Endogenous Growth Without Scale Effects: Comment

Chol-Won Li[*]
Department of Economics
University of Glasgow

August 1998

[*]Correspondence: Dept. of Economics, Univ. of Glasgow, Adam Smith Building, Glasgow G12 8RT, UK; (Tel.) ++44-(0)141-330-4654; (Fax) ++44-(0)141-330-4940; (E-mail) cw.li@socsci.gla.ac.uk. I am grateful to Philippe Aghion, Julia Darby and Elias Dinopoulos for their valuable comments. Needless to say, I am solely responsible for opinions and all remaining errors.

In this Review, Segerstrom (1998) modifies Grossman and Helpman's (1991) R&D-based growth model (hereafter GH) in order to reconcile it with Jones's (1995a) empirical evidence which shows no "scale effects" in growth. Segerstrom's main idea is that R&D becomes progressively more difficult over time, offsetting the effect of population growth.[1]

In this note, we first argue that while this idea is intuitively appealing, it is incorporated in a rather ad hoc manner and generates an unrealistic implication. Second, and more importantly, we generalize Segerstrom's model, parameterizing the elasticity of substitution between any two consumption goods. That is, Segerstrom's Cobb-Douglas with perfect substitutes (CDS) preferences is generalized to the Constant Elasticity of Substitution (Dixit-Stiglitz type) with perfect substitutes (CESS) preferences.[2]

Generalization yields striking results. First, positive and normative results in Segerstrom are overturned, e.g. under certain conditions it is globally optimal to subsidize R&D. This arises because firms' pricing decisions differ depending on whether the CESS or CDS preferences are assumed. Second, it is shown that Segerstrom's idea of R&D being progressively difficult is fully compatible with the GH model with scale effects. A crucial assumption for removing scale effects turns out to be diminishing returns to the knowledge accumulation in R&D technology – the same assumption used in Jones (1995b).

1 Segerstrom's Assumption of R&D

Segerstrom uses $X(t)$ to denote an R&D difficulty index. The Poisson arrival rate of an innovation is $I(t) = AL_I(t)/X(t)$; $A > 0$; which decreases in $X(t)$ and increases in researchers $L_I(t)$: Segerstrom assumes that $X(t)$ grows at a rate proportional to $I(t)$:

[1] Aghion and Howitt (1998, Ch.12) also stress this idea in eliminating scale effects.

[2] The CDS preferences were first introduced by Segerstrom, et al. (1990).

Now suppose that an entrepreneur invests in R&D at time t and successfully invents the state-of-the-art product at $t + dt$. The difficulty index rises to $X(t + dt) > X(t)$, since $I(\cdot) > 0$ (i.e. $L_I > 0$) for the time period dt. This makes perfect sense, as current research success makes future R&D more difficult (e.g. computer chips).

Next suppose that no innovation has occurred for T years. The difficulty index is now $X(t + T) > X(t)$, since $I(\cdot) > 0$ (i.e. $L_I > 0$) for T years. That is, continual failures to innovate render the identical R&D project more difficult! This is simply counter-factual. Causality should run in the opposite direction; a more difficult R&D project causes more failures at least on average.

The problem of Segerstrom's difficulty index is that it effectively depends on the accumulation of the number of R&D workers who are employed, irrespective of research success or failure. A more plausible assumption is that the difficulty index depends only on the past successful innovations. We implement this assumption in the following section.

2 Generalization: CESS Preferences

We maintain Segerstrom's notations and assumptions unless otherwise stated.

2.1 Consumers and Workers

The number of workers is given by $L(t) = L_0 e^{nt}$ where L_0 denotes the population at $t = 0$ ($L_0 = 1$ in Segerstrom). The utility per person takes the form of the CESS preferences:

$$u(t) = \left\{ \int_0^1 \left[\sum_j \lambda^{j_\omega} d(j; \omega; t) \right]^{\alpha} d\omega \right\}^{\frac{1}{\alpha}}; \quad 1 > \alpha > 0; \quad \lambda > 1; \tag{1}$$

where the dependence of j on ω in λ^{j_ω} is made explicit. The CDS preferences used in Segerstrom (and GH) is a special case in which $\alpha = 0$:

From (1), the demand function for the product with the lowest quality-adjusted price in industry ω is given by (see below)

$$d(j;\omega;t) = \frac{\lambda^{\varepsilon j_\omega} p(j;\omega;t)^{-(1+\varepsilon)}}{\int_0^1 \left[\lambda^{j_{\omega'}}/p(j;\omega';t)\right]^{\varepsilon} d\omega'} c(t); \qquad \varepsilon = \frac{\alpha}{1-\alpha}; \tag{2}$$

and other goods in the same industry are not consumed. As in Segerstrom, the intertemporal utility maximization yields $\dot{c}(t)/c(t) = r(t) - \rho.$

2.2 Product Markets

Substituting (2) into (1) yields the indirect utility function per person: $\upsilon(t) = \left\{\int_0^1 \left[p(j;\omega;t)/\lambda^{j_\omega}\right]^{-\varepsilon} d\omega\right\}^{1/\varepsilon} c(t).$ Clearly, $d(j;\omega;t)$ is purchased (i.e. gives a higher utility) if and only if it has the lowest-quality adjusted price, i.e. $p(j;\omega;t)/\lambda^{j_\omega} \leq p(j-1;\omega;t)/\lambda^{j_\omega - 1}$ or $p(j;\omega;t) \leq \lambda p(j-1;\omega;t)$ where $p(j-1;\omega;t) = 1.$ Thus, given that the demand function (2) has the price elasticity of $-1/(1-\alpha)$, a top-quality firm sets $p(j;\omega;t) = 1/\alpha$ for $1/\alpha < \lambda$ or a limit price $p(j;\omega;t) = \lambda$ otherwise; i.e.

$$p(j;\omega;t) = \frac{1}{\theta} \equiv \begin{cases} 1/\alpha & \text{for } 1/\alpha < \lambda \quad \text{(drastic innovation)} \\ \lambda & \text{for } 1/\alpha > \lambda \quad \text{(non-drastic innovation).} \end{cases} \tag{3}$$

Innovation is drastic for $1/\alpha < \lambda$ in the sense that firms' price decisions are not constrained by potential competition from previous incumbent producers. In contrast, innovation is always non-drastic in Segerstrom, since $\alpha = 0.$ This difference has important implications for welfare analysis below. The quality leader earns

$$\pi(j;\omega;t) = (1-\theta)\frac{\lambda^{\varepsilon j_\omega}}{Q(t)} L(t) c(t); \qquad Q(t) = \int_0^1 \lambda^{\varepsilon j_{\omega'}} d\omega' \tag{4}$$

where $Q(t)$ is equivalent to the average quality across industries.[3]

[3] Note that there are two types of the business-stealing effects. Following innovation in industry ω, the former quality leader in the same industry loses all of its profits. At the same time, profits of firms

2.3 R&D Races

Any R&D firm i that uses ℓ_i workers in industry ω will succeed in generating the $(j+1)$th innovation with instantaneous probability of $\frac{A\ell_i}{\lambda^{\varepsilon(j_\omega+1)}}Q(t)^{\phi}$; $1>\phi>0$: This assumption has several features worth mentioning. First, if $\alpha=0$ (i.e. $\varepsilon=0$); it is reduced to $A\ell_i$; which is essentially equivalent to the R&D technology used by GH. Segerstrom simply modifies this into $A\ell_i/X(\omega;t)$ where $X(\omega;t)$ is his difficulty index that grows at a rate proportional to $\sum_i A\ell_i/X(\omega;t)$: Second, $\lambda^{\varepsilon(j_\omega+1)}$ is our difficulty index and depends only on the effects of the past successful R&D, unlike Segerstrom's.[4] Third, $Q(t)^{\phi}$ represents the positive knowledge spillover effect across industries. Fourth, $1>\phi>0$ captures Jones's (1995b) idea that R&D technology exhibits diminishing returns to knowledge accumulation.

Let $v(j+1;\omega;t)$ denote the expected discounted profit for inventing the $(j+1)$th invention in industry ω: Since an R&D firm maximizes $v(j+1;\omega;t)\frac{A\ell_i}{\lambda^{\varepsilon(j_\omega+1)}}Q(t)^{\phi}-\ell_i$; free entry leads to

$$v(j+1;\omega;t)=\frac{\lambda^{\varepsilon(j_\omega+1)}}{AQ(t)^{\phi}} \tag{5}$$

for all ω: This condition makes entrepreneurs indifferent to any R&D projects.

The value of innovation is defined by the "no-arbitrage" condition $\pi(j;\omega;t)/v(j;\omega;t)+\dot{v}(j;\omega;t)/v(j;\omega;t)=r(t)+I(j;\omega;t)$ where

$$I(j;\omega;t)\equiv\frac{AL_I(j;\omega;t)}{\lambda^{\varepsilon(j_\omega+1)}}Q(t)^{\phi};\qquad L_I(j;\omega;t)=\sum_i \ell_i: \tag{6}$$

Note that (i) equation (5) implies $\dot{v}(j;\omega;t)/v(j;\omega;t)=-\phi\dot{Q}(t)/Q(t)$; since $\lambda^{\varepsilon(j_\omega+1)}$ is fixed from the viewpoint of entrepreneurs and investors, and (ii) from (4) and (5),

in other industries fall due to an increase in $Q(t)$:

[4] A similar specification is used in Barro and Sala-i-Martin (1995, Ch.7).

$\pi(j;\omega;t)/v(j;\omega;t) = (1-\theta)AL(t)c(t)/Q(t)^{1-\phi}$ for all j and ω. Therefore, the above no-arbitrage condition implies $I(j;\omega;t) = I(t)$ for all j and ω, so that it can be re-expressed as

$$r(t) + I(t) = -\phi\frac{\dot{Q}(t)}{Q(t)} + \frac{(1-\theta)AL(t)c(t)}{Q(t)^{1-\phi}} \qquad \text{for all } j \text{ and } \omega. \tag{7}$$

2.4 The Labor Market and $Q(t)$

The total employment of research workers is derived from (6): $L_I(t) = \int_0^1 L_I(j;\omega;t)\,d\omega = \frac{\lambda^{\varepsilon} I(t)Q(t)^{1-\phi}}{A}$, as $I(j;\omega;t) = I(t)$. Employment in the manufacturing sector is given by $D(t) = \int_0^1 d(j;\omega;t)\,d\omega = \theta L(t)c(t)$. Thus, the labor full-employment requires

$$1 = \theta c(t) + \frac{\lambda^{\varepsilon} I(t)Q(t)^{1-\phi}}{AL(t)}. \tag{8}$$

A key variable in this model is $Q(t)$. Note that quality improvement $\lambda^{\varepsilon(j_\omega+1)} - \lambda^{\varepsilon j_\omega}$ occurs with arrival rate of $I(j;\omega;t)$. Therefore, the law of large numbers implies

$$\dot{Q}(t) = \int_0^1 I(j;\omega;t)\left[\lambda^{\varepsilon(j_\omega+1)} - \lambda^{\varepsilon j_\omega}\right] d\omega = (\lambda^{\varepsilon} - 1)I(t)Q(t) \tag{9}$$

where the second equality uses (6) (see, e.g. Barro and Sala-i-Martin 1995, p.260).

2.5 Balanced Growth Equilibrium

First define $x(t) = Q(t)^{1-\phi}/L(t)$. Equations (7) and (8) imply that $x(t)$ must be constant in steady sate, which in turn implies[5]

$$I = \frac{n}{(1-\phi)(\lambda^{\varepsilon}-1)}. \tag{10}$$

[5] Given $I(j;\omega;t) = I$, (6) implies that industries which have in the past experienced more innovations devote relatively more resources to R&D. Thus, although the patent rate is the same across industries, R&D employment levels change stochastically around the average over time.

Besides, (1) is reduced to $u(t) = \theta c Q(t)^{1/\varepsilon}$. (9) and (10) imply that the utility grows at

$$\frac{\dot{u}(t)}{u(t)} = \frac{n}{\varepsilon(1-\phi)}. \tag{11}$$

In Segerstrom, equation (10) is replaced with $I = n/\mu$ where μ is a parameter, and a higher μ accelerates an increase in his R&D difficulty index. In our model, μ is endogenized in terms of technology parameters, $(1-\phi)(\lambda^{\varepsilon}-1)$. Another interesting difference lies in utility growth, which is increasing in λ in Segerstrom (see equation (19) of his paper) but is independent of λ in our model (equation (11)). Segerstrom mentions a possible extension of endogenizing utility growth through endogenizing λ. Our model suggests that such an extension endogenizes the rate of technical progress but not utility growth.

Equilibrium conditions in steady-state ($\dot{c} = \dot{x} = 0$) are obtained from (7) and (8):

$$\frac{(1-\theta)Ac}{x} = \rho + \left(\frac{1}{\lambda^{\varepsilon}-1}+\phi\right)\frac{n}{1-\phi}, \tag{12}$$

$$1 = \frac{\lambda^{\varepsilon} n}{A(1-\phi)(\lambda^{\varepsilon}-1)}x + \theta c. \tag{13}$$

A figure depicting these two conditions is essentially identical to Figure 3 of Segerstrom.[6] Solving (12) and (13) yields the share of R&D workers $k \equiv L_I(t)/L(t) = 1-\theta c$:

$$k = \frac{1}{1+\frac{\theta}{1-\theta}\left(\Psi - \frac{\Psi-1}{\lambda^{\varepsilon}}\right)}, \qquad \text{where } \Psi = \frac{\rho}{n}(1-\phi)+\phi > 1. \tag{14}$$

Segerstrom finds that this share is monotonically increasing λ, because innovation is always non-drastic, and as a result, a higher λ means a higher monopoly mark-up with a greater incentive for R&D. In our model, in contrast, k is increasing in λ for $\lambda < 1/\alpha$ but decreasing for $\lambda > 1/\alpha$ due to two opposing effects. The monopoly mark-up effect is captured by a term $\frac{\theta}{1-\theta} = \frac{1}{\lambda-1}$ for $\lambda < 1/\alpha$. But this effect disappears for $\lambda > 1/\alpha$. The second effect arises from increasing difficulty of R&D and is captured by $1/\lambda^{\varepsilon}$. A

[6] One can easily establish that equilibrium is saddle-path stable.

higher λ means a lower arrival rate of the next innovation, which tends to reduce the R&D incentive. This effect is dominated by the monopoly mark-up effect for $\lambda < 1/\alpha$. This result has important implications for welfare analysis below.

Following Segerstrom, we explicitly introduced the idea that R&D becomes more difficult. However, this assumption is not sufficient to eliminate scale effects. To show this, consider the case of $n = 0$ and $\phi = 1$, which is essentially equivalent to the case of GH. Solving the model, it is easy to verify that R&D intensity is now given by

$$I = \frac{(1 - \theta) A L_0 - \theta\rho}{\varepsilon\lambda}, \tag{15}$$

which is strictly increasing in L_0 for $(1 - \theta) L_0 A > \theta\rho$.[7] This suggests that a key to eliminating scale effects is not the assumption of R&D becoming more difficult per se. Scale effects can be eliminated if and only if $1 > \phi > 0$. This is the same assumption used by Jones's (1995b) variety model.[8] In this sense, our model shows a sharp parallel between R&D-based growth models with quality innovations and those with variety innovations.

2.6 Social Optimum

Next we compare the market and socially optimal outcomes. Appendix A shows that (i) the optimal R&D intensity is the same as (10), and (ii) the optimal share of R&D workers

[7] The rate of utility growth is $\frac{\dot{u}(t)}{u(t)} = \frac{1}{\varepsilon}(\lambda^{\varepsilon} - 1) I$. (15) is comparable with equation (13) of GH (p.50).

[8] In Jones (1995b), R&D technology is given by $\dot{N} = L_I N^{\phi}$, $1 > \phi > 0$, where N is the number of varieties. Given L_I, frequency of innovation, $\dot{N}$, increases over time, since N rises. (R&D becomes progressively more difficult if and only if $\phi < 0$.) Thus, Jones's assumption is essentially different from Segerstrom's assumption of increasingly difficult R&D, since the industry-wide frequency of innovation in Segerstrom, AL_I/X, decreases over time for a given L_I.

is

$$k^S = \frac{1}{1 + \frac{\rho\varepsilon(1-\phi)}{n}}. \tag{16}$$

Somewhat surprisingly, k^S is independent of the size of an innovation λ.

Figure 1 depicts (14) and (16). If $k > k^S$ at $\lambda = 1/\alpha$ (the dotted line), it is optimal to tax R&D for $\lambda \in \left(\underline{\lambda}, \bar{\lambda}\right)$ and to subsidize it otherwise. On the other hand, if $k < k^S$ at $\lambda = 1/\alpha$ (the solid line), R&D should always be subsidized. When does this case arise? Inequality $k < k^S$ at $\lambda = 1/\alpha$ can be rearranged into $\frac{1-\phi}{\phi}\left(\frac{\rho}{n} - 1\right) < \left(\frac{1}{\alpha}\right)^{\varepsilon}$. An R&D subsidy is globally optimal if (i) knowledge accumulation exhibits sufficiently small diminishing returns, (ii) consumers are sufficiently patient, (iii) population grows sufficiently fast, or (iv) the elasticity of substitution of variety goods is sufficiently large.[9]

The intuition behind this result can be gained by identifying the individual externality effects in the utility metric in (1) (see Appendix B). First, the positive consumer-surplus effect is given by $\frac{\lambda^{\varepsilon}-1}{\varepsilon(\rho-n)}$. Second, $-\frac{1-\theta}{\theta}\frac{\Delta}{\rho-n}$ where $\Delta = \frac{(1-\phi)\lambda^{\varepsilon}}{a + \frac{1}{\lambda^{\varepsilon}-1}}$ combines the positive knowledge spillover effect within and across industries and the negative intertemporal spillover effect due to increasing R&D difficulty which is industry-specific (the latter effect dominates, as the sign is negative). Third, the negative creative destruction effect within and across industries is represented by $-\frac{1-\theta}{\theta}\frac{\Delta}{n}$. Fourth, there is no monopoly distortion effect, given the CESS preferences. For $k > k^S$ at $\lambda = 1/\alpha$, the combined negative externality effects outweigh the combined positive effects for $\lambda \in \left(\underline{\lambda}, \bar{\lambda}\right)$, and the reverse

[9] In the case of $n = 0$ and $\phi = 1$ (comparable with GH), the socially optimal R&D intensity is $I^S = \frac{(1-\alpha)AL_0 - \alpha\rho\frac{\varepsilon}{\lambda^{\varepsilon}-1}}{(1-\alpha)\lambda^{\varepsilon}}$. Using (15), it is easy to verify that $\lim_{\lambda \to 1^+}\left[I^S - I\right] = -1$ and $I^S - I$ is (i) monotonically increasing in λ or takes a $\cap$ shape for $\lambda < 1/\alpha$, and (ii) monotonically increasing in λ for $\lambda > 1/\alpha$, indicating several possibilities. For example, if $I^S > I$ at $\lambda = 1/\alpha$, it is optimal to tax R&D for a small λ but to subsidize it for a large λ. These results differ from GH where R&D should be taxed for a small and large λ, but subsidized for an intermediate range.

holds otherwise. For $k < k^S$ at $\lambda = 1/\alpha$; the positive externalities always dominate irrespective of the value of λ. This happens if, e.g., ρ is sufficiently low.

In Segerstrom, it is optimal to subsidize R&D for a small λ and to tax it for a large λ. As we have demonstrated, his result is overturned in our more general framework.

Appendix A: Social Optimum

We first consider the problem of the static labor allocation across consumption goods industries, taking total workers in manufacturing as constant. Dropping the time argument, the social planner solves $\max_{d(j;\omega)} \log u$ s.t. $D = \int_0^1 d(j;\omega)\,d\omega$ where u is given in (1). With δ denoting the Lagrangian multiplier, the first-order conditions are given by $\delta = \lambda^{\alpha j\omega} d(j;\omega)^{\alpha-1} / \int_0^1 \left[\lambda^{j\omega'} d(j;\omega')\right]^{\alpha} d\omega'$; which gives rise to $d(j;\omega') = \left(\lambda^{j\omega'}/\lambda^{j\omega}\right)^{\varepsilon} d(j;\omega)$ for $\omega \neq \omega'$. Substituting the latter expression into the denominator of the right-hand side of the former equation yields $d(j;\omega) = \lambda^{\varepsilon j\omega}/Q\delta$. Substituting this back into $D = \int_0^1 d(j;\omega)\,d\omega$ gives $D = 1/\delta$; which enables us to rewrite the above equation as $d(j;\omega) = \frac{\lambda^{\varepsilon j\omega}}{Q} D$. Substituting this into (1) gives $u = DQ^{\frac{1}{\varepsilon}}$.

Using this result and defining $z = D/L$ as the share of workers in manufacturing, the dynamic optimization problem that the social planner solves is equivalent to

$$\max \int_0^{\infty} e^{-(\rho-n)t} \ln z x^{\eta} L^{1+\eta}\,dt$$

$$\text{s.t.} \begin{cases} \dot{x} = (1-\phi)\left(1-\frac{1}{\varepsilon}\right)Ak - nx \\ 1 = z + k \end{cases} \tag{17}$$

where $\eta = \frac{1}{\varepsilon(1-\phi)}$. The Hamiltonian is $\ln(1-k)x^{\eta}L^{1+\eta} + \xi\left[(1-\phi)\left(1-\frac{1}{\varepsilon}\right)Ak - nx\right]$ where ξ is a costate variable. By Pontryagin's maximum principle,

$$\frac{1}{1-k} = \xi(1-\phi)\left(1-\frac{1}{\varepsilon}\right)A; \tag{18}$$

$$\dot{\xi} = (\rho-n)\xi - \frac{\eta}{x} + \xi n. \tag{19}$$

In steady state, (18) implies that $\dot{\xi} = 0$, so that (19) becomes $\xi = \frac{\eta}{\rho x}$. This in turn implies $\dot{x} = 0$ and leads to (10). Substituting $\xi = \frac{\eta}{\rho x}$ into (18) and using the resulting equation and another equation from $\dot{x} = 0$ generates (16).

Appendix B: Identifying Externality Effects

This Appendix calculates the external effects in the utility metric. Following GH and Segerstrom, we consider that an external agent invents an extra innovation in an industry ω and its associated profits disappear from the system (see GH and Segerstrom for detailed explanations).

First define $\Phi = \int_0^t I_\tau d\tau$; so that $\ln Q = (\lambda^{\varepsilon} - 1)\,\Phi$ from (9). An extra innovation is represented by an infinitesimal increase in Φ. Second, rewrite the consumption index as $u = \frac{\theta}{L} E Q^{1/\varepsilon}$. Using these equations, the impact of an increase in Φ on the intertemporal utility function (1) is given by

$$\frac{dU}{d\Phi} = \int_0^\infty e^{-(\rho-n)t}\frac{\lambda^{\varepsilon}-1}{\varepsilon}dt + \int_0^\infty e^{-(\rho-n)t}\frac{1}{E}\frac{dE}{d\Phi}dt. \qquad \text{(B.1)}$$

The first integral is reduced to $\frac{\lambda^{\varepsilon}-1}{\varepsilon(\rho-n)}$; which measures the consumer-surplus effect. The second integral incorporates other external effects.

Note that the income identity is $E = L + \Pi - \frac{\lambda^{\varepsilon} I}{A}Q^{1-\phi}$ where Π is aggregate profits and $\frac{\lambda^{\varepsilon} I}{A}Q^{1-\phi}$ is aggregate savings. Thus,

$$\frac{dE}{d\Phi} = \frac{d\Pi}{d\Phi} - (1-\phi)\frac{\lambda^{\varepsilon} I}{A}Q^{1-\phi}\frac{1}{Q}\frac{dQ}{d\Phi}. \qquad \text{(B.2)}$$

The first term captures the negative business-stealing effect within and across industries. The second term combines the positive knowledge spillover effect and the negative intertemporal spillover effect due to increasingly difficult R&D.

Through the income identity with $\Pi = (1-\theta)E$; there is a multiplier effect on E through Π as Φ rises, which is captured by $(1-\theta)\frac{dE}{d\Phi}$. In addition, profits which accrue to the external agent disappears from the system. First, $(1-\theta)\frac{\lambda^{\varepsilon j_{\omega}}}{Q}E$ is lost in an industry where an innovation occurs. Second, as an extra innovation raises Q; profits of producers in other industries fall, i.e. each producer loses $(1-\theta)\frac{\lambda^{\varepsilon j_{\omega}}}{Q^2}\frac{dQ}{d\Phi}E_t = (1-\theta)\frac{\lambda^{\varepsilon j_{\omega}}}{Q}E\,(\lambda^{\varepsilon}-1)$.

Note that the impacts of these effects on welfare are proportional to $\lambda^{\varepsilon j \omega}$. Note also that research costs for all these varieties were proportional to $\lambda^{\varepsilon j \omega}$. Therefore, "true" impacts on welfare of a rise in Φ are obtained by deflating those losses by $\lambda^{\varepsilon j \omega}$. Moreover, these negative welfare effects last only until another innovation occurs in an industry ω. Given that the probability of the welfare losses remaining is e^{-It}, overall changes in aggregate profits due to an extra innovation by the external agent are given by

$$\frac{d\Pi}{d\Phi} = -\left[(1-\theta)\frac{E}{Q} + \int_0^1 (1-\theta)\frac{E}{Q}(\lambda^{\varepsilon}-1)\,d\omega\right] e^{-It} + (1-\theta)\frac{dE}{d\Phi}. \qquad \text{(B.3)}$$

Substituting (B.3) into (B.2) yields

$$\frac{dE}{d\Phi} = -\frac{1-\theta}{\theta}\frac{E}{Q}\lambda^{\varepsilon} e^{-It} - \frac{1-\phi}{\theta}\frac{\lambda^{\varepsilon I}}{A} Q^{1-\phi}(\lambda^{\varepsilon}-1). \qquad \text{(B.4)}$$

Substituting this back into (B.1), evaluating the second integral and rewriting the resultant equation with (10) and (V) generates

$$\int_0^{\infty} e^{-(\rho-n)t}\frac{1}{E}\frac{dE}{d\Phi}dt = -\frac{1-\theta}{\theta}\frac{\Delta}{n} - \frac{1-\theta}{\theta}\frac{\Delta}{\rho-n} \qquad \text{(B.5)}$$

where $\Delta = \frac{(1-\phi)\lambda^{\varepsilon}}{a+\frac{1}{\lambda^{\varepsilon}-1}}$. The first term on the right-hand side represents the negative creative destruction effect, and the second term captures both the positive knowledge spillover effect and the negative intertemporal spillover effect due to increasingly difficult R&D. In the second term, the negative externality dominates the positive one as long as $1 > \phi$.

Thus, (B.1) becomes

$$\frac{dU}{d\Phi} = \frac{\lambda^{\varepsilon}-1}{\varepsilon(\rho-n)} - \frac{1-\theta}{\theta}\frac{\Delta}{n}\left(1+\frac{n}{\rho-n}\right). \qquad \text{(B.6)}$$

Finally it is easy to check that $\text{sign}\left(k^S - k\right) = \text{sign}\left(\frac{dU}{d\Phi}\right)$.

References

[1] Aghion, Philippe and Howitt, Peter, Endogenous Growth Theory, Cambridge, MA: MIT Press, 1998.

[2] Barrow, Robert and Sala-i-Martin, Xavier, Economic Growth, New York: McGraw-Hill, 1995.

[3] Grossman, Gene and Helpman, Elhanan, "Quality Ladder in the Theory of Growth," Review of Economic Studies, January 1991, 58, 43-61.

[4] Jones, Charles I., "Time Series Tests of Endogenous Growth Models," Quarterly Journal of Economics, May 1995a, 110, 495-525.

[5] ______, "R&D-based Models of Economic Growth," Journal of Political Economy, August 1995b, 103, 759-784.

[6] Segerstrom, Paul S., "Endogenous Growth Without Scale E¤ects," American Economic Review, forthcoming 1998.

[7] Segerstrom, Paul S., Anant, T.C.A., and Dinopoulos, Elias, "A Schumpeterian Model of the Product Life Cycles," American Economic Review, December 1990, 80, 1077-1092.

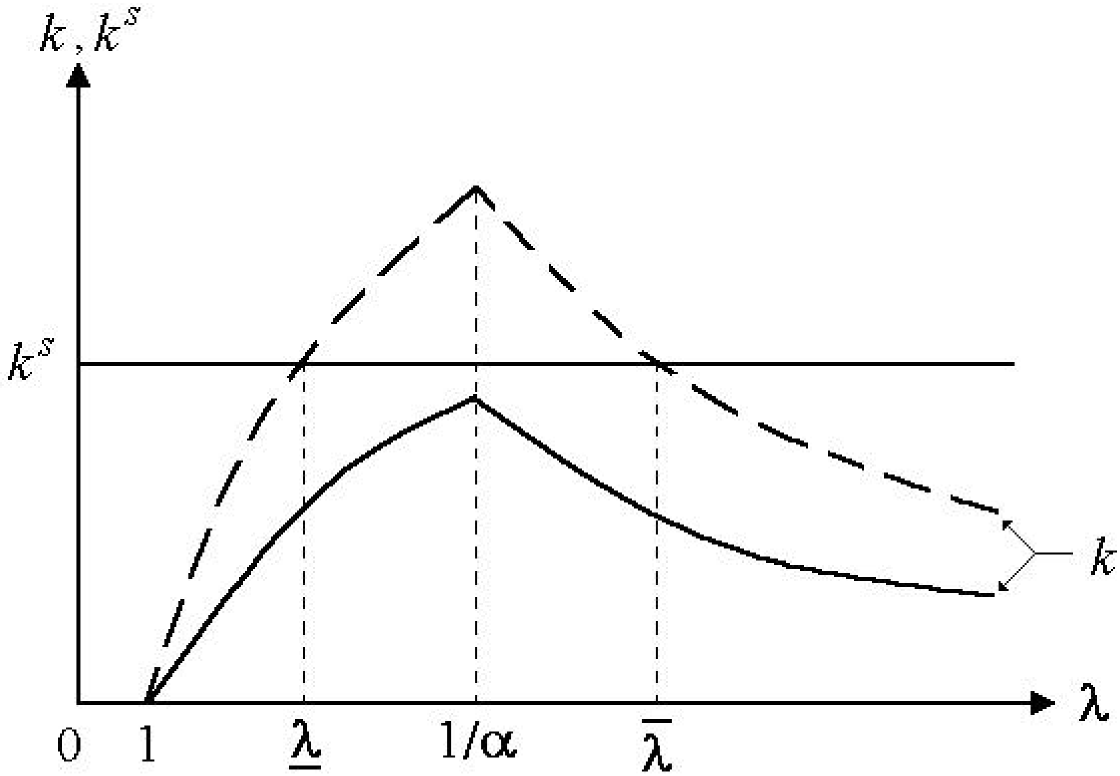

Figure 1: The share of R&D workers.