Measuring the Economic Significance of Structural Exchange Rate Models

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Abstract:

This paper examines both the in-sample and out-of-sample performance of three monetary fundamental models of exchange rates and compares their out-of-sample performance to that of a simple Random Walk model. Using a data-set consisting of five currencies at monthly frequency over the period January 1980 to December 2009 and a battery of newly developed performance measures, the paper shows that monetary models do better (in-sample and out-of- sample forecasting) than a simple Random Walk model.

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1. Introduction

This paper examines both the in-sample and out-of-sample performance of three monetary fundamental models (also called structural models) of exchange rates and compares their performance, by which we mean predictive ability (especially from an economic as opposed to a purely statistical viewpoint) to that of a simple Random Walk model.

Meese and Rogoff (1983), using a variety of structural exchange rate models, show that these models cannot beat a simple Random Walk model when used to forecast exchange rates. The Meese and Rogoff (1983) paper has motivated many researchers to study exchange rate forecasting but without a significant consensus in favour of structural models (see for example, Diebold and Nason, 1990, Engel and Hamilton, 1990, West et al., 1993).

However, the academic literature cited earlier (apart from West et al., 1993) has focused on statistical measures of the accuracy of exchange rate forecasting (for example, root-mean-square errors). Although an empirical model might be statistically relevant, it may not be appropriate when viewed from the standpoint of whether investors or corporate treasures can use it, in practice, as a decision-support tool. Therefore, a second line of research, beginning with West et al. (1993), has focused on finding empirical evidence in support of structural models when used for asset allocation and portfolio management (see Abhyankar et al., 2005 and Della Corte et al., 2009).

This second line of research, based on assessing the economic significance of structural exchange rate models, has produced empirical evidence that structural models can do better (both in-sample and out-of-sample) than a simple Random Walk model. However, it is possible that this empirical evidence might be sensitive to the performance measures used. The most common performance measure has been the Sharpe ratio (Della Corte et al, 2009 is a notable exception). We note that: Sharpe ratios have limited validity, as a performance measure, if: i) portfolio returns are not normally distributed (see Hodges, 1998 and Goetzmann et al., 2007); ii) portfolios are dynamically adjusted (see Marquering and Verbeek, 2004, Han, 2006 and Della Corte et al., 2009).

Cherny and Madan (2009) recently introduced a set of new measures which can be used for assessing the performance of a portfolio and which are entirely valid when returns are not normally distributed. These new measures are computed after shocking portfolios returns using some appropriate distortion function. We utilise these measures to evaluate our portfolios after using different econometric methodologies (specifically Bayesian Linear Regression and Bayesian GARCH) to compute the mean and variance of exchange rate returns. This represents an important departure from the literature cited earlier and a significant contribution.

We also evaluate our forecasts after employing a trading strategy which dynamically rebalances our portfolios. This is consistent with market practice (and also with Abhyankar et al., 2005, who discuss the possibility that empirical results in the extant literature may be impaired by only considering static portfolio strategies when computing asset allocations).

Finally, we use an extended data-set (both in terms of currencies considered and time span of the data) compared to the one used in Della Corte et al. (2009), and different performance measures. We aim to shed some light on whether the main results in Della Corte et al. (2009) are driven by the sample selection, time-span of data and/or performance measures used. Previewing our conclusions, we find that monetary fundamental (structural) models of exchange rates have good forecasting power compared with a simple Random Walk model, when the economic significance of the forecasts are the basis for comparison, which confirms the main conclusion of Della Corte et al. (2009).

2. Economic Fundamentals and Exchange Rate: Theoretical Background

Monetary fundamental (or structural) models of exchange rates are frequently used in the literature on exchange rates forecasting[2]. These models suggest that an increase in domestic money supply will lead to an increase in the level of an exchange rate (measured as the number of units of domestic currency per unit of foreign currency), resulting in a depreciation of the domestic currency. Consider the following model:

$$x_t = z_t - s_t$$

$$z_t = (m_t - m_t^*) - (y_t - y_t^*) \qquad (1)$$

where s_t is the log of the nominal exchange rate (always expressed as the domestic price of the foreign currency), m is the log of money supply in the domestic country and y is the log of national income in the domestic country. Asterisks denote the same quantities in the foreign country. In equation (1), z measures the "disequilibrium" of the economic fundamentals between the domestic and the foreign country and therefore it can be interpreted as the relative velocity between the two countries, while x is the gap between nominal exchange rates and the economic fundamentals. The larger the gap x, the further is the exchange rate away from the level suggested by economic fundamentals and the further it will have to move in the future in order to converge towards its long-run equilibrium level. In this case, z describes this convergence. The monetary fundamental model described by equation (1) is widely accepted in the empirical finance literature.[3]

3. Empirical Evidence on the Economic Significance of Exchange Rate Forecasts

Following Mussa (1979), Cornell (1977)[4] and Frenkel (1981), according to whom exchange rates are unpredictable, Meese and Rogoff (1983) investigated the forecasting power of structural exchange rate models as opposed to a simple Random Walk model, using observations from March 1973 to June 1981 for dollar/yen, dollar/pound, dollar/mark and a

[2] See, for example: West et al. (1993), Abhyankar et al. (2005), Della Corte et al. (2009), Mark (1995), Mark and Sul (2001).

[3] See, for example: West et al. (1993), Mark (1995), Abhyankar et al. (2005), Della Corte et al. (2009).

[4] Mussa (1979) stated that "The natural logarithm of the spot exchange rate follows approximately a Random Walk" and concluded that the correlation found between the exchange rate and the economic fundamental in-sample tests is likely to be unstable in the long run.

traded-weighted dollar exchange rate. Their forecasts were mainly assessed in terms of root-mean- square error (RMSE), after using univariate and multivariate[5] time series models. They found that forecasts from a simple Random Walk model have lower RMSE than a variety of univariate and multivariate models and concluded that:

> "We find that a Random Walk model performs as well as any estimated model at one to twelve month horizons" Meese and Rogoff (1983)

Some potential reasons for the failure of the structural models could be that these models did not account, for example, for nonlinearities, sampling error or simultaneous equation bias. This lead researchers to consider these issues but with very little success (i.e. structural models were not able to perform better than a simple Random Walk model). Diebold (1988), for example, studied seven nominal dollar spot rates and found little evidence of linearities, whereas they found strong evidence that all exchange rate returns demonstrated strong auto regressive conditional heteroskedasticity. Diebold and Nason (1990) used nonparametric techniques to forecast[6] the spot exchange rates for ten major currencies against the USD dollar for the period after the 1973 float. However, these techniques were not able to do better (in terms of forecasting power) than a simple Random Walk model. Engel and Hamilton (1990) studied the Deutsche mark, French franc, and UK pound for the period from 1984 to 1988 using quarterly data. But again, they found that their model is out performed by a simple Random Walk model in the case of 4-quarter forecasts for Deutsche mark and French franc.

Mark (1995) studied Deutsche mark, Canadian dollar, yen and Swiss franc exchange rates for the period 1973 to 1991 to investigate the long-run predictability of these currencies given a set of economic fundamentals. He reported evidence that economic fundamentals do have a role when forecasting exchange rates in the long-run. Mark and Sul (2001) studied the long-run relationship between the nominal exchange rates and the monetary fundamentals of 19 currencies covering the period from January 1973 to January 1997. They performed tests of co-integration between exchange rates and monetary fundamentals and found significant empirical evidence supporting the hypothesis of co-integration.

More recently, Clarida et al. (2003) set up a three-regime Markov-switching vector equilibrium correction model for the spot exchange rate and the term structure of forward

[5] Unconstrained Vector Auto Regression
[6] In-sample and out-of-sample nonparametric forecasts.

interest rates, using weekly data for four major dollar exchange rates, and found that nonlinearities in exchange rate dynamics and the term structure of forward premia play a significant role in predicting future exchange rates. Clarida et al. (2006) used weekly observations for Euro-deposit rates for Germany, Japan and the US for the period February 1982 to December 2000 in a Markov switching model framework, focusing on the out-of-sample forecast of the term structure of interest rates, and found robust evidence of asymmetries and nonlinearities in the term structure of interest rates, which are accommodated by a multivariate asymmetric two-regime Markov-Switching model. They found that the term structure of interest rates contains significant information in out-of-sample forecasting.

The vast majority of the academic literature cited above focuses on statistical measures of the accuracy of exchange rate forecasting, whereas only a small proportion of the academic literature studies the evaluation of the economic significance of the exchange rate predictability. Indeed, even when an empirical model is statistically appropriate for exchange rate forecasts, this does not mean that investors can employ it for asset allocation or portfolio management. West et al. (1993) presented a study where the focus was on evaluating the economic performance of the forecasts as opposed to the statistical significance. They evaluate (weekly) out-of-sample exchange rate volatility for the Canadian dollar, France franc, Deutsche mark, Japanese yen, and British pound for the period from 1973 to 1989, and Euro-deposits from 1981 to 1989, using mean-variance criteria based on the expected mean and volatility from a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and report some evidence favoring structural models.

Abhyankar et al. (2005) investigated the forecasting ability of structural models over a long time horizon using Bayesian econometric models. Based on 10 year forecast horizons and using data covering a significant proportion (January 1977 to December 2000) of the period of floating exchange rates for the Canadian dollar, Japanese yen and British pound vis-à-vis the US dollar, they found that predictability varies substantially depending upon the assumed level of the risk of the representative agent in the market. Their main objective was the out-of-sample predictability measured on the basis of the economic value of the optimal allocation of a portfolio constructed from exchange rate forecasts. They concluded that the allocations based on structural models performed better than the allocations based on a Random Walk model. Della Corte et al. (2009) used a total of 15 different exchange rates models under the assumption of constant, time-dependent and stochastic volatility. After using Bayesian linear regression, Bayesian GARCH and Bayesian stochastic volatility models, they report robust evidence of the predictability of structural models when compared with a Random Walk model.

4. Methodological Issues

In this paper we follow the main literature cited earlier. We use four competing models (a simple Random Walk model and three monetary fundamental models) to assess the forecasting ability, conditional on a set of economic fundamentals, of exchange rates. We begin with the structural model in equation (1). We write the model as:

$$\Delta s_t = \beta_1 + \beta_2 x_{t-1} + u_t \qquad u_t = \sigma_t \varepsilon_t \quad \varepsilon_t \sim NID(0,1) \qquad (2)$$

where β_1 and β_2 are the parameters to be estimated. In a simple Random Walk model, we set $\beta_2 = 0$. The purpose of considering the Random Walk (RW) model is, of course, to allow us to have a benchmark with no predictive ability in exchange rate returns. Following Della Corte et al. (2009), we also consider three monetary fundamental models: Monetary Fundamental I (abbreviated to MF I) uses the model in (1). The other two models (termed Monetary Fundamental II (MF II) and Monetary Fundamental III (MF III)) are obtained from the OLS regression $s_t = c_0 + c_1 z_t + \pi_t$ and $s_t = c_0 + c_1 t + c_2 z_t + \pi_t$ where π_t is an error term. We set $x_t = -\pi_t^*$ in equation (1) where π_t^* denotes the estimated residuals and t is a time trend. Therefore, MF II adjusts the deviation of the nominal exchange rate from monetary fundamentals z_t by including an intercept, while MF III includes an intercept and a time-trend[7]. Following West et al. (1993) and Della Corte et al. (2009), we model the conditional variance using a simple GARCH(1,1) model (Bollerslev, 1986, Engle, 1982):

$$\sigma_{t|t-1}^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1|t-2}^2 \qquad (3)$$

[7] As discussed in Della Corte et al. (2009), the motivation for using MF II and MF III comes from the empirical evidence showing that cointegration between nominal exchange rates and fundamentals can only be found after correcting the model for deterministic components.

5. Economic Significance of Empirical Exchange Rate Models

We evaluate the economic performance of the four models (MF I, MF II, MF III and a simple Random Walk (RW)) presented in the previous section using several criteria. The next two sub-sections describe these criteria.

5.1 Investment Decision: Mean-Variance

We use a mean-variance approach to determine the optimal allocation of funds between a (foreign exchange rate) risky asset and a (domestic) risk-free asset. The strategy used is dynamic and revised monthly. We consider a representative investor and we assume that the investor's utility function is an exponential utility function with coefficient of absolute risk aversion denoted by γ. Therefore, the utility of the end of month wealth is given by

$$U(W) = -\exp(-\gamma W), \qquad \gamma > 0 \tag{4}$$

where W denotes the possible outcome of wealth at the end of the month.

The expected portfolio return is given by:

$$\mu = w'E(r) \tag{5}$$

where w is a vector of portfolio weights and r is the vector of returns from the two classes of assets while the portfolio variance is:

$$\sigma^2 = w'Vw \tag{6}$$

where V is the covariance matrix of asset returns.

Assuming that returns follow a normal distribution with mean μ and standard deviation σ, one can show (see for example, Hodges, 1998) that the certainty equivalent CE of the investment is given by:

$$CE = \mu - \frac{1}{2}\gamma\sigma^2 \tag{7}$$

The optimal (in the sense of maximising the certainty equivalent CE) allocation for an investor with an exponential utility function can be obtained from the optimization:

$$\max_{w}\left\{w'E(r)-\frac{1}{2}\gamma w'Vw\right\} \tag{8}$$

The Sharpe ratio SR, used as a measure to rank portfolio performance, is defined as:

$$SR=\frac{E(r_x)-Rf}{\sigma_x}$$

where $E(r_x)$ (respectively, σ_x) is the expected return (respectively, standard deviation) of portfolio x and Rf is the risk-free return.

5.2 Performance: index of acceptability

In general (leaving aside for now the issue of static versus dynamic portfolio strategies), if portfolios have normally distributed returns, then Sharpe ratios are a valid logical measure for ranking their relative performance. However, when returns follow general (i.e. non-normal) distributions, Sharpe ratios lead to unsatisfactory "paradoxes" which render them unsuitable for ranking relative portfolio performances or more generally for ranking investment opportunities. Specifically, for general distributions of returns, Sharpe ratios are not consistent with no arbitrage and are not consistent with second order stochastic dominance. For more background and some specific illustrative examples, see Cherny and Madan (2009), Bernardo and Ledoit (2000), Hodges (1998), Goetzmann et al. (2007) and Cerny (2003).

This lead Cherny and Madan (2009) to introduce what they termed *indices of acceptability*. Essentially, these are a class of performance measures which satisfy all of a series of properties including consistency with no arbitrage and second order stochastic dominance. These indices of acceptability provide a consistent and logical way of comparing the performance of different portfolios even when the returns on the portfolios are not even closely approximated by a normal distribution. They are also consistent with no arbitrage and with second order stochastic dominance. In short they overcome the limitations of Sharpe ratios. The approach has already been used in asset pricing theory (to price and optimally hedge complex contingent claims, see Madan 2010) and in corporate finance (to price corporate securities, see Madan and Schoutens, 2011). In this paper, we extend this novel approach to measure the economic performance of our dynamically rebalanced portfolio. The extension of the indices of acceptability approach of Cherny and Madan (2009) to portfolio analysis, and more generally to monetary economics, is (to our best knowledge) completely new.

We cannot hope to describe the Cherny and Madan (2009) approach in full here. We content ourselves with just describing the brief outline. The objective of an index of acceptability is to give a performance measure or relative ranking which describes whether and by how much a return on a portfolio is *acceptable* to a liquid financial market. Given a portfolio return *X*, modelled as a random cash flow with end of period distribution function $F_X(X)$, we say that it is *acceptable at a given level* μ if the following condition is satisfied:

$$E(\mu, X) \geq 0 \text{ where } E(\mu, X) = \int_{-\infty}^{\infty} x d(\Psi_{\mu}(F_X(x))) \qquad (9)$$

where $\Psi_{\mu}(F_X)$ is termed a distortion function (and is parameterised by some constant μ). Note that in the special case that $\Psi_{\mu}(F_X) = F_X(X)$, then $E(\mu, X)$ in equation (9) is simply the expected value of X i.e. the expected portfolio return. By contrast, if the distortion function $\Psi_{\mu}(F_X)$ is concave, the effect is to reweight losses upwards when $F_X(X)$ is near zero and discounts gains when $F_X(X)$ is near unity (which, intuitively speaking, is consistent with the behaviour of risk-averse agents). Cherny and Madan (2009) consider four different concave distortion functions. This leads to four indices of acceptability labelled MINVAR, MAXVAR, MAXMINVAR and MINMAXVAR. We now consider each of these indices of acceptability, in turn.

The first index is called MINVAR and is defined by choosing:

$$\Psi_{\mu}(y) = 1 - (1-y)^{\mu_1 + 1}, \qquad \mu_1 \in R_+, \quad y \in [0,1] \qquad (10)$$

The intuition behind MINVAR is two-fold (see section 3.8 of Cherny and Madan, 2009, for full details). Firstly, the condition $E(\mu_1, X) \geq 0$ turns out to be the same as saying that the expectation computed using the minimum of ($\mu_1 + 1$) draws from the distribution of the portfolio return X is still positive. The intuition here is that even using the worst case of ($\mu_1 + 1$) draws is still an acceptable investment opportunity or portfolio return. Secondly, Cherny and Madan (2009) also show that large gains are discounted to zero while large losses are exaggerated by a factor ($\mu_1 + 1$). This points to a possible disadvantage of MINVAR:

One would possibly like large losses to be exaggerated to infinity – not by a factor which is a fixed constant.

The second index is called MAXVAR and, by contrast, it does exaggerate large losses to infinity. MAXVAR is defined by choosing:

$$\Psi_{\mu}(y) = y^{\frac{1}{\mu_2+1}}, \qquad \mu_2 \in R_+ , \qquad y \in [0,1] \qquad (11)$$

For MAXVAR, large losses are exaggerated to infinity but large gains are discounted by a maximum proportional factor of ($\mu_2 + 1$). This points to a possible disadvantage of MAXVAR: One would possibly like large gains to be discounted to zero.

This leads to the consideration of the third and fourth indices termed MAXMINVAR and MINMAXVAR. Both these indices discount large gains to zero and simultaneously exaggerate large losses to infinity.

Specifically, MAXMINVAR is defined by choosing:

$$\Psi_{\mu}(y) = (1-(1-y^{\mu_3+1})^{\frac{1}{\mu_3+1}} \quad , \quad \mu_3 \in R_+ \ , \quad y \in [0,1] \qquad (12)$$

while MINMAXVAR is defined by choosing:

$$\Psi_{\mu}(y) = 1-(1-y^{\mu_4+1})^{\frac{1}{\mu_4+1}}, \qquad \mu_4 \in R_+ \ , \qquad y \in [0,1]$$

All four indices of acceptability produce valid logical measures for ranking portfolio performance: The larger the index of acceptability, the better the portfolio performance. Portfolio returns which are not acceptable at a given level μ (where $\mu \in \{\mu_1, \mu_2, \mu_3, \mu_4\}$) are assigned an index of acceptability identically equal to zero.

Finally, we mention one additional advantage of indices of acceptability. While, intuitively speaking, they are consistent with the notion of risk-aversion and, more specifically, with classical ideas of utility functions, they do not actually require the specification of a particular utility function. This is useful because corporate treasurers and portfolio managers are not typically acting on their own personal account and hence a personalised utility function may not be appropriate. Instead, indices of acceptability (see Cherny and Madan, 2009 for more details) attempt to de-personalise portfolio selection and to measure the acceptability of portfolio returns to a wide-range of agents who collectively constitute the "market" (or a large sub-section of it).

6. Estimation and Forecasting: Bayesian Method

We use Bayesian methods to estimate the parameters of the models discussed in Section 3. In the empirical finance literature, Bayesian methods have been used by, for example, Kandel and Stambaugh (1996) to forecasts stock returns. They were also used by West et al. (1993), Abhyankar et al. (2005) and Della Corte et al. (2009) to assess the economic value of exchange rate forecasts. A detailed discussion of the algorithms used in this paper is provided in the Appendix.

To implement the Bayesian linear regression methodology within a mean-variance framework and compute the optimal weights, we implement a buy and hold strategy, where the investor buys and holds for a period of one month, and rebalances the portfolio in each subsequent month. We specify the following (log) likelihood function:

$$\log l = \sum_{t=1}^{T} \log f(\Delta s_t \mid \sigma_t, \theta)$$

The parameters of interest are contained in the set $\theta = \{\theta_1, \theta_2\}$,where $\theta_1 = (\beta_1, \beta_2)$ $\theta_2 = \{h\}$ where h is the error precision i.e. the inverse of the variance: $h = 1/\sigma^2$. Normally distributed priors are assumed for $\theta_1 = (\beta_1, \beta_2)$. Prior gamma $\left(\frac{v}{2}, \frac{2s^{-2}}{\underline{v}}\right)$ is assumed for

$\theta_2 = \{h\}$ with mean $s^{-2} = 1$ and degree of freedom $\underline{v} = 2$. The Gibbs sampler is applied to obtain the posterior distributions. We also compute the empirical standard errors.

7. Data Description

The empirical data-set which we use consists of industrial production[8], money supply and spot (end of month) exchange rates for UK, Germany, Japan, Australia and Canada, relative to the US dollar. We use monthly observations from January 1980 to December 2009 (i.e. 360 observations). The spot exchange rates are taken from the *Bloomberg terminal*. The Euro-rate is taken as proxy to the Deutsche mark after the introduction of Euro in January 1999. The descriptive statistics for the (log) spot exchange rates are presented in Table 1. The Jarque-Bera statistics indicate that the null hypothesis of normally distributed exchange rate returns is rejected with $100(1 - 0.044947) \approx 95.51\%$ confidence for AUDUSD and at confidence levels well in excess of 99.9% for GBPUSD, for DEM/EURUSD and for JPYUSD.

[8] We have used industrial production rather than GDP since the latter is not typically available on a monthly basis. Della Corte et al. (2009) note that the correlation between the quarterly industrial production index and GDP over the time period they consider is more than 0.95.

Table 1: Descriptive statistics of exchange rates

	AUD/USD	GBP/USD	CAD/USD	DEM/USD	JYP/USD
Mean	-0.282734	0.506769	-0.2517	-0.352312	-4.913858
Median	-0.283292	0.490021	-0.250772	-0.470084	-4.81066
Maximum	0.1667	0.890563	0.051168	0.457994	-4.434837
Minimum	-0.716825	0.076035	-0.471124	-1.203306	-5.624325
Std. Dev.	0.187375	0.131461	0.109553	0.431541	0.302632
Skewness	0.285364	0.319968	0.072433	0.209664	-0.927171
Kurtosis	3.296492	3.735125	2.527672	1.966252	2.621851
Jarque-Bera	6.204564	14.24889	3.661206	18.66707	53.72366
Probability values (Jarque-Bera)	0.044947	0.000805	0.160317	0.000088	0.00000
Kurtosis (portfolio)	4.37 (MF I)	3.11 (MF II)	4.32 (MF III)	3.20 (RW)	
Observations	360	360	360	360	360

Note: The descriptive statistics of (log) exchange rates AUD/USD, CAD/USD, DEM (EURO)/USD, GBP/USD and JPY/USD from January 1980 to December 2009. We have also computed the standardized kurtosis using exchange rates returns and the portfolio kurtosis from the four models presented in section 3.

8 Empirical Results

8.1 Statistical Measures

Throughout this paper, for *in-sample* forecasting all the observations are used to estimate the parameters, whereas, for *out-of*-sample the data is split into two halves, (180 observations each) the first half is used to estimate the parameters and these parameters are used to forecast the second half observations in a recursive fashion. The forecasts are compared with the benchmark Random Walk (RW) model. To save space we do not report all the estimation results but only the RMSE ratio between the structural models and the Random Walk model (see Table 2 below)[9]. The results presented in table 2 are in line with the existing literature (except for the UK when out-of-sample forecasts are considered): Based on statistical measures of performance, the Random Walk model appears to perform as well as if not better than structural models.

Table 2: RMSE ratio between the structural and Random Walk model

		RMSE Ratio				
		Australia	Canada	Germany	Japan	UK
1 Month ahead	In-Sample	0.951148	1.000787	1.000215	0.999416	1.011661152
	Out-of-Sample	1.122199	1.250154	1.222334	1.004709	0.813991947

Note: This table presents the ratios of RMSE between the structural and Random Walk model, a value greater than or equal to one represents a better performance for the Random Walk model.

8.2 Bayesian Linear Regression: Statistical Analysis

Statistical analysis of forecasts using Bayesian linear regression is presented in the tables below. Tables 3 and 4 report the log-likelihood of the models:

Table 3: Log likelihood of the Models (In-Sample)

	UK	Germany	Japan	Canada	Australia
RW	-912	-1064	-952	-751	-934
MF I	-921	-1068	-955	-757	-941
MF II	-914	-1068	-956	-756	-938
MF III	-914	-1060	-955	-755	-938

Note: This table represents the log likelihood for the Random Walk (RW) model and structural models.

[9] Results are available upon request. However we note that coefficients of determination from the Random Walk model are in large part greater than 90% while the ones from structural models are very low.

Table 4: Log likelihood of the Models (Out-of-Sample)

	UK	Germany	Japan	Canada	Australia
RW	-488	-477	-479	-306	-455
MF I	-495	-482	-484	-312	-460
MF II	-491	-481	-483	-310	-458
MF III	-491	-476	-483	-311	-458

Note: This table represents the log likelihood for the Random Walk (RW) model and structural models.

The statistics are generally of the same order of magnitude which implies that no model is superior to the others[10]. Tables 5 and 6 below report the estimates of the models.

Table 5: Bayesian linear regression results (In-Sample)

Monetary Fundamental I	β_1	β_2	$\bar{h}^{-2}$
UK	-.0953*** (0.1611) (0.0005093)	0.0003*** (0.0032) (0.01)	0.108 (0.0245)
Germany	0.2604*** (0.2424) (0.0007661)	0.0003*** (0.0044) (0.0137)	0.0477 (0.0163)
Japan	0.2767*** (0.1773) (0.0005634)	0.0084*** (0.0047) (0.0147)	0.0891 (0.0222)
Canada	0.0258*** (0.1022) (0.0003233)	-0.003*** (0.0028) (0.0089)	0.2684 (0.0386)
Australia	-0.0557*** (0.1704) (0.0005385)	-0.0002*** (0.0048) (0.0152)	0.0965 (0.0231)
Monetary Fundamental II	β_1	β_2	$\bar{h}^{-2}$
UK	-0.0946*** (0.1586) (0.0005006)	-0.0407*** (0.0121) (0.0383)	0.1114 (0.0249)
Germany	0.2588*** (0.2423) (0.0007657)	-0.0036*** (0.0059) (0.0187)	0.0477 (0.0163)
Japan	0.2735*** (0.1777) (0.0005631)	-0.0089*** (0.0065) (0.0204)	0.0887 (0.0222)
Canada	0.024*** (0.1022) (0.0003232)	-0.0102*** (0.0095) (0.03)	0.2684 (0.0386)
Australia	-0.0598*** (0.1692) (0.0005346)	-0.0235*** (0.0102) (0.0324)	0.0979 (0.0233)

[10] To check these results we have also calculated Bayes information criterion obtaining similar results. To save space, we do not report these results but these are available upon request.

Monetary Fundamental III	A	B	$\bar{h}^{-2}$
	-0.0946***	-0.0411***	
UK	(0.1585)	(0.0121)	0.1115
	(0.0005028)	(0.0381)	(0.0249)
	0.262***	-0.0582***	
Germany	(0.2376)	(0.0153)	0.0496
	(0.0007495)	(0.0481)	(0.0166)
	0.2753***	-0.019***	
Japan	(0.1773)	(0.0105)	0.0891
	(0.0005622)	(0.033)	(0.0222)
	0.024***	-0.0132***	
Canada	(0.1021)	(0.01)	0.2689
	(0.000322)	(0.0317)	(0.0386)
	-0.0596***	-0.0203***	
Australia	(0.1695)	(0.0103)	0.0975
	(0.0005376)	(0.0328)	(0.0233)
Random Walk Model	β_1	β_2	$\bar{h}^{-2}$
	1.389***	-2.9416***	
UK	(0.5843)	(1.114)	0.1101
	(0.0018)	(0.0035)	(0.0247)
	0.1822***	-0.2219***	
Germany	(0.3125)	(0.5592)	0.0477
	(0.001)	(0.0018)	(0.0163)
	-2.5847***	-0.584***	
Japan	(2.1831)	(0.4441)	0.0887
	(0.0069)	(0.0014)	(0.0222)
	-0.2235***	-0.9877***	
Canada	(0.2347)	(0.839)	0.2686
	(0.0007)	(0.0026)	(0.0386)
	-0.5634***	-1.7905***	
Australia	(0.3)	(0.8735)	0.0976
	(0.0009)	(0.0028)	(0.0233)

*Note: The table presents the Bayesian MCMC estimates of the posterior means of the in-sample linear regression, for the USD/GBP, DEM/USD, JPY/USD, AUD/USD and CAD/USD monthly percent FX returns. The MCMC chain runs for 100,000 iterations after an initial burn-in of 10,000 iterations. The numbers in parenthesis indicates standard deviation and the Numerical Standard Error (NSE) respectively. The superscripts *, ** and *** indicate that the 90%, 95% and 99% highest posterior density (HPD) regions, respectively, do not contain zero. The HPD region for each MCMC parameter estimate is the shortest interval that contains 95% of the posterior distribution.*

Table 6: Bayesian linear regression results (Out-of-Sample)

Monetary Fundamental I	β_1	β_2	$\bar{h}^{-2}$
UK	-0.3573*** (0.4361) (0.0014)	0.0037*** (0.0082) (0)	0.079 (0.0295)
Germany	-0.1055*** (0.3945) (0.0012)	0.0042*** (0.0065) (0)	0.0914 (0.0318)
Japan	0.4289*** (0.3107) (0.0009815)	0.0033*** (0.007) (0.022)	0.0895 (0.0314)
Canada	-0.1021*** (0.1861) (0.0005896)	-0.0004*** (0.0054) (0.017)	0.5922 (0.0809)
Australia	-0.5345*** (0.4123) (0.0013)	0.012*** (0.0128) (0)	0.1153 (0.0357)
Monetary Fundamental II	β_1	β_2	$\bar{h}^{-2}$
UK	-0.195*** (0.2621) (0.00083)	-0.043*** (0.0167) (0.053)	0.0818 (0.0301)
Germany	-0.2798*** (0.4514) (0.0014)	-0.0135*** (0.0138) (0)	0.0917 (0.0318)
Japan	0.4554*** (0.2811) (0.0008924)	-0.0043*** (0.0091) (0.0288)	0.0895 (0.0314)
Canada	-0.0905*** (0.1004) (0.0003176)	-0.0148*** (0.0155) (0.0489)	0.5952 (0.0811)
Australia	-0.1944*** (0.2191) (0.0006935)	-0.028*** (0.0146) (0.0462)	0.1171 (0.036)
Monetary Fundamental III	β_1	β_2	$\bar{h}^{-2}$
UK	-0.1932*** (0.2621) (0.0008259)	-0.0431*** (0.0167) (0.053)	0.0818 (0.0301)
Germany	0.0883*** (0.2409) (0.0007616)	-0.0474*** (0.0144) (0.0455)	0.0968 (0.0327)
Japan	0.5499*** (0.253) (0.0007979)	-0.012*** (0.013) (0.041)	0.0898 (0.0315)
Canada	-0.1096*** (0.1) (0.0003165)	-0.0029*** (0.0128) (0.0404)	0.5923 (0.0809)
Australia	-0.1945*** (0.2191) (0.0006908)	-0.029*** (0.015) (0.0473)	0.1172 (0.036)

Random Walk Model	β_1	β_2	$\bar{h}^{-2}$
	1.293***	-2.9946***	
UK	(0.7956)	(1.504)	0.0806
	(0.0025)	(0.0048)	(0.0298)
	-0.3934***	-0.7047***	
Germany	(0.7729)	(1.0679)	0.0915
	(0.0024)	(0.0034)	(0.0318)
	-0.5255***	-0.2049***	
Japan	(2.5862)	(0.5068)	0.0894
	(0.0082)	(0.0016)	(0.0314)
	-0.2699***	-0.6918***	
Canada	(0.2403)	(0.981)	0.5938
	(0.0008)	(0.0031)	(0.081)
	-0.5836***	-1.8499***	
Australia	(0.3223)	(1.1552)	0.1164
	(0.001)	(0.0036)	(0.0359)

*Note: The table presents the Bayesian MCMC estimates of the posterior means of the out-of-sample Linear Regression, for the USD/GBP, DEM/USD, JPY/USD, AUD/USD and CAD/USD monthly percent FX returns.. The MCMC chain runs for 100,000 iterations after an initial burn-in of 10,000 iterations. The numbers in parenthesis indicates standard deviation and the Numerical Standard Error (NSE) respectively. The superscripts *, ** and *** indicate that the 90%, 95% and 99% highest posterior density (HPD) regions, respectively, do not contain zero. The HPD region for each MCMC parameter estimate is the shortest interval that contains 95% of the posterior distribution.*

The estimated parameters are, generally, statistically significant and overall MF I seems to be more accurate than the Random Walk model. This is evident if we consider the error precision h. Tables 7, 8 and 9 show the root-mean-square errors:

Table 7: RMSE ratio between the structural and Random Walk model (Monetary Fundamental I)

		RMSE Ratio				
		UK	Germany	Japan	Canada	Australia
1 Month ahead	In-Sample	1.011323	1.000215	0.998742	1.000691	1.006315
	Out-of-Sample	1.02226	0.999384	0.995745	1.001627	1.040908
3 Month ahead	In-Sample	1.015824	1.000178	0.998426	1.000545	1.009242
	Out-of-Sample	1.0253	0.998904	0.994277	1.001872	1.041046
6 Month ahead	In-Sample	1.018348	1.000275	0.998643	1.00212	1.009671
	Out-of-Sample	1.02953	0.998984	0.994731	1.002934	1.042853

Note: This table presents the ratios of RMSE between the monetary fundamental I and Random Walk model.

Table 8: RMSE ratio between the structural and Random Walk model (Monetary Fundamental II)

		RMSE Ratio				
		UK	Germany	Japan	Canada	Australia
1 Month ahead	In-Sample	0.995635	0.999712	1.000631	1.000697	0.998993
	Out-of-Sample	0.995948	1.007837	0.998377	1.000656	1.001637
3 Month ahead	In-Sample	0.996658	0.999691	1.000685	1.000564	0.999185
	Out-of-Sample	0.992259	1.007778	0.997841	1.000212	1.00009
6 Month ahead	In-Sample	1.001659	0.999577	1.002361	1.001739	1.00175
	Out-of-Sample	0.99615	1.006804	0.998649	1.001184	1.003945

Note: This table presents the ratios of RMSE between the monetary fundamental II and Random Walk model.

Table 9: RMSE ratio between the structural and Random Walk model (Monetary Fundamental III)

		RMSE Ratio				
		UK	Germany	Japan	Canada	Australia
1 Month ahead	In-Sample	0.995337	0.980477	0.998688	0.999899	1.022264
	Out-of-Sample	0.995187	0.978805	0.999825	1.001288	1.006425
3 Month ahead	In-Sample	0.996354	0.98229	0.997893	1.000576	1.027413
	Out-of-Sample	0.991438	0.981653	0.997664	1.001443	1.004388
6 Month ahead	In-Sample	1.001489	0.978477	1.001501	1.001355	1.026715
	Out-of-Sample	0.995439	0.97875	1.000846	1.002436	1.007085

Note: This table presents the ratios of RMSE between the monetary fundamental III and Random Walk model.

The root-mean-square errors are substantially of the same order of magnitude across the different models. Overall, the results in tables 7, 8 and 9 reiterate the previous results and are in line with the extant empirical literature: Structural models appear to not be able to do better than a simple Random Walk model when statistical measures are used as the basis for comparison.

8.3. Economic Evaluation of Forecasts: Mean-Variance Analysis

Although the statistical evaluation of a model provides important pieces of information about the empirical validity of that model, it says little about whether the same model can be used to profitably exploit investment opportunities. The recent contributions of Abhyankar et al. (2005) and Della Corte et al. (2009) have started to address this issue and we will do likewise in the following sections.

We start with the mean-variance approach discussed earlier. The aim is to maximise the certainty equivalent of the utility of an investor conditional on our proposed models. We implement a very simple dynamic trading strategy where a domestic (US) investor, will invest in a portfolio consisting of two assets: a (foreign exchange rate) risky asset and a (domestic) risk-free asset, which we take to be a one month certificate of deposit denominated in US dollar. Thus, the only risk involved is currency risk.

We compare the out-of-sample predictability of the competing models (i.e. the three structural models versus the Random Walk model). The variance is analysed in two different ways. First, we consider the case where the variance is constant. Thereafter, it is assumed that the variance is time varying and one month ahead forecast of variance is estimated using Bayesian GARCH. We set the risk aversion coefficient γ (defined via equations (4) and (7)) equal[11] to 20. The out-of-sample forecasts are based on a recursive approach where at the end of each month a new set of weights are determined based on the portfolio expected return. Thus, our portfolios are dynamically rebalanced according to the new computed weights.

In Tables 10, 11, 12 (and also in Tables 17 and 18), we report the average optimal portfolio weights. We stress these are average weights – the actual weights are changing dynamically through time.

[11] We have also considered different values for the risk aversion coefficient but the results were qualitatively the same.

Table 10: Mean-variance analysis results (Bayesian Linear Regression)-In-sample

		Foreign Exchange	Risk Free	Portfolio Mean	Portfolio Sigma	Sharpe Ratio
Monetary Fundamental 'I'	UK	-1.000	2.000	0.217	0.338	0.462
	Germany	2.000	-1.000	0.460	0.436	0.914
	Japan	1.229	-0.229	0.569	0.486	0.951
	Canada	0.249	0.751	0.218	0.746	0.220
	Australia	-1.000	2.000	0.178	0.322	0.362
Monetary Fundamental 'II'	UK	0.299	0.701	0.617	0.503	1.173
	Germany	1.656	-0.656	0.477	0.398	0.997
	Japan	1.732	-0.732	0.533	0.547	0.850
	Canada	0.349	0.651	0.198	0.756	0.198
	Australia	0.098	0.902	0.502	0.457	0.978
Monetary Fundamental 'III'	UK	0.257	0.743	0.624	0.507	1.170
	Germany	0.877	0.123	1.163	0.392	2.735
	Japan	1.095	-0.095	0.656	0.500	1.105
	Canada	0.508	0.492	0.223	0.786	0.229
	Australia	-0.020	1.020	0.422	0.398	0.949
Random Walk	UK	0.081	0.919	0.455	0.477	0.887
	Germany	2.000	-1.000	0.460	0.436	0.917
	Japan	1.958	-0.958	0.510	0.577	0.775
	Canada	0.324	0.676	0.200	0.753	0.199
	Australia	-0.045	1.045	0.422	0.459	0.801

Note: The table shows the proportion of the portfolio which is invested, on average, in foreign exchange and the proportion which is invested, on average, in the risk-free asset when using a GARCH model. Bayesian Regression is used. We report the portfolio return, risk and Sharpe ratios respectively.

Table 10 shows the in-sample results based on Bayesian linear regression while Table 11 shows the out-of-sample results. The results from the GARCH model are reported in Table 12. The columns denoted Portfolio Mean and Portfolio Sigma denote the return and risk (standard deviation) respectively.

Table 11: Mean-variance analysis results (Bayesian Linear Regression)-Out-of-Sample

		Foreign Exchange	Risk Free	Portfolio Mean	Portfolio Sigma	Sharpe Ratio
Monetary Fundamental 'I'	UK	1.187	-0.187	0.246	0.533	0.420
	Germany	-0.313	1.313	0.463	0.400	0.197
	Japan	-0.313	1.313	0.359	0.399	0.858
	Canada	1.187	-0.187	0.549	1.020	0.573
	Australia	-0.313	1.313	0.253	0.420	0.484
Monetary Fundamental 'II'	UK	-0.313	1.313	0.637	0.390	1.880
	Germany	0.194	0.806	0.428	0.451	0.942
	Japan	-0.053	1.053	0.573	0.425	1.521
	Canada	1.187	-0.187	0.606	1.019	0.611
	Australia	0.408	0.592	0.314	0.501	0.591
Monetary Fundamental 'III'	UK	0.215	0.785	0.496	0.443	1.125
	Germany	0.194	0.806	0.428	0.451	0.939
	Japan	0.450	0.550	0.599	0.475	1.178
	Canada	1.187	-0.187	0.795	1.021	0.732
	Australia	-0.313	1.313	0.260	0.420	0.504
Random Walk	UK	0.240	0.760	0.510	0.445	1.156
	Germany	-0.003	1.003	0.372	0.437	0.830
	Japan	-0.204	1.204	0.400	0.410	0.992
	Canada	1.187	-0.187	0.649	1.019	0.639
	Australia	0.307	0.693	0.440	0.462	0.881

Note: The table shows the proportion of the portfolio which is invested, on average, in foreign exchange and the proportion which is invested, on average, in the risk-free asset when using a GARCH model. Bayesian Regression is used. We report the portfolio return and Sharpe ratios respectively.

Consider Table 10 as an example. In the case of GBPUSD, the Monetary Fundamental II model suggests that, on average through time, about 29.9% of the principal should be invested in the (foreign exchange) risky asset and, on average through time, about 70.1% should be in the (domestic) risk-free asset. Overall it appears that structural models tend to allocate a larger proportion of wealth to the risky asset in comparison to the Random Walk model.

Table 12: Mean-variance analysis results (GARCH)

		Foreign Exchange	Risk Free	Portfolio Mean	Portfolio Sigma	Sharpe Ratio
Monetary Fundamental 'I'	UK	-0.490	1.490	0.137	0.125	0.614
	Germany	1.002	-0.002	0.261	0.452	0.442
	Japan	0.590	0.410	0.310	0.424	0.527
	Canada	0.141	0.859	0.139	0.258	0.297
	Australia	-0.479	1.479	0.117	0.109	0.515
Monetary Fundamental 'II'	UK	0.161	0.839	0.337	0.479	0.552
	Germany	0.818	0.182	0.266	0.327	0.665
	Japan	0.847	0.153	0.293	0.411	0.533
	Canada	0.185	0.815	0.129	0.246	0.284
	Australia	0.065	0.935	0.280	0.399	0.520
Monetary Fundamental 'III'	UK	0.140	0.860	0.341	0.477	0.559
	Germany	0.446	0.554	0.613	0.634	0.711
	Japan	0.545	0.455	0.358	0.434	0.627
	Canada	0.269	0.731	0.142	0.257	0.311
	Australia	-0.019	1.019	0.243	0.341	0.546
Random Walk	UK	0.052	0.948	0.257	0.390	0.500
	Germany	0.996	0.004	0.260	0.361	0.562
	Japan	0.967	0.033	0.283	0.411	0.520
	Canada	0.173	0.827	0.130	0.246	0.288
	Australia	0.004	0.996	0.238	0.355	0.467

Note: The table shows the proportion of the portfolio which is invested, on average, in foreign exchange and the proportion which is invested, on average, in the risk-free asset when using GARCH model.

We notice that optimal weights from structural models are sometimes of opposite sign compared to optimal weights from the Random Walk model. This result is in line with Abhyankar et al. (2005). The change in sign suggests that the Random Walk model may indicate shorting an asset when structural models indicate the opposite. Overall, there is evidence suggesting that the monetary fundamental models (particularly Monetary Fundamental II and III) perform better than a simple Random Walk model. This result also holds in the case of out-of-sample forecasts and seems to be stronger in the case when returns are modelled using a GARCH process (see Table 12). This may suggest that, in modelling portfolio returns, allowing for GARCH processes may be important. However, our results thus far are based on the use of Sharpe ratios. As already discussed, Sharpe ratios are subject to various criticisms (see, for example Cherny and Madan, 2009, and Bernardo and Ledoit, 2000) and hence results based on Sharpe ratios should be interpreted with caution, especially in the ranking of non-normal portfolio return distributions and dynamic portfolio strategies. This leads us to consider indices of acceptability for ranking portfolio performance.

8.4. Indices of Acceptability

We now turn to indices of acceptability. In contrast with Sharpe ratios, indices of acceptability are consistent with no arbitrage and with second order stochastic dominance. The results are reported in tables 13 and 14:

Table 13: index of acceptability (in-sample)

		MINVAR	MAXVAR	MINMAXVAR	MAXMINVAR
Monetary Fundamental 'I'	UK	0.000	0.000	0.000	0.000
	Germany	0.170	0.168	0.107	0.154
	Japan	0.195	0.189	0.112	0.165
	Canada	0.037	0.037	0.023	0.034
	Australia	0.000	0.000	0.000	0.000
Monetary Fundamental 'II'	UK	0.100	0.104	0.070	0.097
	Germany	0.168	0.171	0.114	0.097
	Japan	0.187	0.187	0.121	0.172
	Canada	0.037	0.038	0.026	0.038
	Australia	0.083	0.083	0.052	0.074
Monetary Fundamental 'III'	UK	0.111	0.105	0.057	0.083
	Germany	0.313	0.308	0.186	0.266
	Japan	0.213	0.206	0.119	0.176
	Canada	0.045	0.046	0.030	0.043
	Australia	0.066	0.069	0.046	0.065
Random Walk Model	UK	0.007	0.066	0.039	0.057
	Germany	0.172	0.169	0.103	0.151
	Japan	0.188	0.185	0.113	0.164
	Canada	0.039	0.038	0.023	0.034
	Australia	0.060	0.063	0.037	0.053

Note: The different columns of the table represent different measures for the index. The rows represent the models and the corresponding. μ_3 (MAXMINVAR) and μ_4 (MINMAXVAR) are set to 0.2, while we set μ_1 (MINVAR) and μ_2 (MAXVAR) equal to 2.

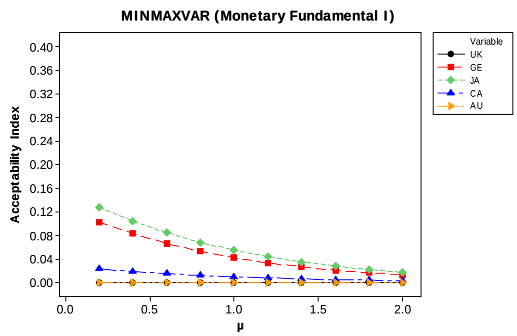
Table 14: Index of acceptability (out-of-sample)

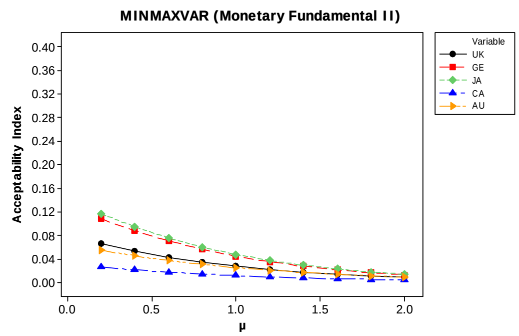
		MINVAR	MAXVAR	MINMAXVAR	MAXMINVAR
Monetary Fundamental 'I'	UK	0.000	0.000	0.000	0.000
	Germany	0.159	0.163	0.104	0.148
	Japan	0.032	0.034	0.023	0.033
	Canada	0.000	0.000	0.000	0.000
	Australia	0.000	0.000	0.000	0.000
Monetary Fundamental 'II'	UK	0.050	0.340	0.050	0.022
	Germany	0.095	0.099	0.066	0.093
	Japan	0.077	0.078	0.049	0.069
	Canada	0.012	0.011	0.000	0.033
	Australia	0.000	0.027	0.025	0.000
Monetary Fundamental 'III'	UK	0.070	0.080	0.090	0.020
	Germany	0.022	0.012	0.013	0.219
	Japan	0.049	0.052	0.037	0.051
	Canada	0.005	0.005	0.003	0.004
	Australia	0.044	0.032	0.022	0.000
Random Walk Model	UK	0.000	0.000	0.000	0.000
	Germany	0.113	0.121	0.084	0.118
	Japan	0.100	0.112	0.064	0.093
	Canada	0.000	0.000	0.000	0.000
	Australia	0.000	0.000	0.000	0.000

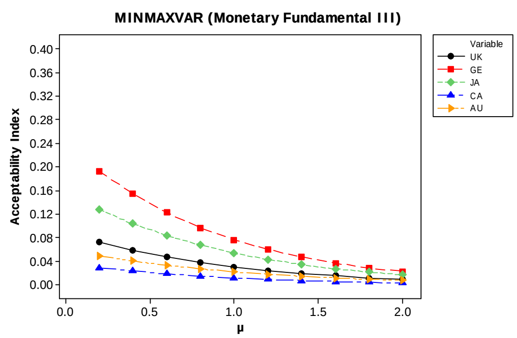
Note: The different columns of the table represent different measures for the index. The rows represent the models and the corresponding μ_3 *(MAXMINVAR) and* μ_4 *(MINMAXVAR) are set to 0.2, while we set* μ_1 *(MINVAR) and* μ_2 *(MAXVAR) equal to 2.*

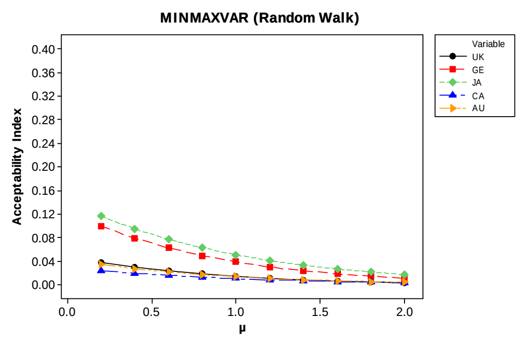
The results in Table 13 and Table 14 confirm the results shown in Tables 10, 11 and 12 and suggest that monetary models for the exchange rate (specifically MFII and MFIII) perform better than the Random Walk model. For example, in-sample results (Table 13) show that Monetary Fundamental III (MFIII) has a higher index of acceptability, for all four indices and for all five exchange rates, than that of the Random Walk model. The margin of out-performance is greatest for GBPUSD, DEM/EURUSD and JPYUSD which (from Table 1) are the three exchange rates for which the Jarque-Bera statistics indicate the greatest departure from normally distributed exchange rate returns. The JPYUSD and DEM/EURUSD show the highest indices of acceptability.

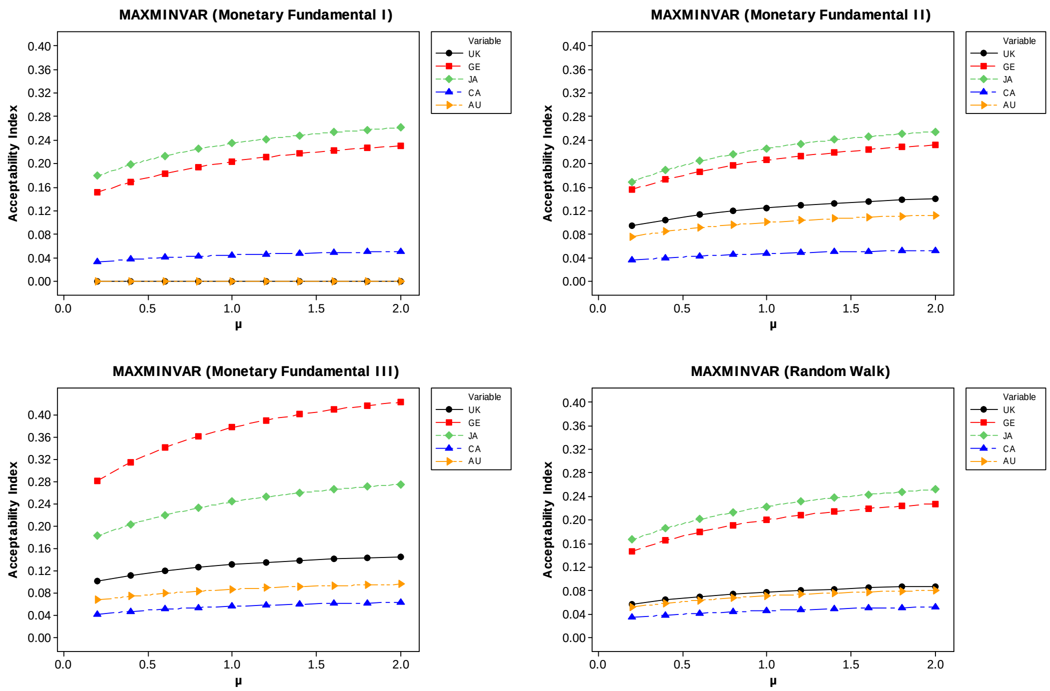
We have considered different values of levels μ ranging from 0.2 to 2 with an increment of 0.2. The results are showed in the following figures:











The MAXMINVAR *acceptability index* increases as the level of μ_3 increases whilst the MINMAXVAR *index* decreases as the level of μ_4 increases. The figures confirm the results in Tables 13 and 14: Forecasts obtained from the monetary fundamental models perform better than the forecasts obtained from the Random Walk model. In particular, we notice that MF II and MF III (i.e. the ones considering deterministic trends in the model) perform the best. These results are true for a wide range of values of μ_3 and μ_4.

9. Robustness Tests

In this section we present some robustness tests to support our empirical results. We re-run all the tests presented above after excluding the financial crisis period. Thus, our empirical results refer to the period January 1980 to December 2007. To save space we only present a few selected results[12].

[12] Full analysis is available upon request.

9.1 Statistical Analysis: Bayesian linear regression.

We start with the statistical analysis (in-sample and out-of-sample) of the models discussed above and use Bayesian linear regression. Results are reported in the tables below:

Table 15: Log likelihood of the Models (In-Sample)

	UK	Germany	Japan	Canada	Australia
RW	-841.987	-837.576	-837.622	-833.674	-841.987
MF I	-986.404	-985.714	-978.886	-981.395	-986.404
MF II	-877.058	-876.854	-876.883	-873.070	-877.058
MF III	-622.486	-622.749	-622.667	-618.048	-622.486

Note: This table shows the log likelihood of the models using Bayesian linear regression

Table 16: Log likelihood of the Models (Out-of-Sample)

	UK	Germany	Japan	Canada	Australia
RW	-459.251	-456.674	-456.677	-452.67	-459.251
MF I	-445.097	-445.073	-440.793	-440.44	-445.097
MF II	-447.265	-447.148	-446.584	-443.19	-447.265
MF III	-287.457	-285.767	-286.186	-281.9	-287.457

Note: This table represents the loglkelihood of the models using Bayesian linear regression.

There seems to be more evidence favoring the Monetary Fundamental III model with respect to a simple Random Walk model. It is possible that the results in Section 8 were adversely impacted by the financial crisis of 2008 and the correspondingly high FX volatilities. To save space, we do not report the full analysis but overall results are similar to the ones for the full sample. We now turn to the economic evaluation of the models.

9.2. Economic Analysis: Mean-Variance Approach.

Tables 17 and 18 show the (in-sample and out-of-sample) results from the mean-variance approach.

Table 17: Mean-variance analysis results (Bayesian Linear Regression) In-sample

		Foreign Exchange	Risk Free	Portfolio Mean	Portfolio Sigma	Sharpe Ratio
Monetary Fundamental 'I'	UK	0.182	0.818	0.417	0.491	0.771
	Germany	1.791	-0.791	0.462	0.426	0.946
	Japan	1.472	-0.472	0.404	0.536	0.623
	Canada	0.039	0.961	0.174	0.844	0.179
	Australia	-0.120	1.120	0.386	0.494	0.685
Monetary Fundamental 'II'	UK	-0.455	1.455	0.224	0.416	0.427
	Germany	1.791	-0.791	0.461	0.433	0.928
	Japan	0.634	0.366	0.518	0.433	0.932
	Canada	0.115	0.885	0.272	0.869	0.292
	Australia	-0.950	1.950	0.226	0.365	0.452
Monetary Fundamental 'III'	UK	0.332	0.668	0.502	0.515	0.895
	Germany	1.531	-0.531	0.544	0.402	1.166
	Japan	1.028	-0.028	0.540	0.478	0.908
	Canada	-0.413	1.413	0.139	0.745	0.143
	Australia	0.039	0.961	0.385	0.501	0.673
Random Walk	UK	0.307	0.693	0.500	0.513	0.893
	Germany	0.777	0.223	1.103	0.397	2.573
	Japan	0.852	0.148	0.510	0.487	0.860
	Canada	-0.128	1.128	0.149	0.807	0.152
	Australia	-0.070	1.070	0.297	0.480	0.534

Note: The table shows the proportion of the portfolio which is invested, on average, in foreign exchange and the proportion which is invested, on average, in the risk-free asset. Bayesian Regression is used. We report the portfolio return, risk and Sharpe ratios respectively.

Table 18: Mean-variance analysis results (Bayesian Linear Regression) out-of-sample

		Foreign Exchange	Risk Free	Portfolio Mean	Portfolio Sigma	Sharpe Ratio
Monetary Fundamental 'I'	UK	-0.413	1.413	0.501	0.396	1.473
	Germany	-0.413	1.413	0.536	0.403	1.524
	Japan	0.936	0.064	0.319	0.536	0.529
	Canada	-0.028	1.028	0.196	0.715	0.304
	Australia	-0.413	1.413	0.697	0.429	1.782
Monetary Fundamental 'II'	UK	-0.061	1.061	0.321	0.432	0.728
	Germany	0.047	0.953	0.243	0.449	0.492
	Japan	0.936	0.064	0.544	0.536	0.912
	Canada	0.441	0.559	0.278	0.835	0.364
	Australia	-0.036	1.036	0.390	0.474	0.771
Monetary Fundamental 'III'	UK	-0.053	1.053	0.322	0.435	0.724
	Germany	0.223	0.777	0.494	0.466	0.996
	Japan	0.911	0.089	0.614	0.535	1.026
	Canada	0.081	0.919	0.226	0.745	0.330
	Australia	-0.003	1.003	0.380	0.477	0.756
Random Walk	UK	-0.212	1.212	0.308	0.419	0.729
	Germany	-0.413	1.413	0.401	0.404	1.059
	Japan	0.936	0.064	0.515	0.536	0.862
	Canada	0.098	0.902	0.220	0.748	0.324
	Australia	0.081	0.919	0.296	0.483	0.535

Note: The table shows the proportion of the portfolio which is invested, on average, in foreign exchange and the proportion which is invested, on average, in the risk-free asset. Bayesian Regression is used. We report the portfolio return, risk, certain equivalent and Sharpe ratios respectively.

Overall these results (Tables 17 and 18) are in line with the results presented earlier and show that Monetary Fundamental models do better than a Random Walk model when the economic significance of the model forecasts is considered. We have also considered a GARCH model as before and results remain substantially unchanged[13].

[13] These results are available upon request.

9.3 Economic Analysis: Acceptability Index

In this section we further assess the empirical results obtained earlier by using the indices of acceptability. To make sure that our previous results are not affected by the choice of the parameters μ_1, μ_2, μ_3 and μ_4, we consider different values compared to those used in the previous section.

Table 19: index of acceptability in-sample

		MINVAR	MAXVAR	MINMAXVAR	MAXMINVAR
Monetary Fundamental 'I'	UK	0.012	0.013	0.006	0.0013
	Germany	0.187	0.119	0.081	0.184
	Japan	0.181	0.183	0.079	0.176
	Canada	0.041	0.042	0.018	0.041
	Australia	0.000	0.000	0.000	0.000
Monetary Fundamental 'II'	UK	0.108	0.108	0.041	0.0100
	Germany	0.218	0.217	0.086	0.206
	Japan	0.200	0.205	0.090	0.199
	Canada	0.021	0.020	0.008	0.019
	Australia	0.063	0.066	0.030	0.066
Monetary Fundamental 'III'	UK	0.106	0.107	0.004	0.009
	Germany	0.326	0.340	0.153	0.337
	Japan	0.175	0.177	0.074	0.172
	Canada	0.024	0.025	0.011	0.024
	Australia	0.038	0.0040	0.018	0.0040
Random Walk Model	UK	0.008	0.009	0.003	0.008
	Germany	0.190	0.188	0.107	0.179
	Japan	0.162	0.163	0.067	0.156
	Canada	0.028	0.030	0.013	0.028
	Australia	0.054	0.054	0.002	0.050

Note: The different columns of the table represent different measures for the index. The rows represent the models and the corresponding μ_3 *(MAXMINVAR) and* μ_4 *(MINMAXVAR) are set to 0.5 while we set* μ_1 *(MINVAR) and* μ_2 *(MAXVAR) equal to 3.*

Table 20: index of acceptability out-of-sample

		MINVAR	MAXVAR	MINMAXVAR	MAXMINVAR
Monetary Fundamental 'I'	UK	0.000	0.000	0.000	0.000
	Germany	0.000	0.000	0.000	0.000
	Japan	0.148	0.150	0.058	0.138
	Canada	0.015	0.016	0.006	0.014
	Australia	0.000	0.000	0.000	0.000
Monetary Fundamental 'II'	UK	0.041	0.042	0.015	0.037
	Germany	0.029	0.032	0.012	0.026
	Japan	0.314	0.332	0.139	0.326
	Canada	0.082	0.086	0.036	0.081
	Australia	0.073	0.077	0.031	0.071
Monetary Fundamental 'III'	UK	0.043	0.042	0.012	0.034
	Germany	0.189	0.200	0.084	0.183
	Japan	0.380	0.390	0.151	0.369
	Canada	0.035	0.036	0.013	0.033
	Australia	0.069	0.072	0.026	0.068
Random Walk Model	UK	0.015	0.016	0.006	0.015
	Germany	0.000	0.000	0.000	0.000
	Japan	0.299	0.310	0.120	0.290
	Canada	0.030	0.032	0.013	0.032
	Australia	0.068	0.070	0.002	0.059

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The rows represent the models and the corresponding μ_3 (MAXMINVAR) and μ_4 (MINMAXVAR) are set to 0.5, while we set μ_1 (MINVAR) and μ_2 (MAXVAR) equal to 3.

The results in Tables 19 and 20 are in line with the results from the full sample and show empirical support for the monetary fundamental models (specifically MFII and MFIII). In particular, compare the out-of-sample results of Table 18 and Table 20. Note that in Table 18 (where we consider Sharpe ratios), while MFII and MFIII generally perform rather better than the Random Walk model, this is not actually true for every individual entry in the table. By contrast in Table 20 (where we consider indices of acceptability), MFII and MFIII out-perform the Random Walk model in every single entry in the table for all four indices of acceptability. The margin of out-performance is particularly large for DEM/EURUSD and JPYUSD which, as we observed earlier (see Table 1), are the exchange rates for which the Jarque-Bera statistics indicate the greatest departure from normally distributed exchange rate returns.

10. Conclusion

This paper assesses the forecasting performance of widely used monetary fundamental models of exchange rates. We find evidence in their support by evaluating the economic significance of their forecasting ability. Specifically, we compare the performance of portfolios, consisting of a (foreign exchange rate) risky asset and a (domestic) risk-free asset, constructed using model predictions. We utilize new measures (indices of acceptability) to evaluate portfolio performance which are robust to non-normally distributed portfolio returns. We find that structural models perform better than Random Walk models in generating "profitable" trading signals. This conclusion is particularly important because, while it is in line with Della Corte et al. (2009), it is in contrast to the majority of papers (which have evaluated predictive ability based on purely statistical measures) in the extant literature.

Appendix

In this appendix we briefly explain the main algorithms for the Bayesian linear regression and the Bayesian GARCH (1,1) algorithms used in this paper.

Bayesian Linear Regression:

We are interested in the estimation of the parameters that are contained in the set Θ , these are $\theta = \{\theta_1, \theta_2\}$, where $\theta_1 = \{\beta_1, \beta_2\}$ and $\theta_2 = \{h\}$ where h is the error precision, that is the inverse of the variance $h = 1/\sigma^2$. The Normal priors for θ_1 have zero mean and variance equal to one. Prior gamma $\left(\frac{v}{2}, \frac{2s^{-2}}{\underline{v}}\right)$ is assumed for $\theta_2 = \{h\}$ with mean $h^{-2} = 1$ and degree of freedom $\underline{v} = 2$. The following algorithm shows the steps of the Monte Carlo simulation:

1. The Monte Carlo integration used is $\hat{g}s = \frac{1}{S}\sum_{S=1}^{S} g\left(\beta^{(S)}\right)$. where S is the number of simulations. $(\beta \mid y)$
2. First obtain a random draw of $\beta^{(s)}$ from the posterior is generated by MATLAB random number generator for *t* distribution..
3. Thereafter calculate the function $g\left(\beta^{(S)}\right)$ and retain the result.
4. Repeat steps 2 and 3 for *S*=100000.
5. Finally take the average of *S* draws in order to obtain the mean of the posterior distribution of β.

We also compute the empirical standard errors as follows: Let $\theta^{(S)}$ for *s* = *1,....,S* be a random sample from $p(\theta \mid y)$, and define

$$\hat{g}s = \frac{1}{S}\sum_{S=1}^{S} g\left(\theta^{(S)}\right) \tag{13}$$

Then $\hat{g}s$ converges to $E\left[g\left(\theta\right) \mid y\right]$ as S goes to infinity

$$\sqrt{S\left\{\hat{g}s - E\left[g\left(\theta\right)\right]\right\}} \rightarrow N(0, \sigma_g^2) \tag{14}$$

where $\sigma_g^2 = \text{var}\left[g\left(\theta\right) | \, y \right]$. The number of replications S is set to 100,000.

Bayesian GARCH:

The GARCH algorithm follows Ardia (2008) and assumes $\sigma_{t|t-1}^2 = \omega + \alpha u_{t-1}^2 + \beta\sigma_{t-1|t-2}^2$ The conditional volatility is recursive in nature; hence it restricts the use of conjugacy between prior density and the likelihood function. Therefore, the Metropolis-Hastings algorithm is used to draw samples from the posterior distribution. The algorithm is the modified version of the algorithm described by Nakatsuma (1998), (2000). Truncated Normal distribution with zero mean and unit variance is selected as prior. Using Baye's rule, the joint posterior probability distribution is $p(\theta | y) \propto p(y | \theta) p(\theta)$.

We applied the Bayesian GARCH estimations on the returns calculated from the three monetary fundamental models and the Random Walk model. These estimations are obtained by the *bayesGARCH* function of the *R* language by the CRAN project. The *bayesGARCH* function is is provided by Ardia (2008). As an input argument we provided the prior parameters and the length of each MCMC chain, that are $\omega = 0.01$, α=0.1, β=0.7, υ=20 and the MCMC chain of 100000. The sampler convergence is controlled by the Gelman and Rubin (1992) diagnostic test. The first 10000 draws are discarded from the MCMC draws.

1. First initial values of the prior are drawn for θ^0 from the parameter space of θ.
2. For each iteration j, draw a (multivariate) realization, θ^* from the density conditional on θ^{j-1}, that is the parameter value at the previous step.
3. Compute the acceptance probability as $\min\left\{ \frac{p(\theta^* | y)}{p(\theta^{[j-1]} | y)} \frac{q(\theta^{[j-1]} | \theta^*)}{q(\theta^* | \theta^{[j-1]})}, 1 \right\}$. After drawing *U* from a uniform distribution $U(0,1)$ check if $U \leq$ acceptance probability. If it is, set $\theta^{[j]} = \theta^*$, otherwise, set $\theta^{[j]} = \theta^{[j-1]}$.
4. Iterate from step 2 until convergence is obtained

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