Revisiting the Dollar-Euro Permanent Equilibrium Exchange Rate: Evidence from Multivariate Unobserved Components Models

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Abstract

We propose an alternative approach to obtaining a permanent equilibrium exchange rate (PEER), based on an unobserved components (UC) model. This approach offers a number of advantages over the conventional cointegration-based PEER. Firstly, we do not rely on the prerequisite that cointegration has to be found between the real exchange rate and macroeconomic fundamentals to obtain non-spurious long-run relationships and the PEER. Secondly, the impact that the permanent and transitory components of the macroeconomic fundamentals have on the real exchange rate can be modelled separately in the UC model. This is important for variables where the long and short-run effects may drive the real exchange rate in opposite directions, such as the relative government expenditure ratio. We also demonstrate that our proposed exchange rate models have good out-of-sample forecasting properties. Our approach would be a useful technique for central banks to estimate the equilibrium exchange rate and to forecast the long-run movements of the exchange rate.

Key words: Permanent Equilibrium Exchange Rate; Unobserved Components Model; Exchange rate forecasting.
JEL Classifications: F31; F47

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1. Introduction

There are a large variety of methods available for calculating a country's equilibrium exchange rate, from the internal-external balance approach, to the behavioural and permanent equilibrium approaches, through to the new open economy approach (see MacDonald (2000) and Driver and Westaway (2004) for a survey of the literature). All of these approaches have their own advantages and disadvantages which is why, perhaps, end users (such as central banks and practitioners) use a range of estimates in coming to a view as to whether an exchange rate is misaligned or not.

In this paper we focus on an extension to the so-called permanent equilibrium approach, which has appeared under different guises in the literature. In sum, this approach relies on decomposing an actual real exchange rate into its permanent and transitory components and then using the permanent component as a measure of the equilibrium exchange rate. A variety of time series methods have been used to extract the permanent component, ranging from Beveridge-Nelson (1981) decompositions (Huizinga, 1986; Cumby and Huizinga, 1990) to structural vector autoregression (Clarida and Gali, 1994) and cointegration-based methods (Clark and MacDonald, 2004).

This paper proposes an alternative approach to obtain the PEER, which is based on an unobserved components model specification and offers a number of advantages over the conventional cointegration-based PEER proposed by Clark and MacDonald (2004). The first advantage is that we do not rely on the prerequisite that cointegration has to be found between the real exchange rate and macroeconomic fundamentals to obtain non-spurious long-run relationships and estimates of the PEER. Our approach facilitates the estimation of the long-run relationship between the integrated variables using maximum likelihood, and the use of the likelihood ratio test to identify the significance of these long-run coefficients, even if cointegration is rejected. Secondly, the impact that the permanent and transitory components of the macroeconomic fundamentals have on the real exchange rate can be modelled separately in the UC model. This is important for variables whose the long and short-run effects may drive the real exchange rate in opposite directions, such as the relative government expenditure ratio. Additionally, although our UC model does not require cointegration amongst the real exchange rate and the fundamentals as a prerequisite in obtaining the PEER, the UC model can easily accommodate a cointegration analysis using the methods of Nyblom and Harvey (2000). We also provide an out-of-sample forecasting exercise to test the validity of multivariate UC models against a random walk process of the real exchange rate.

The outline of the remainder of this paper is as follows. In the next section we outline our unobserved component models of the PEER. In section 3 we discuss the data set and the time series properties of the data. Section 4 contains our estimates of the unobserved component models while Section 5 contains cointegration based tests of the models. Our out-of-sample forecasting results are contained in Section 6 and Section 7 concludes.

2. Unobserved component models of the PEER

This section presents the UC model used to obtain the dollar-euro permanent equilibrium exchange rate (PEER). We construct four commonly used macroeconomic variables (the macroeconomic fundamentals), which are the terms of trade, tot_t, the productivity differential, pd_t, the relative government expenditure ratio, gov_t, and the real interest rate differential, rid_t, to track the underlying values of the dollar-euro exchange rate.[1] In contrast to the first three fundamentals, which are expected to have a long-run impact on the real exchange rate, the fourth variable, the real interest rate differential, is thought to affect the real exchange rate in the medium to short-run.

According to Clark and MacDonald (1999), the actual exchange rate may be determined by the following equation

$$q_t = \theta_1 tot_t + \theta_2 pd_t + \theta_3 gov_t + \theta_4 rid_t + \varepsilon_t, \qquad (1)$$

[1] In the portfolio balance models discussed by Faruqee (1995), Fell (1996) and MacDonald (1997), net foreign assets (NFA) are a central determinant of the real exchange rate. Therefore, we tried to incorporate this variable in our model specification. We tried both the euro NFA, which is measured as the cumulated current account balance as a percentage of GDP, and the overall relative NFA between the US and the euro area. However, neither variable could be used successfully. This problem, as explained in Maeso-Fernandez et al. (2001), is that the euro NFA is aggregated on the basis of national data, so that it does not correct for intra-euro area current positions. Moreover, for the bilateral exchange rate analysed in this study, one should use the NFA position between the two countries involved. However, this data is not available and cannot be accurately proxied by the overall relative NFA between the euro area and the US. Therefore, the following analysis uses the above four fundamentals.

where q_t denotes the real exchange rate, the coefficients θ_j, for $j = 0,1,2,4$, indicate the effects that individual fundamentals have on the real exchange rate and ε_t may contain a set of short-run variables and a random error. The behavioural equilibrium exchange rate (BEER) approach of Clark and MacDonald (2004) employs Johansen's cointegration approach (Johansen and Juselius, 1990) to determine whether there is a cointegration or long-run relationship amongst the real exchange rate and macroeconomic fundamentals. If there is one cointegration relationship, the current equilibrium exchange rate, $\overline{q}_t$, can be calculated using the estimated long-run coefficients as follows:

$$\overline{q}_t = \hat{\theta}_1 tot_t + \hat{\theta}_2 pd_t + \hat{\theta}_3 gov_t + \hat{\theta}_4 rid_t, \tag{2}$$

and the error correction term, ε_t, measures the current misalignment.

One way of calculating the PEER, $\overline{\overline{q}}_t$, is to substitute the sustainable (equilibrium) values of the fundamentals (i.e. $\overline{tot_t}$, $\overline{pd_t}$, $\overline{gov_t}$ and $\overline{rid_t}$), into the pre-estimated long-run relationship in equation (2).[2]

$$\overline{\overline{q}}_t = \hat{\theta}_1 \overline{tot_t} + \hat{\theta}_2 \overline{pd_t} + \hat{\theta}_3 \overline{gov_t} + \hat{\theta}_4 \overline{rid_t}. \tag{3}$$

Therefore, the total misalignment is given by

$$\begin{aligned} \tilde{\varepsilon}_t &= \varepsilon_t + \hat{\theta}_1 \left(tot_t - \overline{tot_t} \right) + \hat{\theta}_2 \left(pd_t - \overline{pd_t} \right) + \hat{\theta}_3 \left(gov_t - \overline{gov_t} \right) + \hat{\theta}_4 \left(rid_t - \overline{rid_t} \right) \\ &= \varepsilon_t + \hat{\theta}_1 tot_t^C + \hat{\theta}_2 pd_t^C + \hat{\theta}_3 gov_t^C + \hat{\theta}_4 rid_t^C \quad , \end{aligned} \tag{4}$$

which is the sum of the current misalignment and the transitory components of the fundamentals (i.e. tot_t^C, pd_t^C, gov_t^C and rid_t^C). The calibration of the fundamentals at their sustainable levels is usually achieved by using some sort of statistical filter, such as the Hodrick-Prescott filter (1997, HP henceforth). However, these filters are known to produce spurious cycles for non-stationary data (Cogley and Nason, 1995; Murray 2003; Doorn, 2006). In addition, we question whether the long-run coefficients should remain the same

[2] See the discussion in Égert (2003).

when the actual values of the macroeconomic fundamentals are replaced by their sustainable values. That is to say, we do not believe the sustainable level (i.e. the permanent component) of each fundamental should be given the same weight, $\hat{\theta}_i$, as its transitory component, as suggested by the conventional cointegration-based PEER approach in equations (3) and (4).

Alternatively, Clark and MacDonald (2004) use the Granger and Gonzalo (1995) decomposition to estimate the PEER, which is derived from a vector error correction model (VECM). This approach identifies the common trends shared by the variables in the model, and their impact on the real exchange rate, but offers no direct measure of how the permanent component of each fundamental drives the movements of the real exchange rate. Instead the UC model proposed in this paper focuses on the impact the permanent and transitory components of each fundamental has on the real exchange rate. This provides a clearer economic interpretation than analysing the common permanent and transitory components in the macroeconomic fundamentals and the real exchange rate.

We also argue that the main constraint of the cointegration-based PEER approach is that it requires the presence of a cointegrating relationship amongst the real exchange rate and the fundamentals. If this prerequisite is lacking, we cannot proceed with the estimation of the PEER. However, in the following UC setting, we can relax this constraint.

The multivariate UC model used in this paper to estimate the PEER is specified as follows

$$\begin{bmatrix} tot_t \\ pd_t \\ gov_t \\ rid_t \\ q_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \theta_1 & \theta_2 & \theta_3 & \theta_4 & 1 \end{bmatrix} \begin{bmatrix} \overline{tot}_t \\ \overline{pd}_t \\ \overline{gov}_t \\ \overline{rid}_t \\ \tilde{q}_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \theta_{10} & \theta_{20} & \theta_{30} & \theta_{40} & 1 \end{bmatrix} \begin{bmatrix} tot_t^C \\ pd_t^C \\ gov_t^C \\ rid_t^C \\ \varepsilon_{q,t} \end{bmatrix}. \tag{5}$$

It can overcome a number of the drawbacks in conventional estimates of the PEER, as outlined above. Firstly, in the UC model, each macroeconomic fundamental is decomposed into its permanent and transitory components. We do not arbitrarily set the signal-noise ratio as in the HP filter. Instead, the weights used for signal extraction are estimated from the data by maximising the likelihood function. The permanent and transitory components of each fundamental are specified in equations (6) and (7), respectively

$$\overline{\mathrm{Y}}_t = \overline{\mathrm{Y}}_{t-1} + \beta_{\overline{\mathrm{Y}}} + \eta_{\overline{\mathrm{Y}}}, \quad \eta_{\overline{\mathrm{Y}}} \sim \mathrm{NID}\left(0, \sigma^2_{\eta\mathrm{Y}}\right), \tag{6}$$

$$\mathrm{Y}_t^C = \phi_{\hat{\mathrm{Y}}}(L)\,\mathrm{Y}_{t-1}^C + \kappa_{\hat{\mathrm{Y}}},\ \ \kappa_{\hat{\mathrm{Y}}} \sim \mathrm{NID}\left(0, \sigma_{\kappa \mathrm{Y}}^2\right). \tag{7}$$

To save space, $\overline{\mathrm{Y}}_t$ represents the permanent components of tot_t, pd_t, gov_t and rid_t, (i.e. $\overline{tot}_t$, $\overline{pd}_t$, $\overline{gov}_t$ and $\overline{rid}_t$) and Y_t^C denotes the transitory components (i.e. tot_t^C, pd_t^C, gov_t^C and rid_t^C). The permanent component, $\overline{\mathrm{Y}}_t$, is often referred to as the sustainable (equilibrium) value of the fundamental, and is generally specified as having the same order of integration as the fundamental. For example, if a fundamental appears to be stationary, the permanent component should also not contain a unit root, therefore

$$\overline{\mathrm{Y}}_t = \rho \overline{\mathrm{Y}}_{t-1} + \beta_{\overline{\mathrm{Y}}} + \eta_{\overline{\mathrm{Y}}}, \qquad \eta_{\overline{\mathrm{Y}}} \sim \mathrm{NID}\left(0, \sigma_{\eta \mathrm{Y}}^2\right) \tag{8}$$

where the damping factor ρ satisfies $0 < \rho < 1$. However, if Y_t is integrated of order two, $I(2)$, a stochastic drift, $\beta_{\overline{\mathrm{Y}},t}$, can be included in the random walk process

$$\begin{aligned} \overline{\mathrm{Y}}_t &= \overline{\mathrm{Y}}_{t-1} + \beta_{\overline{\mathrm{Y}},t} + \eta_{\overline{\mathrm{Y}}}, \qquad \eta_{\overline{\mathrm{Y}}} \sim \mathrm{NID}\left(0, \sigma_{\eta \mathrm{Y}}^2\right) \\ \beta_{\overline{\mathrm{Y}},t} &= \beta_{\overline{\mathrm{Y}},t-1} + \xi_{\overline{\mathrm{Y}},t}, \qquad \beta_{\overline{\mathrm{Y}},t} \sim \mathrm{NID}\left(0, \sigma_{\xi \mathrm{Y}}^2\right) \end{aligned} \tag{9}$$

Therefore, $\overline{\mathrm{Y}}_t$ becomes an $I(2)$ process. In the special case that $\sigma_{\eta \mathrm{Y}}^2 = 0$, $\overline{\mathrm{Y}}_t$ becomes a smoothed $I(2)$ process. On the other hand, the transitory component, Y_t^C, measures the extent to which the actual fundamental differs from its sustainable level, which is modelled as a stationary autoregressive process. In this paper, we adopt the stationary AR(2) specification used by Clark (1987) to model the transitory components for each macroeconomic fundamental.

Since the macroeconomic fundamentals are related to the real exchange rate, as shown in equation (1), the last row of equation (5) is a generalisation of equation (1), where we allow the permanent and transitory components of each fundamental to have different coefficients. This generalisation is important, as the permanent and transitory components of some fundamentals may have opposite effects on the real exchange rate. For example, according to Frenkel and Mussa (1988), high government expenditure today may lead to tax distortion and monetisation of government debt in the future. This in turn can produce a real depreciation of a currency in the long-run. However, in the short to medium-run, the real exchange rate can be positively affected if a rise in government spending increases net domestic demand, especially for non-traded goods. Therefore, it is important for researchers to be able to separate the permanent and transitory effects of this variable on the real exchange rate. However, this cannot be achieved using the cointegration-based BEER/PEER approach. Therefore, as stated by Osbat *et al*. (2003), who used this approach to analyse the yen-euro exchange rate, the sign on the long-run coefficient of the relative government expenditure ratio can be ambiguous as the estimated coefficient is an average of both the permanent and transitory impact. However, using the UC model outlined above can overcome this problem and we expect it to show θ_4 (the coefficient on the permanent component) to be negative, and θ_{40} (the coefficient on the transitory component) to be positive.

In the last row of equation (5), the term $\theta_1 \overline{tot}_t + \theta_2 \overline{pd}_t + \theta_3 \overline{gov}_t + \theta_4 \overline{rid}_t$ is that part of the PEER that can be explained by the four fundamentals at their sustainable values. This corresponds to equation (3). However, we also include an unobserved random walk component, $\tilde{q}_t$

$$\tilde{q}_t = \tilde{q}_{t-1} + \eta_{q,t}\ ,\ \eta_{q,t} \sim \mathrm{NID}(0, \sigma^2_{q,\eta}).$$

This is intended to model any remaining non-stationarity that is not captured by the fundamentals used. Therefore, the PEER is measured by $\theta_1 \overline{tot}_t + \theta_2 \overline{pd}_t + \theta_3 \overline{gov}_t + \theta_4 \overline{rid}_t + \tilde{q}_t$ in the UC model. The random walk component, $\tilde{q}_t$, can also be thought of as being applied in the manner of Harvey *et al*. (1986) and Sarantis and Stewart (2001), who used an unobserved component to model the variables that are omitted from the cointegration relationship.[3] If the standard deviation of the innovation to the random walk process, $\sigma_{q,\eta}$ is zero, $\tilde{q}_t$ will reduce to a constant and the real exchange rate is said to be cointegrated with the fundamentals (Nyblom and Harvey, 2000). However, a significant advantage of the UC model over the cointegration-based PEER approach is that we do not need the real exchange rate to be cointegrated with the fundamentals as a prerequisite to obtain non-spurious long-run relationships, and in turn to calculate the PEER.

[3] Harvey *et al.*(1986) add an unobserved component to the employment-output relation to account for the underlying productivity trend. Sarantis and Stewart (2001) use an unobserved component to capture any omitted variables from the consumption-income relationship such as wealth.

Instead, omitted variables from the cointegration relationship can be treated as an unobserved component that can be estimated from the observed data using Kalman filter. This allows the estimation of long-run relationships between integrated variables (i.e. the real exchange rate and permanent components of macroeconomic fundamentals) using maximum likelihood and the significance of these long-run coefficients can be tested using the likelihood ratio test. Finally, as with equation (4), the total misalignment is a linear combination of transitory components, $\theta_{10} tot_t^C + \theta_{20} pd_t^C + \theta_{30} gov_t^C + \theta_{40} rid_t^C$, plus an irregular term, $\varepsilon_{q,t} \sim \mathrm{NID}(0, \sigma_{q,\varepsilon}^2)$.

The UC model used in this paper can be recast into state-space form for estimation.[4] The hyperparameters in the UC model can be estimated by maximum likelihood using the prediction error decomposition produced by the Kalman filter. Since non-stationary variables, such as $\overline{tot_t}$, $\overline{pd_t}$, $\overline{gov_t}$ and $\tilde{q}_t$, appear in the state vector, the Kalman filter requires a diffuse initialisation and we use the initialisation method developed by Koopman and Durbin (2003). Estimating the multivariate UC model, we can obtain the unobserved permanent and transitory components and the coefficients on the real exchange rate equation simultaneously.[5]

3 Data

The quarterly data used in this paper covers the period 1975Q1 to 2008Q4. This sample encompasses the period of floating exchange rates between the euro area countries and the US after a brief 'adjustment phase' following the collapse of the Bretton Woods system. The real dollar-euro exchange rate (*LQ*) for twelve euro area members prior to 1999 (based on consumer prices) is computed as a weighted geometric average of the bilateral exchange rates of the eleven currencies (Belgium and Luxembourg already had a common currency) against the dollar. The weights are given by the share of external trade of each euro area country in total euro area trade (taking into account third market effects) for the period 1995-1997.[6] The consumer price indices and the bilateral nominal exchange rates are re-based to 2005=100. The real exchange rate used in the analysis is in its natural log-form. The consumer price

[4] The state-space representation of the model is available upon request.

[5] All the computations were preformed using the library of state-space functions in SsfPack 3.0 developed by Koopman et al. (2008) and Ox 5 by Doornik (2006).

[6] Weights used for each currency are 34.49% for Deutsche mark, 17.75% for French franc, 13.99% for Italian lira, 9.16% for Dutch guilder, 7.98% for Belgian and Luxembourg franc, 4.90% for Spanish peseta, 3.76% for Irish pound, 3.27% for Finnish markka, 2.89% for Austrian schilling, 1.07% for Portuguese escudo, 0.74% for Greek drachma.

indices and bilateral nominal exchange rates were taken from IMF International Financial Statistics (IFS), lines 64 and rf, respectively.

A country's terms of trade (*LTOT*) is computed as the ratio of its export prices to import prices (for some countries export unit value and import unit value are used). The same weights used to construct the synthetic euro-dollar exchange rate prior to 1999 are used to compute the ratio for the euro area. Finally, the terms of trade differential is computed as the ratio of the euro terms of trade relative to the US. All variables are rebased so that 2005=100 and the log of the terms of trade differential is used. Export and import prices for Austria, Finland, Germany and the US were obtained from IFS, lines 76 and 76.x, respectively. Since the data are not available for the remaining countries, export and import unit values taken from IFS, lines 74 and 75, are used.[7]

The productivity differential (*LPD*) is measured as the ratio of real GDP to the number of employed persons in the euro area compared with the same ratio for the US. This is a direct measure of the Balassa-Samuelson effect. This variable is also rebased to 2005=100 and logged. Real GDP series were obtained from IFS, line 99bv; Annual employment was from the OECD Labour productivity growth dataset. Quarterly data have been converted from the annual data.

Relative government expenditure (*LGOV*) is computed as the ratio of government expenditure to GDP in the euro area relative to the same ratio for the US. Consistent with the above variables, this series is also rebased and logged. For the euro area, the government expenditure to GDP ratio is obtained from the Area Wide Model constructed by Fagan et al. (2001). The corresponding variable for the US is constructed using GDP and government expenditure at current prices taken from IFS, lines 99b and 91f.

Finally, the real interest rate differential (*RID*) is the difference between real interest rates for the euro area and the US. Data on bond yields for the US and a geometric weighted average of long-run interest rates of countries constituting the euro area are used. The expected rate of inflation is proxied by the annual rate of consumer price inflation in the previous year. The nominal long-term interest rates for the euro area countries and the US were taken from IFS lines 61.

[7] For France, the data series of export and import unit values are available from 1990Q1 onwards, therefore the terms of trade data taken from the OECD database is used before 1990Q1.

3.1 Unit root tests

The stationarity of the data used in this paper are examined using the conventional Augmented Dickey-Fuller (ADF) test, which tests the null hypothesis that an $\mathrm{AR}(p)$ process contains a unit root against the alternative that it is stationary. The test proposed by Kwiatkowski *et al*. (1992, hereafter KPSS), based on structural time series models, is also used to cross-check the results of the ADF tests. In contrast to the ADF test, the KPSS tests the null hypothesis that the innovation of a random walk process has zero variance against the alternative that the innovation variance is positive. Both test statistics presented in the upper panel of Table 1 suggest that the level of the real interest rate differential is a stationary variable, while the levels of the terms of trade and productivity differential are non-stationary around either a level or a trend. However, the results are inconclusive for the levels of the real dollar-euro exchange rate and the relative government expenditure ratio. The same unit root tests are also conducted for the first differenced variables. Although the ADF tests strongly indicate that all first differenced variables are stationary, the KPSS tests reject stationarity in the first differenced series of the productivity differential and the relative government expenditure ratio at the 5% and 10% levels, respectively.

Given the results obtained from the unit root tests, apart from the real interest rate differential, we treat all the other variables as non-stationary. The evidence for a stationary interest rate differential has been widely suggested in the literature, including Hoffmann and MacDonald (2009).

3.2 Model modification

Given the properties of the data suggested by the unit root tests, we ensure that the permanent component to has the same order of integration as the logged actual data. Since both the ADF and KPSS suggest that the terms of trade is an $I(1)$ variable, we model its permanent component as a random walk with a constant drift, as specified in equation (6). However, as the interest rate differential appears to be stationary, as indicated by both the ADF and KPSS statistics, the permanent component of this variable should also not contain a unit root and therefore equation (8) is used.

In addition, evidence of $I(2)$ processes for the productivity differential and relative government expenditure is supported by the KPSS test statistics, but rejected by the ADF tests. This is explained by Nyblom and Harvey (2001), who argue that the ADF test too often rejects an $I(2)$ null because the process followed by the second differences of the observations is close to being non-invertible. Furthermore, they support the use of an $I(2)$ permanent component for series such as real GDP, as it can give a good fit when modelling within an unobserved components framework. In this paper, we used the $I(2)$ specification in equation (9) to model the permanent components of the productivity differential and the relative government expenditure ratio. This specification is supported by the KPSS statistics and using a stochastic drift allows us to model the gradual slowdown of euro area productivity and government expenditure relative to the US, as observed in Figure 1 where the estimated permanent components are plotted against the observed data.

{Table 1 about here}

4 Estimation results

The parameter estimates of the five-variate UC model (hereafter, Model 1) outlined in Section 2 are reported in Table 2. Inspection of the auxiliary residuals allows us to detect two outliers occurring during 1980Q1 and 2008Q3 for the interest rate differential and the terms of trade, respectively. Two dummies are used for these outliers and both dummy variable coefficients are statistically significant. In addition, both the Ljung-Box statistics for autocorrelation in one-step-ahead prediction errors and the Jarque-Bera statistics for normality are insignificant.

The upper panel of Table 2 presents the coefficients for the real exchange rate equation. The positive and significant parameter estimate of θ_1 suggests that an increase in the terms of trade differential of the euro area relative to the US will result in the euro appreciating against the dollar. This can be by a substitution effect generated by higher prices of exported goods relative to imported goods. Since higher export prices will initially lead to higher wages in the tradable sector relative to the non-tradable sector, this will raise the overall price level in the domestic economy in the long-run, which makes the domestic currency appreciate. A negative parameter estimate is found for θ_{10}. This may reflect the income effect generated by growing export revenues that may induce higher demand for non-traded goods. To restore internal equilibrium in this situation, the real exchange rate needs to depreciate. However, the negative income effect appears to be less predominant than the positive substitution effect, as θ_{10} is both smaller than θ_1 and statistically insignificant.

According to the Balassa-Samuelson hypothesis (Balassa, 1964 and Samuelson, 1964), higher productivity in the domestic relative to the foreign economy is usually expected to result in an appreciation of the domestic currency. Therefore, we expect θ_2 to be positive. Whilst θ_2 has the expected sign (positive), the coefficient is statistically insignificant. This seems to contradict Alquist and Chinn (2002) and Schnatz *et al.* (2004), who both find that the direct measure of the Balassa-Samuelson effect has a significant and positive long-run relationship with the dollar-euro exchange rate between 1985 and 2001. However, this difference may be due to the sample used in this paper being extended to 2008. It can be observed from Figure 1 that the real dollar-euro exchange rate and relative productivity move in different directions over the extended sample period, 2002-2008. The euro appreciated against the dollar from 2002 onwards, whilst the decline in the relative productivity of the euro area with respect to the US continued.

The coefficient of the permanent component of the relative government expenditure ratio, θ_3, is negative and significant. This is consistent with the argument of Frenkel and Mussa (1988) that a higher government expenditure ratio will lead to a real depreciation in the long-run. However, the short-run positive effect of this variable on the real exchange rate remains unclear as θ_{30} appears to be positive but insignificant. As outlined in Section 2, one of the advantages of using the UC model is that it can separate the negative long-run effect of the relative government expenditure ratio from its potentially positive short-run impact on the real exchange rate. As demonstrated in Section 5, this cannot be achieved using the VECM-based BEER/PEER approach, as it only estimates the average of these two effects.

As the interest rate differential appears to be stationary, the permanent component of this series should not contain a unit root, and it is thus specified to follow a stationary AR(1) process as in equation (8). However, we find that the estimate of the AR(1) component appears to be very small and less persistent than the AR(2) transitory component of this variable. In addition, the damping factor, ρ_{rid}, and the constant, β_{rid}, are insignificant. Therefore, in the results presented in Table 2, we restrict θ_4 to be zero a priori.[8] On the other hand, a positive estimate of θ_{40} is found, which is in line with the theoretical prediction that higher demand for the currency with a relatively higher interest rate will create an appreciation pressure on that currency. However, θ_{40} appears to be insignificant, suggesting

[8] The log-likelihood value for the unrestricted model is 1994.508. The null hypothesis that $\theta_4 = 0$ cannot be rejected by the log-likelihood ratio test at the 10% level.

there is no strong evidence that an increase in the interest rate differential in the euro area relative to the US leads to a real appreciation of the euro against the dollar for the sample period studied.

{Table 2 about here}

Finally, there are signs of a cointegrating relationship between the macroeconomic fundamentals and the real exchange rate, as the standard deviation of the innovation to the unobserved random walk process, $\sigma_{q,\eta}$, appears to be small. Although the cointegration between the real exchange rate and the fundamentals is not required by the UC model to obtain the PEER, the UC model can easily accommodate the cointegration analysis. We employ the test proposed in Nyblom and Harvey (2000) to determine whether $\sigma_{q,\eta} = 0$. This test can be regarded as an assessment of the validity of the pre-specified cointegrating vector, which is discussed in detail in Section 5. Section 5 also performs more general tests of cointegration proposed in Nyblom and Harvey (2000) and Johansen and Juselius (1990) to examine the possible cointegration relationship among these variables.

The permanent component of each series is plotted against its actual value in Figure 1. The estimated PEER plotted against the actual exchange rate is a linear combination of four non-stationary components, $\theta_1 \overline{tot_t} + \theta_2 \overline{pd_t} + \theta_3 \overline{gov_t} + \tilde{q}_t$. The subsequent misalignment is then given by $\theta_{10} tot_t^C + \theta_{20} pd_t^C + \theta_{30} gov_t^C + \theta_{40} rid_t^C + \varepsilon_{q,t}$. The results show that the euro is undervalued during the mid-1980s, as a result of the dollar's strength prior to the plaza agreement. However, strong signs of overvaluation of the euro became apparent in 1995-1998, which corresponds to a period of weakness of the dollar against major European currencies. Later on, the euro was broadly at its equilibrium level in 1999, but considerably undervalued in the years immediately after its launch as a result of financial market uncertainty. Nevertheless, the euro moves closely around its equilibrium value from 2003 onwards. Figure 1 also shows that apart from the real interest rate differential, which is primarily driven by its AR(2) transitory component, the fluctuations in the other variables are mainly attributable to their permanent components.

{Figure 1 about here}

4.1 AR(2) specification for the transitory component of the real exchange rate

Since none of the coefficients of the transitory components are significant in Model 1 at the 5% level suggested by the likelihood ratio tests reported in Table 2, we model the transitory component of the real exchange rate independently to follow a stationary AR(2) process. As such the last row of equation (5) becomes

$$q_t = \theta_1 \overline{tot}_t + \theta_2 \overline{pd}_t + \theta_3 \overline{gov}_t + \theta_4 \overline{rid}_t + \tilde{q}_t + q_t^C + \varepsilon_{q,t}, \tag{10}$$

where $q_t^C = \phi_{1,q} q_{t-1}^C + \phi_{2,q} q_{t-2}^C + \kappa_{q,t}$ and $\kappa_{q,t} \sim \text{NID}(0, \sigma_{q,\kappa}^2)$. All other rows in equation (5) remain the same. The parameter estimates of the modified model (hereafter, Model 2) are reported in Table 3. Some changes are observed when comparing the parameter estimates of Model 2 with those of Model 1. Firstly, the standard deviations of innovations to the AR(2) transitory components are increased in Model 2. This is accompanied by a decline in the volatility of the permanent components of the macroeconomic fundamentals. The largest change is observed in the decomposition of the productivity differential. Its AR(2) component becomes large and persistent, while the permanent component becomes a smoothed $I(2)$ process. Secondly, a moderate increase in the size of θ_1, θ_2 and θ_3 is identified in Model 2 compared to Model 1. However, the significance of these coefficients does not alter.

{Table 3 about here}

Figure 2 plots the unobserved components estimated from Model 2. The estimate of the PEER resembles the one obtained from Model 1. Consequently, the AR(2) specification of the total misalignment is also consistent with that obtained from Model 1.

{Figure 2 about here}

Although Models 1 and 2 are not nested, they are defined over the same five variables. Therefore, the Bayes factor methodology, discussed by Kass and Raftery (1996), can be used to indicate which model is favored by the data.[9] This approach compares the log-likelihood values of the two models, taking into account the number of parameters used in each. Kass and Rafter suggest using the Schwarz criterion for comparing models

$$S = \ell(D|M_J) - \ell(D|M_I) - 0.5(d_J - d_I)\log n,$$

where $\ell(\ )$ is the maximised log likelihood, d is the number of parameters, and n is the sample size. $2S$ can then be used to judge the strength of evidence against M_I with respect to M_J. If $2S$ lies between zero and two, Kass and Raftery suggest that the evidence against M_I is "not worth more than a bare mention". If $2S$ is between two and six, the evidence against M_I is 'strong' and if it is greater than ten the evidence against M_I is 'very strong'. In our case, M_J is Model 1 and M_I is Model 2. Model 1 has a slightly higher likelihood value, but it also has one more parameter than Model 2. The value of $2S$ is calculated to be 0.33, which suggests that the Bayes factor does not go against Model 2 with respect to Model 1.

4.2 Two models excluding the real interest rate differential

It is important to note that the interest rate differential did not play any role in the exchange rate equation in Model 2. Therefore, we removed this variable in Model 3 and instead conducted the estimation using four non-stationary variables (the real exchange rate, the terms of trade, the productivity differential, and the relative government expenditure ratio). The parameter estimates of Model 3 are reported in Table 4. It can be seen that excluding the real interest rate differential does not significantly alter the parameter estimates of the other variables in the model.

{Table 4 about here}

Furthermore, to obtain a more parsimonious model than Model 3, we subsequently restricted θ_2 to be zero, as it is insignificant in all of the previous estimations conducted. Setting θ_2 to zero indicates that the permanent component of relative productivity has no impact on

[9] The Bayes factor approach has also been used by Basistha and Startz (2008) and Basistha (2009) to compare different models used to estimate the output gap, core inflation and the non-accelerating inflation rate of unemployment.

determining the PEER. As a result, Model 4 cannot be rejected at the 10% level with respect to Model 3. In addition, θ_1 and θ_3 remain significantly positive and negative, respectively, as shown in Table 5.[10]

{Table 5 about here}

5 Testing for cointegration

In the previous section, we found the standard deviation of the innovation to the random walk process, $\sigma_{q,\eta}$, to be small. This may indicate that the four non-stationary variables in the UC model are cointegrated. In this section, we perform three cointegration tests. The first two are based on the structural time series models proposed by Nyblom and Harvey (2000). The third test is Johansen's cointegration test (Johansen and Juselius, 1990), which is a vector autoregression based test used in the BEER/PEER approach to determine the long-run relationship between the real exchange rate and the fundamentals. It is worth noting that the difference between Nyblom and Harvey (2000) and Johansen's cointegration test is analogous to the difference between the KPSS and the ADF test.

5.1 Testing for pre-specified cointegrating vectors

In this subsection, we perform the first cointegration test proposed by Nyblom and Harvey (2000) to determine whether $\sigma_{q,\eta} = 0$. This test can be regarded as testing the validity of a set of pre-specified cointegrating vectors. It is based on a multivariate local level model that can be written as

$$\mathbf{y_t} = \begin{bmatrix} \mathbf{y}_{1t} \\ \mathbf{y}_{2t} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_K & \mathbf{0} \\ \mathbf{\Theta} & \mathbf{I}_r \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu}_t^{\dagger} \\ \bar{\boldsymbol{\mu}}_t \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_{1t} \\ \boldsymbol{\varepsilon}_{2t} \end{bmatrix}, \tag{11}$$

where $\mathbf{y}_t$ is partitioned into a $K\times1$ vector $\mathbf{y}_{1t}$ and an $r\times1$ vector $\mathbf{y}_{2t}$ with $r = N - K$. $\boldsymbol{\varepsilon}_t$ is also similarly partitioned. The $K\times1$ vector $\boldsymbol{\mu}_t^{\dagger}$ and $r\times1$ vector $\bar{\boldsymbol{\mu}}_t$ follow multivariate random walk processes, with their disturbance vectors $\boldsymbol{\eta}_t^{\dagger}$ and $\bar{\boldsymbol{\eta}}_t$ having positive definite

[10] Complete parameter estimates of Model 4 are available upon request.

covariance matrices of $\boldsymbol{\Sigma}_{\boldsymbol{\eta}}^{\dagger}$ and $\bar{\boldsymbol{\Sigma}}_{\boldsymbol{\eta}}$, respectively. Finally, $\boldsymbol{\Theta}$ is an $r \times K$ matrix of coefficients.

If $\mathbf{y}_{1t}$ is cointegrated with $\mathbf{y}_{2t}$ in equation (11), there will be r linear combinations of the observations, $\mathbf{Ay}_t$, that are stationary. The rows of $\mathbf{A}$ constitute a set of r cointegrating vectors and can be partitioned as $\mathbf{A}=\left(\mathbf{A}_1, \mathbf{A}_2\right)$ with $\mathbf{A}_2$ being $r \times r$. Then

$$\mathbf{Ay}_t = \mathbf{A}_1\mathbf{y}_{1t} + \mathbf{A}_2\mathbf{y}_{2t} = (\mathbf{A}_1 + \mathbf{A}_2\boldsymbol{\Theta})\boldsymbol{\mu}_t^{\dagger} + \mathbf{A}_2\bar{\boldsymbol{\mu}}_t + \mathbf{A}_1\boldsymbol{\varepsilon}_{1t} + \mathbf{A}_2\boldsymbol{\varepsilon}_{2t}, \tag{12}$$

where $\mathbf{A}_1 + \mathbf{A}_2\boldsymbol{\Theta} = \mathbf{0}$ and $\bar{\boldsymbol{\Sigma}}_{\boldsymbol{\eta}} = \mathbf{0}$.

The $r \times N$ matrix $\mathbf{A}$ can be formed according to economic theory or through $\boldsymbol{\Theta}$. If we choose $\mathbf{A}$ such that $\mathbf{A}_1 + \mathbf{A}_2\boldsymbol{\Theta} = \mathbf{0}$, the test applied to $\mathbf{Ay}_t$ is the locally best invariant (LBI) test of the null hypothesis that $\bar{\boldsymbol{\Sigma}}_{\boldsymbol{\eta}} = 0$ against the alternative that $\bar{\boldsymbol{\Sigma}}_{\boldsymbol{\eta}}$ is proportional to $\boldsymbol{\Theta}\Sigma_{11}\boldsymbol{\Theta}' + \boldsymbol{\Theta}\Sigma_{12} + \Sigma_{21}\boldsymbol{\Theta}' + \Sigma_{22} = \mathbf{0}$, where the Σ_{ii}'s are the blocks of $\Sigma_{\boldsymbol{\varepsilon}}$. The test statistic is given by

$$\eta\left(r;\mathbf{A}\right) = \mathrm{tr}\left[\left(\mathbf{ASA}'\right)^{-1}\mathbf{ACA}'\right], \tag{13}$$

where

$$\mathbf{C} = T^{-2}\sum_{j=1}^{T}\left[\sum_{t=1}^{j}\left(\mathbf{y}_t - \bar{\mathbf{y}}\right)\right]\left[\sum_{t=1}^{j}\left(\mathbf{y}_t - \bar{\mathbf{y}}\right)\right]',$$

and

$$\mathbf{S} = T^{-1}\sum_{t=1}^{T}\left(\mathbf{y}_t - \bar{\mathbf{y}}\right)\left(\mathbf{y}_t - \bar{\mathbf{y}}\right)'.$$

The limiting distribution of $\eta\left(\mathrm{r};\mathbf{A}\right)$ is the Cramér-von Mises, $\mathrm{CvM}\left(r\right)$.

In this subsection, we wish to test the null hypothesis that $\sigma_{q,\eta} = 0$ against the alternative that $\sigma_{q,\eta} > 0$. The cointegrating vector $\mathbf{A}$ is formed such that $\mathbf{A}_1 + \mathbf{A}_2\boldsymbol{\Theta} = \mathbf{0}$, where

$\mathbf{\Theta} = [\theta_1, \theta_2, \theta_3]$ is estimated from Models 1 and 3.[11] $\mathbf{y_t} = [tot_t, pd_t, gov_t, q_t]'$ is partitioned into $\mathbf{y}_{1t} = [tot_t, pd_t, gov_t]'$ and $\mathbf{y}_{2t} = q_t$. If the null that $\sigma_{q,\eta} = 0$ cannot be rejected, $\mathbf{y}_{1t}$ is said to be cointegrated with $\mathbf{y}_{2t}$. As the coefficient of the permanent component of the productivity differential, θ_2, appears insignificant in all estimations in Section 4, we also test whether cointegration can be found between $\mathbf{y}_{1t} = [tot_t, gov_t]'$ and $\mathbf{y}_{2t} = q_t$. In this case, $\mathbf{A}$ is formed using $\mathbf{\Theta} = [\theta_1, \theta_3]$ estimated from Model 4 where θ_2 is restricted to zero.

Since the stationary components in our UC models are not white noise processes (i.e. the transitory components follow stationary AR(2) processes), the test statistic in equation (13) is modified to allow for serial correlation. The modification adapted here is in line with the KPSS test, which replaces $\mathbf{S}$ with a consistent estimator of the long-run variance

$$\mathbf{S}(m) = \sum_{\tau=-m}^{\tau=m} \omega_{\tau m} \hat{\Gamma}(\tau),$$

where $\hat{\Gamma}(\tau)$ is the sample autocovariance matrix at lag τ, that is

$$\hat{\Gamma}(\tau) = T^{-1} \sum_{t=\tau+1}^{T} (\mathbf{y}_t - \bar{\mathbf{y}})(\mathbf{y}_{t-\tau} - \bar{\mathbf{y}})',$$

and $\omega_{\tau m}$ is a weighting matrix function such as $\omega_{\tau m} = 1 - \tau/(1+m)$, $\tau = 1,..,m.$ Therefore, as with the KPSS test, the selection of lag length, m, may affect the conclusion reached. Although an increase in the lag length leads to a test closer to the desirable size, it sacrifices the power of the test. Therefore, we use $m \leq 4$ for the test statistics presented in Table 6. The tests cannot reject the null hypothesis that $\sigma_{q,\eta} = 0$ in Models 1 and 3 when $m \geq 2$, implying $\mathbf{y}_{1t} = [tot_t, pd_t, gov_t]'$ is cointegrated with $\mathbf{y}_{2t} = q_t$. In addition, there is weaker evidence that $\sigma_{q,\eta} = 0$ in Model 4 when the test statistics are adjusted for serial correlation with $m > 3$. This suggests that cointegration may be found amongst the real exchange rate, the terms of trade and the relative government expenditure ratio.

[11] We found that excluding the real interest rate differential from Model 2 to obtain Model 3 does not alter the parameter estimates in the rest of the model. Therefore, we only present the cointegration test using parameters estimates from Model 3.

{Table 6 about here}

5.2 Testing for a specified number of common trends

In this subsection, we performed a more general test outlined in Nyblom and Harvey (2000) to cross-check the above cointegration results. To do so, we first test whether $\mathbf{y}_{1t} = [tot_t, pd_t, gov_t]'$ is a set of variables that are indeed not cointegrated themselves. Second, we examine whether more cointegration relationships can be found in $\mathbf{y_t} = [tot_t, pd_t, gov_t, q_t]'$.

In these tests, we do not pre-specify the matrix $\mathbf{A}$. Instead, we test the null hypothesis that $\text{rank}\left(\mathbf{\Sigma_\eta}\right) = K$ against the alternative that $\text{rank}\left(\mathbf{\Sigma_\eta}\right) > K$ for $K < N$. The test statistic is given by

$$\xi_{K,N} = \lambda_{K+1} + ... + \lambda_N, \tag{14}$$

which is the sum of the $\left(N-K\right)$ smallest eigenvalues of $\mathbf{S}\left(m\right)^{-1}\mathbf{C}$. $\mathbf{C}$ and $\mathbf{S}\left(m\right)$ are defined as in equation (13), except that in this case they are formed from the OLS residuals from regressing $\mathbf{y_t}$ on the vectors of constants and time.

The test statistics and critical values tabulated from Nyblom and Harvey (2000) are presented in Table 7. The null hypothesis that $\text{rank}\left(\mathbf{\Sigma}_\mathbf{\eta}^\dagger\right) = 2$ against the alternative that $\text{rank}\left(\mathbf{\Sigma}_\mathbf{\eta}^\dagger\right) = 3$ amongst $\mathbf{y}_{1t} = [tot_t, pd_t, gov_t]'$ is strongly rejected at the 1% level. This confirms the validity of the first cointegration test. However, evidence of a single cointegrating vector amongst $\mathbf{y_t} = [tot_t, pd_t, gov_t, q_t]'$ appears to be much weaker than the previous test suggested, as the null hypothesis that $\text{rank}\left(\mathbf{\Sigma_\eta}\right) = 3$ cannot be rejected at the 1% level after the test statistics have been adjusted for serial correlation with $m \geq 4$.

{Table 7 about here}

5.3 Johansen cointegration test

Given the second Nyblom and Harvey test offers only weak support for the presence of one cointegrating vector among $\mathbf{y_t} = [tot_t, pd_t, gov_t, q_t]'$, Johansen's cointegration approach is performed to cross-check whether the variables are indeed cointegrated.[12] If cointegration is found using Johansen's test, we can then compare the coefficients on the cointegrating vector with θ_1, θ_2 and θ_3 estimated from the UC model.

The starting point of this approach is a VAR model of dimension four and order p, which can be written in vector-error correction (VEC) form as follows

$$\Delta \mathbf{y}_t = \mathbf{m} + \mathbf{\Pi(1)}\mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \mathbf{\Gamma}_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \ , \tag{15}$$

where $\mathbf{y}_t = [tot_t, pd_t, gov_t, q_t]'$ and $\mathbf{\Pi(1)}$ is the 4×4 long-run matrix that can be factored as $\alpha\beta'$ if $\mathbf{\Pi(1)}$ has reduced rank $r < 4$. $\mathbf{\Gamma}_i$ represents the 4×4 matrix of short-run coefficients, $\mathbf{m}$ is a 4×1 vector of constants, and $\boldsymbol{\varepsilon}_t$ denotes a 4×1 vector of white noise residuals. If there are r linear combinations of the variables in $\mathbf{y}_t$ that yield stationary series, $\mathbf{y}_t$ is said to have $k = 4 - r$ common trends.

We set p equal to two and the constant is restricted to the cointegrating space. The trace test statistics, reported in Table 8, indicate the presence of a single cointegrating vector at the 1% level. This offers much stronger support for cointegration than the second Nyblom and Harvey (2000) test. Therefore, we set the cointegration rank equal to one and a standard set of long-run exclusion and weak exogeneity tests are conducted. The exclusion test indicates that the relative productivity variable can be excluded from the long-run relationship given that $\chi^2(1) = 0.231$. This is broadly in line with the results from the UC model which found that this variable does not have a permanent impact on the real exchange rate. In addition, the relative productivity and terms of trade variables are also found to be weakly exogenous with the joint test statistic $\chi^2(3) = 1.22$. Consistent with the result from the UC models, a significant and positive long-run relationship is found between the terms of trade and the real exchange rate. However, the positive relationship identified between the relative government expenditure ratio and the real exchange rate seems to contradict the conclusion drawn from UC models that a higher government expenditure ratio will eventually lead to a depreciation of the real exchange rate. Nevertheless, this contradiction may be attributed to the UC model separating the negative and permanent effect of the relative government expenditure ratio

[12] We treat all non-stationary variables as $I(1)$ processes in the VAR model, as suggested by the ADF tests.

from its potentially positive transitory impact on the real exchange rate, while the long-run coefficient obtained from the VECM just reflects the average of these two effects. In addition, the adjustment term for the real exchange rate is negative and significant. This implies that the real dollar-euro exchange rate is one of the variables in the system that adjusts to exogenous shocks. However, the speed of adjustment is found to be slow, the half-life of deviations from equilibrium being about three years.[13]

{Table 8 about here}

Finally, we calculate the PEER and the total misalignment using the Granger and Gonzalo (1995) decomposition. The results are plotted against the corresponding components obtained from Model 3 in Figure 3. Although these models use the same set of fundamentals to track the long-run movement of the dollar-euro exchange rate, they produce significantly different results. The PEER estimated from Model 3 indicates that the euro is closer to its fundamental value than the cointegration-based approach implied. Despite the differences observed, both models suggest that the euro was undervalued against the dollar during the mid-1980s and early 2000s.

{Figure 3 about here}

6 Out-of-sample forecasting

Ever since Meese and Rogoff (1983), conditional out-of-sample forecasting has become a standard procedure for testing the validity of exchange rate models. In this section, we compare the out-of-sample forecasting ability of multivariate UC models 1, 3 and 4 with the random walk process of the real exchange rate.[14] To conduct a transparent comparison between UC models with the random walk process, we first compare the out-of-sample forecasting ability of a univariate UC model of the real exchange rate, specified as follows

[13] The half-life is computed as log(0.5)/log(1- α), where α is the adjustment term in the equation for the real exchange rate.

[14] Model 2 is not considered here as it has the same exchange rate equation as Model 3 specified in equation (11).

$$
\begin{aligned}
&q_t = q_t^T + q_t^C \\
&q_t^T = m_q + q_{t-1}^T + \eta_{q,t}, \qquad \eta_{q,t} \sim \text{NID}\left(0, \sigma_{\eta,q}^2\right) \\
&q_t^C = \phi_{q,1} q_{t-1}^C + \phi_{q,2} q_{t-2}^C + k_{q,t}, \quad k_{q,t} \sim \text{NID}\left(0, \sigma_{k,q}^2\right)
\end{aligned}
\tag{16}
$$

where the real exchange rate is decomposed into a permanent $\left(q_t^T\right)$ and a transitory component $\left(q_t^C\right)$. If the univariate UC model is no better than the random walk process in terms of forecasting, while the multivariate UC models are better, this suggests that inclusion of the fundamentals helps to predict future real exchange rates. A rolling sample approach is used, with the full-sample period first divided into a pre-forecasting period from 1975Q1 to 1995Q4 and a forecasting period from 1996Q1 to 2008Q4. Although the choice of 1996Q1 is ad hoc, it provides a sufficiently large sample for initial estimation and for evaluating out-of-sample forecasting performances of the multivariate UC models. The pre-forecasting sample moves forward quarter by quarter and the model's hyperparameters are re-estimated at each step until the end of the sample is reached. In total, 53 one-quarter-ahead forecasts and 42 twelve-quarter-ahead forecasts are calculated.

Table 9 reports the ratios of root-mean-squared errors (RMSE) of both the univariate and multivariate UC models relative to the random walk process. One striking result revealed from Table 9 is that the relative RMSEs of the univariate UC model with respect to the random walk process are very close to one and remain relatively constant across different forecasting horizons. However, for multivariate UC models, the longer the forecasting horizons, the smaller the RMSE produced by these models relative to the random walk process. Diebold and Mariano's (1995, DM henceforth) test of equal forecast accuracy is preformed to determine whether differences in forecasting errors between a UC model and the random walk process are significant. The DM statistic is specified as

$$
DM = \bar{d} \Big/ \sqrt{\sigma_d^2} \, ,
$$

where $\bar{d}$ is the sample mean of a differential loss function, such that $\bar{d} = n^{-1} \sum_{j=1}^{n} d_j = n^{-1} \sum_{j=1}^{n} (e_{\mathrm{A},j}^2 - e_{\mathrm{B},j}^2)$, where $e_{\mathrm{A},j}$ and $e_{\mathrm{B},j}$ are the $j-$th, $h-$step-ahead forecast errors obtained from models A and B. σ_d^2 is the variance of $\bar{d}$ estimated using the heteroskedastic-autocorrelation consistent (HAC) estimator.

The values of the DM statistic with the small sample modification proposed by Harvey, Leybourne and Newbold (1997, HLN henceforth) are also calculated as:

$$DM^* = DM\left\{\left[n+1-2H+H\left(H-1\right)/n\right]/n\right\}^{1/2},$$

and reported in Table 9, where n and H denote the number of forecasts and the forecast horizon respectively.

Both the DM and HLN statistics are calculated under the null hypothesis that the UC model is equivalent in forecasting accuracy to the random walk process. The alternative hypothesis varies depending on the sign of the test statistic. If the sign on the test statistic is positive, the alternative hypothesis is that the multivariate model is better than the random walk process in terms of forecasting accuracy. If the test statistic has a negative sign, the alternative hypothesis is inverted. The calculated statistics are compared to the critical values of the Student's t-distribution with $n-1$ degree of freedom.

The test statistics highlighted in bold indicate that the null hypothesis of equivalent forecasting accuracy is rejected. It can be seen that none of the null hypotheses that the univariate UC model is equivalent in forecasting accuracy to the random walk process can be rejected. However, Model 1 (the multivariate UC model) is significantly better in terms of forecasting future exchange rates than the random walk process from eight-quarter-ahead forecasting horizons onwards. This indicates that the multivariate models which include the macroeconomic fundamentals will help to predict the long-run movement of real exchange rates. Furthermore, Model 3, which allows the transitory component of the real exchange rate to be modelled independently to follow an AR(2) process, improves the short-run forecasting accuracy with respect to Model 1. It is also interesting to note that when the productivity differential is restricted to have zero impact on the real exchange rate in Model 4, the quadratic loss differential sequence, d_j, which is calculated using the forecasting errors produced by this model and the random walk process, presents a stronger autocorrelation pattern, and in turn a larger value of σ_d^2. This partly leads to less significant test statistics than those of Model 3.

{Table 9 about here}

7 Conclusions

This paper proposes an alterative approach to estimating the PEER based on a UC model specification. We believe our approach offers a number of advantages over the conventional cointegration-based PEER proposed by Clark and MacDonald (2004). First, we do not rely on the prerequisite that cointegration has to be found among the real exchange rate and macroeconomic fundamentals to obtain non-spurious long-run relationships and estimates of the PEER. Instead, in the UC model specifications, an unobserved random walk process is used to capture any missed variables from the cointegration relationship. This allows the estimation of the long-run relationships between the integrated variables using maximum likelihood, and the use of the likelihood ratio test to identify the significance of these long-run coefficients, even if cointegration is rejected.

Second, the impact that the permanent and transitory components of the macroeconomic fundamentals have on the real exchange rate can be modelled separately in the UC model. This is important for variables where the long and short-run effects may drive the real exchange rate in opposite directions, such as the relative government expenditure ratio. However, the long-run coefficient on the cointegrating vector estimated using the VECM just reflects the average of these two effects. This is illustrated in subsection 5.3.

In addition, although the UC model outlined above does not require cointegration amongst the real exchange rate and the fundamentals as a prerequisite for obtaining the PEER, the UC model can also accommodate the cointegration analysis as outlined in the first two subsections of Section 5. Following Nyblom and Harvey (2000), using the pre-specified cointegrating vector formed from the estimated UC models, we found that the real exchange rate is cointegrated with the macroeconomic fundamentals.

Finally, a forecasting exercise was conducted to test the validity of our multivariate UC models against a random walk process of the real exchange rate. In general, the longer the forecasting horizons, the smaller the RMSE produced by the multivariate UC models relative to the random walk process. As suggested by the DM and HLN statistics, Model 1 is significantly better at forecasting future exchange rates than the random walk process from eight-quarter-ahead periods onwards. This indicates that the multivariate models which include the macroeconomic fundamentals will help to predict the long-run movement of real exchange rates. However, Model 3, which allows for the transitory component of the real exchange rate to be modelled independently to follow an AR(2) process, considerably improves the short-run forecasting accuracy with respect to Model 1. In short, we demonstrate that the method proposed in this paper can be a useful technique for central banks to estimate the equilibrium exchange rate and to predict long-run exchange rate movements.

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Table 1: Unit root tests (1975Q1-2008Q4)

	ADF		***KPSS***			
	Variables in level					
	Constant	*Constant +Trend*	*Constant*		*Constant +Trend*	
			KPSS(4)	*KPSS(12)*	*KPSS(4)*	*KPSS(12)*
LQ	-2.443	-2.485	0.196	0.102	0.121*	0.063
LTOT	-1.867	-3.019	1.447***	0.684**	0.123*	0.066
LPD	-1.069	-1.378	0.646**	0.287	0.572***	0.256***
LGOV	-2.839*	-1.759	2.054***	0.886***	0.339***	0.166**
IRD	-3.765***	-3.809**	0.118	0.068	0.113	0.065
	Variables in first differences					
	Constant	*Constant +Trend*	*Constant*		*Constant +Trend*	
DLQ	-8.246***	-8.212***	0.054	0.054	0.048	0.048
DLTOT	-9.045***	-8.991***	0.037	0.046	0.036	0.046
DLPD	-9.513***	-9.946***	0.558**	0.470**	0.038	0.044
DLGOV	-11.479***	-11.915***	0.457*	0.442*	0.065	0.083
DIRD	-8.365***	-8.362***	0.059	0.056	0.045	0.043

Notes: the ADF tests use a log length of 4, while KPSS test uses lag truncation parameters of 4 and 12; Significant test statistics are marked using stars with *, ** and *** denoting the 10%, 5% and 1% significance level respectively.

Table 2: Parameter estimates of Model 1

Coefficients on the exchange rate equation							
θ_1	θ_2	θ_3	θ_4	θ_{10}	θ_{20}	θ_{30}	θ_{40}
2.572***	0.320	-1.081***	0.000	-1.213	-4.703	3.993	1.249
(0.516)	(0.467)	(0.288)	-	(1.144)	(3.241)	(4.797)	(1.067)
AR terms, constant drifts and damping factor							
$\phi_{1,tot}$	$\phi_{2,tot}$	$\phi_{1,pd}$	$\phi_{2,pd}$	$\phi_{1,gov}$	$\phi_{2,gov}$	$\phi_{1,rid}$	$\phi_{2,rid}$
0.946***	-0.260**	1.399***	-0.561**	1.926***	-0.946***	1.651***	-0.708***
(0.132)	(0.105)	(0.283)	(0.270)	(0.120)	(0.116)	(0.049)	(0.046)
β_{tot}	β_{ird}	ρ_{rid}					
0.001	0.004	0.365					
(0.001)	(0.003)	(0.322)					
Standard deviations of the shocks							
$\sigma_{\eta,tot}$	$\sigma_{\kappa,tot}$	$\sigma_{\eta,pd}$	$\sigma_{\kappa,pd}$	$\sigma_{\xi,pd}$	$\sigma_{\eta,gov}$	$\sigma_{\kappa,gov}$	$\sigma_{\xi,gov}$
1.345	0.902	0.713	0.289	0.041	1.178	0.057	0.054
(0.200)	(0.253)	(0.092)	(0.176)	(0.028)	(0.120)	(0.039)	(0.047)
$\sigma_{\eta,rid}$	$\sigma_{\kappa,rid}$	$\sigma_{q,\eta}$	$\sigma_{\varepsilon,q}$				
0.251	0.303	0.011	0.008				
(0.073)	(0.071)	(0.370)	(0.443)				
Dummy variables							
D80q1	*D08q3*						
-0.013***	0.040***						
(0.003)	(0.011)						

Likelihood ratio tests

$H_0:\theta_1=0$, $\text{LogL}^R=1980.80$, $\chi^2(1)=25.639$***

$H_0:\theta_2=0$, $\text{LogL}^R=1993.35$, $\chi^2(1)=0.536$

$H_0:\theta_3=0$, $\text{LogL}^R=1985.75$, $\chi^2(1)=15.734$***

$H_0:\theta_{10}=0$, $\text{LogL}^R=1993.13$, $\chi^2(1)=0.974$

$H_0:\theta_{20}=0$, $\text{LogL}^R=1992.47$, $\chi^2(1)=2.294$

$H_0:\theta_{30}=0$, $\text{LogL}^R=1992.92$, $\chi^2(1)=1.394$

$H_0:\theta_{40}=0$, $\text{LogL}^R=1993.20$, $\chi^2(1)=0.834$

Residual diagnostics

LogL	$Q(12)_{tot}$	$Q(12)_{pd}$	$Q(12)_{rid}$	$Q(12)_{gov}$	$Q(12)_q$
1993.617	12.689	15.863	13.449	10.765	10.267
JB_{tot}	JB_{pd}	JB_{rid}	JB_{gov}	JB_q	
3.507	0.091	0.955	1.842	2.586	

Notes: Numbers in the parentheses are standard errors computed using the delta method; The standard deviations of variations of variances parameters are multiplied by 100; *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively.

Table 3: Parameter estimates of Model 2

Coefficients on the exchange rate equation

θ_1	θ_2	θ_3	θ_4
3.034***	0.507	-1.251**	0.000
(1.166)	(0.794)	(0.617)	-

AR terms, constant drifts and damping factor

$\phi_{1,tot}$	$\phi_{2,tot}$	$\phi_{1,pd}$	$\phi_{2,pd}$	$\phi_{1,gov}$	$\phi_{2,gov}$	$\phi_{1,rid}$	$\phi_{2,rid}$
0.994***	-0.273**	1.122***	-0.183**	1.453***	-0.490**	1.658***	-0.715***
(0.132)	(0.142)	(0.095)	(0.087)	(0.269)	(0.256)	(0.108)	(0.104)
$\phi_{1,q}$	$\phi_{2,q}$	β_{tot}	β_{ird}	ρ_{rid}			
1.340***	-0.428*	0.001	0.004	0.337			
(0.272)	(0.258)	(0.001)	(0.003)	(0.277)			

Standard deviations of the shocks

$\sigma_{\eta,tot}$	$\sigma_{\kappa,tot}$	$\sigma_{\eta,pd}$	$\sigma_{\kappa,pd}$	$\sigma_{\xi,pd}$	$\sigma_{\eta,gov}$	$\sigma_{\kappa,gov}$	$\sigma_{\xi,gov}$
1.105	1.160	0.014	0.772	0.039	0.868	0.579	0.054
(0.221)	(0.243)	(0.508)	(0.051)	(0.022)	(0.283)	(0.272)	(0.040
$\sigma_{\eta,rid}$	$\sigma_{\kappa,rid}$	$\sigma_{q,\kappa}$	$\sigma_{q,\eta}$	$\sigma_{\varepsilon,q}$			
0.250	0.299	2.179	0.028	0.007			
(0.061)	(0.062)	(1.366)	(0.509)	(1.283)			

Dummy variables

D80q1	*D08q3*
-0.013***	0.039***
(0.003)	(0.010)

Likelihood ratio tests

$H_0:\theta_1=0$, LogL^R =1974.555, $\chi^2(1) = 35.660$***

$H_0:\theta_2=0$, LogL^R =1992.159, $\chi^2(1) = 0.452$

$H_0:\theta_3=0$, LogL^R =1988.013, $\chi^2(1) = 8.744$***

Residual diagnostics

LogL	$Q(12)_{tot}$	$Q(12)_{pd}$	$Q(12)_{rid}$	$Q(12)_{gov}$	$Q(12)_q$
1992.385	12.758	16.365	10.265	10.770	9.663
JB_{tot}	JB_{pd}	JB_{rid}	JB_{gov}	JB_q	
3.309	0.045	0.774	1.661	2.049	

Notes: Numbers in the parentheses are standard errors computed using the delta method; The standard deviations of variations of variances parameters are multiplied by 100; *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively.

Table 4: Parameter estimates of Model 3

Coefficients on the exchange rate equation							
θ_1	θ_2	θ_3					
3.059***	0.519	-1.263**					
(1.210)	(0.804)	(0.636)					
AR terms, constant drifts and damping factor							
$\phi_{1,tot}$	$\phi_{2,tot}$	$\phi_{1,pd}$	$\phi_{2,pd}$	$\phi_{1,gov}$	$\phi_{2,gov}$	$\phi_{1,q}$	$\phi_{2,q}$
0.994***	-0.271**	1.122***	-0.183**	1.453***	-0.490**	1.348***	-0.435
(0.132)	(0.142)	(0.095)	(0.087)	(0.267)	(0.254)	(0.298)	(0.283)
β_{tot}							
0.001							
(0.001)							
Standard deviations of the shocks							
$\sigma_{\eta,tot}$	$\sigma_{\kappa,tot}$	$\sigma_{\eta,pd}$	$\sigma_{\kappa,pd}$	$\sigma_{\xi,pd}$	$\sigma_{\eta,gov}$	$\sigma_{\kappa,gov}$	$\sigma_{\xi,gov}$
1.101	1.164	0.004	0.773	0.039	0.864	0.581	0.053
(0.226)	(0.248)	(0.627)	(0.050)	(0.022)	(0.285)	(0.272)	(0.040)
$\sigma_{q,\kappa}$	$\sigma_{q,\eta}$	$\sigma_{\varepsilon,q}$					
2.144	0.001	0.007					
(1.440)	(0.116)	(0.910)					
Dummy variables							
D08q3							
0.039***							
(0.010)							
Likelihood ratio tests							
$H_0:\theta_1=0$, $\text{LogL}^R=1446.051$, $\chi^2(1)=35.744$***							
$H_0:\theta_2=0$, $\text{LogL}^R=1463.698$, $\chi^2(1)=0.450$							
$H_0:\theta_3=0$, $\text{LogL}^R=1459.886$, $\chi^2(1)=8.074$***							
Residual diagnostics							
LogL	$Q(12)_{tot}$	$Q(12)_{pd}$	$Q(12)_{gov}$	$Q(12)_q$			
1463.923	12.821	16.363	10.263	9.661			
JB_{tot}	JB_{pd}	JB_{gov}	JB_q				
3.307	0.045	0.776	2.045				

Notes: Numbers in the parentheses are standard errors computed using the delta method; The standard deviations of variations of variances parameters are multiplied by 100; *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively.

Table 5: Parameter estimates of Model 4

Coefficients on the exchange rate equation				
θ_1	θ_2	θ_3		
2.749***	0.000	-1.076**		
(0.853)	-	(0.451)		
Likelihood ratio tests				
$H_0:\theta_1$ =0, LogL^R =1980.80, $\chi^2(1)$ = 39.450***				
$H_0:\theta_3$ =0, LogL^R =1985.75, $\chi^2(1)$ = 7.892***				
Residual diagnostics				
LogL	$Q(12)_{tot}$	$Q(12)_{pd}$	$Q(12)_{gov}$	$Q(12)_q$
1463.698	12.188	16.506	10.375	9.649
JB_{tot}	JB_{pd}	JB_{gov}	JB_q	
3.524	0.085	0.758	2.363	

Notes: Numbers in the parentheses are standard errors computed using the delta method; The standard deviations of variations of variances parameters are multiplied by 100; *, ** and *** indicate significance at the 10%, 5% and 1% level, respectively.

Table 6: Nyblom and Harvey (2000) test for pre-specified cointegrating vectors

	m=0	*m=1*	*m=2*	*m=3*	*m=4*
Model 1	0.399*	0.278	0.222	0.186	0.164
Model 3	0.514**	0.361*	0.291	0.246	0.220
Model 4	0.767***	0.533**	0.426*	0.356*	0.313

Note: the critical values with 1 degree of freedom at the 10%, 5% and 1% level are 0.347, 0.461 and 0.743, respectively; Significant test statistics are marked using stars with *, ** and *** denoting the 10%, 5% and 1% significance level respectively.

Table 7: Nyblom and Harvey's (2000) test for common trends

Hypotheses		***Test statistics***					***Critical values***		
H_0	H_1	*m=0*	*m=1*	*m=2*	*m=4*	*m=6*	*10%*	*5%*	*1%*
Testing cointegration amongst LTOT, LPD and LGOV									
K=2	*K=3*	0.384***	0.267***	0.212***	0.155***	0.130***	*0.061*	*0.075*	*0.113*
K=1	*K=2*	1.015***	0.704***	0.558***	0.403***	0.333***	*0.151*	*0.180*	*0.245*
Testing cointegration amongst LTOT, LPD, LGOV and LQ									
K=3	*K=4*	0.162***	0.117***	0.097***	0.077**	0.069**	*0.046*	*0.055*	*0.081*
K=2	*K=3*	0.651***	0.451***	0.358***	0.262***	0.222***	*0.11*	*0.128*	*0.176*
K=1	*K=2*	1.285***	0.888***	0.703***	0.507***	0.419***	*0.215*	*0.246*	*0.321*

Note: Significant test statistics are marked using stars with *, ** and *** denoting the 10%, 5% and 1% significance level respectively.

Table 8: Parameter Estimates of the VECM

		Cointegration test			
r	*Trace*	*Trace[a]*			
None	68.427***	64.60***			
1	26.529	21.228			
		Selected parameter estimates			
	LQ	*LTOT*	*LPD*	*LGOV*	*constant*
Long-run coefficients	1.000	-1.598***	0.000	-0.713**	5.995***
	-	[-4.09]	-	[-2.36]	[3.75]
Adjustment terms	-0.062**	0.000	0.000	0.045***	
	[-2.10]	-	-	[5.33]	
		Multivariate residual diagnostics			
LogL		2267.740		2267.132	
LM(8)		22.538 [0.12]		22.764[0.12]	
LM(12)		16.816 [0.40]		17.849[0.33]	
NM(8)		12.949 [0.11]		13.375[0.10]	

Notes: [a]Trace statistic is adjusted for small sample bias according to Reimers (1992); Numbers in the parentheses are t-statistics; LM(8) and LM(12) are multivariate Godfrey (1991) LM-type statistics for the eighth and twelfth-order autocorrelation; NM(8) is Doornik and Hansen's (1994) multivariate normality test; *, ** and *** denote the 10%, 5% and 1% significance level respectively.

Table 9: Out-of-Sample Forecasts (1996-2008)

	H=1	H=2	H=4	H=6	H=8	H=9	H=10	H=11	H=12
				Relative Root-Mean-Squared Errors					
				Univariate UC model					
Vs. RW	0.951	0.969	0.931	0.935	0.941	0.934	0.933	0.930	0.919
DM	1.05	0.91	1.02	0.82	0.65	0.62	0.58	0.60	0.66
HLN	1.04	0.88	0.95	0.73	0.54	0.50	0.45	0.45	0.48
				Model 1					
Vs. RW	0.956	0.956	1.000	0.933	0.808	0.789	0.780	0.769	0.748
DM	0.60	0.44	0.37	1.15	**2.14****	**2.43*****	**2.55*****	**2.62*****	**2.75*****
HLN	0.59	0.45	0.35	1.02	**1.79****	**1.97****	**2.00****	**1.98****	**2.00****
				Model 3					
Vs. RW	0.932	0.897	0.826	0.760	0.724	0.711	0.706	0.701	0.701
DM	1.58	**2.13****	**1.88****	**2.27****	**2.66*****	**2.76*****	**2.85*****	**2.88*****	**2.90*****
HLN	1.57	**2.06****	**1.75****	**2.01****	**2.22****	**2.24****	**2.23****	**2.18****	**2.11****
				Model 4					
Vs. RW	0.947	0.930	0.874	0.809	0.767	0.753	0.747	0.742	0.745
DM	1.21	1.42	1.29	**1.71****	**2.09****	**2.19****	**2.26****	**2.28****	**2.28****
HLN	1.20	1.38	1.20	1.52	**1.75****	**1.77****	**1.77****	**1.72****	**1.66****
				Root-Mean-Squared Errors of the random walk process					
RW	0.041	0.064	0.102	0.138	0.169	0.181	0.193	0.201	0.209

Note: the first row of each model (***Vs. RW***) denotes the relative RMSE of the multivariate model with respect to the RW model; The rows (*DM and HLN*) present the *DM* and *HLN* statistics; ** and *** indicate significance at the 5% and 1% level, respectively.

Figure 1: Unobserved components from Model 1

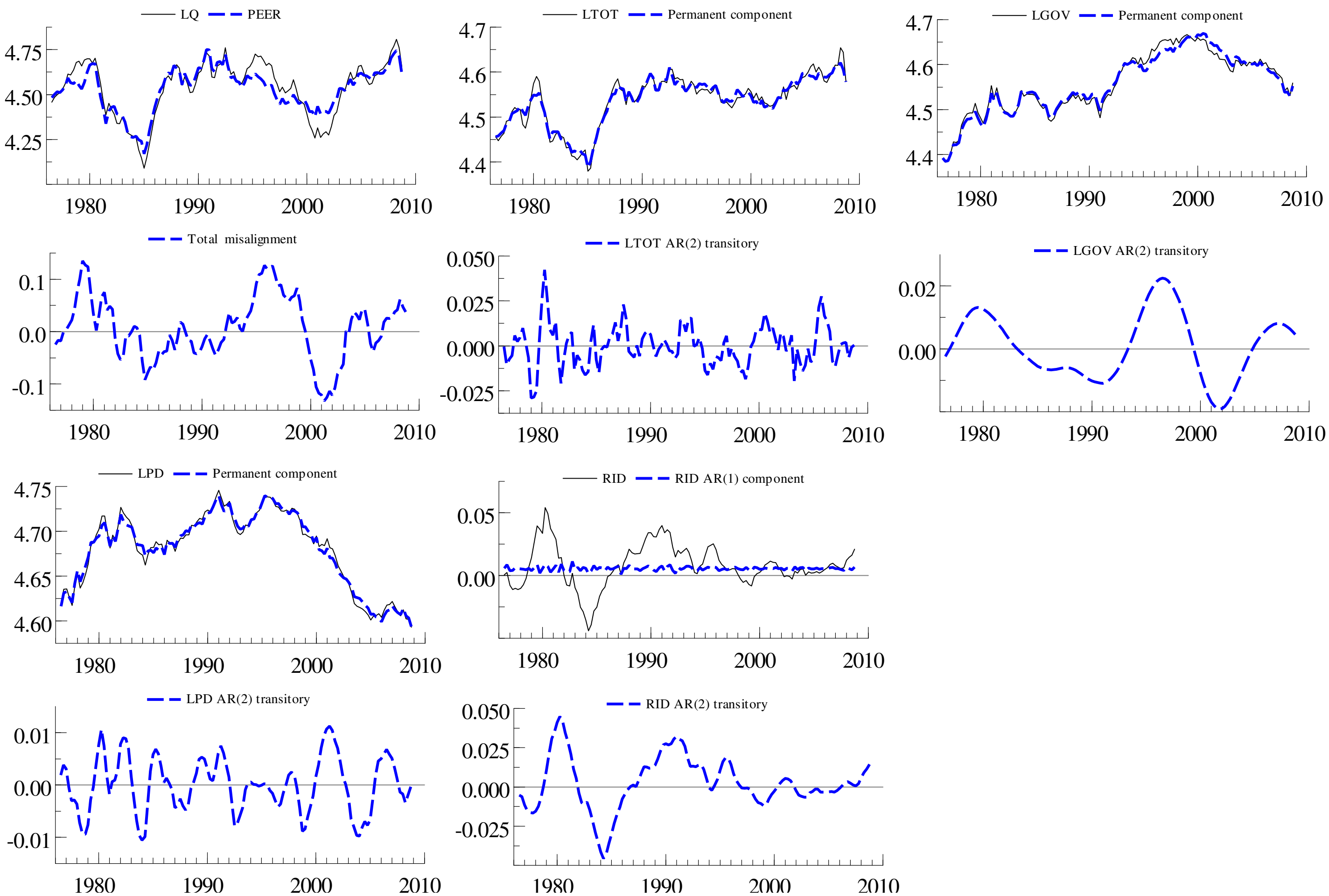

Figure 2: Unobserved components from Model 2

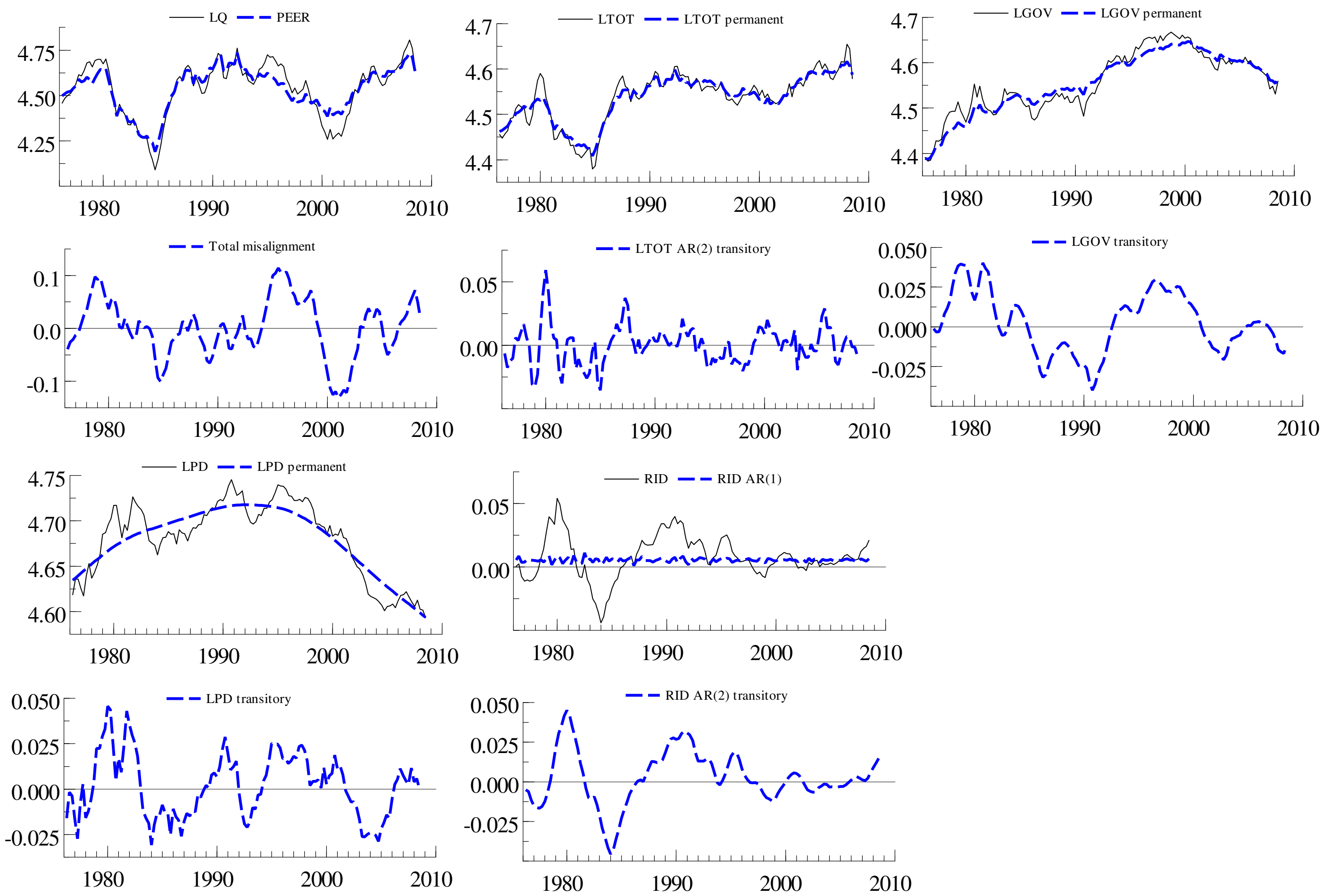

Figure 3: The estimates of PEER and total misalignment obtained from the VECM-based approach and Model 3