First-and second-best allocations under economic and environmental uncertainty

by

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October 29, 2010

Abstract: This paper uses a micro-founded DSGE model to compare second-best optimal environmental policy and the resulting allocation to first-best allocation. The focus is on the source and size of uncertainty, and how this affects optimal choices and the inferiority of second best vis-à-vis first best.

Keywords: General equilibrium; uncertainty; environmental policy; second best.
JEL classification: C68; D81; H23.

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Acknowledgements: We thank Nick Hanley, Saqib Jafarey, Christos Kotsogiannis, Elissaios Papyrakis, Hyun Park and Tassos Xepapadeas for comments and discussions. We also thank seminar participants at the CESifo workshop on the Fiscal implications of climate change, held in Venice, July, 2010. Any errors are ours.

1. Introduction

It is known that in the presence of environmental externalities, the resulting allocation is not efficient and hence there is need for government intervention. It is also known that if the government has at its disposal extra lump-sum (Pigouvian) policy instruments, then it is able to reproduce the first-best allocation. However, in the real world where only distorting policy instruments are available, the government has to choose a second-best optimal policy and hence a second-best allocation. In the relevant literature, such second-best policy often takes the form of distorting taxes on polluting generating activities like output.[1]

The goal of this paper is to study the importance of the source and size of uncertainty to optimal choices and the associated welfare under both a first-best and a second-best setup. As far as we know, this is novel. We focus on uncertainty because it is a big concern in environmental policy. In assessing the risks from climate change and the costs of averting it, there is a variety of uncertainties that contribute to big differences of opinion as to how, and how much, to limit emissions (on uncertainty and the environment, see e.g. the Congressional Budget Office paper prepared for the Congress of the US, 2005).[2]

Our setup is the basic stochastic neoclassical growth model augmented with the assumptions that pollution occurs as a by-product of output produced and environmental quality has a public good character. Within this setup, there is reason for policy intervention. There are two exogenous stochastic processes that create uncertainty about future outcomes and drive the stochastic dynamics of the model. The first is uncertainty about production technology (standard shocks to total factor productivity) and the second arises from uncertainty about the impact of economic activity on the environment. Loosely speaking, we call the former shock "economic" and the latter "environmental".

We study the implications of uncertainty for macroeconomic outcomes, environmental quality and, ultimately, social welfare in both second-best and first-best allocations. Social welfare is defined as the conditional expectation of the discounted sum of household's lifetime utility. Regarding second-best allocations, and since the decentralized equilibrium solution depends on the value of the tax rate

[1] For environmental policy instruments, see the survey by Bovenberg and Goulder (2002). For environmental tax rates in growth models, see the survey by Xepapadeas (2004).
[2] There is a rich literature on the role of uncertainty in environmental policy that goes back to Weitzman (1974). For a review of this literature, see Bovenberg and Goulder (2002).

employed, we study the case in which this tax rate takes its welfare-maximizing value. We then compare second-best optimal tax policy and the resulting allocation to the first-best outcome derived by solving a fictional social planner's problem. To solve the model and compute the associated welfare in each case, we approximate both the equilibrium solution and the welfare criterion to second-order around their non-stochastic long-run by using the methodology of Schmitt-Grohé and Uribe (2004). A second-order approximation allows us, among other things, to take into account the risk that economic agents face.

Regarding the second-best regime, it should be noted that when choosing its distorting policy, the government aims at the following: First, to correct for pollution externalities. Second, to create revenues to finance public abatement in the least distorting way. Third, since there is also uncertainty, the risk-averse government aims to reduce volatility. Second-best optimal tax policy will reflect all these tasks.

The main results of the paper are as follows. First, while higher economic uncertainty reduces welfare, higher environmental uncertainty can increase welfare under optimal policies. Thus, environmental uncertainty stimulates a type of precautionary behavior that can improve welfare. Second, the relative inferiority of second best relative to first best depends on the source and size of extrinsic uncertainty. When economic uncertainty is dominant, the relative inferiority of second best decreases with uncertainty. By contrast, when environmental uncertainty is dominant, the relative inferiority of second best further deteriorates with uncertainty.

The rest of the paper is as follows. Section 2 presents a decentralized economy with taxes. Section 3 presents the social planner case (first-best). Section 4 compares welfare under second best and first best. Section 5 closes the paper.

2. Decentralized economy given taxes

We augment the basic stochastic neoclassical growth model with natural resources and environmental policy. The economy is populated by a private agent, who derives utility from private consumption and the stock of environmental quality, and the government. The private agent consumes, saves and produces a single good. Output produced generates pollution and this damages environmental quality. Since the private agent does not internalize the effect of his/her actions on the environment, the decentralized equilibrium is inefficient. Hence, there is room for government intervention, which here takes the form of taxes on polluting-generating output (see also e.g. Economides and Philippopoulos, 2008).

Private agent

The private agent's expected utility is defined over stochastic sequences of private consumption, c_t, and the economy's beginning-of-period environmental quality, Q_t:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, Q_t) \tag{1a}$$

where $0 < \beta < 1$ is a time preference rate and E_0 is an expectations operator based on the information available at time zero.

Without loss of generality, we use for instantaneous utility:

$$u(c_t, Q_t) = \frac{[(c_t)^{\mu}(Q_t)^{1-\mu}]^{1-\sigma}}{1-\sigma} \tag{1b}$$

where $0 < \mu, 1-\mu < 1$ are the weights given to consumption and environmental quality respectively and $\sigma \geq 1$ is a measure of risk aversion.

The private agent's within-period budget constraint is:

$$k_{t+1} - (1-\delta^k)k_t + c_t = (1-\tau_t)y_t = (1-\tau_t)A_t k_t^{\alpha} \tag{2}$$

where $y_t = A_t k_t^{\alpha}$ is current output,[3] k_{t+1} is the end-of-period capital stock, k_t is the beginning-of-period capital stock, A_t is a standard index of production technology (whose stochastic motion is defined below), $0 < \alpha < 1$ and $0 \leq \delta^k \leq 1$ are usual parameters, and $0 \leq \tau_t < 1$ is the tax rate on (polluting) output.

The agent chooses $\{c_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize (1a-b) subject to (2) taking policy variables and environmental quality as given. The latter is justified by the open-access and public-good features of the environment.

[3] We abstract from labor-leisure choices to keep the model simpler. This is not important.

Natural resources and pollution

The stock of environmental quality evolves over time according to:[4]

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - p_t + \nu g_t \tag{3}$$

where the parameter $\bar{Q} \geq 0$ represents environmental quality without pollution, p_t is the current pollution flow, g_t is public spending on abatement activities, and $0 \leq \delta^q \leq 1$ and $\nu \geq 0$ are parameters measuring respectively the degree of environmental persistence and how public spending is translated into actual units of renewable natural resources.

The flow of pollution, p_t, is modeled as a by-product of output produced, y_t:

$$p_t = \phi_t y_t = \phi_t A_t k_t^{\alpha} \tag{4}$$

where ϕ_t is an index of pollution technology or a measure of emissions per unit of output. We assume that ϕ_t is stochastic (its motion is defined below).

Government budget constraint

Assuming a balanced budget for the government, we have in each period:

$$g_t = \tau_t y_t = \tau_t A_t k_t^{\alpha} \tag{5}$$

Exogenous stochastic variables

The two technologies, A_t and ϕ_t, follow $AR(1)$ stochastic processes of the form:

$$A_{t+1} = A^{(1-\rho_a)} A_t^{\rho_a} e^{\varepsilon_{t+1}^a} \tag{6a}$$

$$\phi_{t+1} = \phi^{(1-\rho_\phi)} \phi_t^{\rho_\phi} e^{\varepsilon_{t+1}^\phi} \tag{6b}$$

where A and ϕ are constants, $0 < \rho_a, \rho_\phi < 1$ are auto-regressive parameters and $\varepsilon_t^a, \varepsilon_t^\phi$ are Gaussian i.i.d. shocks with zero means and known variances, σ_a^2 and σ_ϕ^2.

[4] The motion of natural resources in (3) is as in Jouvet et al. (2005); see p. 1599 in their paper for further details. The inclusion of the parameter $\bar{Q} \geq 0$ is helpful when we solve the model numerically.

Decentralized competitive equilibrium (given output taxes)

The Decentralized Competitive Equilibrium (DCE) of the above economy is summarized by the following equations at $t \geq 0$ (see Appendix A for details):

$$k_{t+1} - (1-\delta^k)k_t + c_t = (1-\tau_t)A_t k_t^{\alpha} \tag{7a}$$

$$\frac{\partial u_t}{\partial c_t} = \beta E_t \left[\frac{\partial u_{t+1}}{\partial c_{t+1}} [1-\delta^k + (1-\tau_{t+1})\alpha A_{t+1} k_{t+1}^{\alpha-1}] \right] \tag{7b}$$

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - (\phi_t - \nu\tau_t)A_t k_t^{\alpha} \tag{7c}$$

where $\frac{\partial u_t}{\partial c_t} = \mu(c_t)^{\mu(1-\sigma)-1}(Q_t)^{(1-\mu)(1-\sigma)}$. This is a three-equation system in $\{c_t, k_{t+1}, Q_{t+1}\}_{t=0}^{\infty}$ given the tax rates, $\{\tau_t\}_{t=0}^{\infty}$, initial conditions for the stock variables, k_0 and Q_0, and stochastic processes for the exogenous variables, A_t and ϕ_t. Section 4 will choose the optimal tax rate.

3. Social planner's solution (first best)

This will serve as a benchmark. The social planner chooses allocations $\{c_t, g_t, k_{t+1}, Q_{t+1}\}_{t=0}^{\infty}$ directly to maximize (1a-b) subject to resource constraints only. The solution is (see Appendix B for details):

$$k_{t+1} - (1-\delta^k)k_t + c_t + g_t = A_t k_t^{\alpha} \tag{8a}$$

$$\frac{\partial u_t}{\partial c_t} = \beta \frac{\partial u_{t+1}}{\partial c_{t+1}}(1-\delta^k + \alpha A_{t+1} k_{t+1}^{\alpha-1}) - \beta\xi_{t+1}\phi_{t+1}\alpha A_{t+1} k_{t+1}^{\alpha-1} \tag{8b}$$

$$Q_{t+1} = (1-\delta^q)\bar{Q} + \delta^q Q_t - \phi_t A_t k_t^{\alpha} + \nu g_t \tag{8c}$$

$$\xi_t = \beta \frac{\partial u_{t+1}}{\partial Q_{t+1}} + \beta\delta^q \xi_{t+1} \tag{8d}$$

$$\frac{\partial u_t}{\partial c_t} = \nu\xi_t \tag{8e}$$

where $\dfrac{\partial u_t}{\partial c_t}=\mu(c_t)^{\mu(1-\sigma)-1}(Q_t)^{(1-\mu)(1-\sigma)}$, $\dfrac{\partial u_{t+1}}{\partial Q_{t+1}}=(1-\mu)(c_{t+1})^{\mu(1-\sigma)}(Q_{t+1})^{(1-\mu)(1-\sigma)-1}$ and $\xi_t>0$ is a dynamic multiplier associated with (8c). We thus have a five-equation system in $\{c_t,g_t,k_{t+1},Q_{t+1},\xi_t\}_{t=0}^{\infty}$ given initial conditions for the stock variables, k_0 and Q_0, and stochastic processes for the exogenous variables, A_t and ϕ_t.

4. Second-best optimal allocation and comparison with the first best

This section compares social welfare under the decentralized economy in section 2 to social welfare under the first-best solution in section 3. Social welfare is defined as the conditional expectation of the discounted sum of household's lifetime utility in (1a-b) above.

How we work

Since the DCE solution and the associated welfare in section 2 depend on the value of the output tax rate, we search for the welfare-maximizing value of the output tax rate and the associated maximum second-best welfare. In particular, focusing on flat tax rates, namely, tax rates that remain constant over time (see also e.g. Lucas, 1990, Stokey and Rebelo, 1995, Mendoza and Tesar, 1998, and Ortigueira, 1998), we compute welfare for a wide range of values of the flat tax rate and thus find the welfare-maximizing value of the tax rate and the associated maximum welfare in the second-best case. We then compare this second-best welfare to the welfare derived from the social planner's first-best solution in section 3.

To solve the system of non-linear expected difference equations that form the DCE in (7a-c) and then search for the welfare-maximizing value of the output tax rate, we approximate the equilibrium equations (7a-c), as well as the welfare criterion in (1a-b), to second-order around the non-stochastic second-best steady state solution. We work similarly in the case of the social planner; namely, we approximate the equilibrium equations (8a)-(8e), as well as (1a-b), to second-order around the non-stochastic first-best steady state solution. Regarding the equilibrium solutions, (7a-c) and (8a-e), we solve and simulate their second-order approximations following the methodology and using the Matlab codes made available by Schmitt-Grohé and Uribe (2004). Regarding the welfare criterion, (1a-b), its second-order approximation is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, Q_t) \cong \frac{u(c,Q)}{1-\beta} + E_0 \sum_{t=0}^{\infty} \beta^t \{a_1 \hat{c}_t + a_2 \hat{Q}_t + a_3 (\hat{c}_t)^2 + a_4 (\hat{Q}_t)^2 + a_5 (\hat{c}_t \hat{Q}_t)\} \qquad (9)$$

where, for any variable x_t, $\hat{x}_t \equiv \ln(x_t / x) \cong (x_t - x)/x$ and x is the long-run value of x_t. Also, $a_1 \equiv \mu(1-\sigma)u(c,Q)$, $a_2 \equiv (1-\mu)(1-\sigma)u(c,Q)$, $a_3 \equiv \frac{\mu^2(1-\sigma)^2 u(c,Q)}{2}$, $a_4 \equiv \frac{(1-\mu)^2(1-\sigma)^2 u(c,Q)}{2}$, $a_5 \equiv \mu(1-\mu)(1-\sigma)^2 u(c,Q)$. The values of $\hat{c}_t$ and $\hat{Q}_t$ follow from the second-order approximation to the equilibrium solutions in (7a)-(7c) and (8a)-(8c) respectively as said above.

Finally, we need a measure of comparison of welfare gains/losses when we move from first-best to second-best regimes. This measure, denoted as ζ_{ij} in what follows, is obtained by computing the percentage compensation in private consumption that the private agent would require in each time-period under regime j so as to be equally well off between regimes i and $j \neq i$. This is a popular measure in dynamic general equilibrium models (see e.g. Lucas, 1990).

Parameter values used

We keep all parameter values the same across different regimes, so that the evaluation of different regimes is not blurred by parameter differences. The parameter values used are reported in Table 1.

Table 1 around here

The values of the economic parameters are standard. In particular, the time preference rate (β), the depreciation rate of capital (δ^k), the capital share in output (α), the intertemporal elasticity of substitution ($1/\sigma$), the constant term (A) and the persistence parameter (ρ_a) in the TFP process, are as in most dynamic stochastic general equilibrium calibration and estimation studies (see e.g. King and Rebelo, 1999). We will experiment with different values of the standard deviation of the TFP process, σ_α, in equation (6a).

There is less empirical evidence and consensus on the value of environmental parameters. Regarding the value of μ, which is the weight given to private consumption vis-à-vis environmental quality in the utility function, we set it at 0.6, so that the weight given to environmental quality is 0.4. The latter is higher than the value given usually to simple public goods in related utility functions. Regarding the parameters in the exogenous stochastic process for environmental technology in (6b), we choose a high persistence parameter, $\rho_{\phi} = \rho_{\alpha} = 0.933$, normalize its constant term, ϕ, at 0.5, and experiment with different values of its standard deviation, σ_{ϕ}. Regarding the parameters in the motion for environmental quality in (3), we choose a relatively high persistence parameter, $\delta^{q} = 0.9$, and normalize its constant term (i.e. the level of environmental quality without economic activity) to unity, $\overline{Q} = 1$. Finally, we set v (i.e. how public abatement spending is translated into actual units of environmental quality) at 5; this parameter value helps us to match the units in the environmental quality equation (3) and obtain a well-defined trade-off in second-best optimal policy. Notice that since v is the same across regimes, its value does not matter for the comparison of these two regimes.

Welfare results under uncertainty

We assume that the economy is initially at its steady state and, starting from $t = 0$, there are shocks to A_t and ϕ_t, which are the exogenous stochastic autoregressive processes for production and pollution technologies in (6a)-(6b). We compute the discounted expected lifetime utility for a varying degree of uncertainty as summarized by the standard deviations of the production and pollution technologies, σ_a and σ_{ϕ}. We do so for both the first-best and the second-best case, and then compare them, as explained above.

Welfare results for the two cases are reported in Table 2, while Tables 3a-b present first and second moments of the arguments in the utility function under the first-best and the second-best case respectively. For expositional reasons, we study: (i) a deterministic economy ($\sigma_a = \sigma_{\phi} = 0$); (ii) when there is only one source of uncertainty ($\sigma_{\alpha} = 0.01$ and $\sigma_{\phi} = 0$; $\sigma_{\alpha} = 0.05$ and $\sigma_{\phi} = 0$; $\sigma_{\alpha} = 0$ and $\sigma_{\phi} = 0.01$; $\sigma_{\alpha} = 0$ and $\sigma_{\phi} = 0.05$); (iii) a scenario of relatively low uncertainty in both stochastic variables ($\sigma_{\alpha} = \sigma_{\phi} = 0.01$); (iv) two scenarios representing high levels of uncertainty in one of the two stochastic variables ($\sigma_{\alpha} = 0.01$ and $\sigma_{\phi} = 0.05$; and $\sigma_{\alpha} = 0.05$, $\sigma_{\phi} = 0.01$); (v) a scenario with relatively high uncertainty in both stochastic variables ($\sigma_a = \sigma_{\phi} = 0.05$).

Tables 2 and 3a-b around here

The first result to observe in Table 2 is that when environmental uncertainty increases, welfare increases. The reason why this happens can be understood by looking at the first and second moments in Tables 3a-b. In particular, regarding the first-best allocation, as environmental uncertainty increases, both consumption and environmental quality increase. This seemingly paradoxical result happens because the social planner wishes to stabilize the environmental process, and the way to achieve this is through higher abatement. The latter necessitates an increase in output, so that more resources are made available for both consumption and abatement. In other words, environmental uncertainty creates a form of precautionary behavior that is welfare increasing. The increases in consumption and output more than compensate for the welfare losses due to higher volatility of consumption and environmental quality. On the other hand, the model generates the expected result that higher TFP uncertainty decreases welfare, for a standard parameterization of the risk aversion in the utility function. The same pattern is observed under second-best optimal policy. However, because of the distortions introduced by the output tax rate, now the increases in consumption and environmental quality are smaller. Nevertheless, the negative correlation between these two variables helps to increase welfare.

We next compare first best to second best. This is summarized by the values of ζ_{12} reported in the last column of Table 2. These values are all positive, as expected, meaning that the first best is superior or, equivalently, the household who lives in the second-best world would require an extra consumption subsidy in each time period to be indifferent between second best and first best.

In the deterministic case, $\sigma_a = \sigma_\phi = 0$, social welfare rises by 4.65%, when we move from second best to first best. In a stochastic world, the results are similar. For instance, when $\sigma_\phi = 0.05$ and $\sigma_\alpha = 0.01$, a welfare gain of 4.79% can be obtained if we move from second best to first best.

It is interesting to note that the degree of relative superiority of the first-best allocation depends on the source and size of uncertainty. When there is only economic uncertainty ($\sigma_\alpha > 0$ and $\sigma_\phi = 0$), the relative superiority of first best over second best decreases with the degree of uncertainty. For instance, when $\sigma_\alpha = 0.01$, a welfare gain of 4.6% is obtained if we move from the second best to first best, whereas the welfare gain decreases to 4.5% when $\sigma_\alpha = 0.05$. This is despite the fact that in absolute terms, the welfare difference between the two regimes increases. As the economy moves to a worse equilibrium, it matters less, in relative terms, whether first- or second-best optimal policies are chosen. On the other hand, when there is only environmental uncertainty ($\sigma_\alpha = 0$ and $\sigma_\phi > 0$), the relative superiority of first best over second best increases with the degree of uncertainty. For instance, when $\sigma_\phi = 0.01$, a welfare gain of 4.65% can be obtained if we move from second best to first best, whereas the welfare gain rises to 4.83% when $\sigma_\phi = 0.05$. As the economy moves to a better equilibrium, it matters more, in relative terms, whether first- or second-best optimal policies are chosen.

In all other more general cases in which both variances are positive, the same message applies. Namely, given σ_ϕ (resp. σ_α), the superiority of first best decreases (resp. increases) as σ_α (resp. σ_ϕ) increases. Also, if both σ_α and σ_ϕ increase, but it is the increase in the latter (resp. former) that dominates, the relative superiority of first best increases (resp. decreases) as uncertainty increases.

5. Concluding remarks and possible extensions

We compared second-best optimal environmental policies and the resulting allocations to first-best allocations in a micro-founded dynamic stochastic general equilibrium model. We focused on the role of uncertainty and showed that the source and magnitude of extrinsic uncertainty matter to optimal choices and welfare under both first- and second-best optimal allocations. In this paper, we focused on taxes in the second-best case. An extension could be to study other policy instruments, like pollution permits and emission rules.

Table 1: Baseline parameter values

Parameter	Description	Value
α	Capital share in production	0.33
δ^k	capital depreciation rate	0.1
σ	curvature parameter in utility function	2
β	Time discount factor	0.97
μ	Consumption weight in utility function	0.6
$\overline{Q}$	environmental quality without pollution	1
δ^q	persistence of environmental quality	0.9
A	Long-run total factor productivity	1
ρ_a	persistence of total factor productivity	0.933
ϕ	long-run pollution technology	0.5
ρ_ϕ	Persistence of pollution technology	0.933
ν	transformation of spending into units of nature	5

Table 2: Expected lifetime utility (ELU)

σ_a	σ_ϕ	(1) First-best ELU	(2) Second-best ELU	ELU maximizing tax rate (τ)	ζ_{12} (%)
0	0	- 13.41	- 14.30	0.3319	4.65
0	0.01	- 13.41	- 14.30	0.3321	4.65
0	0.05	-13.32	-14.24	0.3319	4.83
0.01	0	- 13.55	- 14.44	0.3318	4.60
0.05	0	- 16.97	- 18.06	0.3288	4.50
0.01	0.01	- 13.55	- 14.45	0.3320	4.65
0.01	0.05	- 13.46	- 14.38	0.3318	4.79
0.05	0.01	- 16.98	- 18.07	0.3290	4.50
0.05	0.05	-16.89	- 18.00	0.3289	4.61

Notes: (i) The value of ζ_{ij} is given by $\zeta_{ij} \cong \frac{1}{\mu(1-\sigma)} \log(V_t^i / V_t^j) X 100$, where V_t^i and V_t^j denote the discounted sums of second-order approximations to welfare in equation (9) and averaged over 1000 simulations. (ii) To evaluate the expectation in the welfare calculations, we use numerical integration with 1000 simulations. As said, we use 300 years in our evaluation of life-time welfare since, because of discounting, there is practically a zero weight attached to later outcomes.

Table 3a: First and second moments for c_t and Q_t under first best

σ_a	σ_ϕ	$E(c_t)$	$E(Q_t)$	$\sigma(c_t)$	$\sigma(Q_t)$	$\rho(c_t, Q_t)$
0	0	0.68095	17.337	0	0	1
0	0.01	0.68095	17.336	0.0019836	0.059818	0.81934
0	0.05	0.68166	17.355	0.0099509	0.30013	0.81871
0.01	0	0.68065	17.34	0.018118	0.54766	0.82251
0.05	0	0.67275	17.408	0.089786	2.7623	0.81975
0.01	0.01	0.68065	17.34	0.018226	0.55094	0.82218
0.01	0.05	0.68136	17.359	0.020698	0.62571	0.82074
0.05	0.01	0.67274	17.408	0.08981	2.763	0.8197
0.05	0.05	0.67346	17.428	0.090444	2.7831	0.81946

Table 3b: First and second moments for c_t and Q_t under second best

σ_a	σ_ϕ	$E(c_t)$	$E(Q_t)$	$\sigma(c_t)$	$\sigma(Q_t)$	$\rho(c_t, Q_t)$
0	0	0.64593	15.988	0	0	1
0	0.01	0.64565	15.998	0.0003	0.11943	- 0.41974
0	0.05	0.64593	16.027	0.0015	0.59828	- 0.4188
0.01	0	0.64594	15.984	0.020051	0.43533	0.84781
0.05	0	0.64616	15.828	0.10056	2.1679	0.84631
0.01	0.01	0.64565	15.994	0.020043	0.45187	0.81163
0.01	0.05	0.64594	16.023	0.020106	0.74281	0.45248
0.05	0.01	0.64588	15.838	0.10051	2.1731	0.84464
0.05	0.05	0.64602	15.874	0.10054	2.2602	0.80892

APPENDICES

Appendix A: DCE with taxes

The first-order conditions of the individual's problem include the budget constraint in (2) and the Euler equation (7b). Then, using (4)-(5) into (3), we get (7c). All this gives (7a-c) which is a three-equation system in $\{c_t, k_{t+1}, Q_{t+1}\}_{t=0}^{\infty}$ in terms of $\{\tau_t\}_{t=0}^{\infty}$. The long-run DCE follows if we simply drop time subscripts.

Appendix B: Social planner's solution

The planner chooses $\{c_t, g_t, k_{t+1}, Q_{t+1}\}_{t=0}^{\infty}$ to maximize (1a-b) subject to the resource constraints:

$$k_{t+1} - (1-\delta^k)k_t + c_t + g_t = A_t k_t^{\alpha} \tag{B.1a}$$

$$Q_{t+1} = (1-\delta^q)\overline{Q} + \delta^q Q_t - \phi_t A_t k_t^{\alpha} + \nu g_t \tag{B.1b}$$

The optimality conditions include (D.1a), (D.1b) and:

$$\frac{\partial u_t}{\partial c_t} = \beta \frac{\partial u_{t+1}}{\partial c_{t+1}}(1-\delta^k + \alpha A_{t+1} k_{t+1}^{\alpha-1}) - \beta \xi_{t+1} \phi_{t+1} \alpha A_{t+1} k_{t+1}^{\alpha-1} \tag{B.2a}$$

$$\xi_t = \beta \frac{\partial u_{t+1}}{\partial Q_{t+1}} + \beta \delta^q \xi_{t+1} \tag{B.2b}$$

$$\frac{\partial u_t}{\partial c_t} = \nu \xi_t \tag{B.2c}$$

where $\xi > 0$ is a dynamic multiplier associated with (B.1b), $\frac{\partial u_t}{\partial c_t} = \mu (c_t)^{\mu(1-\sigma)-1} (Q_t)^{(1-\mu)(1-\sigma)}$, and $\frac{\partial u_{t+1}}{\partial Q_{t+1}} = (1-\mu)(c_{t+1})^{\mu(1-\sigma)} (Q_{t+1})^{(1-\mu)(1-\sigma)-1}$. (B.1a)-(B.1b) and (B.2a)-(B.2c) constitute a five-equation system in $\{c_t, k_{t+1}, Q_{t+1}, g_t, \xi_t\}_{t=0}^{\infty}$. The long-run follows if we drop time subscripts.

References

Bovenberg A. L. and L. H. Goulder (2002): Environmental taxation and regulation, in *Handbook of Public Economics*, volume 3, edited by A. Auerbach and M. Feldstein, North-Holland, Amsterdam.

Congressional Budget Office (2005): Uncertainty in analyzing climate change: policy implications, The Congress of the United States, *Congressional Budget Office Paper*, January, Washington.

Cooley T. F. and G. D. Hansen (1992): Tax distortions in a neoclassical monetary economy, *Journal of Economic Theory*, 58, 290-316.

Economides G. and A. Philippopoulos (2008): Growth enhancing policy is the means to sustain the environment, *Review of Economic Dynamics*, 11, 207-219.

Jouvet P. A., P. Michel and G. Rotillon (2005): Optimal growth with pollution: how to use pollution permits, *Journal of Economic Dynamics and Control*, 29, 1597-1609.

Lucas R. E. (1990): Supply-side economics: an analytical review, *Oxford Economic Papers*, 42-293-316.

Mendoza E. and L. Tesar (1998): The international ramifications of tax reforms: supply side economics in a global economy, *American Economic Review*, 88, 226-245.

Ortigueira S. (1998): Fiscal policy in an endogenous growth model with human capital accumulation, *Journal of Monetary Economics*, 42, 323-355.

Schmitt-Grohé S. and M. Uribe (2004): Solving dynamic general equilibrium models using a second-order approximation to the policy function, *Journal of Economic Dynamics and Control*, 28, 755-775.

Stokey N. and S. Rebelo (1995): Growth effects of flat-rate taxes, *Journal of Political Economy*, 103, 519-550.

Weitzman M. (1974): Prices vs. quantities, *Review of Economic Studies*, 41, 477-491.