Inequality and Growth: A Schumpeterian Perspective*

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Abstract

The aim of this paper is to analyse the effect of income inequality on long-run economic growth when it is driven by endogenous technological advance. The model employed emphasises the strategic interactions between competing firms in the product market and indivisibility which characterises many high-tech goods. Widening inequality is harmful for growth in a monopoly but beneficial in a Bertrand duopoly, giving a V-shaped relationship between growth and inequality. This also implies that the growth rate is not necessarily higher in a concentrated market structure than in a less concentrated one.

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Key words: inequality, product market, strategic interactions, R&D, growth.

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1 Introduction

The aim of this paper is to analyse the effect of income inequality on long-run economic growth when it is driven by endogenous technological advance. The model employed emphasises the strategic interactions between competing rms in the product market and indivisibility which characterises many high-tech goods such as computers. Our model relies on two key traits: (i) income inequality determines the qualities demanded of goods, and (ii) the demand for quality and price competition among rms determines the pro t incentive for their R&D efforts to improve product quality and hence growth. Therefore, the present model shares the feature of the so-called Schumpeterian growth models that pro ts drive R&D, but additionally considers the link between income inequality and pro ts.

The importance of this analysis is three-fold. First, there is a common trend among developed economies that income inequality has risen dramatically in recent decades. For example, the Gini coefficient of household income inequality increased by 10 percentage points between 1977 and 1991 in the UK and by three and a half percentage point in the US (Atkinson, 1997). Second, expenditure on R&D is empirically con rmed to have a positive impact on productivity growth (see, for example, Griliches, 1986), and it has become increasingly important in raising our standard of living.[1] Third, despite these observations, the literature on inequality and growth predominantly focuses on liquidity constraints, political and other issues to study a connection between equity and efficiency in a dynamic context. When growth is driven by technological innovations, strategic interactions of rms in the product market following a success in research crucially determine the pro t incentive for R&D and hence growth. This potential channel through which inequality

[1] R&D intensity (the ratio of R&D expenditure to sectoral production) is often taken as a measure of the technological sophistication of an industry, and its steady increase is commonly observed among developed economies. According to OECD (1996), the average R&D intensity of manufacturing industry between 1978 and 1980 stood at 1.68% in the UK, 1.72% in Sweden and 2.2% in the US. It increased to 2.5%, 3.2% and 3.15% between 1990 and 1992. The OECD average has risen from 1.19% to 1.98% between two periods.

affects growth remains relatively unexplored.

Our starting point is the so-called quality-ladders growth model of Aghion and Howitt (1992) and Grossman and Helpman (1991), in which pro t-seeking R&D improves the quality of products. As higher quality products replace lower quality goods, the standard of living in the economy improves over time. However, due to the assumption of consumers homothetic preferences, the growth rate is independent of income distribution in those models.

To overcome this problem, we stress indivisibility of consumption goods which is particularly appropriate for modern high-tech products. This is consistent with Lancaster (1980) who argues that vertical product differentiation of consumption goods becomes most relevant when they are indivisible and consumers differ in their income. Once indivisibility is assumed, consumer preferences are no longer homothetic, and inequality comes to have an impact on growth through a product market, since the degree of inequality endogenously determines the number of price-competing rms with positive market shares. In the Industrial Organisation literature, the same assumption is used in the static price competition models of Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). Therefore, the present paper combines the standard endogenous growth model and the IO model on price competition in vertically differentiated markets. An additional gain from this exercise is a richer realism. In the benchmark quality-ladders models, only the top-quality products are consumed (or used if they are inputs). This aspect does not t reality where, for example, computers with inferior microprocessors are on demand along with superior ones. In our mode, in contrast, some consumers purchase inferior quality products simply because they cannot afford superior ones.

The key results of this study are as follows. (i) When the degree of inequality is relatively small, a monopoly endogenously arises in the product market. In this circumstance, as inequality exogenously widens, the value of innovation (an R&D incentive) drops and the growth rate falls. (ii) When the degree of inequality is relatively large, on the other hand, the product market is characterised by Bertrand duopoly. Widening inequality raises the value of innovation and hence the growth rate. This suggests that the growth rate has a V-shaped relationship with inequality. The intuition is as follows. The question of how inequality affects growth in our model boils down to the link between inequality and a profit incentive for R&D. Widening inequality intensifies competition in the form of increased entry threat, thereby reducing profits, in a monopoly, but relaxes it in a duopoly, generating higher profits. The V-shaped relation also implies that the intensity of innovative activity is not necessarily higher in a concentrated market and can be higher in a less concentrated market. This finding is consistent with empirical studies of Geroski (1990) and Blundel, *et al.* (1995), who show a negative link between growth and the degree of a market concentration.

It is only recently that attention has started to be paid to the product market as a potential channel through which inequality affects growth. Closest to our study is Zweimüller and Brunner (1996) who independently develop a similar framework. They show that inequality is always harmful for growth. However, this result is obtained due to the assumption that there are three groups of consumers in terms of wealth (i.e. rich, middle-income and poor), in each of which the number of consumers is fixed. A consequence is that when different quality products are consumed, the demand for each product is not affected by price changes for a given market structure. It seems unnecessarily restrictive for modelling strategic interactions of firms when the market is oligopolistic. Once this assumption is relaxed, as we do in the present paper, the V-shaped relation emerges. A similar result is reported in Glass (1996). But, due to homothetic preferences of consumers and divisibility of quality goods, she has to assume that consumers subjective evaluation of the quality level differs between the rich and the poor. It is this extra assumption that makes them purchase different quality goods and income distribution matter for growth in her model. If consumers place the same evaluation on quality goods, growth becomes independent of the income inequality. Garćia-Peñalosa (1995) also considers the equity-efficiency link within the Schumpeterian framework. But her analysis focuses on the credit market rather than the product market.

Other studies in the literature highlight different potential channels other than the product market. Alesina and Rodrik (1994), Bénabou (1996), Bertola (1993), Glomm and Ravikumar (1992), Perotti (1993), Persson and Tabellini (1994) and Saint-Paul and Verdier (1993) develop models with endogenous scal policy through voting. Since taxation is distortionary, it adversely affects investment and growth. Imperfection of the capital market is stressed by Bénabou (1996), Galor and Zeira (1993), Garía-Peñalosa (1995). Since some agents are credit-constrained, productive activities are adversely affected. Benhabib and Rustichini (1991) and Krussel and Rios-Rull (1996) focus on the redistribution between social groups with different interests. Expropriative activities create the disincentive for investment and hence discourages growth. Galor and Tsiddon (1997) focus on the parental externality in the formation of human capital. In a different context, Murphy, Shleifer and Vishny (1989) consider the interactions between the income distribution and demand externality to examine their effect on industrialisation. Although this list is by no means exhaustive for the vast literature, many of the theoretical models predict that inequality is harmful for growth. Among the above-cited studies, however, Saint-Paul and Verdier (1993), Garćia-Peñalosa (1995) and Perotti (1993) demonstrate that inequality can be bene cial for growth in some circumstances. In this sense, the conclusion of our study is similar to theirs. But the mechanism behind our V-shaped result is different from theirs.

The plan of our paper is as follows. In Section 2, we rst explain the source of inequality and examine consumers utility maximisation in the presence of indivisibility of goods. On the basis of this, we turn to the supply side, in which the product market is characterised by monopoly and Bertrand duopoly. Section 3 examines the steady state equilibrium of the two cases and analyses the effect of widening inequality on growth. Some recent empirical evidence is also brie y discussed there. Conclusions are given in

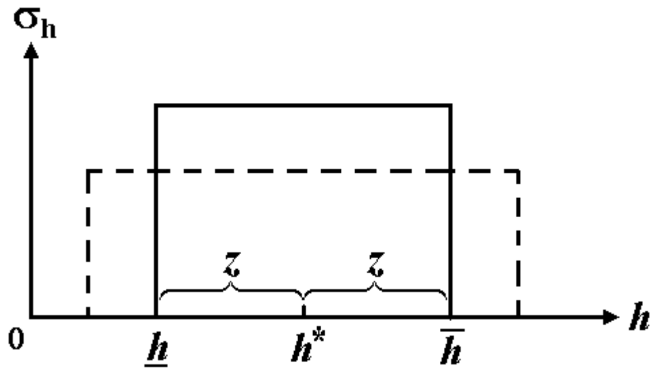

Figure 1: The mean-preserving spread of labour efficiency.

Section 4.

2 The Model

2.1 Consumers

The economy is populated by L consumers who live in nitely. They supply one unit of ow labour service without its disutility in each period. To introduce inequality, consumers are assumed to differ in labour efficiency denoted by h. It is assumed to be uniformly distributed between $\underline{h}$ and $\overline{h}$, $0 < \underline{h} < \overline{h}$, with the density of $\sigma_h = L/(\overline{h} - \underline{h})$. This is represented by a solid schedule in Figure 1. Now de ne h^* as the average labour efficiency, so that

$$\overline{h} = h^* + z, \qquad \underline{h} = h^* - z \tag{1}$$

where z is a measure of dispersion. This enables us to rewrite the density as $\sigma_h = L/2z$. An increase in z means more income inequality in our model, since consumers are remunerated according to labour efficiency. Thus, we are interested in the effect on growth of an increase in z. Such a mean-preserving spread is illustrated by the movement from the solid schedule to the dotted one in Figure 1.

There are two types of final goods in the economy. One group of products, indexed by j, $j \in [0,1]$, are subject to quality innovation over time. Consumers consume a single unit of those quality products in each period. The other good is a homogenous product, denoted by x, which is a numeraire.

The consumers' intertemporal utility function is

$$U = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \ln u_t \tag{2}$$

where ρ is the subjective rate of time preference. The static utility function is

$$\ln u_t = \int_0^1 \ln q_{nt}(j)\, dj + \ln x_t \tag{3}$$

where q_n is the quality index defined as

$$q_{nt}(j) = \gamma^{n(t,j)}, \quad \gamma > 1, \quad n(t,j) = 0, 1, 2, ... \tag{4}$$

In (4), γ represents the increase in quality from each innovation and $n(t,j)$ is the cumulative number of innovations in the jth industry up to time t. We use N to denote the highest number of innovation achieved at time t, so that $q_{Nt}(j)$ is the state-of-the-art, $q_{N-1t}(j)$ the second highest on the quality ladder and so on.

The flow budget constraint of consumers with efficiency h is $k_{t+1}^h = hw_t - E_t^h + (1+r_t)\, k_t^h$ where w_t is a wage per unit efficiency, r_t is an interest rate, E_t^h is expenditure and k_t^h is an asset holding. After purchasing a single unit of quality products, consumers spend the remaining income on the homogenous products. Hence a utility-maximizing demand for x_t is

$$x_t^h = E_t^h - P_{nt} \tag{5}$$

where $P_{nt} = \int_0^1 p_{nt}(j) dj$ and $p_{nt}(j)$ is the price of quality products.

Given the utility function and the budget constraint, consumers choose $\{E_t^h\}_{t=0}^{\infty}$. Once they optimally determine expenditure, the choice of $q_{nt}(j)$ is made simultaneously along with the decisions on $p_{nt}(j)$ by profit-maximising firms. Hence, setting aside the choice of quality on the part of consumers for a moment, we rst consider the optimal choice of expenditure. Since E_t^h and x_t^h are linearly related due to (5), the choice of E_t^h is equivalent to choosing x_t^h for a given P_{nt}. So, we let consumers maximize (2) subject to the ow budget constraint by choosing x_t^h. The rst order condition is

$$\frac{x_{t+1}^h}{x_t^h} = \frac{1 + r_{t+1}}{1 + \rho}. \tag{6}$$

Since the distribution of expenditure is important for the following analysis, we establish the following lemma.

Lemma 1 *If consumers have the same initial asset holding (i.e., $k_0^h = k_0 > 0$)*[2] *and consume the state-of-the-art products for all j s (i.e., a monopoly), then their expenditure is linearly and positively related to their labour efficiency, so that expenditure is uniformly distributed for any sequence of* $\{w_t, r_t, P_{Nt}\}_{t=0}^{\infty}$.

Proof: See Appendix A.

We also analyse a duopoly market structure. In this case, as will become clear, the state-of-the-art and second highest quality products are consumed.

Lemma 2 *Suppose that the initial asset holding is the same for all consumers. If there is a unique threshold expenditure level which divides consumers into two groups such that one group consume q_{Nt} and the other purchase q_{N-1t} for all j s (i.e., duopoly), then consumers expenditure is linearly and positively related to their labour efficiency, so that expenditure is uniformly distributed for any sequence of* $\{w_t, r_t, P_{Nt}, P_{N-1t}\}_{t=0}^{\infty}$.

Proof: It is evident from applying Lemma 1 to each group of consumers. ■

The two lemmas imply that the population density can be rewritten in terms of expenditure as $\sigma_{Et} = L/(\overline{E}_t - \underline{E}_t)$ where $\overline{E}_t$ and $\underline{E}_t$ are the highest and lowest expenditures and correspond to $\overline{h}$ and $\underline{h}$ respectively. In the following section, it proves convenient

[2]This assumption removes any other source of inequality apart from efficiency differences.

to use consumers expenditure rather than labour efficiency to describe the supply side of the model. But readers may nd it helpful to bear in mind that E has a one-to-one correspondence with h.

2.2 Quality Goods Industry: Monopoly[3]

Since all quality industries are assumed to be identical in every respect, we focus upon one representative quality industry, dropping the argument j. The degree of inequality plays a crucial role in determining the market structure. Thus, for the time being, we suppose that the distribution of consumers efficiency is such that only the state-of-the-art products are consumed.

2.2.1 Production

It is assumed that producing a unit of quality products requires $c > 0$ units of labour efficiency. When a monopoly prevails in a market, the best response of other rms is marginal cost pricing, i.e. $p_n = wc$, $n = 0, 1, ..., N-1$. It follows that the monopoly market arises if $E_N \leq \underline{E}$ where E_N is the expenditure of threshold consumers who are indifferent between q_N and q_{N-1}. A threshold expenditure E_N is derived from equating the static utility (3) in each period of consuming q_N to that of consuming q_{N-1}, i.e. $\int_0^1 \ln q_N dj + \ln(E_N - P_N) = \int_0^1 \ln q_{N-1} dj + \ln(E_N - P_{N-1})$. Since $p_N(j) = p_N$, this leads to[4]

$$E_N = \Gamma p_N - (\Gamma - 1) p_{N-1}, \qquad \Gamma = \frac{\gamma}{\gamma - 1} > 1. \tag{7}$$

Given this, one can easily establish the following lemma:

[3] To avoid an excessive notation, we drop the time subscripts in what follows unless it causes ambiguity.

[4] We have assumed that all consumers buy one unit of quality goods. But if c is sufficiently small, all consumers always choose to consume a quality product. To show this, consider the poorest consumers. If they buy q_n, $n = 1, 2, ...N$, their utility level is $\int_0^1 \ln q_n dj + \ln(\underline{E} - P_n)$, while they can attain $\ln \underline{E}$ without consuming it. All consumers purchase quality goods if and only if $\int_0^1 \ln q_n dj + \ln(\underline{E} - P_n) > \ln \underline{E}$. But, since $p_n = wc$ for $n = 1, 2, ...N-1$ for a monopoly, the poorest consumers buy q_N or q_{N-1} if they are to consume quality goods. Thus, the above inequality constraint can be rewritten as $(1 - 1/\gamma^{b_t})\underline{E}/w > c$ where $b_t = \int_0^1 N_j dj$ or $\int_0^1 (N_j - 1) dj$ depending on whether q_N or q_{N-1} is purchased. This inequality holds if c is sufficiently small.

Lemma 3 *A monopoly price and pro t are*

$$p_N^M = \frac{\underline{E} - wc}{\Gamma} + wc, \qquad \pi_N^M = \frac{L}{\Gamma} \left(\underline{E} - wc \right). \tag{8}$$

Proof. Monopoly pro ts are $L(p_N - wc)$. Since L is given, pro t maximisation implies that a monopoly rm chooses the highest price such that $E_N \leq \underline{E}$. This inequality and $p_{N-1} = wc$ imply $p_N \leq (\underline{E} - wc)/\Gamma + wc$. Hence we obtain p_N^M in (8), and π_N^M is evident from p_N^M. ■

A monopoly price depends upon $\underline{E}$, since it is a limit-price such that $E_N = \underline{E}$, i.e. a monopoly rm serves the whole market.[5] As the poorest consumers become poorer (i.e. $\underline{h}$ and $\underline{E}$ fall), p_N^M and π_N^M decrease. This leads to a drop in the incentive for R&D and discourages growth, as we will see.

2.2.2 When a Monopoly Arises?

The demand schedule facing the monopolist can be written as $\sigma_E(\overline{E} - E_N)$ where E_N is de ned by (7). Since it is linear in p_N and a marginal cost is constant, a monopolist chooses output which is the half of the output he would obtain under marginal cost pricing, i.e. $\sigma_E(\overline{E} - wc)/2$. Thus, a monopoly arises if $L \leq \sigma_E(\overline{E} - wc)/2$, which leads to $(\overline{E} - wc)/(\underline{E} - wc) \leq 2$. It follows that a quality product market is monopolised if

$$1 \leq \frac{\overline{E} - wc}{\underline{E} - wc} \leq 2. \tag{9}$$

2.2.3 R&D

R&D is conducted in every period and its outcome is realised at the beginning of each period. We assume that in each period nature picks up one successful entrepreneur at most or no innovation occurs. More precisely, for an entrepreneur k conducting ξ_k units

[5] Having reached this point, the readers may have realised that Lemma 3 is robust to any distribution of labour efficiency, since the poorest consumers expenditure only matters. Thus, a uniform distribution of labour efficiency is virtually redundant. But we have assumed it to maintain consistency with the next section of duopoly in which this assumption becomes important.

of R&D, the probability of succeeding in his R&D is $\Xi^k = \xi^k/(\mu+\xi)$ where $\xi = \sum \xi^{k'}$ and $\mu > 0$. Thus, de ning $\Xi_t = \sum_k \Xi_t^k$, a single innovation occurs in an industry with a probability of

$$\Xi_t \equiv \Xi(\xi_t) = \frac{\xi_t}{\mu+\xi_t} \tag{10}$$

or no innovation occurs with a complementary probability of $1-\Xi$. A parameter μ is interpreted as difficulties which cannot be overcome even if all resources are devoted to R&D.

A successful entrepreneur starts selling his products from the period when innovation occurs. If he succeeds in R&D at the tth period, he achieves the value of his innovation V_t^M, which is de ned as

$$V_t^M = \sum_{m=t}^{\infty} \theta_m \pi_{Nm}^M \tag{11}$$

where $\theta_t = 1$ and $\theta_m = \prod_{i=t}^m (1-\Xi_i)/(1+r_i)$ for $m \geq t+1$. The innovator earns pro ts in the tth period with certainty. But, from the next period on, there is always a positive probability of the rm being displaced. Hence pro ts earned after period $t+1$ are discounted using the effective cumulative discount factor θ_m. Equation (11) is also interpreted as the no-arbitrage condition which ensures that consumers are indifferent between bonds and equities of research rms as a means of saving, and the interest rate r_t is determined in the competitive nancial market (see Grossman and Helpman (1991)).

It is assumed that one unit of R&D requires $a > 0$ efficiency units of labour services. Thus, free entry in R&D leads to

$$V^M = wa(\mu+\xi) \qquad \text{for } \xi > 0. \tag{12}$$

So far, we have implicitly assumed that an incumbent rm does not conduct R&D to improve its own product. This is due to what is called the replacement effect, i.e. an incremental gain for an incumbent from an extra innovation is strictly smaller than the gain for outside rms. For simplicity, we continue to assume this throughout the paper.[6]

[6] For a model in which incumbent rms conduct R&D, see Ulph (1991), who analyses the relation

2.3 Quality Goods Industry: Bertrand Duopoly

2.3.1 Production

As before, we start by supposing that the degree of inequality is such that only the two highest quality products are consumed. Consider consumers who are indifferent between q_N and q_{N-1}. They are characterised by (7). Note that E_N is unique given p_N and p_{N-1}: consumers with expenditure larger than E_N buy the q_N products and others consume the q_{N-1} goods. Since consumers are divided into two distinct groups, Lemma 2 now applies and duopoly pro ts are given by $\pi_N = \sigma_E(\overline{E} - E_N)(p_N - wc)$ and $\pi_{N-1} = \sigma_E(E_N - \underline{E})(p_{N-1} - wc)$. Simultaneously solving these two equations, we can establish the following lemma:

Lemma 4 *The Bertrand prices* $\{p_N^B, p_{N-1}^B\}$ *and pro ts* $\{\pi_N^B, \pi_{N-1}^B\}$ *are*

$$p_N^B = \frac{2\left(\overline{E} - wc\right) - \left(\underline{E} - wc\right)}{3\Gamma} + wc, \quad p_{N-1}^B = \frac{\left(\overline{E} - wc\right) - 2\left(\underline{E} - wc\right)}{3\left(\Gamma - 1\right)} + wc, \tag{13}$$

$$\pi_N^B = \frac{\sigma_E\left[2\left(\overline{E} - wc\right) - \left(\underline{E} - wc\right)\right]^2}{9\Gamma}, \quad \pi_{N-1}^B = \frac{\sigma_E\left[\left(\overline{E} - wc\right) - 2\left(\underline{E} - wc\right)\right]^2}{9\left(\Gamma - 1\right)}. \tag{14}$$

Proof. See Appendix B.

Note that duopoly prices are increasing in $\overline{E}$ and decreasing in $\underline{E}$. This contrasts with a monopoly price in (8) which is independent of $\overline{E}$ and increasing in $\underline{E}$. As a result, duopoly and monopoly pro ts change differently as the distribution of expenditure alters. As we will see, these differences give rise to the V-shaped relation between inequality and growth in our model.

2.3.2 When a Duopoly Arises?

If the two top rms are only to have positive market shares, the best response of producers of quality lower than q_{N-1} is marginal cost pricing, i.e. $p_n = wc$, $n = 0, 1, ..., N-2$. Thus,

between growth and industrial structure.

duopoly requires $E_{N-1} \leq \underline{E}$ where E_{N-1} is the expenditure of the consumers indifferent between q_{N-1} and q_{N-2}. Invoking the same logic used in deriving (7), we can write $E_{N-1} = \Gamma p_{N-1} - (\Gamma - 1)wc$. Using this and the second equation of (13), $E_{N-1} \leq \underline{E}$ is rearranged to yield $(\overline{E} - wc)/(\underline{E} - wc) \leq 2 + 3/\gamma$. As long as this inequality holds, at most two top rms have positive market shares for equilibrium prices (13). An intuition is that Bertrand equilibrium prices are so low that consumers purchase products of either q_N or q_{N-1} and other goods are not consumed. On the other hand, as $(\overline{E} - wc)/(\underline{E} - wc)$ approaches 2 from above, the demand for the q_{N-1} products fall to zero. Therefore, the condition for Bertrand duopoly is

$$2 < \frac{\overline{E} - wc}{\underline{E} - wc} \leq 2 + \frac{3}{\gamma}. \tag{15}$$

Given (15), one can verify that $p_N^B > p_{N-1}^B$ and $\pi_N^B > \pi_{N-1}^B$.

2.3.3 R&D

Turning to the value of innovation which occurs in the tth period, V_t^B, it is given by

$$V_t^B = \sum_{m=t}^{\infty} \theta_m \pi_{Nm}^B + \sum_{m=t}^{\infty} \theta_m \delta_{m+1} \Gamma_{m+1}, \tag{16}$$

where

$$\Gamma_{m+1} = \pi_{N-1m+1}^B + \sum_{s=m+2}^{\infty} \theta_s \pi_{N-1s}^B, \tag{17}$$

$\theta_m = \prod_{i=t}^{m}(1 - \Xi_i)/(1 + r_i)$ with $\theta_t = 1$, $\theta_s = \prod_{l=m+2}^{s}(1 - \Xi_l)/(1 + r_l)$ and $\delta_{m+1} = \Xi_{m+1}/(1 + r_{m+1})$. First consider pro ts arising from the production of the top-quality products. Just like V^M, the expected discounted value of those pro ts is obtained by multiplying π_{Nm}^B by the effective cumulative discount factor θ_m, and its sum is given by the rst term on the RHS of (16). The second term represents the value of innovation arising from producing the second-highest goods on the quality ladder. In (17), Γ_{m+1} is the sum of the expected values of pro ts due to producing those goods which are discounted back to the $(m+1)$th period. Viewed from the mth period when π^B_{Nm} is still earned, its expected discounted value is $\delta_{m+1}\Gamma_{m+1}$, and hence $\theta_m\delta_{m+1}\Gamma_{m+1}$ is its expected value discounted back to the tth period. Summing it over m gives the second term on the RHS of (16). Free entry in R&D leads to

$$V^B = wa\left(\mu+\xi\right) \qquad \text{for } \xi > 0. \tag{18}$$

2.4 Homogeneous Goods

The homogeneous products are produced with the production function of

$$X^l = \left(L^l_x\right)^\alpha, \quad 0<\alpha<1, \quad l = M, B \tag{19}$$

where L^l_x is the labour force employed in efficiency units. In this sector, perfect competition prevails and hence the labour demand is given by $L^l_x = \alpha X^l/w$.

2.5 Labour Market

There are three sources of labour demand: R&D and manufacturing of quality and homogeneous products. Full employment requires

$$a\xi + cL + \frac{\alpha X^l}{w} = h^* L, \qquad l = M, B. \tag{20}$$

where $X^l = \int_{\underline{E}}^{\overline{E}} \sigma_E x(E^h)dE^h$ and $x(E^h)$ is given by (5). For (20) to make sense, we require $h^* > c$.

2.6 Growth Rate

In the present model, there is neither physical capital accumulation or productivity improvement. But consumers become better-off over time due to quality improvement of consumption goods. The steady-state growth rate of individual utility is calculated as follows:

$$g\ \left(=\ln u_t - \ln u_{t-1}\right) = \int_0^1 \ln\left(\frac{q_{\tilde{n}t}(j)}{q_{\tilde{n}t-1}(j)}\right) dj = \Xi\left(\xi\right)\ln\gamma \tag{21}$$

where $\widetilde{n}$ is N or $N-1$ depending upon the market structure and labour efficiency with which consumers are endowed. The integral term is due to technological innovation in the quality goods market. If innovation occurs in a product line j, the utility gain is $\ln \gamma dj$, and it is 0 otherwise. Since we have a continuum of industries, the Law of Large Numbers applies. That is, the fraction of industries with innovations occurring in each period is $\Xi(\xi)$, leading to the second equality in (21). The growth rate of consumer utility g is strictly increasing in ξ, although its level differs depending upon consumers efficiency endowment.

3 Steady-State Equilibrium

In this section, we focus upon a steady-state equilibrium. Setting $w_t = w$, $E_t^h = E^h$ and $P_t = P$, consumers intertemporal budget constraint becomes $E^h = wh + \rho k_0$.[7]Next de ne $E^* = wh^* + \rho k_0$ as the average consumer expenditure. Then, subtracting it from the extreme expenditures $\overline{E}$ and $\underline{E}$ and making use of (1), we obtain

$$\frac{\overline{E} - wc}{w} = \omega + z - c, \qquad \frac{\underline{E} - wc}{w} = \omega - z - c, \qquad \omega = \frac{E^*}{w}. \tag{22}$$

Using (1) and (22), we also rewrite the density of expenditure distribution as

$$\sigma_E = \frac{\sigma_h}{w}. \tag{23}$$

3.1 Monopoly

First let us consider the range of ω consistent with a monopoly in a steady state. Rewriting (9) gives

$$3z + c \leq \omega. \tag{24}$$

[7]This equation can be obtained from (29) in Appendix A by imposing $w_t = w$, $E_t^h = E^h$ and $P_t = P$.

The steady state equilibrium in the case of a monopoly is analysed by rewriting the two equations (11) and (20) as

$$a\left(\mu+\xi\right)=\frac{1+\rho}{\rho+\Xi\left(\xi\right)}\Pi_N^M\left(\omega\right), \tag{V^M}$$

$$a\xi+\alpha\left[L\omega-\Pi_N^M\left(\omega\right)\right]=\left[h^*-\left(1-\alpha\right)c\right]L, \tag{L^M}$$

where Π_N^M $(\equiv \pi_N^M/w)=L(\omega-z-c)/\Gamma$. Equation (V^M) is the equilibrium condition in the R&D sector and (L^M) is the full-employment condition in the factor market. They constitute the system of the two equations with two unknowns ξ and ω.

The right-hand side of (V^M) is the value of innovation. It is decreasing in ξ, since a higher ξ implies, on average, a shorter period during which a product remains the state-of-the-art. The left-hand side of (V^M) represents research costs. It is increasing in ξ. A greater total R&D in the jth industry reduces the probability that an individual entrepreneur succeeds in R&D in that sector, raising R&D cost for a given probability of her success.

Proposition 1 *There exists a unique equilibrium in which the economy grows with a positive growth rate when a monopoly operates in the quality goods market.*

Proof: See Appendix C.

What this proposition demonstrates is shown in Figure 2 in which the V^MV^M and L^ML^M schedules represent (V^M) and (L^M) respectively. They are monotonic, so that they intersect at a unique equilibrium labelled A for the range of (24).

We are interested in the impact on ξ of exogenously raising the degree of inequality. As explained before, this is done by increasing z, i.e. the mean-preserving spread of labour efficiency which will make the income distribution more unequal.

Proposition 2 *When the quality goods market is characterised by a monopoly, increasing inequality (a higher z) leads to a lower growth rate.*

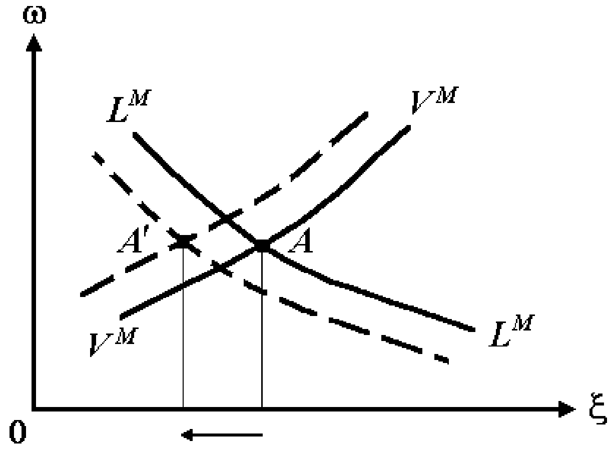

Figure 2: Monopoly: inequality is harmful for growth.

Proof: See Appendix D.

The intuition for this proposition is as follows. Given the monopoly market structure, changes in z do not alter the rm s total demand. Their effect comes only through prices. A monopolist uses limit-pricing to ensure that the whole market is captured and lower quality goods are not consumed. But, as inequality widens, poor consumers become poorer and nd the q_{N-1} products which are priced at marginal cost more attractive. This tends to make the entry of the rm producing them easier, i.e. the threat of entry increases or competition intensi es. In order to persuade those consumers to continue to buy state-of-the-art goods, the monopolist reduces his limit price, decreasing pro ts. Note that widening inequality is equivalent to *intensifying* competition in monopoly.

As pro t rates fall, the incentive for R&D diminishes. This reduces ξ for a given ω, shifting the $V^M V^M$ schedule leftward in Figure 2. On the other hand, a fall in p_N^M raises the demand for homogenous products, boosting the labour demand in that sector. As upward-pressure on wages builds up in the homogeneous goods sector, it attracts labour from R&D. This decreases ξ for a given ω, moving $L^M L^M$ curve leftward. The net effect is a movement of the equilibrium from A to A' with an unambiguous drop in ξ.

3.2 Bertrand Duopoly

We first derive the range of ω consistent with Bertrand duopoly from (15):

$$\frac{3(\gamma+1)}{\gamma+3}z+c \leq \omega < 3z+c. \tag{25}$$

The equilibrium conditions are now derived from (16) and (20):

$$a(\mu+\xi)=\frac{1+\rho}{\rho+\Xi(\xi)}\left[\Pi_N^B(\omega)+\frac{\Xi(\xi)}{\rho+\Xi(\xi)}\Pi_{N-1}^B(\omega)\right] \tag{V^B}$$

$$a\xi+\alpha\left[L\omega-\Pi_N^B(\omega)-\Pi_{N-1}^B(\omega)\right]=\left[h^*-(1-\alpha)c\right]L \tag{L^B}$$

where Π_N^B $(\equiv \pi_N^B/w)=(L/2z)(\omega+3z-c)^2/9\Gamma$ and Π_{N-1}^B $(\equiv \pi_{N-1}^B/w)=(L/2z)(3z-\omega+c)^2/9(\Gamma-1)$. Conditions (V^B) and (L^B) are for equilibrium in the R&D sector and the labour market respectively.

The right-hand side of (V^B) is the value of innovation. A term Π_N^B represents profits which are earned as long as the product remains the state-of-the-art. Once its quality is improved upon, those profits are lost. Its expected present value is decreasing in ξ just as in a monopoly. Now consider Π_{N-1}^B which accrues to an innovator as long as her product is the second highest on the quality ladder. Its expected present value is decreasing in ξ if $\rho<\Xi(\xi)$ but increasing for $\rho>\Xi(\xi)$. This picks up two opposing effects. The first is the same as above: a higher ξ makes the period of earning Π_{N-1}^B shorter on average, leading to a negative relation between the expected present value and ξ. On the other hand, as ξ rises, the product becomes the second highest more quickly than otherwise, realising Π_{N-1}^B much earlier in time on average. This tends to increase the expected present value of Π_{N-1}^B.

Proposition 3 *There exists a unique equilibrium in which the economy grows with a positive growth rate when the market for quality goods is characterised by Bertrand duopoly.*

Proof: See Appendix E.

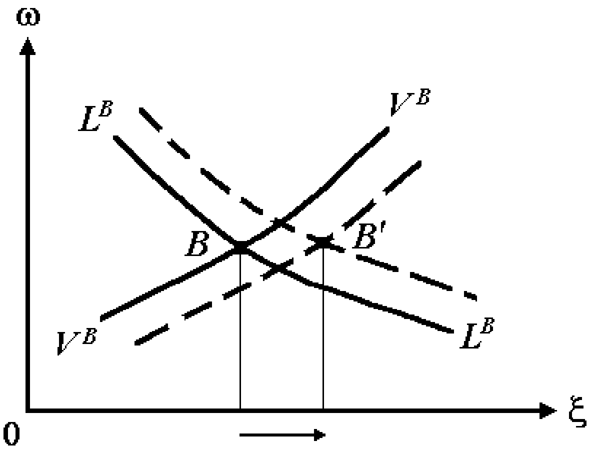

Figure 3: Bertrand duopoly: inequality is bene cial for growth.

Figure 3 describes what this proposition demonstrates. The monotonic V^BV^B and L^BL^B schedules represent (V^B) and (L^B) respectively, and their intersection point B gives a unique equilibrium for the range of (25).

Proposition 4 *When the quality product market is characterised by a Bertrand duopoly, the growth rate rises with the degree of inequality (a higher z).*

Proof: See Appendix F.

There are two effects operating. The mean-preserving spread of labour efficiency implies that the rich become richer and the poor become poorer. The former nd q_N more appealing and the latter see q_{N-1} as more attractive. That is, wider inequality reinforces the tendency that consumers with higher labour efficiency purchase higher quality products and lower-efficiency consumers buy lower quality products. This makes it easier for duopolists to segment the market into two, and as a result, price competition does not need to be as intensive as before, giving them scope to increase their prices.[8]This competition-easing effect tends to raise their pro ts. It is true that a higher z helps to

[8]An alternative explanation is that for the both qualities, the price elasticity of demand decreases with the degree of inequality, so that their prices become higher.

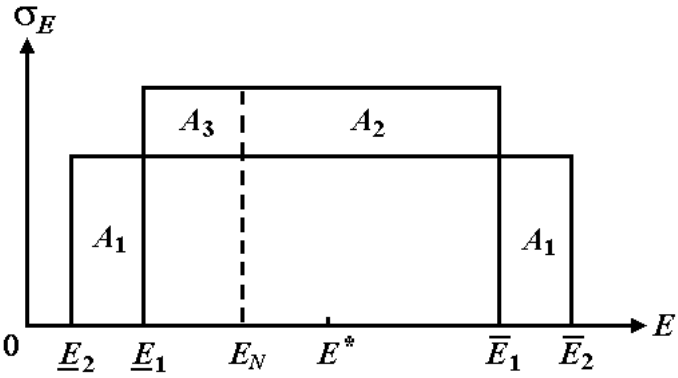

Figure 4: Changes in the composition of the total demand.

ease the entry of the q_{N-2} rm, but it has little impact on the rms price decision.[9] Note that, in contrast with the monopoly case, widened inequality leads to the *relaxation* of competition in Bertrand duopoly.

The second effect is due to changes in the composition of the total demand for quality products. This is best explained by using Figure 4 where the horizontal and vertical axes represent expenditure and the population density. The initial consumer distribution is given by the rectangle associated with $\overline{E}_1$ and $\underline{E}_1$. Consumers between E_N and $\overline{E}_1$ purchase the q_N product and consumers between $\underline{E}_1$ and E_N buy the q_{N-1} product. Note that E_N is always smaller than E^*.[10] As inequality widens, the distribution becomes wider with the extreme expenditures moving to $\overline{E}_2$ and $\underline{E}_2$. Each of the duopolists, ceteris paribus, gains the extra demand represented by the areas A_1, which is the same for the both rms. However, the density σ_E decreases following a rise in z, and this causes the loss of demand: A_2 for the top-quality rm and A_3 for the q_{N-1} rm. Since E_N and E^* do not change and $E_N < E^*$, an area A_2 is larger than A_3. The net effects are that the demand for q_N falls as $A_1 < A_2$, and the demand for q_{N-1} rises as $A_1 > A_3$.

Because the price and demand of the second highest rm on the quality ladder increase,

[9] This effect becomes important for $\frac{\overline{E}-wc}{\underline{E}-wc} > 2 + \frac{3}{\gamma}$ (see equation (15)).

[10] Indeed, $E_N = [(\overline{E} - wc) + (\underline{E} - wc)]/3 + wc$ and $E^* = [(\overline{E} - wc) + (\underline{E} - wc)]/2 + wc$.

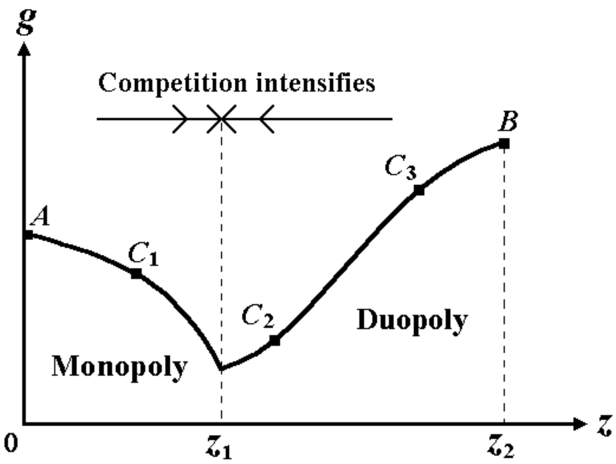

Figure 5: The V-shaped relationship between growth and inequality.

its pro t rates unambiguously rise with the degree of inequality. As for the top-quality rm, its price rises whereas its demand shrinks. But the competition-easing effect always dominates the demand composition effect, pushing up the top rm s pro t rates.

Increased pro ts lead to a greater incentive for R&D, raising ξ for a given ω. Thus, the $V^B V^B$ schedule in Figure 3 shifts rightward. On the other hand, rises in p_N and p_{N-1} reduce the demand for homogeneous goods and hence the labour demand in that sector. As wages tend to drop in the homogeneous products sector, more resources are diverted to R&D. This boosts ξ for a given ω, moving the $L^B L^B$ schedule rightward. A net effect is an unambiguous rise in ξ.

3.3 Discussion

The main implication of the present study is that (i) the growth rate is a function of the measure of income inequality, i.e. $g = f(z)$, and (ii) this function has a V-shaped form, as depicted in Figure 5 where z_1 is associated with $\frac{\overline{E}-wc}{\underline{E}-wc} = 2$ and z_2 with $\frac{\overline{E}-wc}{\underline{E}-wc} = 2 + 3/\gamma$, given other parameters. As we move towards z_1 from either points A or B, the degree of competition increases and the value of innovation falls. Since it reaches the lowest at z_1, we obtain the V-shaped relation between g and z. Thus, whether inequality is bene cial or harmful for the growth of the R&D-driven economy crucially depends upon the market structure in which innovative rms compete.

Existing empirical studies on inequality and growth mainly focus on channels other than the product market. Alesina and Rodrik (1994), Persson and Tabellini (1994) and Perotti (1992, 1994) estimate reduced form equations for the models with endogenous s-cal policy. Perotti (1994) examines the impact of imperfect capital markets, and Alesina and Perotti (1996) and Alesina, *et al.* (1996) analyse the effect of socio-political instability. Bénabou (1996) also explores similar lines of research. These studies suggest that inequality is harmful for growth.[11] However, our model cannot be literally subject to this evidence, since they typically use investment or growth of physical output as dependent variables in their estimating equations, whereas in our model real output remains constant and the growth rate is in terms of consumers utility.

In the absence of evidence directly related to the theme of the present paper, most relevant empirical studies for our purpose are those on the traditional Schumpeterian hypothesis that larger rm size and more concentrated market structure are bene cial for innovative activities. This literature examines the underlying theoretical structure of the R&D-based growth model, including ours. As for studies up to the 1980 s, Cohen and Levin (1989) conclude that evidence is inconclusive on the whole. But more recent studies, such as Geroski (1990) and Blundel, *et al.* (1995) present evidence against the hypothesis. We argue that the present model is consistent with the more recent empirical evidence for three reasons. First, in the R&D-based models including ours, the incentive for R&D is *anticipated* pro ts. Geroski (1990) shows that this expected pro t and successful research output are positively correlated, although he admits some difficulties in estimating it with precision.

[11] Empirically the endogenous policy models predict an inverse relationship between the share of the third quantile of income distribution and the share of public investment, transfers and education depending upon models. But, Perotti (1994) shows that such prediction is not supported by the data. Hence, the predicted negative linkage between inequality and growth are supported in the reduced form estimations, whereas the evidence does not favour the underlying political process which characterises them.

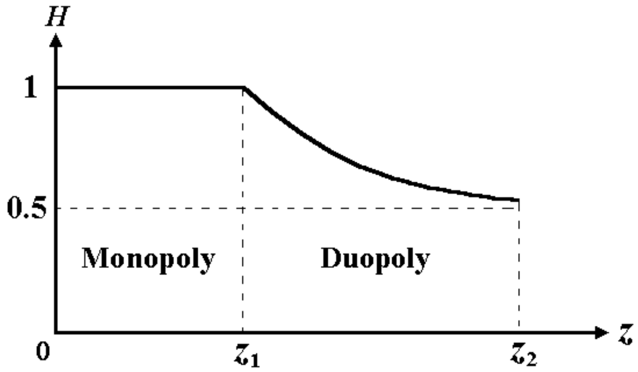

Figure 6: The Her ndahl index.

Second, those empirical studies typically use the market share of a few top-ranking rms to measure the concentration of the market structure. In their evidence, successful research output falls as the market becomes more concentrated. It implies that the growth rate falls with an increase in the Her ndahl concentration index, which is de ned as $H = \sum_{k=1}^{m} (S_k)^2$ where m is the number of rms in an industry and S_k is the kth rm s market share. It is evidently one for a monopoly. For the duopoly case, the index can be rewritten as

$$H = \frac{1}{18} \left[\frac{\left(\overline{E} - wc\right) / \left(\underline{E} - wc\right) + 1}{\left(\overline{E} - wc\right) / \left(\underline{E} - wc\right) - 1} \right]^2 + \frac{1}{2}. \tag{26}$$

Since $(\overline{E} - wc)/(\underline{E} - wc)$ is monotonically increasing in z, the right-hand side is strictly decreasing in z, as depicted in Figure 6. Comparing it with Figure 5 con rms that the growth rate falls as the index rises when the market structure is characterised by a duopoly. Third, the non-monotonic relationship described in Figure 5 predicts that more vigorous innovative research activity can be observed in a less concentrated market. To show this, consider two economies at C_1 and C_2 in Figure 5. Evidently, a monopoly at C_1 is associated with a higher innovative intensity. Next compare the economies at C_1 and C_3. A higher research intensity is now observed with a duopoly at C_3.

It should also be noted that our result depends on a negative link between growth and competition. This is seen in Figure 6 where the growth rate falls with the degree of competition in the product market. But, empirical evidence for this view is not strong. For example, Nickell (1993) finds evidence to the contrary, although he admits that it is not "overwhelming". But, his study does not directly estimate the effect of competition on innovative activity, which is the engine of growth in our model. Moreover, his regression equations use the value of sales or value-added as dependent variables to estimate the effect of competition on growth of physical output. Arguably these variables do not take into account quality improvement of products, which increases consumers' welfare in our model.

4 Conclusion

There are many potential chains of causality through which the income distribution affects growth. Of these, this paper highlights a relatively unexplored mechanism: income distribution affects demand for quality products, which in turn determines the profit incentive for technological innovations and growth. In equilibrium, the market is served by a monopoly if the income distribution is relatively less dispersed, or two firms can share the market, if inequality is relatively large. In a monopoly, all consumers purchase the-state-of-the-art goods. On the other hand, the rich consume higher quality goods and the poor purchase lower quality goods in the duopoly case. These equilibrium configurations seem quite plausible even on the basis of casual observation.

The source of the presence of inequality is consumers' endowment of labour efficiency. To analyse the effect of inequality, we conducted the mean-preserving spread of such efficiency, holding everything else constant. When a monopoly prevails, a wider income distribution discourages growth because entry threat increases or competition intensifies. In contrast, widening inequality relaxes price competition in a duopoly and promotes growth.

Robustness of this result may be checked by changing strategies taken by competing firms. Obvious candidates are Stackelberg duopoly and collusion.[12] In a Stackelberg duopoly, it can be established that the U-shaped relationship still arises, at least when the size of innovation is relatively small.[13] When it comes to collusion, widening inequality proves to be harmful for growth. It is intuitively clear, as collusion can be seen as a multi-plant monopoly. An additional implication of such an extension is that the growth rate is also affected by the types of competition strategy adopted by firms. It can be shown that the economy grows faster when firms compete in a Stackelberg fashion than in a Bertrand duopoly, because profit rates and prices are strictly higher in a Stackelberg than Bertrand duopoly.[14] This suggests that strategic behaviour may constitute a potentially important factor which contributes to the divergence of growth rates among economies or industries.[15]

Finally, one limitation of the present paper is that analysis is restricted to monopoly and duopoly. As inequality becomes wider beyond z_2 in Figure 5, the relationship between inequality and growth may become less clear-cut. Besides, our analysis did not consider the general case of oligopoly. However, we believe that our present study is sufficient to shed intriguing light on the possible link between inequality and growth from the Schumpeterian perspective.

[12] In Cournot competition, the first-order conditions for the duopolists would become linearly dependent due to a fixed total demand for quality goods. The details of these cases can be obtained from the author upon request.

[13] When the size of innovation is sufficiently large, the growth rate falls as inequality widens.

[14] The result is not affected by which firm plays the role of a leader or follower.

[15] Aghion, Harris and Vickers (1996) examine the similar issue but from a different perspective.

Appendixes

In the following appendixes, we use the notations of $\overline{I} = \frac{\overline{E}-w}{w}$ and $\underline{I} = \frac{\underline{E}-w}{w}$.

A. Proof of Lemma 1

Applying successive substitution to (6) yields $x_{t+\tau}^h = x_t^h / R_\tau^1 (1+\rho)^\tau$, $\tau = 0, 1, 2, ...$ where $R_\tau^1 = [\prod_{s=1}^{\tau}(1+r_{t+s})]^{-1}$. This is rewritten as

$$E_{t+\tau}^h = \frac{E_t^h - P_{Nt}}{R_\tau^1 (1+\rho)^\tau} + P_{Nt+\tau,}, \quad \tau = 1, 2, ... \tag{27}$$

using (5). By successive substitution, the flow budget constraint gives

$$(1+r_t) k_t^h = \sum_{\tau=1}^{t} R_\tau^2 h w_{t-\tau} - \sum_{\tau=1}^{t} R_\tau^2 E_{t-\tau}^h + R_t^3 k_0 \tag{28}$$

where $R_\tau^2 = \prod_{s=1}^{\tau}(1+r_{t-s+1})$ and $R_t^3 = \prod_{s=0}^{t}(1+r_{t-s})$. Now consider the intertemporal budget constraint $E_t^h + \sum_{\tau=1}^{\infty} R_\tau^1 E_{t+\tau}^h = hw_t + \sum_{\tau=1}^{\infty} R_\tau^1 hw_{t+\tau} + (1+r_t) k_t^h$. Eliminating $E_{t+\tau}^h$ by using (27) and $(1+r_t)k_t^h$ by (28), we obtain

$$\begin{aligned} E_t^h \;\; = \;\; & \frac{\rho}{1+\rho}\Bigg[hw_t + \sum_{\tau=1}^{\infty} R_\tau^1 hw_{t+\tau} + \sum_{\tau=1}^{t} R_\tau^2 hw_{t-\tau} - \sum_{\tau=1}^{t} R_\tau^2 E_{t-\tau}^h \\ & + R_t^3 k_0 + \frac{P_{Nt}}{\rho} - \sum_{\tau=1}^{\infty} R_\tau^1 P_{Nt+\tau}\Bigg]. \end{aligned} \tag{29}$$

This constitutes the optimal expenditure in the tth period. Note that the terms on the second line is independent of h. Besides the first three terms inside the bracket on the first line are linear in h. Hence E_t^h is linear in h if and only if $\{E_{t-\tau}^h\}_{\tau=1}^t$ is also linear in h. To show that this is the case, let us set $t = 0$ in (29). The last two terms on the first line disappear and other terms are still linear in or independent of h. Thus, E_0^h is linear in h. When $t = 1$, the last term on the first line of (29) becomes $(1+r_1)E_0^h$ with other terms being linear in or independent of h. Hence E_1^h is linear in h, since so is E_0^h. Applying the same procedure, $\{E_t^h\}_{t=0}^{\infty}$ are linear in h. Moreover, linearity implies that expenditure is uniformly distributed for $t = 0, 1, 2, ...$ ■

B. Proof of Lemma 4

Maximising $\pi_N = \sigma_E(\overline{E} - E_N)(p_N - wc)$ and $\pi_{N-1} = \sigma_E(E_N - \underline{E})(p_{N-1} - wc)$ where E_N is de ned in (7) gives the rst-order conditions

$$\frac{\partial \pi_N}{\partial p_N} = 0: \quad 2\Gamma p_N - (\Gamma - 1) p_{N-1} = \overline{E} + \Gamma wc, \tag{30}$$

$$\frac{\partial \pi_{N-1}}{\partial p_{N-1}} = 0: \quad \Gamma p_N - 2(\Gamma - 1) p_{N-1} = \underline{E} - (\Gamma - 1) wc. \tag{31}$$

The second-order conditions are $\frac{\partial \pi_N}{\partial p_N} = -2\sigma_E \Gamma < 0$ and $\frac{\partial \pi_{N-1}}{\partial p_{N-1}} = -2\sigma_E(\Gamma - 1) < 0$. Solving (30) and (31) simultaneously yields (13). Using (13), the demand for each quality product is

$$\sigma_E\left(\overline{E} - E_N\right) = \frac{\sigma_E\left[2\left(\overline{E} - wc\right) - \left(\underline{E} - wc\right)\right]}{3}, \tag{32}$$

$$\sigma_E\left(E_N - \underline{E}\right) = \frac{\sigma_E\left[\left(\overline{E} - wc\right) - 2\left(\underline{E} - wc\right)\right]}{3}. \tag{33}$$

Bertrand duopoly pro ts (14) are obtained from (13), (32) and (33). ■

C. Proof of Proposition 1

From (V^M) and (L^M), we can derive

$$\left.\frac{d\omega}{d\xi}\right|_{V^M} = \frac{a\Gamma\left[\rho + \Xi + (\mu + \xi)\Xi'\right]}{L(1+\rho)} > 0, \qquad \left.\frac{d\omega}{d\xi}\right|_{L^M} = -\frac{a\Gamma}{\alpha L(\Gamma - 1)} < 0 \tag{34}$$

where $\Xi' = \mu/(\mu + \xi)^2 > 0$. They are the slopes of the $V^M V^M$ and $L^M L^M$ schedules in Figure 2. Now de ne $\omega|_s^{\xi=0}$, $s = V^M$, L^M, as the values of ω when $\xi = 0$ for (V^M) and (L^M). Thus, a unique equilibrium with a positive growth rate exists if and only if $\omega|_{L^M}^{\xi=0} > \omega|_{V^M}^{\xi=0}$ or $h^* > \frac{(\Gamma-1)\alpha a \mu}{L}\frac{\rho}{1+\rho} + \alpha z + c$. ■

D. Proof of Proposition 2

From (V^M) and (L^M), we obtain

$$\left.\frac{\partial \xi}{\partial z}\right|_{V^M} = -\frac{L(1+\rho)(\mu+\xi)(\rho+\Xi)}{a\Gamma\left[\rho + \Xi + (\mu+\xi)\Xi'\right]} < 0, \quad \left.\frac{\partial \xi}{\partial z}\right|_{L^M} = -\frac{\alpha L}{a\Gamma} < 0, \tag{35}$$

implying that the two curves shift leftward in Figure 2, reducing ξ. ■

E. Proof of Proposition 3

From (V^B) and (L^B), we obtain

$$\left.\frac{\partial\omega}{\partial\xi}\right|_{L^B} = -\frac{a9\Gamma\left(\Gamma-1\right)/\alpha\sigma_h}{\left(\begin{array}{c}\left[9\left(\Gamma-1\right)+2\right]\Gamma\left(\overline{I}-2\underline{I}\right)\\ +4\left(\Gamma-1\right)\underline{I}\left[\frac{(\gamma-1)^2+3}{4\gamma(\gamma-1)}+\left(2+\frac{3}{\gamma}\right)-\frac{\overline{I}}{\underline{I}}\right]\end{array}\right)} < 0, \tag{36}$$

$$\left.\frac{\partial\omega}{\partial\xi}\right|_{V^B} = \frac{a\left(\rho+\Xi\right)^2+\left(1+\rho\right)\Xi'\left[\Pi_N^B-\Pi_{N-1}^B+\frac{2\Xi}{(\rho+\Xi)^2}\Pi_{N-1}^B\right]}{\left(1+\rho\right)\left(\rho+\Xi\right)\left(\frac{\partial\Pi_N^B}{\partial\omega}+\frac{\partial\Pi_{N-1}^B}{\partial\omega}-\frac{\rho}{\rho+\Xi}\frac{\partial\Pi_{N-1}^B}{\partial\omega}\right)} > 0 \tag{37}$$

where $\Xi' = \mu/\left(\mu+\xi\right)^2 > 0$,

$$\frac{\partial\Pi_N^B}{\partial\omega} = \frac{2\sigma_h\left(2\overline{I}-\underline{I}\right)}{9\Gamma} > 0, \quad \frac{\partial\Pi_{N-1}^B}{\partial\omega} = -\frac{2\sigma_h\left(\overline{I}-2\underline{I}\right)}{9\left(\Gamma-1\right)} < 0, \tag{38}$$

$$\frac{\partial\Pi_N^B}{\partial\omega}+\frac{\partial\Pi_{N-1}^B}{\partial\omega} = \frac{2\sigma_h}{9\Gamma\left(\Gamma-1\right)}\left[2\left(\Gamma-1\right)\underline{I}\left(\frac{\overline{I}}{\underline{I}}-2\right)+\Gamma\underline{I}\left(2+\frac{3}{\gamma}-\frac{\overline{I}}{\underline{I}}\right)\right] > 0, \tag{39}$$

$$\begin{aligned}\Pi_N^B-\Pi_{N-1}^B = \sigma_h\left[\frac{2\overline{I}-\underline{I}}{3\Gamma^{1/2}}+\frac{\overline{I}-2\underline{I}}{3\left(\Gamma-1\right)^{1/2}}\right]\Bigg\{&\left[\Gamma^{1/2}-\left(\Gamma-1\right)^{1/2}\right]\frac{2\overline{I}-\underline{I}}{3\Gamma}\\ &+\left(\Gamma-1\right)^{1/2}\Gamma\underline{I}\left[\frac{2}{\gamma}\left(\frac{\overline{I}}{\underline{I}}-2\right)+2+\frac{3}{\gamma}-\frac{\overline{I}}{\underline{I}}\right]\Bigg\} > 0.\end{aligned} \tag{40}$$

Equations (36) and (37) are the slopes of the L^BL^B and V^BV^B curves in Figure 3. Thus, a unique equilibrium with positive a growth rate exists if and only if $\omega|_{L^B}^{\xi=0} > \omega|_{V^B}^{\xi=0}$ where $\omega|_{V^B}^{\xi=0} = \left(\frac{18z\Gamma a\mu}{L}\frac{\rho}{1+\rho}\right)^{\frac{1}{2}} - 3z + c$ and $\omega|_{L^B}^{\xi=0}$ is implicitly determined in (V^B) with $\xi = 0$. ■

F. Proof of Proposition 4

From (V^B) and (L^B), we obtain

$$\left.\frac{\partial\xi}{\partial z}\right|_{V^B} = \frac{\left(1+\rho\right)\left(\rho+\Xi\right)\left(\frac{\partial\Pi_N^B}{\partial z}+\frac{\Xi}{\rho+\Xi}\frac{\partial\Pi_{N-1}^B}{\partial z}\right)}{a\left(\rho+\Xi\right)^2+\left(1+\rho\right)\Xi'\left[\Pi_N^B-\Pi_{N-1}^B+\frac{2\Xi}{(\rho+\Xi)^2}\Pi_{N-1}^B\right]} > 0 \tag{41}$$

$$\left.\frac{\partial\xi}{\partial z}\right|_{L^B} = \frac{\alpha}{a}\left(\frac{\partial\Pi_N^B}{\partial z}+\frac{\partial\Pi_{N-1}^B}{\partial z}\right) > 0 \tag{42}$$

where

$$\frac{\partial\Pi_N^B}{\partial z} = \frac{\sigma_h\left(2\overline{I}-\underline{I}\right)\left(\overline{I}-2\underline{I}\right)}{9\Gamma z} > 0, \quad \frac{\partial\Pi_{N-1}^B}{\partial z} = \frac{\sigma_h\left(2\overline{I}-\underline{I}\right)\left(\overline{I}-2\underline{I}\right)}{9\left(\Gamma-1\right)z} > 0. \tag{43}$$

They show that the two schedules shift rightward in Figure 3, increasing ξ. ■

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