon - 1)[\delta a + (1 - \delta)(1 - \kappa)] + a\kappa} \qquad (RA)$$

where $a = (1 - s^q) a^q / (1 - s^v) a^v$, $1 > \kappa + a$, and ρ is consumers subjective rate of time preference. Equilibrium values of g_N and g_Q along the balanced growth path are determined by (RA) and (BG). As Figure 1 shows, a unique equilibrium with g_N, $g_Q > 0$ exists if consumers are sufficiently patient.[13]

In Figure 1, an iso-growth contour is also drawn (see (7)). The BG line is atter than the iso-g_y contour for $\phi > \delta$ and steeper for $\phi < \delta$. Consider the effect of changes in ρ. For $\phi > \delta$, a higher ρ shifts the RA line upward, raising g_Q but reducing g_N. A net effect is that the growth rate becomes lower. The reverse result holds for $\phi < \delta$.

Other comparative static results for the case of $\phi > \delta$ are summarised in Table 1, and results are reversed for $\phi < \delta$, except for column (11).[14]Columns (2)~(5) concern the

[13]For $1 > \phi \geq 0$ and $\delta = 1$, our model becomes essentially equivalent to the existing two-R&D-sector models.

[14]$\phi > \delta$ seems more plausible, because more patient nations (or economies with a low interest rate

effects of public policies on the long-run growth. Targeted R&D subsidies are considered in columns (2) and (3). As expected, a subsidy stimulates the research to which the policy is applied. An interesting observation is that research subsidies have permanent effects on growth, but do *not* necessarily raise the long-run growth.[15] Moreover, column (4) shows that untargeted subsidies to both types of R&D do not change anything, since it does not affect the relative pro tability of research activities. Column (5) shows the effect of widening the scope of patent of variety innovators. Growth is promoted, as κ rises. In this sense, fundamental research, e.g. scienti c or basic research, matters more for the long-run growth, if knowledge it creates is more important in the total stock of the technological knowledge (i.e. $\phi > \delta$).

The above analysis con rms that the economy can grow at a *constant* endogenous rate in the long run, which is independent of the size of the economy L_t. However, this result requires two knife-edge conditions $\phi^v = \phi^q$ and $\delta^v = \delta^q$, which are obtained only on measure-zero subsets of relevant parameter values (e.g. $\phi^v, \phi^q, \delta^v, \delta^q \in (0, 1)$ for $\phi^v, \phi^s, \delta^v, \delta^s < 1$). Those conditions are clearly less general than those which generates semi-endogenous growth, i.e. $\phi^v \neq \phi^q$ or/and $\delta^v \neq \delta^q$.

Next, we turn to the case of $\phi = \delta < 1$. In fact, this parameter restriction gives rise to semi-endogenous growth. This can easily be understood by imposing $\phi = \delta$ in (BG). The equation is reduced to $\lambda = (1 - \phi)(g_N + g_Q)$. Substituting this into (7), we have

$$g_y = \frac{\lambda}{\varepsilon (1 - \phi)}. \tag{14}$$

The result can also be con rmed in Figure 1. Note that the BG line has the same slope as the iso-g_y contour for $\phi = \delta$, so that g_y is independent of the RA line. That is, long-run

policy) are thought to grow faster. In fact, this is the common prediction of most growth models. However, Dinopoulos and Thompson (1998) and Peretto (1998) predict the opposite.

[15] This point is stressed by Seterstrom (1998b).

growth is independent of how the economy devotes resources to two types of R&D.

Note that if ϕ is only slightly smaller or larger than δ, endogenous growth arises. Of course, this does not imply that endogenous growth is more general than semi-endogenous growth, because endogenous growth needs $\phi^v = \phi^q$ and $\delta^v = \delta^q$ in the first place. However, if these parameter restrictions are accepted for whatever reasons, one can make an interesting observation about the *quantitative* effects of public policy. Long-run effects of, e.g. R&D subsidies are large, if $|\phi - \delta|$ (an absolute difference) is large, since the iso-growth contour is much steeper or flatter than the BG line in Figure 1. On the other hand, if $|\phi - \delta|$ is very small but not zero, long-run effects on growth turn out very modest. This suggests two things. First, it is important to know the values of ϕ and δ for effective public policies. Second, it is off the point to criticise endogenous growth models on the quantitative basis of policy impact, since its long-run effect can be very small.

4 Conclusion

Is R&D-driven growth endogenous or semi-endogenous in the long run? This question has important policy implications. If it is endogenous, public policy is effective in affecting long-run growth. If it is semi-endogenous, on the other hand, the government is powerless in changing the trend growth (as long as population growth is exogenous).

The present paper has contributed to the endogenous versus semi-endogenous growth debate by establishing the generality of semi-endogenous growth even in the two-R&D-sector model. The main mechanisms driving this result operate through knowledge spillovers between quality and variety R&D, and this route is largely neglected in existing studies. Our finding resurrects the stark policy conclusion of semi-endogenous growth, which is challenged by the recent two-R&D-sector models.

Appendix: Mathematical Details of the Model

Since the original models of Grossman and Helpman (1991, Ch.3 & 4) are sufficiently familiar, the exposition of mathematical details of our model are made concise.

A.1 Consumers The utility function of the household is $U = \int_0^\infty e^{-(\rho-\lambda)t} \ln y_t dt$ where $\rho > \lambda$ is the rate of time preference. Utility maximisation generates the familiar condition: $\dot{c}_t/c_t = r_t - \rho$ where r_t is the rate of interests. See Barro and Sala-i-Martin (1995, Ch.2) for more details.

A.2 Final Output Sector Given the production function (1), the demand for x_{nit} is $x_{nit} = \frac{q_{nit}^{\varepsilon} p_{nit}^{-(1+\varepsilon)} c_t L_t}{\int_0^{N_t} (q_{nit}/p_{ni't})^{\varepsilon} di'}$ where p_{nit} is the price of x_{nit}. Moreover, using this demand function, it is easy to verify that the unit cost function of final output is $p_t = \left[\int_0^{N_t} (q_{nit}/p_{nit})^{-\varepsilon} di\right]^{1/\varepsilon}$. Clearly, x_{nit} is used (i.e. gives a lower unit cost) if and only if it has the lowest-quality adjusted price in industry i, i.e. $p_{nit}/q_{nit} \leq p_{n-1it}/q_{n-1it}$ or $p_{nit} \leq \gamma$, since $p_{n-1it} = 1$ due to Bertrand competition. We assume that as long as this weak inequality holds, final output producers use the top-quality inputs only.

A.3 Intermediate Goods Sector Maximising $\pi_{nit} = (p_{nit} - 1)\, x_{nit}$ (x_{nit} is defined in Section A.2) gives $p_{nit} = \frac{1+\varepsilon}{\varepsilon}$ for $\frac{1+\varepsilon}{\varepsilon} \leq \gamma$. This is the case of drastic innovation in the sense that the size of innovation γ is so large that firms price decisions are not constrained by potential competition from lower-quality-goods producers. $\frac{1+\varepsilon}{\varepsilon} \leq \gamma$ is assumed throughout the paper. The *total* profit flow arising from selling the ith variety at time t is $\pi_{nit} = \frac{q_{nit}^{\varepsilon} c_t L_t}{(1+\varepsilon) N_t Q_t}$.

A.4 R&D Activities R_t^v and R_{it}^q are determined by solving $\max_{R_t^v} V_t^v \frac{R_t^v K_t^v}{a^v Q_t} - (1 - s^v) R_t^v$ and $\max_{R_{it}^q} V_{nit}^q \frac{R_{it}^q K_t^q}{a^q q_{nit}} - (1 - s^q) R_{it}^q$. Free entry implies $V_t^v = \frac{(1-s^v) a^v Q_t}{K_t^v}$ and

$V^q_{nit} = \frac{(1-s^q)a^q q^\varepsilon_{nit}}{K^q_t}$ whenever R^v_t, $R^q_{it} > 0$.

A.5 Stockmarket Values V^q_{nit} is de ned by the no-arbitrage condition $r_t = \frac{\pi^q_{nit}}{V^q_{nit}} + \frac{\dot{V}^q_{nit}}{V^q_{nit}} - \xi_{n+1it}$ where the right-hand side is the rate of return from equities of quality innovative rm (the dividend rate, the capital gains and the risk of losing pro ts in future). Note that $\pi^q_{nit}/V^q_{nit} = \frac{(1-\kappa)K^q_t c_t L_t}{(1+\varepsilon)(1-s^q)a^q N_t Q_t}$ which is independent of i and n_i. Moreover, the R&D free entry condition (derived in Section A.4) implies $\dot{V}^q_{nit}/V^q_{nit} = -\dot{K}^q_t/K^q_t$ for all i and n_i, since q^ε_{nit} is xed when the rm is producing the state-of-the-art product. Substituting these into the above no-arbitrage condition gives result (6) in the text. These results enable us to rewrite the quality R&D no-arbitrage condition as

$$r_t = \frac{1-\kappa}{(1+\varepsilon)(1-s^q)a^q}\frac{c_t L_t K^q_t}{N_t Q_t} - \frac{\dot{K}^q_t}{K^q_t} - \xi_t \tag{15}$$

which holds in all industries. Next consider V^v_t. Note that the variety innovator loses $(1-\kappa)\pi_{0it}$ for ever when the rst quality innovation occurs. Using V^q_{0it} to denote the expected present value of $(1-\kappa)\pi_{0it}$, V^q_t is de ned by the no-arbitrage condition $r_t = \frac{\pi_{0it}}{V^v_t} + \frac{\dot{V}^v_t}{V^v_t} - \frac{V^q_{0it}}{V^v_t}\xi_t$. On the right-hand side are the dividend rate, the capital gain (or loss) and the term which captures the capital loss due to the rst quality innovation. Note that we have $V^v_{0it} = \frac{Q_t}{q^\varepsilon_{nit}}V^q_{nit}$,[16] and the R&D free entry conditions (see Section A.4) give $V^q_{nit}/V^v_t = \frac{a q^\varepsilon_{nit} K^v_t}{Q_t K^q_t}$. Using these results, the variety R&D no-arbitrage condition can be rewritten as

$$r_t = \frac{1}{(1+\varepsilon)(1-s^v)a^v}\frac{c_t L_t K^v_t}{N_t Q_t} + \frac{\dot{Q}_t}{Q_t} - \frac{\dot{K}^v_t}{K^v_t} - a\xi_t\frac{K^v_t}{K^q_t}. \tag{16}$$

A.6 Labour Market The labour supply L_t must be equal to the total demand

[16] Consider the value of quality innovation which occurs at t. It can be rewritten as

$$V^q_{nit} = \int_t^\infty e^{-\int_t^T (r_s+\xi_s)ds}\pi^q_{niT}dT = \frac{q^\varepsilon_{nit}}{Q_t}\int_t^\infty e^{-\int_t^T (r_s+\xi_t)ds}(1-\kappa)\pi_{0iT}dT = \frac{q^\varepsilon_{nit}}{Q_t}V^q_{0it},$$

since $\pi^q_{niT} = \frac{(1-\kappa)q^\varepsilon_{nit}c_T L_T}{(1+\varepsilon)N_T Q_T} = \frac{q^\varepsilon_{nit}}{Q_t}(1-\kappa)\frac{Q_t c_T L_T}{(1+\varepsilon)N_T Q_T} = \frac{q^\varepsilon_{nit}}{Q_t}(1-\kappa)\pi_{0iT}$ where q^ε_{nit} and Q_t are determined at t and xed for $T > t$.

of workers. Workers in manufacturing is $\int_0^{N_t} x_{nit} di = \frac{\varepsilon}{1+\varepsilon} c_t L_t$. (9) gives the number of researchers in variety and quality R&D: $R_t^v = a^v g_{Nt} L_t / \chi_t^v$ and $R_t^q = a^q g_{Qt} L_t / \left(1 - 1/\gamma^\varepsilon\right) \chi_t^q$. Thus, full-employment of workers requires

$$1 = \frac{a^v g_{Nt}}{\chi_t^v} + \frac{a^q g_{Qt}}{\left(1 - 1/\gamma^\varepsilon\right) \chi_t^q} + \frac{\varepsilon}{1+\varepsilon} c_t. \tag{17}$$

A.7 Growth Rate of Q_t(equation (8)) First write $N_t Q_t = \int_0^1 q_{nit}^\varepsilon di + \int_1^{N_t} q_{nit}^\varepsilon di$ where $N_0 = 1$. We write the rst term as $\int_0^1 q_{nit}^\varepsilon di = E\left(q_{nit}^\varepsilon \left|\tau = 0\right.\right)$ ($E(.)$ is the expectation operator), using the Law of Large Numbers. The second term is approximated with the average of q_{nit}^ε such that $\int_1^{N_t} q_{nit}^\varepsilon di \simeq \int_1^{N_t} E\left(q_{nit}^\varepsilon \left|\tau > 0\right.\right) di$, so that

$$N_t Q_t \simeq E\left(q_{nit}^\varepsilon \left|\tau = 0\right.\right) + \int_1^{N_t} E\left(q_{nit}^\varepsilon \left|\tau > 0\right.\right) di. \tag{18}$$

Moreover, given the Poisson distribution of quality innovation, $E\left(q_{nit}^\varepsilon \left|\tau \geq 0\right.\right) = \sum_{n_i=0}^{\infty} \frac{\left(\int_\tau^t \xi_s ds\right)^{n_i} e^{-\int_\tau^t \xi_s ds}}{n_i!} \gamma^{\varepsilon n_i} Q_\tau = e^{(\gamma^\varepsilon - 1)\int_\tau^t \xi_s ds} Q_\tau$ where $Q_0 = 1$. Using this and changing the variable i in the integral of (18) with τ with the use of $i = e^{\int_0^\tau g_{Ns} ds}$, $\tau \leq t$, we rewrite (18) as $N_t Q_t \simeq e^{(\gamma^\varepsilon - 1)\int_0^t \xi_s ds}\left(1 + \int_0^t g_{N\tau} e^{-(\gamma^\varepsilon - 1)\int_0^\tau \xi_s ds} N_\tau Q_\tau d\tau\right)$. Differentiating $\ln Q_t$ with respect to t (and replacing $\simeq$ with $=$) yields (8).

A.8 Equilibrium Conditions and Balanced Growth Path Equilibrium conditions consists of the labour market condition (17), the two no-arbitrage conditions (15) and (16), which are rewritten as

$$\frac{\dot{c}_t}{c_t} + \rho = \frac{(1-\kappa) c_t \chi_t^q}{(1+\varepsilon)(1-s^q) a^q} - \phi^q g_{Nt} - \left(\delta^q + \frac{1}{\gamma^\varepsilon - 1}\right) g_{Qt}, \tag{19}$$

$$\frac{\dot{c}_t}{c_t} + \rho = \frac{c_t \chi_t^v}{(1+\varepsilon)(1-s^v) a^v} - \phi^v g_{Nt} - \left(\delta^v - 1 + \frac{a}{\gamma^\varepsilon - 1}\frac{\chi_t^q}{\chi_t^v}\right) g_{Qt}, \tag{20}$$

and two equations obtained from differentiating χ_t^q and χ_t^v with respect to time t :

$$\dot{\chi}_t^s = \chi_t^s \left[\lambda - (1-\phi^s) g_{Nt} - (1-\delta^s) g_{Qt}\right], \qquad s = q, v. \tag{21}$$

These conditions clearly indicate that $\dot{c}_t = \dot{\chi}_t^q = \dot{\chi}_t^v = 0$ and g_N and g_Q are constant in the balanced growth path where the labour share is constant.

A.9 Semi-endogenous Growth Along this path, (21) is collapsed to (10). Given g_N and g_Q in (11), conditions (17), (19) and (20) (where $\dot{c}_t = 0$) determine c, χ^q and χ^v.

A.10 Endogenous Growth Note that $\chi_t \equiv \chi_t^q = \chi_t^v$ for $\phi \equiv \phi^v = \phi^q$ and $\delta \equiv \delta^v = \delta^q$. Using this, (RA) is derived by combining (19) and (20) to eliminate $c_t\chi_t$. Given g_N and g_Q determined by (RA) and (BG), c and χ are determined by (17) and either (19) or (20) (where $\dot{c}_t = 0$).

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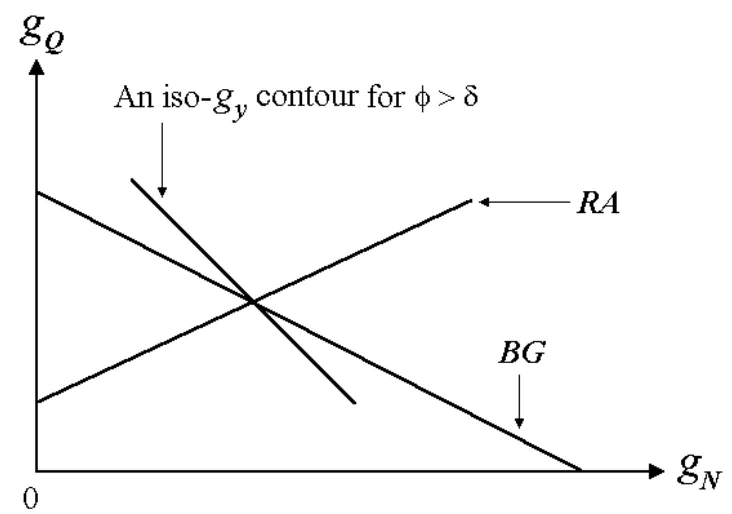

Figure 1: The special case of $\phi \equiv \phi^v = \phi^q$ and $\delta \equiv \delta^v = \delta^q$.