Science, Diminishing Returns, and Long Waves*

Chol-Won Li†
Department of Economics
University of Glasgow

March 1998

Abstract

This paper constructs a growth model with endogenous cycles, underlining the distinction between science and technology. Scienti c progress accelerates the rate of technological progress, but diminishing returns to technological research decelerates it. This process repeats itself with endogenous clustering of innovations. A higher long-run trend growth rate is associated with more frequent cycles or a larger amplitude of uctuations.

JEL Classi cation: O30, O40.
Key words: science, technology, diminishing returns, growth, long-waves.

*I am grateful to Philippe Aghion for his constant encouragement and perceptive comments. My thanks also go to Dan Maldoom, John Van Reenen, Andrew Stevenson and participants of the Royal Economic Society Conference and the European Meeting of the Econometric Society in 1997. Needless to say, I am solely responsible for opinions and all remaining errors.

†Correspondence: Dept. of Economics, Univ. of Glasgow, Adam Smith Building, Glasgow G12 8RT, UK; (Tel.) ++44-141-330-4654; (Fax) ++44-141-330-4940; (E-mail) cw.li@socsci.gla.ac.uk.

Science is defined as a systematic study of the nature and behaviour of the material and physical Universe, and technology as the practical application of this knowledge especially in industry and commerce (HMSO, 1996, p.1). Given these definitions, we quote Dasgupta and David (1995, p.487)

> To say that economic growth in the modern era has been grounded on the exploitation of scientific knowledge is to express a truism

This represents a widely-held view on the contribution of scientific progress to improving welfare. This view is strongly supported by empirical studies, such as Griliches (1986) and Mansfield (1980). Surprisingly, however, the endogenous growth literature pays little attention to the role of science. For example, the models of endogenous technological changes treat scientific and technological research equally under a single heading R&D (see Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990)). This may be because of the impression that scientific and technological research differ only in the degree of non-excludability of knowledge they create, and hence distinguishing them sheds little additional light on the understanding of economic growth.

The present paper aims to show that such an impression is quite misleading and demonstrates that distinguishing between science and technology on the basis of the types of knowledge created generates intriguing insights into the link between growth and technology-induced cycles.

There are two observations which are particularly important for our purposes. First, on the basis of his time-series evidence, Jones (1995) rejects a strong form of knowledge externality as assumed in the standard R&D-based growth model. To be consistent with the evidence, we assume a weak knowledge externality. Due to this more realistic assumption, diminishing returns to technological innovations arise. Second, in his influential work, Kuhn (1962) classifies scientific progress into normal and revolutionary. The former consists of incremental additions to the scientific knowledge on the basis of a paradigm within which scientists engage in research, and the latter represents a transformation of such a paradigm. Obvious examples of scientific revolutions are the work of Copernicus, Newton and Einstein, which necessitated the rejection of then-prevailing scientific theories. Kuhn stresses that scientific revolutions are a non-cumulative process of scientific advance. A similar theory is also proposed by Lacatos (1978). These studies suggest that the frontier of scientific knowledge occasionally expands in a *discontinuous* way. Since we are interested in the effect of science on technology, the present paper interprets scientific revolutions broadly as more wide-spread phenomena, such as Bell Laboratory's discovery of properties of semi-conductors which led to the microelectronic revolution.

In the model presented here, scientific research is conducted in both the public and private sectors; the government finances it through tax revenues and firms invest part of their profits in it. When scientific breakthroughs stochastically occur, scientific knowledge discontinuously expands, triggering a series of technological innovations. But, between two major scientific discoveries, diminishing returns to technological research set in, and as a result, technological research intensity falls over time until another scientific discovery occurs. This process repeats itself over an infinite horizon, so that an economy grows with endogenous cycles in terms of the level and growth rate of output.

A prominent feature of fluctuations in our model is that technological innovations arrive in clusters. This resembles the Schumpeterian version of Kondratiev s long-waves. In the work published in 1925, Kondratiev saw long cycles as an expression of the internal regulating mechanism of an economy and technological innovations passively respond to these endogenous forces. In contrast, Schumpeter (1939) viewed long-waves as being caused by innovations which occur in clusters, so that the growth rate accelerates and decelerates in response. This latter theory received much attention in the 1980s, since it could potentially give a coherent explanation of the productivity slowdown of developed economies since the mid-1960s, although some economists remain sceptical.[1] Endogenous bunching of innovations is also found in the theoretical models of Shleifer (1986) and Stein (1997) with different underlying mechanisms. Supportive evidence for clustered innovations are given by, for example, Mensch (1975) and Kleinknecht (1987).

There are several studies related to the work presented here. Within a general equilibrium framework, R&D-based growth models of Aghion and Howitt (1992), Cheng and Dinopoulos (1992), Corriveau (1994), Helpman and Trajtenberg (1994, 1996) generate endogenous cycles in the growth rate as well as in the level of output.[2] A common mechanism goes as follows. When returns from R&D are expected to be high for endogenous or exogenous reasons, resources are switched to the research sector from the production sector, causing a rise in the growth rate but a fall in the level of output. The reverse happens when returns from R&D are expected to be low. Thus, uctuations of output are created through reallocation of workers *between* the research and production sectors.

[1] See, for example, the August issue of *Futures*, 1981, for pro-long-waves studies. See also Rosenberg and Frischtak (1983) and Mans eld (1983) for studies which are sceptical about it.

[2] For studies which treat uctuations as exogenous in endogenous growth models, see King and Rebelo (1986), Stadler (1990), Aghion and St.Paul (1991) and Caballero and Hammour (1994).

However, a crucial limitation of such a mechanism is that, as Aghion and Howitt (1998) point out, only 2 or 3 percent of the labour force are allocated to research in modern economies. This fact calls into question the plausibility of this mechanism in creating large aggregate fluctuations.

In contrast, the present model is not subject to such a criticism, since we assume that skilled workers are exclusively used for scientific and technological research and unskilled workers are employed only for a manufacturing purpose. This assumption removes the possibility of reallocating workers between the research and production sectors. Employment fluctuations *within* the research sector, which is tiny compared with the production and service sectors, is the propagating mechanism to generate aggregate cycles. Such fluctuations make the rate of technological progress fluctuate in an endogenous way, and as a result, output grows in waves rather than in a smooth exponential fashion. The model of Amable (1995) exhibits a similar time-profile of output. But his main concern is expectations-driven fluctuations of research employment. By contrast, we are interested in cycles of output induced by science and technology.

The plan of this paper is as follows. Section 1 develops the model with a close attention to the structure of the general knowledge and firms' decision on private scientific research. Section 2 examines the equilibrium dynamics when scientific knowledge is constant and expands. We shall examine the cyclical movement of some key economic indicators and the effect of an industrial policy. Section 3 concludes.

1 The Model

Since our model is based on a familiar framework of Grossman and Helpman (1991, Ch.3), we outline it brie y. Interested readers are referred to their work. To facilitate the presentation, we abbreviate technological research as TR and scienti c research as SR. We also use terms technologists and scientists for skilled workers engaged in TR and SR respectively.

1.1 Consumers and Final Output Sector

There are two types of consumers who act as suppliers of labour services: H skilled workers and L unskilled workers. The former are exclusively used for TR and SR, and the latter are employed in manufacturing only. Their intertemporal utility function is time-separable and the instantaneous utility function is logarithmic in homogeneous nal output. Under this assumption, Grossman and Helpman (1991) shows that the interest rate is always equal to consumers time preference rate ρ if aggregate consumer expenditure is normalised.

Final output y_t is produced under competitive conditions with the CES aggregate production function:

$$y_t = \left(\int_0^1 \int_0^{n_{it}} x_{jit}^{\alpha} dj di \right)^{1/\alpha}, \quad 0 < \alpha < 1 \tag{1}$$

where x_{jit} denotes intermediate products and n_{it} is the number of varieties in the ith industry. This speci cation implies that there is a continuum of industries indexed by $i \in [0, 1]$ and their associated sub-industries indexed by $j \in [0, n_{it}]$. Technological innovations take the form of increasing n_{it}. Given the production technology (1), output producers demand for x_{it} has the price elasticity of $-1/(1-\alpha)$.

1.2 Intermediate Goods Sector

The intermediate goods sector is monopolistically competitive. It is assumed that a single rm monopolises the ith industry. Those incumbent rms engage in TR to expand the variety in its industry and conduct SR. This captures the observations of Rosenberg (1990) that (i) SR capabilities are complementary to technological research activities in the sense that the former may provide guidance to how the latter is conducted,[3] and (ii) private SR is highly concentrated in the sense that a small number of large rms with strong market position dominate basic research in industry.[4]

Producing one unit of inputs is assumed to require one unskilled labourer. Given a constant price elasticity demand, input producers maximise their pro ts by setting their prices at $p_{it} \equiv p_t = w_t^l/\alpha$ where w_t^l is the wage for unskilled workers and the subscript i is dropped due to symmetry. We assume that the government taxes pro ts at the rate of $0 < \tau < 1$ to nance public SR. Moreover, monopoly rms invest a fraction $0 < \kappa_i < 1$ of after-tax pro ts in SR. Under these assumptions, their *net* pro ts arising from each variety is

$$\pi_{jit} = \pi_{it} = \frac{\kappa_i (1-\tau)(1-\alpha)}{N_t}, \qquad N_t = \int_0^1 n_{it} di. \tag{2}$$

[3] An example he cites is Bell Labs search for a substitute for the vacuum tube which eventually led to the invention of transistors. SPRU (1996) cites several other forms of bene ts of scienti c research to private rms.

[4] He also refers to the fact that a large number of small rms conduct basic research, especially in the realm of biotechnology. But he notes that they seem to be engaged in basic research which is close to the commercialisaton stage.

For a monopolist in the ith industry, total pro ts are $n_{it}\pi_{it}$.

Now suppose that an outside rm successfully generates an innovation by means of TR in the ith industry. Its impact on the incumbent s pro ts is approximated by a small increase in N_t in the denominator of (2). But it can avoid this loss, i.e. $\partial(n_{it}\pi_{it})/\partial N_t < 0$, if he innovates by himself. Thus, he takes $-\partial(n_{it}\pi_{it})/\partial N_t$ as rewards for innovation.[5] As we will see, the equilibrium is characterised by symmetry of all industries, so that

$$n_{it} = n_t = N_t. \tag{3}$$

Thus, we have

$$-\frac{\partial\left(n_{it}\pi_{it}\right)}{\partial N_t} = \frac{\kappa_i\left(1-\tau\right)\left(1-\alpha\right)}{N_t}\frac{n_{it}}{N_t} = \pi_{it} \tag{4}$$

On the other hand, outside rms can attain pro ts $(1-\tau)\left(1-\alpha\right)/N_t$ if they succeed in TR and do not invest in SR. Pro ts are greater than π_{it}. But we assume that incumbent rms TR productivity is sufficiently greater than that of outside rms to the extent that the latter do not have an incentive for TR in equilibrium.[6]

In order to nance its TR, the ith rm issues shares, which are freely traded in the stock market. All *net* pro ts generated by its differentiated inputs are distributed as dividends. We use v_{it} to denote the discounted value of ow pro ts π_{it}. It obeys the following asset equation

$$E\left[\frac{\dot{v}_{it}}{v_{it}}\right] + \frac{\pi_{it}}{v_{it}} = \rho. \tag{5}$$

[5]This explanation is based on the so-called efficiency effect: since competition destroys pro ts, an incumbent has an incentive to deter entry of new rms.

[6]Grossman and Helpman (1991, p.564) note that incumbent rms are likely to have acquired substantial industry-speci c information as a result of their successful innovation. Such information may be valuable in inventing newer variety, and it may not be readily apparent to outside rms.

From the viewpoint of investors, on the left-hand side is the return to equity of monopoly rms which consists of an expected capital gain and a dividend rate. It is equated to the return on safe bonds.

Technological innovations increase the number of variety according to

$$\dot{n}_{it} = \delta R_{it} K_t, \qquad \delta > 0 \tag{6}$$

where R_{it} is the number of technologists used and K_t is the stock of *general* knowledge in the economy. Since rms can generate $\dot{n}_{it}$ of new varieties through TR in each instance, they solve $\max_{R_{it}} v_{it} \delta R_{it} K_t - w_t^h R_{it}$, which yields

$$v_{it} = \frac{w_t^h}{\delta K_t} \qquad \text{for } R_{it} > 0 \tag{7}$$

where w_t^h is a wage of skilled workers. This implies that v_{it} and $E\left[\dot{v}_{it}/v_{it}\right]$ in (5) are the same for all industries. It follows that π_{it} should also be the same for all i s in equilibrium with $R_{it} > 0$, con rming (3).

1.3 Knowledge Production

We assume that the general knowledge is produced with *technological* knowledge created by industrial TR and *scienti c* knowledge generated by public and private SR:

$$K_t = N_t^{\varepsilon} Q_t^{\nu}, \quad 1 > \varepsilon, \nu > 0, \quad \varepsilon + \nu < 1, \tag{8}$$

where technological knowledge is equated to N_t and scienti c knowledge is denoted by Q_t. In (8) K_t exhibits decreasing returns to scale to N_t, given Q_t. That is, dynamic learning-by-doing through TR is limited, since the marginal contribution of technological knowledge to TR is decreasing over time, i.e., $\lim_{N_t \to \infty} \partial(v_{it} \delta R_{it} N_t^{\varepsilon} Q_t^{\nu})/\partial N_t = 0$. This fact causes diminishing returns to TR which plays a crucial role in generating long-waves. The assumption (8) is empirically supported by Jones (1995) who rejects the TR-based endogenous growth models with $\varepsilon+\nu=1$ on the basis of his time-series evidence in favour of $\varepsilon+\nu<1$.

We assume that one scienti c breakthrough raises scienti c knowledge by a factor $\lambda>1$:

$$Q_t=\lambda^{m_t}, \qquad m_t=0,1,2,.... \tag{9}$$

where m_t is the cumulative number of scienti c discoveries up to time t. Note that scienti c discoveries discontinuously expand Q_t, and hence K_t. The assumption (9) captures the radical nature of scienti c discoveries.[7] To stress uncertainty, it is assumed that one scientist brings about a discovery with the Poisson arrival rate of

$$q(Q_t,N_t)=\varphi\frac{N_t^{1-\varepsilon}}{Q_t^{\nu}}, \qquad \varphi>0. \tag{10}$$

As a scienti c discovery occurs, the next one will be more difficult to be brought about due to the presence of Q_t in the denominator. But technological innovations will generate positive externalities on science (due to N_t in the numerator), improving the productivity of SR. This speci cation is consistent with the observation of many writers that science and technology interact in shaping the paths of their progress, rejecting a simple linear function in which in uence is unidirectional from science to technology.[8] Thus, a Poisson

[7] It takes several years and even decades before major scienti c breakthroughs have any impact on an economy. Thus, we could assume that following a scienti c discovery at t, the scienti c knowledge stock rises at $t+\Delta$, $\Delta>0$, by a factor λ. Alternatively, we could assume that as new scienti c ideas diffuse throughout the economy, the scienti c knowledge stock gradually rises, following an equation like $\dot{Q}_t=bQ_t(\lambda^{m_t}-Q_t), b>0$. However, these modi cations do not substantially change results to be derived.

[8] For example, the problems addressed by scientists often originate in their links with industry, and

arrival rate of a scienti c breakthrough in the economy as a whole is given by $q(.)S_t$ where S_t is the total number of scientists in the economy.

1.4 Scienti c Research

The total corporation tax revenue is $\int_0^1 \int_0^{n_{it}} \tau(1-\alpha)/N_t dj di$. Since the government pays scientists a wage prevailing the labour market, the total number of scientists in the public sector S_t^G is

$$S_t^G = \frac{\tau (1-\alpha)}{w_t^h}, \tag{11}$$

using (3).

In the private sector, each monopolist devotes a fraction κ_i of pro ts to SR, so that the total expenditure on private SR is $\int_0^1 \int_0^{n_{it}} (1-\kappa_i)(1-\tau)(1-\alpha)/N_t dj di$. Thus, the number of scientists in the private sector S_t^P is

$$S_t^P = \frac{1}{w_t^h} \int_0^1 (1-\kappa_i)(1-\tau)(1-\alpha)\, dj, \tag{12}$$

using (3). By investing in science, rms can stochastically expand the scienti c knowledge which ultimately raises TR productivity. Given scienti c knowledge λ^{m_t}, the ith rm s gain from TR is $\Theta_{im} = v_{it}\dot{n}_{it} = v_{it}\delta R_{it} N_t^{\varepsilon} \lambda^{\nu m_t}$, and it increases to Θ_{im+1} with a Poisson arrival rate of $\varphi(S_t^G + S_t^P)$. It follows that the expected bene t from SR is $\varphi(S_t^G + S_t^P)(\Theta_{im+1} - \Theta_{im})$. The ith rm maximises the expected bene t by choosing κ_i. Denoting $v_{it} = \kappa_i \widetilde{v}_{it}$ (where $\widetilde{v}_{it}$ is the present value of after-tax pro ts before investment

instrumentation invented in technological research proves to be extremely important in bringing about scienti c discovery (Rosenberg (1982)).

funds in SR is deducted), this maximisation problem is equivalent to

$$\max_{\kappa_i} \quad \varphi \left[\tau \left(1-\alpha\right) + \int_0^1 \left(1-\kappa_i\right)\left(1-\tau\right)\left(1-\alpha\right) dj \right] \kappa_i \Gamma \tag{13}$$

where $\Gamma = \widetilde{v}_{it} \delta R_{it} N_t^{\varepsilon} \lambda^{\nu m_t} (\lambda^{\nu} - 1)/w_t^h$. The maximand is concave and the rst-order condition is

$$\kappa_i = \kappa = \frac{1}{2\left(1-\tau\right)}. \tag{14}$$

Note that κ is always larger than $1/2$ for $\tau > 0$. It is also increasing in τ. This is because of free-riding of private rms on public SR. As τ rises, the probability of an extra scienti c discovery does not decrease even if κ is slightly raised.[9] This arises, because public and private SR are substitutes.[10]

1.5 Labour Markets

The combined expenditures on public and private SR is denoted by $\zeta = \tau(1-\alpha) + (1-\kappa)(1-\tau)(1-\alpha) = (1-\alpha)/2$ where the second equality uses (14). Thus, skilled and unskilled labourers are all employed if

$$H = \frac{\dot{N}_t}{\delta K_t} + \frac{\zeta}{w_t^h}, \qquad L = \frac{\alpha}{w_t^l}, \tag{15}$$

On the right-hand side of the rst equation are the demand for technologists and scientists in the public and private sectors.

[9] If we assume $i \in [0, z]$, $z > 1$, in (1), then 2 in the denominator of (14) is replaced with $1 + 1/z$ and κ would be increasing in z. This is again a free-rider problem. With an increase in the number of other rms which can take advantage of public scienti c knowledge, each rm devotes less resources to SR.

[10] Lichtenberg (1984) empirically supports a hypothesis that public- nanced R&D and private R&D are substitutes.

2 Equilibrium Dynamics

To facilitate the following analysis, we de ne

$$\omega_t \equiv \frac{1}{w_t^h}, \qquad \chi_t \equiv \frac{N_t}{\lambda^{\frac{\nu}{1-\varepsilon}m_t}}, \qquad \xi_t \equiv \frac{\dot{N}_t}{N_t} = \frac{\dot{\chi}_t}{\chi_t} \quad \text{for a constant } m_t. \tag{16}$$

Given these de nitions, the growth rate of output for a given m_t is written as

$$\frac{\dot{y}_t}{y_t} \equiv g_t = \frac{1-\alpha}{\alpha}\xi_t, \tag{17}$$

since the production function (1) can be reduced to $y_t = LN_t^{\frac{1-\alpha}{\alpha}}$.

2.1 When Q_t is Constant

First suppose that m_t is held xed to analyse the transitional dynamics when scienti c knowledge is constant. We can re-express the asset market equilibrium condition (5) and the skilled labour market condition in (15) as

$$\frac{\dot{\omega}_t}{\omega_t} = \frac{\delta}{\chi_t^{1-\varepsilon}}\left[(1+\varepsilon)\zeta\omega_t - \varepsilon H\right] - \nu(\ln\lambda)\varphi\zeta\chi_t^{1-\varepsilon}\omega_t - \rho, \tag{A}$$

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{\delta}{\chi_t^{1-\varepsilon}}(H - \zeta\omega_t), \tag{H}$$

respectively.[11] Note that τ does not affect the both equations because the effect of τ is fully accommodated in the determination of κ. Hence, in this paper we do not consider

11 In deriving (A), we used (i) $E[\dot{v}_{it}/v_{it}] = \dot{w}_t^h/w_t^h - \varepsilon\xi_t - E[\dot{Q}_t/Q_t]$ from (7) and (ii) the expected rate of growth of scienti c knowledge is governed by $E[\dot{Q}_t/Q_t] = (\ln\lambda)\, q(Q_t, N_t)S_t$. (ii) can be veri ed by noting that the expected stock of scienti c knowledge at t is given by

$$E[\ln Q_t] = E[m_t]\ln\lambda = (\ln\lambda)\sum_{m=0}^{\infty} f(m;t)\, m = (\ln\lambda)\int_0^t q(Q_s, N_s)\, S_s ds$$

where $f(m;t) = \left[\int_0^t q(Q_s, N_s)\, S_s ds\right]^m e^{\int_0^t q(Q_s,N_s)S_s ds}/m!$ is the Poisson density function.

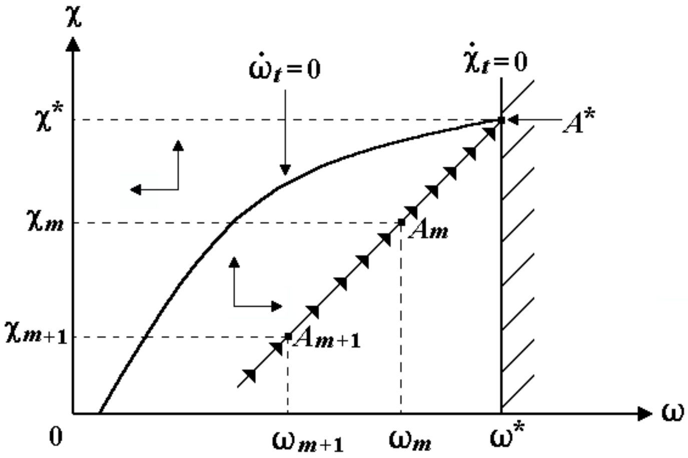

Figure 1: Transitional dynamics.

the optimal determination of τ.

The two conditions are depicted in Figure 1 where the shaded area is an infeasible region. The $\dot{\chi}_t = 0$ line is independent of N_t, since all skilled workers are devoted to SR in a steady state. The $\dot{\omega}_t = 0$ schedule is upward-sloping, because more differentiated inputs are created with a lower w_t^h (i.e. a lower TR cost) in a steady state. The gure shows that a steady state is unique and the economy is saddle-path stable, taking m as given.[12] Note that output stops growing in the steady state A^*, since no technological innovation occurs. This is the same feature as found in the neo-classical growth model, where no growth is sustained in the long-run in the absence of exogenous technical progress.

Note that differential equations (A) and (H) determine the values of χ_t and ω_t in the (χ, ω) plane, and those values are identi ed by a single point like A_m or A_{m+1} on the saddle path in Figure 1. Furthermore, given the equilibrium values determined, equations

[12]The existence of a unique non-trivial steady state requires $\chi^* > n_0/\lambda^{\frac{\nu}{1-\varepsilon}m_0}$ where χ^* is implicitly de ned by (A) with $\dot{\omega}_t = 0$.

(A) and (H) give the proportionate rate of changes of χ_t and ω_t at time t. With this interpretation, we establish the following.

Proposition 1 *Given the scienti c knowledge stock Q_t,*

1. *$\partial \xi_t / \partial t < 0$ and $\partial g_t / \partial t < 0$ along the equilibrium transitional path, and*

2. *$\partial \xi_t / \partial t = \partial g_t / \partial t = 0$ in the steady state.*

Proof. Note that $\dot{\chi}_t / \chi_t = \xi_t$ for a given m_t. Thus, (H) and (17) imply

$$\frac{\partial \xi_t}{\partial t} = -(1-\varepsilon)\left(\frac{\dot{\chi}_t}{\chi_t}\right)^2 - \frac{\delta}{\chi_t^{1-\varepsilon}} \frac{1-\alpha}{2} \dot{\omega}_t < 0, \qquad \frac{\partial g_t}{\partial t} = \frac{1-\alpha}{\alpha} \frac{\partial \xi_t}{\partial t} < 0. \tag{18}$$

They are zero when $\dot{\chi}_t = \dot{\omega}_t = 0$. ■

This proposition indicates that less and less innovation is occurring in transition, and as a result the growth of output gradually evaporates. This is precisely because of the diminishing returns to technological TR. More speci cally, we rewrite the asset equation (5) as

$$-\frac{\dot{\omega}_t}{\omega_t} - \varepsilon \frac{\dot{\chi}_t}{\chi_t} - \nu (\ln \lambda)\, \varphi \zeta \chi_t^{1-\varepsilon} \omega_t + \delta \zeta \frac{\omega_t}{\chi_t^{1-\varepsilon}} = \rho. \tag{19}$$

The rst three terms on the left-hand side represent the expected depreciation of the stock market value of an innovative rm. The fourth term is an earning-price ratio, which is the rate of return from monopoly pro ts distributed as dividends to investors. Since $\dot{\omega}_t$, $\dot{\chi}_t > 0$ in transition, the dividend rate exceeds the rate of interest ρ. The earning-price ratio falls and eventually drops to $\widetilde{\rho} = \rho + \nu (\ln \lambda)\, \varphi \zeta (\chi^*)^{1-\varepsilon} \omega^*$, as the economy approaches the steady state A^* in Figure 1. When this happens, consumers stop investing in new TR projects, since they are unwilling to postpone consumption to the future. This explanation of the diminishing returns to TR is analogous to that of the convergence property of the Ramsey-type neo-classical growth model. The dividend rate in (19) plays exactly the same role as the marginal productivity of capital in that model. However, a crucial difference is that a long-run growth can be sustained due to scienti c discoveries in our model, as we will see. This is re ected in the fact that the dividend rate is always greater than the interest rate even in the steady state, i.e. $\widetilde{\rho} > \rho$ for all t.

As regards scientists, their number rises along the transitional path, because w_t^h is falling. The likelihood of a next scienti c discovery gradually increases along the transitional path, and it is maximised at $\varphi H(\chi^*)^{1-\varepsilon}$ in a steady state A^*.

2.2 When Q_t Expands

Next we consider the instantaneous adjustment when scienti c knowledge expands by using Figure 1. A step-up increase in Q_t leaves the both $\dot{\omega} = 0$ and $\dot{\chi} = 0$ schedules and the saddle path intact. What it changes is the location of the economy on the saddle path. Suppose that the scienti c knowledge stock λ^m rises to λ^{m+1} when the economy is at A_m where $N_t = N_m$, $\chi_m = N_m/\lambda^{\frac{\nu}{1-\varepsilon}m}$ and ω_m. Note that χ_m and ω_m can be equal to χ^* and ω^*. Following a scienti c discovery, the economy jumps to A_{m+1} which is associated with $\chi_{m+1} = N_m/\lambda^{\frac{\nu}{1-\varepsilon}(m+1)}$ and ω_{m+1}. Observe that A_{m+1} is always located southwest of A_m. An instantaneous jump from A_m to A_{m+1} involves drops in χ_t and ω_t. Also note that changes of the endogenous variables, i.e. $\chi_m - \chi_{m+1}$ and $\omega_m - \omega_{m+1}$ depend not only on parameter values but also on the stochastic time interval between two scienti c discoveries.

Proposition 2 *When a scienti c breakthrough occurs, ξ_t and g_t increase.*

Proof. Denote the rate of technological change and the growth rate at A_m and A_{m+1} in Figure 1 as ξ_m, ξ_{m+1}, g_m and g_{m+1}. From (H) and (17), we obtain

$$\xi_{m+1} - \xi_m = \frac{\delta}{\chi_m^{1-\varepsilon}} \left[(\lambda^\nu - 1) \left(H - \frac{1-\alpha}{2} \omega_m \right) + \frac{\lambda^\nu (1-\alpha)}{2} (\omega_m - \omega_{m+1}) \right] > 0, \quad (20)$$

$$g_{m+1} - g_m = \frac{1-\alpha}{\alpha} \left(\xi_{m+1} - \xi_m \right) > 0. \quad (21)$$

■

An intuition for this proposition and the discrete jumps in χ_t and ω_t following a scienti c discovery is the opposite of Proposition 1. With a discontinuous rise in Q_t, the dividend rate in (19) increases, and it makes consumers willing to postpone more consumption to the future, since investment becomes more attractive. In other words, the diminishing returns to TR is overcome due to a scienti c breakthrough. From the viewpoint of entrepreneurs, they nd it pro table to employ more technologists for TR, since TR productivity has improved. This results in a rise in the demand for technologists with the result of a discrete upward jump in the skilled workers wage. It also causes the number of scientists to decline. This explanation accords with Rosenberg s (1974, p.107) view that as scienti c knowledge grows, the cost of successfully undertaking any given, science-based invention declines from in nitely high , in the case of an invention which is totally unattainable within the present state of knowledge, down to progressively lower and lower levels.

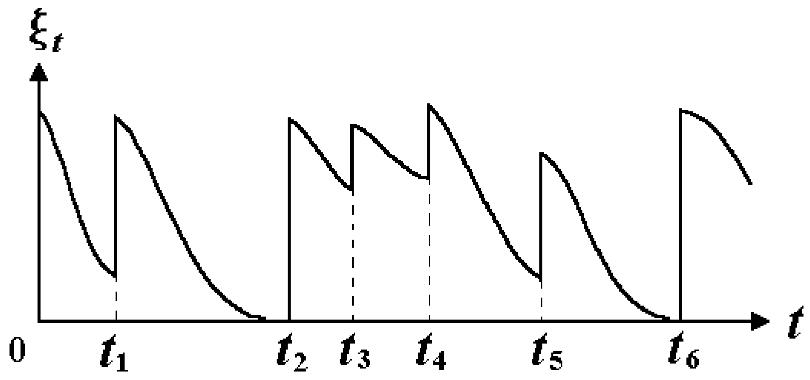

Figure 2: A time-pro le of ξ_t.

2.3 Cyclical Growth

Thus far, we have established that (i) given the scienti c knowledge stock, χ_t and ω_t gradually rise along the saddle path until a steady state A^* is attained or another scienti c discovery takes place, and (ii) χ_t and ω_t discontinuously fall following a rise in Q_t. This process repeats itself for good, sustaining long-run growth.

Propositions 1 and 2 imply that ξ_t repeats the process of monotonous decrease after a discontinuous rise with each expansion of scienti c knowledge, as shown in Figure 2. Declining TR intensity is due to the diminishing returns to technological innovations, and its discrete rises are caused by the expansion of scienti c knowledge at t_s, $s = 1, 2, ..., 6...$ with the stochastic time intervals $t_s - t_{s-1}$. Note that when ξ_t is high, a greater number of innovations are occurring in the economy. In other words, technological innovations tend to cluster after each scienti c breakthrough. This is the feature that resembles the Schumpeterian long-waves. Equation (17) implies that a time pro le of g_t is basically the same as ξ_t.

A possible time-pro le of $\ln y_t = \ln L + \frac{1-\alpha}{\alpha} \ln N_t$ is shown in Figure 3. Since g_t (and

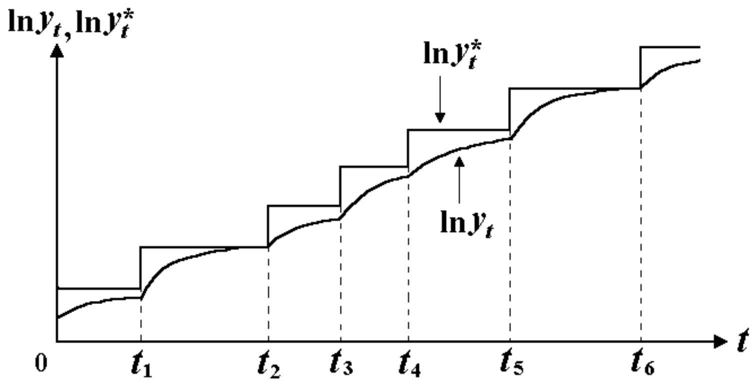

Figure 3: Time pro les of $\ln y_t$ and $\ln y_t^*$.

ξ_t) peaks in each cycle at a time when a scienti c discovery takes place, the slope of the $\ln y_t$ schedule is steepest at t_s, $s = 1, 2, ...6$. As the economy approaches a steady state, g_t (and ξ_t) falls and hence the slope of $\ln y_t$ (and $\ln N_t$) becomes atter and atter over time due to the diminishing returns.[13] A schedule with discrete rises in the gure represents steady-state output, to which y_t converges over time, given λ^{m_t}. It is de ned as

$$\ln y_t^* = \ln L + \frac{1-\alpha}{\alpha} \ln N_t^* \qquad \text{where } \ln N_t^* = \ln \chi^* + \frac{\nu}{1-\varepsilon} m_t \ln \lambda. \qquad (22)$$

Note that the steady-state levels of output y_t^* and technology N_t^* are crucially determined by the stock of scienti c knowledge. Since y_t^* and N_t^* expand only when a scienti c breakthrough occurs, this result highlights the importance of SR in determining the standard of living in the long run. When the two schedules coincide, the economy stagnates temporarily.

[13]This explanation accords with studies in which diminishing returns to technology are attributed to productivity slowdown since the 1960s. See Nordhaus (1972) for example.

2.4 Key Economic Indicators

It can be easily veri ed that relative wages are given by $w_t^h/w_t^l = L/\alpha\omega_t$. They rise discretely when a scienti c discovery occurs, and gradually decreases for a given scienti c knowledge stock. Thus, relative wages increase when TR intensity is high. This is consistent with one of the explanations for widening wage inequality in recent years, namely that the impact of technological innovations on the labour demand is biased towards skilled workers.

GDP at factor cost consists of real labour incomes and aggregate after-tax pro ts: $G_t = \left[\frac{H}{\omega_t} + \frac{1+\alpha}{2}\right] LN_t^{\frac{1-\alpha}{\alpha}}$. It initially rises with a scienti c breakthrough. This is due to a rise in skilled wages. Its steady state value, $G^* = L(\chi^*)^{\frac{1-\alpha}{\alpha}}\lambda^{\frac{1-\alpha}{\alpha}\frac{\nu}{1-\varepsilon}m}$, also increases. But, whether G_t monotonically rises in transition is ambiguous.

An aggregate stock market value is $N_t v_t = \chi_t^{1-\varepsilon}/\delta\omega_t$. From the explanation of the diminishing returns using equation (19), we know that $\omega_t/\chi_t^{1-\varepsilon}$ expands with a scienti c breakthrough followed by a steady decrease. Thus, the time-pro le of $N_t v_t$ is exactly the opposite of this. Aggregate real after-tax pro ts in terms of output are $\kappa(1-\alpha)LN_t^{\frac{1-\alpha}{\alpha}}$, which mirrors y_t.

2.5 Growth and Cycles

In Figure 3, y_t always converges to its steady state values y_t^*, and the former cannot expand beyond the latter. That is, y_t^* determines the possibility or potential of output at time t that the economy can achieve with the current scienti c knowledge stock. Furthermore, the long-run trend of sustained output level can be recovered by tracing the schedules representing $\ln y_t^*$. Using (22), we can derive the straight line through the schedule of $\ln y_t^*$:

$$E\left[\ln y_t^*\right] = \ln L\left(\chi^*\right)^{\frac{1-\alpha}{\alpha}} + \frac{1-\alpha}{\alpha} E\left[\ln N_t^*\right], \qquad E\left[\ln N_t^*\right] = \frac{\nu \ln \lambda}{1-\varepsilon} \vartheta\left(t\right) t \tag{23}$$

where $\vartheta\left(t\right) = \left(1/t\right) \int_0^t \varphi\zeta\chi_s^{1-\varepsilon}\omega_s ds$ which is the average arrival rate of a scienti c discovery up to time t. That is, the average SR intensity decides the long-run trend of output. Differentiating (23) gives

$$g_t^* = \frac{1-\alpha}{\alpha}\xi_t^*, \qquad \xi_t^* = \frac{\nu \ln \lambda}{1-\varepsilon}\varphi\zeta\chi_t^{1-\varepsilon}\omega_t, \tag{24}$$

which implies that the expected growth rates of *trend* output depend on the current SR intensity. This contrasts with the fact that the *actual* growth rate of output g_t is determined by the current TR intensity ξ_t.

We are now in a position to examine the relation between growth and cycles. This will be done by comparing two hypothetical time-pro les of output, given y_0 and y_0^*. One is shown in Figure 3. Suppose that the other has a higher trend growth rate, i.e. a straight line through $\ln y_t^*$ is steeper. Such a time-pro le is obtained if scienti c discoveries occur more often or if the size of scienti c breakthroughs, λ, is larger. In other words, a higher trend growth rate is associated with more frequent cycles or a higher amplitude of uctuations. Thus, the present model predicts a positive correlation between cycles and long-run growth. This is in line with the nding of Aghion and St.Paul (1991) and Caballero and Hammour (1994), who draw a conclusion that exogenous uctuations may be bene cial for growth, although their underlying mechanisms are quite different.

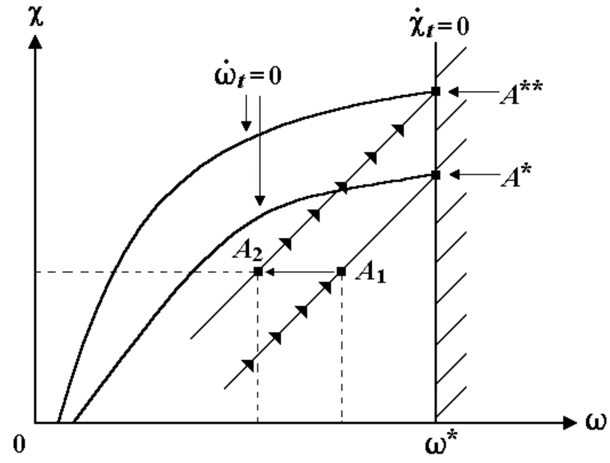

Figure 4: A subsidy to R&D.

2.6 Subsidy to Technological Research

Using the distinction between the actual and trend growth rates drawn above, we examine the effect of a subsidy to TR, which drew much attention in the Schumpeterian growth literature. We start by assuming that the fraction $0 \leq \psi < 1$ of TR costs is subsidised through lump-sum transfer. Figure 4 shows that following a small increase in ψ from zero, the $\dot{\omega} = 0$ schedule shifts upward but the $\dot{\chi} = 0$ schedule is not affected.

Proposition 3 *Following a subsidy to TR, (i) ω_t falls, (ii) g_t and ξ_t increase, but (iii) g_t^* and ξ_t^* decrease.*

Proof. See Appendix A.

To explain this proposition, consider the economy at A_1 on the before-policy saddle path in Figure 4 (of course A_1 can be A^*). If the policy is applied, the entire saddle path shifts upwards with the steady state A^* moving to A^{**}. Since χ_t cannot jump immediately, ω_t falls instead. An intuition is that a TR subsidy reduces TR costs borne by entrepreneurs, inducing them to hire more technologists. This results in an increase in w_t^h, i.e. a fall in ω_t (result (i)).

Result (ii) of the proposition is familiar in the TR-based growth literature. But, recall that g_t and ξ_t monotonically converge to zero in the absence of further scienti c breakthroughs. Thus, unlike the existing Schumpeterian models, a TR subsidy has merely a *temporary* rather than a permanent effect on the actual growth rate. More striking is result (iii), i.e. an TR subsidy actually depresses the long-run trend growth rate. This demonstrates that the government can bene t from a TR subsidy in the form of rises in g_t and ξ_t at the cost of a fall in g_t^* and ξ_t^*. If TR is taxed instead, we obtain exactly the opposite result. The government cannot raise both the actual and trend growth rates. Thus, whether TR is subsided or taxed in an effort to improve an economic performance depends upon the time-horizon of policy makers.

What is crucial in the above result is that SR is nanced out of corporation tax and pro ts rather than saving which nances TR projects. Since expenditure (and pro ts) and saving move in an opposite direction for a given income, TR and SR intensity respond differently to the policy shift in question. If, instead, private saving is taxed and rms use investors funds to nance SR, then its intensity could move along with TR intensity. However, sustainable per capita growth is impossible in the long-run, since consumers do not save in a steady state. Thus, a necessary condition for an endogenous growth in the presence of the diminishing returns to technological innovations is that SR should be at least partially nanced out of output-based tax revenues or rms pro ts. Its inevitable consequence is that the actual and trend growth rates of output differently respond to the policy shift.

3 Conclusion

The neo-classical growth model predict (conditional) convergence due to diminishing returns to capital accumulation. On the other hand, endogenous growth models predict otherwise, because endogenous factors permanently prevent diminishing returns from arising. This fact makes diminishing returns and endogenous growth seemingly incompatible. But the present study demonstrates that if they are combined, growth and cycles are endogenously generated. A propagating mechanism is the reallocation of resources within the research sector. Although that sector is typically very small in comparison with the production and service sectors, uctuations of employment in that sector have signi cant impacts on aggregate variables. Output grows in waves and its growth rate repeats the process of rising and falling over time. One interpretation of such cycles is the Schumpeterian version of Kondratiev s long-waves rather than short-run uctuations studied by the real-business cycle or neo-Keynesian theory, since a weak form of technological externality and scienti c breakthroughs are the main driving force of cyclical growth.

In the present model, endogenous long-run growth is a result of the repetition of phases in which output grows at a decreasing rate. It is scienti c breakthroughs that make this repetitive process possible. A consequence was that the *actual* and *trend* growth rates of output differ. The former is determined by technological research, whereas the latter is governed by scienti c research. This dichotomy was shown to carry several important implications. First, a higher trend growth rate is associated with more frequent cycles or a greater amplitude of uctuations. Second, a subsidy to technological research raises the actual growth rate only at the expense of the trend growth rate.

Appendix: Proof of Proposition 3

Result (i): We rst prove by contradiction that the equilibrium saddle path entirely shifts upward. Suppose that the new and old saddle paths intersect at least at one point. Denote ω_t corresponding to the intersection point closest to ω^* as $\omega^{\#}$. Since A^{**} is located above A^* in Figure 4, it implies

$$\frac{\partial}{\partial \psi}\left(\frac{\partial \chi}{\partial \omega}\right)\Bigg|^{\omega=\omega^{\#}} > 0 \tag{25}$$

where $\partial\chi/\partial\omega$ is the slope of the saddle path for a given m. The system of the differential equations (H) and (A) implies that the slope of the saddle path is $\partial\chi/\partial\omega|^{\omega=\omega^{\#}} = \dot{\chi}_t/\dot{\omega}_t|^{\omega=\omega^{\#}}$ for a given m, which is

$$\frac{\partial \chi}{\partial \omega}\Bigg|_{\psi=0}^{\omega=\omega^{\#}} = \frac{\delta\chi_t^{\varepsilon}\left(H-\zeta\omega^{\#}\right)}{\omega^{\#}\left\{\frac{\delta}{\chi_t^{1-\varepsilon}}\left[\left(\frac{\kappa(1-\tau)(1-\alpha)}{(1-\psi)}+\varepsilon\zeta\right)\omega^{\#}-\varepsilon H\right]- \quad \right\}} > 0 \tag{26}$$

where $\quad = \nu\left(\ln\lambda\right)\varphi\zeta\chi_t^{1-\varepsilon}\omega^{\#}+\rho$. From this, it is evident that

$$\frac{\partial}{\partial \psi}\left(\frac{\partial \chi}{\partial \omega}\right)\Bigg|_{\psi=0}^{\omega=\omega^{\#}} < 0. \tag{27}$$

But (25) contradicts (27). Therefore, the old and new saddle paths cannot intersect, so that the latter is always entirely located above the former. Given this result, it is evident from Figure 4 that $\frac{\partial \omega_t}{\partial \psi}\Big|_{\psi=0} < 0$, since χ_t cannot jump.

Results (ii) and (iii): From (H) and (24), we obtain

$$\frac{\partial \xi_t}{\partial \psi}\Bigg|_{\psi=0} = -\frac{\delta\zeta}{\chi_t^{1-\varepsilon}}\frac{\partial \omega_t}{\partial \psi}\Bigg|_{\psi=0} > 0, \quad \frac{\partial \xi_t^*}{\partial \psi}\Bigg|_{\psi=0} = \frac{\nu\ln\lambda}{1-\varepsilon}\varphi\zeta\chi_t^{1-\varepsilon}\frac{\partial \omega_t}{\partial \psi}\Bigg|_{\psi=0} < 0. \tag{28}$$

Besides, $\frac{\partial g_t}{\partial \psi}\Big|_{\psi=0} > 0$ and $\frac{\partial g_t^*}{\partial \psi}\Big|_{\psi=0} < 0$ are evident from $g_t = \frac{1-\alpha}{\alpha}\xi_t$ and $g_t^* = \frac{1-\alpha}{\alpha}\xi_t^*$. ■

References

[1] Aghion, P. and Howitt, P. (1992). A Model of Growth through Creative Destruction. *Econometrica*, **60**, 323-351.

[2] Aghion, P. and Howitt, P. (1998) *Endogenous Growth Theory.* MIT Press, forthcoming.

[3] Aghion, P. and St.Paul, (1991). On the Virtue of Bad Times: an Analysis of the Interaction between Economic Fluctuations and Productivity Growth. *CEPR* Discussion Paper No. 578, CEPR, London.

[4] Amable, B. (1995). Endogenous Growth and Cycles through Radical and Incremental Innovation. *CEPREMAP* Discussion Paper No. 9504.

[5] Caballero, R.J. and Hammour, M.L. (1994). The Cleansing Effect of Recessions. *American Economic Review*, **84**, 1350-1368.

[6] Cheng, L.K. and Dinopoulos, E. (1992). Schumpeterian Growth and Stochastic Economic Fluctuations. Mimeo, University of Florida.

[7] Corriveau, L. (1994). Entrepreneurs, Growth and Cycles. *Economica*, **61**, 1-15.

[8] Dasgupta, P. and David, P. (1995). Toward a New Economics of Science. *Research Policy*, **23**, 487-521.

[9] Griliches, Z. (1986). Productivity, TR and Basic Research at the Firm Level in the 1970s. *American Economic Review*, **76**, 141-154.

[10] Grossman, M.G. and Helpman, E. (1991). *Innovation and Growth in the Global Economy*, MIT Press.

[11] Helpman, E. and Trajtenberg, M. (1994). A Time to Sow and Time to Reap: Growth Based on General Purpose Technologies. *NBER* Working Paper No. 4854.

[12] Helpman, E. and Trajtenberg, M. (1996). Diffusion of General Purpose Technologies. *NBER* Working Paper No. 5773.

[13] HMSO. (1996). *Science, Engineering and Technology Statistics 1996.* Office of Science and Technology, London.

[14] Jones, G. (1995). Time Series Tests of Endogenous Growth Models. *Quarterly Journal of Economics*, **CX**, 495-525.

[15] King, R.G. and Rebelo, S.T. (1986). Business Cycles with Endogenous Growth. Mimeo.

[16] Kleinknecht, A. (1987). *Innovation Patterns in Crisis and Prosperity: Schumpeter s Long Cycle Reconsidered,* London: Macmillan.

[17] Kuhn, T. (1962). *The Structure of Scienti c Revolutions.* Chicago Univ. Press.

[18] Lacatos, L. (1978). *The Methodology of Scienti c Research Programmes.* Cambridge Univ. Press.

[19] Lichtenberg, F.R. (1984). The Relationship between Federal Contract R&D and Company R&D. *American Economic Review Papers and Proceedings,* **74**, 73-78.

[20] Mans eld, E. (1980). Basic Research and Productivity Increase in Manufacturing. *American Economic Review*, **70**, 863-873.

[21] Mans eld, E. (1983). Long Waves and Technological Innovations. *American Economic Review Papers and Proceedings*, **83**, 141-145.

[22] Mensch, G. (1975). *Stalemate in Technology: Innovations Overcome the Depression.* New York: Ballinger.

[23] Nordhaus, W. D. (1972). The Recent Productivity Slowdown. *Brookings Papers on Economic Activity*, 493-537.

[24] Romer, P.M. (1990) Endogenous Technological Change. *Journal of Political Economy*, **98**, S71-S102.

[25] Rosenberg, N. (1974). Science, Invention and Economic Growth. *Economic Journal*, **84**, 90-108.

[26] Rosenberg, N. (1982). *Inside the Black Box: Technology and Economics.* Cambridge Univ. Press.

[27] Rosenberg, N. (1990). Why Do Firms Do Basic Research (with their Own Money)? *Research Policy*, **19**, 165-174.

[28] Rosenberg, N. and Frischtak, C.R. (1983). Long Waves and Economic Growth: A Critical Appraisal. *American Economic Review Papers and Proceedings*, **83**, 146-151.

[29] Schumpeter, J.A. (1939). *Business Cycles.* New York and London: McGraw-Hill.

[30] Shleifer, A. (1986). Implementation Cycles. *Journal of Political Economy*, **94**, 1163-1190.

[31] SPRU. (1996). *The Relationship between Publicly Funded Basic Research and Economic Performance.* Prepared for HM Treasury, Science policy Research Unit, University of Sussex.

[32] Stadler, G.W. (1990). Business Cycle Models with Endogenous Technology. *American Economic Review*, **80**, 763-778.

[33] Stein, J.C. (1997). Waves of Creative Destruction: Firm-Speci c Learning-by-Doing and the Dynamics of Innovation. *Review of Economic Studies*, **64**, 265-288.