A Bootstrap Neural Network Based Heterogeneous Panel Unit Root Test: Application to Exchange Rates

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Abstract

This paper proposes a bootstrap artificial neural network based panel unit root test in a dynamic heterogeneous panel context. An application to a panel of bilateral real exchange rate series with the US Dollar from the 20 major OECD countries is provided to investigate the Purchase Power Parity (PPP). The combination of neural network and bootstrapping significantly changes the findings of the economic study in favour of PPP.

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1 Introduction

The standard linear autoregressive framework used to test for unit roots in time series is increasingly viewed to be unsatisfactory and, as a result, alternative frameworks within which to test for unit roots are considered. Im et al. [2003] propose a panel testing procedure, denoted IPS test, based on averaging individual unit root test statistics. Blake and Kapetanios [2003] extend the unit root test proposed by Caner and Hansen [2000], Kapetanios and Shin [2000] using artificial neural network. de Peretti et al. [2009] propose

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a panel unit root test, denoted PSC test, which is based on the averages of the individual Blake and Kapetanios [2003] statistics. However, these tests suffer from size distortion.

Then, this paper extends the PSC test by using bootstrap techniques to account for size distortion. The performance of the new bootstrap neural test is investigated and compared to IPS test via Monte Carlo experiments in section 3. An application to a panel of bilateral real exchange rate series with the US Dollar from the 20 major OECD countries is provided to examine the Purchase Power Parity (PPP) property in section 4. The procedures and programs to run the tests and to generate simulated data, written in Gauss software, as well as the exchange rates dataset are available at: http://recherche.univ-lyon2.fr/eric/82-Carole-Siani.html. Section 5 provides some concluding remarks.

2 Bootstrap test

de Peretti et al. [2009] propose several test statistics for panel unit root, depending on whether the sample period T and/or the number of cross sections N is/are large, and the degree of heterogeneity in the number of hidden neural nodes across sections. Let any of these test statistics be denoted by τ.

2.1 Bootstrap DGP under the null hypothesis

The original data are estimated under the null hypothesis, that is an independent panel of unit root processes:

$$y_{it} = y_{i,t-1} + \varepsilon_{it}, \qquad i = 1, \ldots, N; \quad t = 1, \ldots, T, \tag{1}$$

Deterministic regressors, such as constant term or trend, can be included in this regression. In the case of autocorrelated error terms:

$$\varepsilon_{it} = \rho_{i1}\varepsilon_{i,t-1} + \ldots + \rho_{i,p_i}\varepsilon_{i,t-p_i} + \xi_{it}, \tag{2}$$
$$\xi_{it} \sim \text{independent } N(0, \sigma_i^2). \tag{3}$$

the unknown parameters $(\rho_{ij})_{j=1}^{p_i}$ and σ_i^2 are estimated by maximum likelihood. Call these estimates $(\hat{\rho}_{ij})_{j=1}^{p_i}$ and $\hat{\sigma}_i^2$. The optimal number of lags p_i can be assessed using the AIC criterion. Define it as $\hat{p}_i$.

2.2 Simulating bootstrap samples

Simulate B samples, denoted $(y_{it}^b)_{i=1,\ldots,N;t=1,\ldots,T}$ for $b = 1, \ldots, B$:

$$y_{it}^b = y_{i,t-1}^b + \varepsilon_{it}^b, \qquad i = 1, \ldots, N; \quad t = 2, \ldots, T; \tag{4}$$
$$y_{i1}^b = y_{i1}, \qquad i = 1, \ldots, N; \tag{5}$$
$$\varepsilon_{it}^b = \hat{\rho}_{i1}\varepsilon_{i,t-1}^b + \ldots + \hat{\rho}_{i,p_i}\varepsilon_{i,t-p_i}^b + \xi_{it}^b, i = 1, \ldots, N; t = -\infty, \ldots, T; \tag{6}$$

The process ε_{it}^b is an autoregressive (AR) process. Since we cannot generate the process from $t = -\infty$, we proceed as follows:

1. Define a negative integer M small enough (in practice -25 is enough),

2. $\varepsilon_{iM}^b = \xi_{iM}^b$,
3. $\varepsilon_{i,M+1}^b = \hat{\rho}_{i1}\varepsilon_{i,M}^b + \xi_{i,M+1}^b$,
4. $\varepsilon_{i,M+2}^b = \hat{\rho}_{i1}\varepsilon_{i,M+1}^b + \hat{\rho}_{i,2}\varepsilon_{i,M}^b + \xi_{i,M+2}^b$,
5. $\ldots$,
6. $\varepsilon_{i,M+p_i}^b = \hat{\rho}_{i1}\varepsilon_{i,M+p_i-1}^b + \ldots + \hat{\rho}_{i,p_i}\varepsilon_{i,M}^b + \xi_{iM+p_i}^b$,
7. the following ε_{it}^b are generated according Equation 6.

M has to be small enough to allow the AR processes ε_{it}^b to follow their stationary probabilistic distribution (otherwise, the null hypothesis can be rejected even if true). We consider four different ways of generating the residuals ξ_{it}^b: parametric and three three nonparametric bootstrap methods (see Davidson [1998] for details).

2.3 Bootstrap P value

For each simulated sample, the test statistic is computed. Let denote it τ_b. The important point is that the optimal number of hidden nodes in step 1 is not imposed. Indeed the optimal number of hidden nodes is re-assessed using the AIC for each new simulated sample.

The bootstrap P value is then computed as follows:

$$pv = \frac{1}{B}\sum_{b=1}^{B} I(\tau_b > \tau),$$

where τ is computed on the original sample of panel data. For a test at significance level α, the null hypothesis of unit roots is then rejected if $\widehat{pv} < \alpha$.

3 Monte Carlo Experiments

We compare the performance of the asymptotic and bootstrapped neural test with the performance of the IPS test. [1] A detailed Monte Carlo study is provided in an on line appendix [2], using the graphical methods [3] of Davidson and MacKinnon [1998] for investigating the size and the power of hypothesis tests.

3.1 Simulation under the null hypothesis

Simulated samples are generated as follows:

$$\begin{aligned}
y_{it} &= y_{i,t-1} + \varepsilon_{it}, \\
\varepsilon_{it} &= \rho_i \varepsilon_{i,t-1} + e_{it}, \\
e_{it} &\sim i.i.d.N(0, \sigma_i^2), \\
\sigma_i &\sim i.i.d.U[0.5, 1.5],
\end{aligned}$$

Table 1: Size of the tests at the 5% significance level

				Test type / Statistic			
				Asymptotic		Bootstrap	
DGP	ρ_i	T	N	IPS	Neural	IPS	Neural
Null	0	20	5	0.0482	0.0467	0.0530	0.0500
Null	0	20	20	0.0500	0.0354	0.0590	0.0550
Null	0	100	5	0.0476	0.0661	0.0500	0.0500
Null	0	100	20	0.0478	0.0637	0.0500	0.0500
Null	i.i.d.U[0.2,0.4]	20	5	0.1046	0.0854	0.0490	0.0530
Null	i.i.d.U[0.2,0.4]	50	5	0.0632	0.0760	0.0370	0.0480
Null	i.i.d.U[-0.4,-0.2]	20	5	0.0924	0.0572	0.0410	0.0360
Null	i.i.d.U[-0.4,-0.2]	50	5	0.0537	0.0554	0.0560	0.0580

for $i = 1, \ldots, N$ and $t = 1, \ldots, T$. Table 1 presents the test sizes at the 5% significance level. From Table 1 and on line appendix, it is found that in all the cases, the bootstrap tests display almost no size distortion.

3.2 Simulation under the alternative hypothesis

On a significance level correction basis, the power of bootstrap tests should the same as the power of their associated asymptotic tests (see Davidson and MacKinnon [1998]). Thus a power analysis is not required. Some Monte Carlo results using SETAR models are provided in the on line appendix. In the case of serial correlated error terms, the neural test clearly dominates.

4 Application to a panel of bilateral real exchange rates

We apply the bootstrapped IPS and neural panel unit root tests to real exchange rates against the US dollar for twenty OECD countries over the period 1973Q1–1998Q2. The dataset is the same used by Murray and Papell [2002, 2005] so that the results can be compared. Most of the studies show evidence of unit root behaviour in real exchange rates, which has become a puzzle in international finance.

The results are shown in Table 2. The contrast between the two panel statistics is rather strong. IPS test fails to clearly reject the unit root null at all levels of significance: the P values (asymptotic and bootstrap) are close to the 5% level limit, and thus may imply non-mean reversion in the whole panel of real exchange rates. Our bootstrap neural test rejects the null hypothesis of panel unit root at all reasonable significance levels, giving support to the long-run PPP for the whole panel of OECD countries. This evidence of nonlinear mean reversion in the OECD real exchange rates may suggest that

[1]To compare both the test on the same basis, the IPS test is also bootstrapped.

[2]Available at: http://recherche.univ-lyon2.fr/eric/82-Carole-Siani.html.

[3]Size plots, size discrepancy plots, and power-corrected size curves

Table 2: P values for the panel Unit Root Tests for Real Dollar Exchange Rates

Test	Asymptotic	Bootstrap
IPS	0.0413 *	0.0486 *
ANN	0.0248 *	0.0010 ***

* P value significant at 5% level,
*** P value significant at 1% level.

previous evidence in the literature of non-mean reversion in real exchange rates is due to using linear unit root tests.

For a comparison purpose, other panel unit root tests are also applied in the on line appendix. Most of the panel unit root tests find the real exchange rate non-stationary.

5 Concluding Remarks

We have developed a bootstrap procedure to correct for size distortion of heterogeneous panel tests. We investigated the small sample properties of the proposed bootstrap test. It is found that the bootstrap neural test performs well, even when $T = 20$. An application to exchange rates shows the usefulness of combining bootstrap and neural networks. Evidence for PPP, which is generally rejected by classical linear tests, is found when our bootstrapped neural test is employed.

References

Andrew P. Blake and George Kapetanios. Pure significance tests of the unit root hypothesis against nonlinear alternatives. *Journal of Time Series Analysis*, 24(3):253–267, 2003. 1

M. Caner and B. Hansen. Threshold autoregression with a near unit root. Unpublished manuscript, University of Wisconsin, 2000. 1

R. Davidson. Notes on the bootstrap. *GREQAM*, 1998. 2.2

R. Davidson and J. MacKinnon. Graphical methods for investigating the size and the power of hypothesis tests. *The Manchester School*, 66:1–22, 1998. 3, 3.2

C. de Peretti, C. Siani, and M. Cerrato. An artificial neural network based heterogeneous panel unit root test in case of cross sectional independence. In 2009 International Joint Conference on Neural Network, editor, *IJCNN*, pages 2487–2493. IEEE - INNS - ENNS International Joint Conference, 2009. 1, 2

K. Im, H. Pesaran, and Y. Shin. Testing for unit roots in heterogeneous panels. *Journal of Econometrics*, 115:53–74, 2003. 1

G. Kapetanios and Y. Shin. Testing for a unit root against threshold nonlinearity. Unpublished manuscript, University of Edinburgh, 2000. 1

C. J. Murray and D. H. Papell. The purchasing power persistence paradigm. *Journal of International Economics*, 56:1–19, 2002. 4

C. J. Murray and D. H. Papell. The purchasing power puzzle is worse than you think. *Empirical Economics*, 30(3):783–790, October 2005. 4