

# ***Uncertainties Surrounding Natural Rate Estimates in the G7***

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## **Abstract:**

This paper discusses some of the uncertainties surrounding natural rates of unemployment. Point estimates of the natural rate are usually generated as ratios between parameters estimated in regression equations. Thus uncertainties about the estimated parameter values inevitably implies uncertainty about the natural rate of unemployment. Regression packages do not usually report standard errors for non-linear functions of parameters, so standard errors for natural rate estimates are not usually reported. Staiger, Stock and Watson and others have corrected this deficiency by estimating confidence intervals for natural rates in the US. The present paper extends this work to the G7 countries and finds that although a careful specification search yields an improvement in the precision of estimates of the natural rate, there is still a large degree of uncertainty.

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It is thirty years since the natural rate of unemployment hypothesis was unveiled by Phelps (1967) and Friedman (1968). Yet, as recent surveys and symposia have illustrated (Bean 1994, Cross ed. 1995, *Journal of Economic Perspectives* 1997), there remains much uncertainty both about which factors do and do not determine equilibrium rates of unemployment, and about the corresponding numerical values. The present paper follows Staiger, Stock and Watson (1996a), hereafter SSW, in investigating one particular source of such uncertainty, that arising within the context of a given model specification.

Estimates of natural rates of unemployment tend to be presented as single points. Different models or different specifications of a particular model tend to produce different point estimates. For the UK 1988-90, for example, the point estimates from different models ranged from 3.5% to 8.1%, compared to an actual claimant unemployment rate of 6.8% (Cromb 1993, p.28). There is, however, a more fundamental source of uncertainty as to what the natural rate actually is, which arises within the confines of a given model specification. This type of uncertainty exists because it is not possible to observe natural rates directly. Even in periods in which the rate of inflation is constant it does not necessarily follow that actual unemployment is at the natural rate: other factors, including unobservable shocks, affecting the rate of inflation could be countering the effects of deviations from the natural rate on the rate of inflation. If natural rates are estimated on the basis of statistical inference, the uncertainties surrounding the parameter values in the regression equations used to derive natural rate estimates will inevitably be transmitted into uncertainty about the natural rate estimate itself. Natural rates are

usually non-linear functions of the parameters in regression equations, so natural rate confidence intervals cannot be ascertained directly from the test statistics produced by standard regression packages. Thus confidence intervals for natural rate estimates are not usually reported and the uncertainty surrounding the estimates derived from a particular model is masked.

Fuhrer (1995), King, Stock and Watson (1995) and SSW (1996a, 1996b and 1997) have addressed this problem by estimating natural rate confidence intervals for the US. The typical finding is that the intervals are quite large: for example, the 95% confidence interval for a point estimate of the US natural rate as 5.9% in 1994 was 4.3% to 7.3% (SSW 1996b, Table 1). As far as we are aware, the only estimates of natural rate confidence intervals for countries other than the US are the preliminary ones reported in Cross, Darby and Ireland (1996) . DeBelle and Laxton (1996) have used recursive techniques to obtain somewhat similar intervals for the US, Canada and the UK.

The present paper contributes to this literature by employing the SSW technique to obtain natural rate confidence intervals for each of the G7 countries, with the US being included as a yardstick. We provide results for three sets of specifications: the first in which the natural rate is assumed to be constant over time, the second in which we follow SSW by modelling the natural rate using a cubic spline function and the third in which we select a preferred specification for each country from a variety of deterministic functions of time including the cubic spline. We find that by taking this specification search seriously we can improve precision of our estimates

of the natural rate. Similarly, in contrast to SSW we allow our dynamics to be data determined and once again this leads to an improvement in precision. Nevertheless, we conclude that despite these improvements in precision there is a large degree of uncertainty surrounding estimates of the natural rate.

The rest of the paper is organised as follows. Section I outlines some of the layers of uncertainty that surround attempts to represent and test the implications of the natural rate hypothesis. Section II outlines the different ways the natural rate of unemployment hypothesis can be specified. Section III describes how confidence intervals for the natural rate can be estimated within the context of a given model specification. Section IV reports our estimates of natural rate confidence intervals for the G7 countries using quarterly data 1965.1 - 1995.3. Section V concludes.

## **I UNCERTAINTIES REGARDING NATURAL RATES**

The hallmark of the natural rate hypothesis is the application of the axiom of monetary neutrality to the equilibrium rate of employment: "..... management of monetary demand cannot engineer an arbitrary unemployment rate other than the natural level without sooner or later generating a continuing disequilibrium manifested by rising inflation or mounting deflation ... the actual unemployment rate, though occasionally hit by shocks, is constantly homing in on the natural rate ...." (Phelps 1995, p.15).

Let  $p$  be the log of the price level, so  $\Delta p$  is the rate of inflation and  $\Delta^2 p$  is the change in the rate of inflation (and not the acceleration in the rate of inflation, as in the mis-

coined NAIRU acronym), where  $\Delta$  and  $\Delta^2$  are the first and second difference operators, finally let  $u$  and  $u^*$  denote the actual and natural rates of unemployment. The main refutable implication of the natural rate hypothesis (NRH) is that  $\Delta^2 p \lesssim 0$  according to  $u \gtrsim u^*$  with  $u^* = g(r)$ , where  $r$  is a vector of non-monetary, "real" variables. This exemplifies the general problem raised by Duhem (1906), in that the target NRH cannot be tested directly, but only in conjunction with auxiliary hypotheses about what determines  $u^*$ . Thus tests of the NRH will be unfocused in the sense that any refutations could reflect failure in either the NRH implications regarding  $\Delta^2 p$ , or in the auxiliary hypotheses regarding the determinants of  $u^*$ .

Surveys of the various models that offer accounts of the determinants of  $u^*$  are provided in Blanchard (1990), Bean (1994) and OECD (1994). Perhaps the most comprehensive is the *structuralist* model of Phelps (1994), which is based on intertemporal general equilibrium interaction between incentive wages determined in labour markets, prices set by firms with firm-specific capital in output markets, and asset prices. In this framework the natural rate is endogenously determined by real shocks, with real interest rates and exchange rates playing key roles in the transmission process. To date, however, the US literature has been dominated by the *triangle* model (Gordon 1997), and the European literature by the *battle of the mark ups* model (Layard, Nickell and Jackman 1991). In both of these models the natural rate is derived as the ratio between sets of parameters estimated in regression equations, with the coefficients on unemployment appearing in the denominator. This means that confidence intervals for the natural rate estimates cannot be recovered directly from the standard diagnostic test statistics reported. The failure

to calculate and report such confidence intervals has tended to mask the uncertainty surrounding the natural rate estimates derived from such models.

### **The Triangle Model**

The triangle of factors determining inflation in this model are "demand", in the form of  $(u-u^*)$ , that is deviations of the actual from the natural rate of unemployment; "supply", in form of set of  $x$  variables, which provide proxies or dummies for supply shocks; and "inertia", in the form of lagged effects of the rate of inflation,  $\Delta p_{t-i}$ , whose coefficients are constrained to sum to unity so that the NRH property holds (see Gordon 1997). A simplified version of this model (see SSW 1996a), in which the natural rate is a constant  $u^*$ , can be written as:

$$\Delta^2 p_t = \sum_{i=1} \alpha_i (u_{t-i} - u^*) + \sum_{i=0} \beta_i x_{t-i} + \sum_{i=1} \gamma_i \Delta^2 p_{t-i} + v_t \quad (1)$$

where  $v_t$  is the error term. In (1) the constant natural rate enters non-linearly as  $\sum \alpha_i u^*$ , so this model is difficult to estimate. For this reason (1) is usually estimated as:

$$\Delta^2 p_t = \mu + \sum_{i=1} \alpha_i u_{t-i} + \sum_{i=0} \beta_i x_{t-i} + \sum_{i=1} \gamma_i \Delta^2 p_{t-i} + v_t \quad (2)$$

and the estimate of the constant natural rate derived from this "triangle" model is:

$$\hat{u}^* = -\hat{\mu} / \sum_{i=1} \hat{\alpha}_i \quad (3)$$

Thus the estimate of the natural rate is a non-linear function of the  $\mu$  and  $\alpha$  coefficients as estimated in (2). Regression packages do not, as a rule, estimate standard errors for non-linear functions, so it is not possible to see what the confidence intervals for such natural rate estimates are using the diagnostic test statistics usually reported.

### **The Battle of the Mark Ups Model**

This model is based on the idea that the natural rate is the level of unemployment that reconciles the prices set by firms with the wages negotiated or set in labour markets. A simplified version of this model (see Layard, Nickell and Jackman 1991, p.378) can be written:

$$p - w = a_0 - a_1 u - a_2 (p - p^e) + a_3 z_p + v_p \quad (4)$$

$$w - p = b_0 - b_1 u - b_2 (p - p^e) + b_3 z_w + v_w \quad (5)$$

where  $w$  is the money wage level,  $p$  and  $p^e$  are the actual and expected values of the price level,  $z_p$  and  $z_w$  are price and wage push variables respectively, and  $v_p$  and  $v_w$  are error terms. As in the “triangle” model (see Stiglitz 1997), the rate of inflation is usually assumed to follow a unit root process, and the expected price level  $p^e$  is assumed to be geared purely to this process. Hence  $(p - p^e) = \Delta^2 p$ . The natural rate is then “identified” (see Manning 1993 for an account of the identification problem in this context) by setting  $(p - p^e) = \Delta^2 p = 0$  and solving (4) and (5) for the implied natural rate;

$$u^* = (a_0 + a_3 z_p + b_0 + b_3 z_w) / (a_1 + b_1) \quad (6)$$

Again, as in the “triangle” model, the natural estimate derived from (6) is a non-linear function of the parameters estimated in (4) and (5) with the coefficients on unemployment appearing in the denominator of the estimate of  $u^*$ . Thus the confidence intervals for the natural rate estimates yielded by this “battle of the mark ups” model cannot be ascertained from the diagnostic test statistics generally reported.

### **How Reliable are the Estimates?**

At the time of coining the natural rate metaphor Friedman argued that we "... cannot know what the "natural" rate is ... we have as yet devised no method to estimate accurately and readily the natural rate of either interest or unemployment" (Friedman 1968, p.7). The estimates of natural rates of unemployment in the subsequent literature have elicited a puzzingly wide range of reactions.

In relation to "triangle" model estimates for the US, for example, Joseph Stiglitz, the current Chairman of the Council of Economic Advisors view is that "...the natural rate hypothesis passes all three tests I have outlined ...unemployment is an empirically successful way to predict changes in the inflation rate ...economists have good explanations for the now undeniable fact that the NAIRU has fallen in the last 15 years ...the natural rate provides a useful framework for thinking about policy questions even if there is uncertainty about its exact magnitude" (Stiglitz 1997, p.10). Viewed by another commentator, however, the glass is half empty rather than half full: "...the estimated NAIRU in a variety of studies has tracked the actual unemployment rate sluggishly ...when unemployment rises, analysts tend to discover that the demographic characteristics of workers are deteriorating, or that the job-wage and wage-price dynamic has become unstable ...and when the unemployment rate drifts down again, these flaws mysteriously tend to disappear, and a lower NAIRU is estimated" (Galbraith 1997, p.101).



The failure to routinely calculate and report the confidence intervals surrounding natural rate estimates may perhaps help to explain the varied assessments of the reliability of natural rate estimates.

## II NATURAL RATE SPECIFICATIONS

Ignoring time lags, the NRH can be expressed as:

$$\Delta^2 p = f(u - u^*) \quad \text{NRH}$$

Three aspects of the hypothesis can be distinguished:

$$f(u - u^*) < 0 \text{ and } f(0) = 0 \quad \text{NRH1}$$

$$u^* = g(r) \quad \text{NRH2}$$

$$\text{and } u^* A u \quad \text{NRH3}$$

where  $r$  is a vector of "real", non-monetary variables and  $A$  means "is a attractor for".

The NRH1 *inflation change* aspect focuses attention on the significance of the unemployment terms in regression equations for  $\Delta^2 p$  as in the "triangle" model (see SSW 1996a, Gordon 1997). The NRH2 *natural rate specification* aspect suggests attention should be directed at unit root and cointegration or error-correction tests for whether unemployment tends to revert to an equilibrium pinned down by some vector of "real" variables,  $r$ , as suggested by "battle of the mark ups" models (see Darby and Wren-Lewis 1993), though it is not clear that all the variables used in such tests are necessarily non-monetary in nature (see Westaway 1996). In "triangle" models  $u^*$  is often expressed as a deterministic or stochastic function of time, which begs the question of whether the time variation mimics the movements of "real" or of non-monetary variables. The third aspect NRH3, that  $u^*$  is an *attractor* for actual

unemployment, suggests stationarity tests on  $(u-u^*)$  and some form of causality tests which investigate the possibility that  $u^*$  is an attractee for actual unemployment (see Blanchard 1995 and Phelps 1995). In models where  $u^*$  is influenced by actual unemployment in the short run (Layard, Nickell and Jackman 1991, for example) it could prove to be difficult to distinguish between the natural rate as an attractor and an attractee, especially if the search procedure for the  $r$  vector of "real" variables involves the vector being re-specified whenever there is an otherwise anomalous rise or fall in actual unemployment relative to the previously specified  $u^*$ , as claimed by Galbraith (1997, p.101).

### **Univariate, Bivariate and Multivariate Models**

Testing the NRH involves choosing a set of variables on which the tests are conditioned. The most basic approach is to start with a *univariate* representation in which  $u^*$  is estimated purely on the basis of the time path for  $u$ , applying a moving average or employing some sort of filter such as Hodrick-Prescott. The resulting  $(u-u^*)$  deviations are then used to test the NRH1 *inflation change* aspect of the hypothesis.

A *bivariate* approach uses information on both  $\Delta^2p$  and  $u$  to identify time paths for  $u^*$ . A widely used approach (Elmeskov 1993) assumes that  $u^*$  is constant between pairs of data points and that  $\Delta^2p$  is determined purely by  $(u-u^*)$ . A series of  $u^*$  for pairs of data points is then generated, outlying observations are removed, and the resulting series is smoothed by a Hodrick-Prescott filter. The filtering procedure, designed to counter the problem that  $\Delta^2p$  is not determined purely by  $(u-u^*)$ , is

rather arbitrary and is likely to involve some loss of information. In a stochastic environment the more straightforward approach is to estimate time paths for  $u^*$  in the context of regression models for  $\Delta^2 p$ , in which dynamic and long-run effects from lagged values of the  $(u-u^*)$  terms enter as explanatory variables alongside time lags for  $\Delta^2 p$ , and any dummy or control variables,  $x$ , as in the "triangle" model specification in equation (1) above.

With *multivariate* models the data set is extended to allow vectors of "real" variables,  $r$ , to determine the time paths for  $u^*$ . To assess the neutrality restrictions of the NRH, tests of exclusion restrictions regarding vectors of monetary variables,  $m$ , would also be necessary. Again various strategies are possible here. The NRH2 *natural rate specification* could be tested to see if cointegration and error-correction holds for  $u$  and the specified  $g(r)$  vector; the NRH3 *attractor* property could be tested by causality tests; or the estimated  $u^* = g(r)$  relationship could be introduced into a regression model for  $\Delta^2 p$  to test the NRH1 *inflation change* implications.

### **Linear versus Non-Linear Models**

The relationship estimated by Phillips (1958) was a curve. Non-linear specifications can be introduced by transforming  $u$ , as  $\log u$  or  $u^{-1}$  for example, or by postulating different coefficients for  $u < u^*$  and  $u > u^*$ . The nature of the non-linearity is a matter of debate: Akerlof, Dickens and Perry (1996) have the convex property of inflation increasing more when  $u < u^*$  than it falls when  $u > u^*$ , whereas Eisner (1996) has the opposite property. In some non-linear specifications the NRH holds (Clark and

Laxton 1997), whereas in others the monetary neutrality axiom is violated with  $u^*$  depending on the rate of inflation (Akerlof, Dickens and Perry 1996).

### **Time Varying Parameters**

A further area of choice is whether to retain the standard specification where the parameters are fixed with respect to time, or to introduce time variation in parameters. In the recent literature stochastic specifications for  $u_t^*$  such as  $u_t^* = u_{t-1}^* + w_t$ , where  $w$  is a stochastic variable, have been introduced into "triangle" models. The resulting models time-varying parameter model can be estimated with the time variation restricted to a single parameter (SSW 1996a). Without prior restrictions on the variance of  $w$  the implied variance of  $u^*$  can be perceived to be implausibly high, higher than that of  $u$  in some specifications (Clark and Laxton 1997, p.24), so "smoothness" priors tend to be introduced which impose a low value on the variance of  $w$  (Gordon 1997).

### **Reported Specifications**

Attempting to estimate confidence intervals for all the possible specifications of the natural rate in the G7 countries would produce an indigestible list of "one damned specification after another". In due course it will, hopefully, become standard practice for investigators to report the confidence intervals associated with their own natural rate estimates. However, in the meantime we restrict ourselves to a limited number of specifications.

The confidence intervals reported in the present paper are those that arise in the context of the "triangle" model summarised in equation (1) above. The specification is largely bivariate, with observations on  $p$  and  $u$  being supplemented by  $(0, 1, -1, 0)$  dummy variables and/or proxies for temporary shocks,  $x$ , when this appears necessary to produce reliable estimates of the parameters in equation (1), see the discussion of the results in Section IV. The main focus was on producing natural rate estimates that best satisfy the NRH1 *inflation change* aspect of the natural rate hypothesis, so the time paths for  $u_t^*$  were determined by this criterion rather than by univariate unemployment representations or by pre-specifications of  $r$  vectors such that  $u_t^* = g(r_t)$ . This means that the NRH2 *natural rate specification* is that  $u_t^* = g(t)$ , which begs the question of whether there are vectors of  $r$  "real" economic variables that mimic the behaviour of  $u_t^*$  in order to satisfy the NRH. In our framework the NRH3 *attractor* aspect of the natural rate hypothesis is assumed to hold and cannot be tested independently. However, absence of this property would be likely to result in the regression equation being unbalanced, as pointed out in the discussion below.

The specification searches were restricted to linear specifications of  $u$  and the model parameters, so the resulting natural rate estimates pertain to Phillips straight lines rather than kinks or curves. A further limitation is that the specification search was confined to deterministic functions of time, so stochastic time varying models such as  $u_t^* = u_{t-1}^* + w_t$  were not estimated. In subsequent work we intend to extend the

range of specifications considered to include “battle of the mark ups” specifications in which  $u_t^* = g(r_t)$ , and stochastic trend models.

### III ESTIMATING CONFIDENCE INTERVALS

As indicated in expressions (3) and (6) above, the natural rate usually appears as a ratio of coefficients estimated in inflation ("triangle" models) or price and wage equations ("battle of the mark ups" models) equations. Hence the natural rate is a non-linear function of the estimated coefficients. Standard regression packages do not estimate standard errors for non-linear functions, so supplementary methods have to be employed to estimate confidence intervals for natural rates. In the literature to date, which deals with natural rate estimates derived from "triangle" models, two main methods have been used to tackle this problem.

#### **The Delta Method**

In the context of the "triangle" model, the natural rate is estimated as  $-\hat{\mu} / \Sigma\hat{\alpha}_i$ , as in equations (2) and (3) above. Fuhrer (1995) used the “delta” method to generate confidence intervals around these estimates. This involves using a first-order Taylor series expansion to approximate  $-\mu / \Sigma\alpha_i$  with a linear function that is asymptotically normally distributed. The confidence intervals are then calculated using the estimates of the first derivatives of this linear function. Thus if the natural rate is some non-linear function  $j(\mu, \alpha_i)$ , the distribution of  $j(\hat{\mu}, \hat{\alpha}_i)$  is approximated by a normal distribution whose variance depends solely on the first derivative of  $j(\cdot)$  evaluated at  $(\hat{\mu}, \hat{\alpha}_i)$  and the estimated variance-covariance matrix of  $(\hat{\mu}, \hat{\alpha}_i)$ .

### **The Fieller Method**

SSW refer to the second approach as the "Fieller method", it being an extension of the Fieller (1954) approach to constructing a confidence interval for the mean value of the ratio of the means of two random variables. Basically this approach involves performing hypothesis tests on all the possible values of the true mean. The set of possible values of the true mean not rejected at the 5% level constitutes the 95% confidence interval.

The SSW (1996a) procedure for estimating confidence intervals applied in the present paper can be most easily illustrated for the case where the natural rate is a constant. Inserting a trial value, say  $u_t^* = 8\%$ , into (1) yields:

$$\Delta^2 p_t = \sum_{i=1} \alpha_i \mathbf{b}_{t-i} - 8\% \mathbf{g} \sum_{i=0} \beta_i x_{t-i} + \sum_{i=1} \gamma_i \Delta^2 p_{t-i} + v_t \quad (7)$$

If the estimated intercept in this regression equation is insignificantly different from zero at the 5% level, the hypothesis that  $u^* = 8\%$  must lie within a 95% confidence interval. If the errors  $v_t$  in (7) are i.i.d normal, and the regressors are exogenous, the null hypothesis  $u^* = 8\%$  can be tested by computing a standard F-statistic of (7) against the alternative unrestricted estimate in (2) above:

$$\Delta^2 p_t = \mu + \sum_{i=1} \alpha_i u_{t-i} + \sum_{i=0} \beta_i x_{t-i} + \sum_{i=1} \gamma_i \Delta^2 p_{t-i} + v_t \quad (2)$$

where  $\mu = -\sum_{i=1} \alpha_i u^*$ , or  $u^* = -\mu / \sum_{i=1} \alpha_i$

The procedure is then repeated by replacing  $u^* = 8\%$  with a wide range of possible values in (7) above and in each case constructing the relevant F-statistic and its associated p-value. The p-values can then be plotted against the associated values of  $u^*$ , and for example, the 95% confidence interval is calculated as the range of values

for which the restriction cannot be rejected at the 5% significance level, ie. the range of values for which the p-value is less than or equal to 0.05.

In choosing between the two approaches, SSW (1996a) point out that when the estimated coefficients on lagged unemployment are not well determined (as is the case in a number of our equations) the asymptotic normal approximation underlying the Delta method is likely to be poor. On the other hand, although the Fieller method may appear preferable on the grounds that it generates hypothesis tests with exact finite sample properties, one of the necessary conditions for the exact distribution theory to hold will be invalidated due to the presence of lagged dependent variables in the regressions. Since neither asymptotic nor finite distribution theory can be used to justify the choice between the two methods, SSW employed a Monte Carlo experiment, the result of which allowed them to conclude that the Fieller method is likely to prove more accurate than the Delta method in calculating confidence intervals in this particular application.

#### **IV ESTIMATED CONFIDENCE INTERVALS**

The confidence intervals estimated in the present paper are derived from a "triangle" model specification of the NRH, see equations (1) to (3) above, in which the natural rate appears as:

$$u_t^* = -\mu(t) / \sum_{i=1} \alpha_i \tag{8}$$

where  $\mu(t)$  is a deterministic function of time, or a constant  $\mu$  for the case of a fixed natural rate. Thus intuitively we would expect the degree of precision of the natural



rate estimates derived from this model to depend on how well defined the  $\alpha$  coefficients describing the effects of the  $(u_{t-i} - u_{t-i}^*)$  deviations on  $\Delta^2 p_t$  are, and on how good the  $\mu(t)$  or  $\mu$  specifications of the time variation, or lack of it, in  $u_t^*$  are. The search strategy was basically one of choosing values of  $\mu$  or  $\mu(t)$  to generate the best defined  $\alpha$  coefficients that could be found, subject to a requirement that the equation shows no obvious signs of misspecification.

The results were generated for the G7 countries - the US, Canada, Japan, Germany, France, Italy and the UK - using quarterly data over the period 1965:1-1995:3. The change in the rate of inflation  $\Delta^2 p_t$  is defined as  $\Delta^1\{100.(p_t - p_{t-4})/ p_{t-4}\}$  where  $p$  are consumer price indices obtained from OECD Main Economic Indicators. The unemployment rates were taken from the OECD Business Sector Database.

In order to provide comparisons across countries without reporting a long list of different specifications, only three sets of results are reported in detail. The first is the simplest case of a constant natural rate,  $u_t^* = u^* \forall t$ . The second is for the case where the natural rate follows a cubic spline with two equidistant knots,  $u_t^* = S(t)$ . These are included to provide a direct comparison to the specification reported by SSW and to provide a time varying specification based on the same functional form for each country. Finally we consider a set of results which are arrived at after a wider specification search detailed below. This set of results form the econometrically “preferred” specifications of time variation,  $u_t^* = P(t)$ . In every case the equation dynamics have been data determined so that the lags included are

sufficient to avoid serial correlation. In addition, where necessary, a minimal number of (0, 1, -1, 0) dummy variables have been used to eliminate the effects of outliers in the residuals. These dummy variables typically account for *temporary* factors, such as the impact of changes in VAT and import price shocks, on the rate of inflation. The twin concerns with the absence of serial correlation and normality insure that the equation residuals are as close to Gaussian as possible and that the coefficients are as efficiently estimated as possible.

A search for the preferred specification for each country was conducted using standard diagnostic tests over linear, quadratic, cubic, cubic splines with one, two or three equidistant knots. In addition we tested for pre-specified shifts in the level of the natural rate, taking the break points determined by Bianchi and Zoega (1997). As the results below indicate, this specification search can have a dramatic impact on the calculated confidence intervals, and in particular, a far larger impact than that found across specifications involving different measures of the price level and the unemployment rate in SSW.

### **Specification 1: The Constant Natural Rate**

The constant natural rate specification performs reasonably well in the US. Evidence of mean reversion in  $u_t$  is provided both in a strongly significant negative coefficient on  $u_{t-1}$  in the regression and in the variation of actual unemployment around the estimated constant natural rate  $\hat{u}^*$  shown in Chart 1. The equation is able to explain 58% of the variation in  $\Delta^2 p_t$ . Diagnostic tests do not indicate any serial correlation, functional form, non-normality, heteroscedasticity or ARCH problems. In

comparison to SSW (1996a), who consistently used four lags of each variable, the data determined dynamics identify a more persistent effect from past changes in inflation of up to 2 years, with a faster pass through from unemployment for which 2 lags are sufficient. Similarly long dynamics in inflation and faster pass through from  $(u^*-u)$  are evident in other countries too. These differences in the dynamics are probably the main factor explaining the relatively tighter 1.53 percentage point 95% confidence interval reported for the US in Table 1A compared to SSW's 2.65 percentage points.

In the remaining countries the constant natural rate specifications are less satisfactory. Whilst the estimated coefficients on  $u_{t-1}$  take the expected negative sign, these estimates are only significantly different from zero at the 5% significance level in the Canadian and French regressions. In all cases the estimated coefficients are small so the feedback from  $(u-u^*)$  to  $\Delta^2p$  implied by NRH1 is much weaker than in the US regression. This lack of mean reversion is also evident in the variation of actual unemployment around the estimated constant natural rates shown in Chart 1.

Econometrically, a likely cause of the weakness of the estimates is that the constant  $u^*$  is not acting as an attractor for  $u$ , so that  $(u_{t-1} - u^*)$  is non-stationary. Whilst this hypothesis is not separately tested (as mentioned in Section I, the feedback property NH1 is essentially under test conditional on the NH3, the attractor property) the lack of the attractor property implies that the regression is misspecified and more specifically that it is unbalanced. In this case, provided that  $\Delta^2p \sim I(0)$  and  $(u_{t-1} - u^*) \sim I(1)$ , Monte Carlo results reported in Hendry (1995:p129) can be applied. These

results indicate that in regressions involving I(0) and I(1) regressors, the parameter estimates, sizes of the standard t-statistics and correlation coefficients are close to those obtained in the pure I(0) case. The inference provided by the estimates and t-statistics therefore remains useful even if our testing regressions are unbalanced.

Further evidence against the constant natural rate specifications appears in Chart 2 where  $\Delta^2 p_t$  is plotted against the implied  $(u_{t-1}-u^*)$  deviations. In the US the expected broadly negative correlation can be discerned, though even here the relationship is not particularly well-determined, there being a wide range of  $(u_{t-1}-u^*)$  values for which  $\Delta^2 p_t = 0$ . In the other countries the expected negative correlation is absent or difficult to discern.

Table 1A reports the "Fieller" confidence intervals estimated for the constant natural rate specification while Table 1B expresses these confidence intervals as proportions of the estimated natural rate. Perhaps the best illustration of how well or poorly the natural rate is defined is provided in Chart 3. Here we graph the F-test rejection probabilities against the successive values of  $u^*$  under null hypothesis. The search values of  $u^*$  under the null ranged from -5% to +20% although the plots show the restricted range 0 to 4% for all the countries. All the values of  $u^*$  which cannot be rejected at the 5% level form the 95% confidence interval.

The 95% confidence interval is only defined for three countries (US, Canada and France); in the remaining countries the natural rate was insufficiently well determined to allow us to indicate the interval over the range plotted. Even in the

case of France the width of the 95% interval is so large as to encompass virtually all the variation in actual unemployment over the period (the interval covers 2.57 to 12.94% while the actual unemployment rate was 1.44% at its minimum and 12.43% at its peak over the sample period).

The Japanese case is the least satisfactory. Only confidence intervals of around 50% and below are clearly defined at both upper and lower limits: the imprecision here reflects both the poorly determined coefficients in the regression and difficulties with the Fieller method when the null hypothesis is dramatically different to the freely estimated parameter (see the Monte Carlo evidence reported in SSW (1995a) ). The range of variation of actual recorded unemployment in Japan is far smaller than that in the other countries (ranging from a minimum of 1.05% to a maximum of 3.22%). In any case, our general conclusion from estimation of the constant natural rates has to be that the applicability of this specification is questionable in the US and certainly does not extend to countries outside the US.

### **Specification 2: The Cubic Spline with Two Knots**

The cubic spline,  $u_t^* = S(t)$ , provides a flexible approach to investigating time variation in which  $u_t^*$  evolves smoothly and can both rise and fall. The specification reported here is for such a spline with two equidistant knots, see Suits, Mason and Chan (1978). Since the estimation period runs from 1965:1 to 1995:3 the knots occur at 1975:2 and 1985:4. This spline functional form combines flexibility with the need to estimate only five additional parameters. Between the knot points  $u_t^*$  is a cubic function of time, with the same set of three parameters applying to  $t$ ,  $t^2$  and  $t^3$  within

each interval. The additional two parameters ensure that the first and second derivatives are the same on approaching and on leaving the knot points. The specification search for the spline specification was conducted with the dynamics data determined as before, and the requirements that the residuals be well behaved and the spline and key parameters of the "triangle" equation be as well determined as possible for each of the countries.

We followed SSW (1996a) in assuming that since  $u_t^*$  evolves slowly we can legitimately simplify our dynamic specification by including only  $(u_{t-1} - u_{t-1}^*)$  along with necessary lags of  $\Delta u_{t-i}$ . Further, in calculating the confidence intervals it is necessary to reparametise the estimated dynamic equations to allow us to test hypotheses about  $u^*$  in a particular time period without imposing restrictions on the other parameters: see SSW (1996a), section 4, for further details.

The leftmost plots in Chart 4 show the time paths for  $u_t^*$  implied by the estimated cubic spline specifications along with 75% and 95% Fieller confidence intervals. Chart 5 graphs  $\Delta^2 p_t$  against the implied  $(u_{t-1} - u_{t-1}^*)$  deviations and Tables 2A, 2B and Chart 6 show confidence intervals and F-test rejection probabilities against various hypothesised values of the natural rate at four points in the sample (1970:1, 1980:1, 1990:1 and 1994:1).

In general the cubic spline results are more plausible than the constant specifications. The most satisfactory specifications are those for the US, Canada and Germany. In each of these cases Chart 4 shows clear evidence in support of the

attractor property of the estimated  $u_t^*$  and the negative unemployment coefficients are strongly statistically significant and substantially larger than in those obtained for the constant natural rate specification. Negative correlations between  $\Delta^2 p_t$  and  $(u_{t-1} - u_{t-1}^*)$  are more often evident in Chart 5 than they were in Chart 2. In the cases of the US and Germany the spline regressors are jointly significant, offering further support to this specification of the time variation in  $u_t^*$ .

For the US the significance of the spline conflicts with SSW's result, who found that time variation could be rejected on the basis of an equivalent F-test. Once again it seems likely that the increased efficiency resulting from the data determined dynamics has improved the precision of our inference. For comparison Chart 4 and Tables 2A and 2B present two sets of results for the US: a set labelled SSW which use 4 lags of each regressor, and the other showing our preferred dynamic specification. While the estimated natural rate is very similar in both cases, the SSW equation diagnostics identify significant serial correlation, the probability values associated with the relevant LM tests are LM(1) 0.09, LM(4) .001, and LM(8) .000 and the confidence intervals are consistently a factor of 1.5 to 2 times the size of those obtained in our preferred specification. There are no signs of serial correlation or any other forms of misspecification in our preferred specification.

In the Canadian case some doubt is cast on the results in that the spline regressors are only marginally significant (the probability value on the relevant F-statistic is 0.057) and the confidence intervals are wider than those for the US and Germany.

For some of the other countries the cubic spline specification is more obviously unsatisfactory. The Italian case is the least plausible, here the estimated impact of  $(u_{t-1} - u_{t-1}^*)$  is to increase  $\Delta^2 p_t$ , although the coefficient is in fact insignificantly different from zero. It would appear that  $(u_{t-1} - u_{t-1}^*)$  is non-stationary and that therefore the Italian cubic spline specification is unbalanced as in the constant case. Finally, another factor suggesting that the implied Italian natural rate is implausible is that it clearly covers too large a range with the standard deviation of the implied natural rate more than 5 times that of the measured unemployment rate. Since the performance of this specification is clearly far from adequate the Italian two knot cubic spline results are excluded from Tables 2A and 2B and Chart 4.

In some ways the French spline regression performs less well than the constant specification. It was not possible to estimate a statistically significant impact of  $u_{t-1}$  on  $\Delta^2 p_t$ . Despite this, the spline regressors are highly significant. As the leftmost plot in Chart 4 indicates, the estimated natural rate is strongly trended and the 75% confidence interval is reasonably well determined though the 95% interval is much wider and asymmetric. A clear implication of Chart 4 is that actual unemployment was beneath the natural rate in the mid 1970s and above it in the late 1980s. Over these periods the asymmetry in the 95% confidence interval is most evident. In fact, the range of acceptable values of the natural rate expands *away from* actual unemployment so, for example, in the mid 1970s it is difficult to distinguish between a large number of *lower* values of the natural rate. Thus the conclusion that actual unemployment is *above* the natural rate remains robust. However, for much of the rest of the sample the 95% confidence interval encompasses a wider range than



the cyclical fluctuations in the actual unemployment rate so the usefulness of the estimates to policymakers is less clear. Finally, evidence of reversion of the unemployment rate to the estimated natural rate is weak, particularly through the 1970s and the end of sample acceleration in growth in the natural rate is worrying.

The Japanese results are similar to those obtained for France, except that in this case the time variation is statistically insignificant whilst the unemployment coefficient is large and significant. The 95% confidence interval is not well defined, as a wide range of values of the natural rate offer acceptable null hypotheses. In the case of the UK, the spline regressors are again jointly insignificant, and the unemployment coefficient is significant. As with France and Japan, all the other diagnostics fail to indicate any problems. The weak results on the significance of the key spline parameters means that the natural rate estimates for Japan and the UK are imprecise, this is clearly reflected in the both the central estimate and the confidence intervals in Chart 4.

Whilst allowing for a fairly flexible form of time variation, we must conclude that the two knot spline specifications do not produce universally acceptable results. This finding motivates our country by country search for a preferred time varying natural rate specification.

### **The Preferred Specifications**

The procedure here was to search over various alternative deterministic time functions for  $u_t^*$ , and over mean shifts in  $u_t^*$  taking place at the points in time

identified by Bianchi and Zoega (1997) , hereafter BZ, who used exclusively univariate methods. The objective, as above, was to find the best specifications of the "triangle" model in (2) and (3) and once again, the dynamic specifications were data determined and model specific.

The rightmost plots in Chart 4 show the  $u_t^*$  paths implied by the preferred specifications along with the 75% and 95% confidence intervals. Chart 7 graphs  $\Delta^2 p_t$  against the implied  $(u_{t-1} - u_{t-1}^*)$  deviations and Table 3A records the estimated confidence intervals, with Table 3B expressing the size of the intervals as a percentage of the natural rate point estimate at four selected dates. Chart 8 graphs the F-test rejection probabilities at four selected dates in the sample.

(i) USA

In the case of the US, the cubic spline with two knots specification with data determined dynamics proved to outperform all the alternatives considered (including the same specification of time variation with four lags of unemployment and  $\Delta^2 p$ ) as evidenced by the tests summarised in the Appendix (Table A3.?).

(ii) Canada and Japan:

BZ found three significant mean shifts in Canadian unemployment which also appear highly significant in our own univariate unemployment regressions. However, in the context of the "triangle" model, none of the BZ breaks is either individually or jointly significant. Furthermore the coefficient on lagged unemployment is small and insignificantly different from zero which, as noted above, is strongly suggestive of the regression being unbalanced. Given these

results it is not surprising that the implied natural rate from the mean shift specification is both poorly determined and implausible.

As noted in the previous section, the two-knot cubic spline performed reasonably well for Canada. However, a more parsimonious quadratic specification is preferred to the 2 knot cubic spline,  $F(3,107)=1.18$  [.320], and the parameter associated with the unemployment rate is very well determined with a t-statistic of 4.96.

For Japan too the best specification involved quadratic variation in  $u_t^*$ , though in this case the coefficients on the unemployment rate are not particularly well determined. In both the Japanese and Canadian cases the behaviour of the quadratic based  $u_t^*$  is certainly distinct from the spline, in particular the quadratic natural rate levels off in the late 1980s and begins a slow decline in the final two to three years. In contrast, the spline estimates level off much earlier and climb again at the end of the sample. In the Canadian case the preference for the quadratic specification is clear both in the econometric results (see Appendix Table A3.2 for a full set of tests of our preferred Canadian quadratic specification against the rival specifications) and in the improved precision indicated by the tighter estimated confidence intervals.

Preference for the quadratic specification in the Japanese case is less conclusive. Our decision is largely justified by the significance of  $u_{t-1}$ , albeit borderline. The constant, linear trend and BZ mean shift specifications are all rejected in so far as the lagged unemployment rate and mean shifts are statistically insignificant in the relevant

regressions. Despite the significance of lagged unemployment, time variation as specified in the cubic spline with two knots and in the quadratic, is not statistically significant. (Similar rejection of time variation comes in the 3 knot cubic spline and simple cubic.) In terms of fit, the Japanese quadratic specification outperforms the cubic spline with two knots  $F(3,107)=0.479[.70]$ . However the considerable degree of uncertainty which remains is reflected in the widely different behaviour of the natural rate estimates particularly towards the end of the sample, and also in the confidence intervals. For example, where defined the 95% confidence intervals, dwarf the actual variation in unemployment (see Chart 4).

(iii) Germany:

For the continental European countries - Germany, France and Italy - simple linear trends for  $u_t^*$  performed best, though in Germany the linear trend only just outperformed a cubic specification and there is only a marginal difference between these and the spline. BZ identified four significant mean shifts in Germany's unemployment rate. We find three of these significant in an equivalent univariate regression, but they are statistically insignificant in the "triangle" equation  $F(4,112)=.776[.54]$  and  $F(3,113)=.772[.51]$  with the t-statistic on the lagged unemployment rate at 1.69 [.09]. A simple cubic specification was preferred to the two knot cubic spline but was marginally rejected against a linear trend. Tests of the German linear trend model against other time varying natural rate specifications are summarised in the Table A3.? in the Appendix.

(iv) France:

For France, the linear trend specification clearly outperforms the two knot cubic spline and the constant natural rate, see Table A?. The equation diagnostics identify a significant ARCH(1) problem  $F(1,112)=4.35$  [.039] but there is no evidence of non-normality, incorrect functional form, serial correlation, or heteroscedasticity. BZ found a French mean shift specification to be unsatisfactory in the univariate context so this option has not been pursued here.

The confidence intervals about the French estimated natural rate are generally far better determined in the trend specification. An asymmetry in the 95% interval is still evident in the early to mid 1970s and the late 1980s (see Chart 4) but this is far less marked than in the case of the spline discussed earlier. The natural rate estimates are similar over much of the sample although the growth of the spline estimate accelerates in the early 1990s. A comparison of the confidence intervals over this period is interesting, the 75% intervals associated with the spline specification are actually somewhat tighter than those associated with the trend, though both are wide relative to the movement in the actual unemployment rate over the period (see Chart 4 and the comparison of confidence intervals in Tables 2A, 2B and 3A, 3B).

(v) Italy:

While the Italian linear trend specification is preferred to the alternatives tested the comparison is far less clear cut. In the linear trend specification the coefficient on  $u_{t-1}$  is only just significantly different from zero at the 5% level,  $t=2.02$  [.046]; the 95% confidence interval is ill defined (see Tables 3A, 3B and Chart 4); and the correlation

between  $\Delta^2 p_t$  and  $(u_{t-1} - u_{t-1}^*)$  displayed in Chart 7 is also ill defined and little different to that shown for the constant specification in Chart 2. The linear trend is actually insignificant,  $t=1.47$  [.143], so we cannot conclude that there is strong support for the specification. The Ramsey Reset test of functional form is marginal  $F(1,111) = 3.53$  [.063], despite this, the other diagnostics fail to identify any problems.

A quadratic specification produces a broadly similar path for the natural rate but, as with the linear trend, the quadratic regressors are not significant either individually or jointly. While the significance of  $u_{t-1}$  is marginally higher at  $t=2.03$  [.044] the overall fit of the quadratic specification is inferior. In many ways the Italian cubic performs well: the cubic regressors are jointly significant,  $F(3,110)=3.15$  [.028], and the alternative specifications are all rejected when tested against this benchmark. However, once again the coefficient on lagged unemployment is statistically insignificant,  $t=.61$  [.54], and the variation in the implied natural rate is rather too large to be plausible, as found with the spline discussed earlier. Finally, as with France, BZ found a break specification to be unsatisfactory in the univariate context so again this option has not been pursued. In the Italian case we therefore conclude that the linear trend specification is “the best of a bad bunch”.

(vi) UK:

In the case of the UK, as with Italy and Japan, none of the tested specifications perform well on all fronts. Of the set considered, a mean shift specification is most justifiable in terms of the significance of the time variation and of the lagged unemployment rate (t-statistics on the mean shift and  $u_{t-1}$  are 3.07[.003] and -

3.55[.001] respectively). The cubic spline specification, while in some ways plausible, is rather imprecise (see discussion above). A linear trend specification is clearly rejected when tested against the 2-knot cubic spline,  $F(4,107) = .999$  [.41] and the preferred mean shift specification, t-statistic 0.991 [.324]. BZ found evidence of two breaks in their univariate UK regression. However, in the context of our multivariate regressions, while the break in 1975 is statistically significant, it takes an implausible negative coefficient indicating that the natural rate fell substantially in 1975. The multivariate regressions suggest that inflation responded to a temporary shock at this time rather than a permanent shift in the natural rate. A preferable specification incorporates a temporary dummy in 1975 and a single shift in the natural rate in 1980:1. The addition of spline regressors to the single mean shift specification is generally rejected although the case of adding cubic regressors is borderline, and cannot be rejected at the 10% level  $F(3,108) = 2.634$  [.054]. It is the combination of the mean shift and the cubic which seems successful in that a specification with time variation encapsulated in a simple cubic without the mean shift is clearly rejected  $F(3,109) = .26$  [.86]. This suggests that the UK case is distinct from the other countries and that further analysis of the UK in particular is warranted.

### **Discussion**

The search for a set of preferred specifications generally yields an improvement in the precision of estimated natural rates. However, in none of the countries is the natural rate particularly well defined. It might have been thought that the substantial variations in actual unemployment rates in the non-US G7 countries

would have allowed deterministic time specifications of  $u_t^*$  to determine tighter confidence intervals for  $u_t^*$  than those observed in the US. This, however, is not the case: even in Canada and Germany, which have the next tightest confidence intervals, the smallest 95% interval reported in Tables 3A and 3B are 1.5 percentage points wide or 18 and 21% respectively of the point estimate of  $u_t^*$  (compared to a little below 1 percentage point and 12% of the point estimate in the US). These intervals are also generally less consistently small over time, for example, the Canadian confidence interval is nearer 2 percentage points wide or some 29% of the point estimate in 1970 and 1994.

Even if we consider less demanding confidence intervals, on the argument offered by Debelle and Laxton (1996: p21) that “policy makers simply could not afford to be so certain when making policy decisions which could result in potentially larger errors”, the 75% intervals are sufficient to cover the actual unemployment rate in all but the most extreme points in the cycle. Thus despite our narrowing of the confidence intervals via a specification search over dynamic and time varying specifications, the original SSW (1996a) conclusions for the US of “substantial imprecision” in the measurement of the natural rate apply even more powerfully to the rest of the G7, especially in the cases of Italy and Japan where unemployment deviations have at best only marginal significance in explaining changes in the rate of inflation.



## V CONCLUSION

The natural rate hypothesis states that the rate of inflation will rise if actual unemployment is less than the natural rate, and vice-versa. To test this implication we need to know what the natural rate is. In this paper  $u_t^*$  has been specified as a deterministic function of time, with the "preferred" specification for  $u_t^*$  being that which is most consistent with the implications of the natural rate hypothesis regarding changes in the rate of inflation. In the majority of cases, the preferred specifications do identify significant feedback from deviations in the unemployment rate from the natural rate to the change in the rate of inflation, and standard diagnostic tests are supportive, offering considerable support for the hypothesis of a non-vertical short-run Phillips curve. However, the specification of a deterministic time varying specification for  $u_t^*$  obviously begs the question of whether there are economically meaningful variables of the "real", non-monetary type invoked by the natural rate hypothesis that mimic and offer an explanation for the time variations in  $u_t^*$  in question.

Even within our restrictive deterministic framework, there is much uncertainty about what the natural rate actually is. The point estimates for  $u_t^*$  are determined by ratios of parameters estimated in regression equations. Following SSW (1996a) in using the "Fieller method" for estimating confidence intervals for  $\hat{u}_t^*$  reveals that the intervals are large. They tend to be the largest in countries such as Italy, Japan and the UK where the effects of deviations of actual from natural rates of unemployment on changes in the rate of inflation are poorly determined. Even in countries such as

the US and Germany, where the unemployment terms are reasonably well defined, the size of the 95% confidence interval is sufficiently large to encompass all but the most pronounced cyclical variation in the actual unemployment rate, the same is true of the less demanding 75% confidence intervals in the other countries considered. This suggests a large degree of uncertainty by anyone's standards. In the past, the failure of researchers to calculate and publish such confidence intervals has tended to mask this aspect of the uncertainty surrounding the natural rate hypothesis.

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