Measuring the Euro Area Output Gap using a Multivariate Unobserved Components Model Containing Phase Shifts

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Abstract

Several recent studies have used multivariate unobserved components models to identify the output gap and the NAIRU. A key assumption of these models is that one common cycle component, such as the output gap, drives the cyclical fluctuations in all variables included in the model. This paper also uses the multivariate approach to estimate the euro area output gap and the trends and cycles in other macroeconomic variables. However, it adopts a flexible way of linking the output gap to the cycle components in the other variables, in that we do not impose any leading or lagging restrictions between cycle components, as has been done in most previous studies. Our approach also allows us to assess the strength of cycle association and cross-correlation among cycle components using the model's parameter estimates. Finally, we demonstrate that our multivariate model can provide a satisfactory historical output gap estimate and also a 'real-time' estimate for the aggregate euro area.

Keywords: output gap, higher-order cycle, state-space, Kalman filter.
JEL classifications: C32, E32.

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1 Introduction

A fundamental objective of monetary and fiscal policy is to dampen economic fluctuations by keeping key macroeconomic variables, such as output and unemployment, close to their natural rates. To do this, economists need to be able to identify accurately the unobserved features of an economy, such as potential (trend) output, the output gap (cycle) and the Non-Accelerating Inflation Rate of Unemployment (NAIRU), from observed macroeconomic data. It is well known that these unobserved variables are notoriously difficult to measure and, as a consequence, estimates differ widely depending on the methods used (Canova, 1998). A number of univariate approaches, such as the Hodrick-Prescott (HP, 1997) trend filter, the Baxter-King (1999) band-pass filter, the Beveridge-Nelson (1981) decomposition and the univariate unobserved component (UC) model of Harvey (1985) and Clark (1987), are often used because of their ease of computation. However, Orphanides and van Norden (2002) have shown that these univariate approaches are not particularly useful for calculating the real-time output gap for the US, as the gap estimates are subject to large revisions when new information is subsequently incorporated.

An alternative approach uses multivariate models to estimate the output gap and the trends (the natural rates) and cycles in the other variables simultaneously. One crucial assumption in these models is that there is a single common cycle component, either the output gap or the cyclical unemployment rate, which drives cyclical fluctuations across all variables. An early example of this approach was a bivariate UC model of US output and unemployment, based on Okun's law, that was proposed by Clark (1989) to estimate the output gap. Apel and Jansson (1999) also included inflation to systematically estimate the output gap and the NAIRU for the UK, US and Canada. Recent papers, including Rünstler (2002) for the euro area and Doménech and Gomez (2006) for the US, have extended the trivariate model by including additional variables, such as capacity utilisation and investment.[2] The advantage of using multivariate models over univariate approaches has been highlighted by a number of papers. Rünstler (2002) and Doménech and Gómez (2006) showed that the output gaps estimated from their multivariate models were subject to

[2] Other multivariate models incorporating aggregate output along with inflation and the rate of unemployment, for example Basistha and Nelson (2007) and Berger (2010), also allow for a non-zero correlation between the innovations of the trend and cycle components.

smaller revisions over time than those obtained from univariate methods such as the HP and band-pass filters. In addition, Basistha and Startz (2008) demonstrated that a multivariate model that assumes a common cyclical fluctuation in aggregate output and the unemployment rate reduces the uncertainty associated with estimates of the NAIRU.

Most previous research, including that discussed above, has not specifically investigated the relationships between the output gap and the cyclical components contained in other macroeconomic variables, such as inflation and the unemployment rate. The common practice has been to introduce both the current value and one or two lags of the output gap to model the inflation gap and the cyclical unemployment rate. However, this ad hoc way of dealing with cycles in the multivariate model may result in imprecisely estimated cyclical components which, in turn, may affect estimates of trend components. To avoid imposing restrictions of this type and to 'let the data speak', we use the trigonometric cycle specification introduced by Harvey and Jaeger (1993) and Harvey and Trimbur (2003) to model the dynamics of the output gap, along with the phase-shifts mechanism proposed in Rünstler (2004) to link the output gap to the cyclical components of the additional variables. In addition, an idiosyncratic cycle is introduced in each additional variable to capture any cyclical dynamics that cannot be explained by the output gap subject to a phase shift. The advantages of this specification are two-fold. First, it accommodates any leading or lagging relationships between cyclical components and, second, it allows us to analyse how closely these cyclical components are related to each other.

In this paper we estimate a five-variate model incorporating aggregate output and four additional variables: inflation, unemployment, industrial production and investment. These additional variables are all thought to contain relevant information about the output gap. The first two, inflation and the rate of unemployment, are frequently used in multivariate models to identify the output gap via the Phillips curve and Okun's law relationships. Although industrial production and investment are components of aggregate output, they exhibit larger cyclical swings than the aggregate and are often used by the business cycle dating committee of the Centre for Economic Policy Research (CEPR) to date business cycle turning points for the aggregate euro area.

To preview our results, we confirm that these additional variables all contain highly relevant information about the output gap. Furthermore, estimates of cross-correlations indicate that a number of lags and leads of the output gap have reasonably high correlation with the cyclical components of inflation and unemployment. This raises concerns about previous studies that use only the current value and one or two lags of the output gap to model the cyclical components in of these variables. Our five-variate model is also able to identify a better historical (smoothed) output gap estimate than models excluding industrial production and investment. Finally, we demonstrate that our five-variate model can produce a more satisfactory 'real-time' output gap estimate than any of the univariate methods.

The paper is organised as follows. Section 2 presents the five-variate model used to estimate the output gap, the NAIRU and trend inflation. The parameter estimates and phase shifts, cycle associations and cross-correlations are discussed in Section 3. The reliability of the output gap estimate from the five-variate model is assessed in Section 4. Finally, Section 5 concludes.

2 The model

2.1 Output decomposition

Output decomposition plays a central role in the multivariate UC model used to obtain the output gap. A trend-cycle model of output can be set up as

$$y_t = \mu_t + \psi_t + \varepsilon_t, \qquad \varepsilon_t \sim \text{NID}(0, \sigma_\varepsilon^2) \tag{1}$$

where output, y_t, is decomposed into a trend, μ_t, an output gap, ψ_t, and an irregular component, ε_t. The trend component is modelled as a local linear trend (Harvey 1989),

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t, & \eta_t &\sim \text{NID}(0, \sigma_\eta^2), \\ \beta_t &= \beta_{t-1} + \xi_t, & \xi_t &\sim \text{NID}(0, \sigma_\xi^2), \end{aligned} \tag{2}$$

so that μ_t is an $I(2)$ process. Although this specification may not always be supported by unit root tests, it can give a good fit to series such as real GDP when modelling within an unobserved components framework (Nyblom and Harvey, 2001).[3] If $\sigma_\xi^2 = 0$ the local linear trend simplifies to a random walk with drift, while a smoothed trend component is obtained if $\sigma_\eta^2 = 0$, which is a special case of a class of Butterworth filters (Gomez, 2001).

For the output gap, ψ_t, we used both the first-order trigonometric cycle specification introduced by Harvey and Jaeger (1993) and its generalised form proposed by Harvey and Trimbur (2003). The specification of a first-order cycle is

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}. \tag{3}$$

$$\begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix} \sim \mathrm{NID}\left(0, \begin{bmatrix} \sigma_\kappa^2 & 0 \\ 0 & \sigma_\kappa^2 \end{bmatrix} \right),$$

Both κ_t and κ_t^* are serially uncorrelated and mutually uncorrelated with ε_t, η_t and ξ_t. The parameters $0 \le \rho < 1$ and λ_c are the damping factor and cycle frequency, respectively, with values of ρ close to one yielding a more persistent cycle. The autocovariance function (ACF), $\Gamma(s)$, for $\tilde{\psi}_t = \left[\psi_t, \psi_t^*\right]^{\mathsf{T}}$ is given by damped cosine and sine waves of length $2\pi/\lambda_c$,

$$\Gamma(s) = \rho^{|s|} \frac{\sigma_\kappa^2}{\left(1-\rho^2\right)} \begin{bmatrix} \cos\left(s\lambda_c\right) & \sin\left(s\lambda_c\right) \\ -\sin\left(s\lambda_c\right) & \cos\left(s\lambda_c\right) \end{bmatrix}, \tag{4}$$

where $\sigma_\psi^2 = \dfrac{\sigma_\kappa^2}{\left(1-\rho^2\right)}$ is the variance of the first-order cycle.

Smoother cycle processes can be obtained by generalising the first-order cycle to an nth- order cycle,

[3] The ADF test, for example, too often rejects the $I(2)$ null because the process followed by the second differences of the observations is close to being non-invertible.

$$\begin{bmatrix} \psi_t^{(n)} \\ \psi_t^{*(n)} \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1}^{(n)} \\ \psi_{t-1}^{*(n)} \end{bmatrix} + \begin{bmatrix} \psi_t^{(n-1)} \\ \psi_t^{*(n-1)} \end{bmatrix}, \tag{5}$$

for $n = 1,...,k$, where $\kappa_t = \psi_t^{(0)}$ and $\kappa_t^* = \psi_t^{*(0)}$ are two mutually uncorrelated white noise disturbances with zero mean and common variance σ_κ^2. Setting the order n to be greater than one leads to a greater concentration on a particular frequency band, and thus results in smoother cycle components than when $n = 1$. Harvey and Trimbur (2003) demonstrate that when a higher-order cycle is used to model the US output gap, it more clearly illustrates the state of the economy than the first-order cycle. The ACF for $\tilde{\psi}_t = \left[\psi_t^{(n)}, \psi_{t-1}^{*(n)}\right]^\mathsf{T}$ is given in Trimbur (2005), being

$$\Gamma(s) = \sigma_\kappa^2 \rho^{|s|} \begin{bmatrix} \cos(s\lambda_c) & \sin(s\lambda_c) \\ -\sin(s\lambda_c) & \cos(s\lambda_c) \end{bmatrix} g(\rho, s), \tag{6}$$

where

$$g(\rho, s) = \sum_{k=n-s}^{s} \left(\left(1-\rho^2\right)^{1-k-n} \binom{s}{n-k} \sum_{r=0}^{k-1} \binom{k-1}{r} \binom{n-1}{r+n-k} \rho^{2r} \right).$$

The variance of the nth- order cycle is

$$\sigma_\psi^2\left(n, \rho, \sigma_\kappa^2\right) = \sigma_\kappa^2 \frac{\sum_{k=0}^{n-1} \binom{n-1}{k}^2 \rho^{2k}}{\left(1-\rho^2\right)^{2n-1}} \tag{7}$$

2.2 *Additional variables*

As discussed above, the additional variables included in the model are assumed to contain information about the output gap, because they are either functionally related to it or because they are components of aggregate output. To provide a transparent and flexible way of linking the output gap to the cyclical components in these variables we adopt the phase shift mechanism proposed by Rünstler (2004).

Specifically, we use the "Choleski decomposition" of Rünstler (2004), as it allows us to test for the presence of phase shifts between cycle components. In addition, an idiosyncratic cycle is introduced for each additional variable to capture any unique cyclical fluctuations that cannot be explained by the output gap subject to a phase shift. This specification also allows us to examine whether the additional variables do indeed contain information that is related to the output gap.

The decomposition used for each additional variable in our five-variate model is as follows

$$y_{i,t} = \mu_{i,t} + \theta_i \psi_t^{(n)} + \theta_i^* \psi_t^{*(n)} + \psi_{i,t} + \varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim \text{NID}\left(0, \sigma^2_{\varepsilon,i}\right), \qquad (8)$$

$$\mu_{i,t} = \mu_{i,t-1} + \beta_{i,t-1} + \eta_{i,t}, \qquad \eta_{i,t} \sim \text{NID}\left(0, \sigma^2_{\eta,i}\right), \qquad (9)$$

$$\beta_{i,t} = \beta_{i,t-1} + \xi_{i,t}, \qquad \xi_{i,t} \sim \text{NID}\left(0, \sigma^2_{\xi,i}\right),$$

$$\psi_{i,t} = \phi_{i1} \psi_{i,t-1} + \phi_{i2} \psi_{i,t-1} + \kappa_{i,t}, \qquad \kappa_{i,t} \sim \text{NID}\left(0, \sigma^2_{\kappa,i}\right), \qquad (10)$$

in which $i = \pi, ur, ip, in$ denotes inflation, the rate of unemployment, industrial production and investment, respectively. As shown in equation (8), each variable $y_{i,t}$ is the sum of three components, a trend, $\mu_{i,t}$, a cycle, $\theta_i \psi_t^{(n)} + \theta_i^* \psi_t^{*(n)} + \psi_{i,t}$, and an irregular term, $\varepsilon_{i,t}$. As with the output trend, the trend component in each variable is modelled by a local linear trend, while the cycle is a linear combination of the output gap, its adjacent auxiliary component, and an idiosyncratic cycle modelled as a stationary AR(2) process as in equation (10). In subsection 2.3 we demonstrate that phase shifts, cycle associations and cross-correlations between cyclical components can be analysed using the model's parameter estimates. Finally, a seasonal component is included in the inflation equation to pick up the seasonal pattern in inflation.[4]

The five-variate UC model can be recast into state-space form for estimation.[5] The hyperparameters in the UC model can be estimated by maximum likelihood using the prediction error decomposition produced by the Kalman filter. Since non-

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[4] All other variables used in this paper are seasonally adjusted, so that there is no need to include such a component in their equations.
[5] The state space representation of the model is available upon request.

stationary variables appear in the state vector, the Kalman filter requires a diffuse initialisation and we use the method developed by Koopman and Durbin (2003).[6]

2.3 Modelling the phase shift and cycle association

This subsection illustrates that phase shifts, cycle associations and cross-correlations between the cyclical components in the five-variate model can be revealed through analysing the model's parameter estimates. In order to facilitate the following discussion, we first consider a special case where the AR(2) idiosyncratic cycle, $\psi_{i,t}$, is set to zero. The cycle component in each variable, denoted ψ_t^i, then reduces to a linear combination of just $\psi_t^{(n)}$ and $\psi_t^{*(n)}$. The corresponding ACF for the vector of cycle components, $x_t^C = \left(\psi_t^{(n)}, \psi_t^{\pi}, \psi_t^{ur}, \psi_t^{ip}, \psi_t^{in}\right)^{\mathsf{T}}$, is given by

$$\Gamma_x(s) = \tilde{\Theta}\Gamma(s)\tilde{\Theta}^{\mathsf{T}} \tag{11}$$

where $\Gamma(s)$ is the ACF of $\tilde{\psi}_t = \left[\psi_t^{(n)}, \psi_t^{*(n)}\right]^{\mathsf{T}}$ specified in equation (6) and

$$\tilde{\Theta} = \begin{bmatrix} 1 & \theta_\pi & \theta_{ur} & \theta_{ip} & \theta_{in} \\ 0 & \theta_\pi^* & \theta_{ur}^* & \theta_{ip}^* & \theta_{in}^* \end{bmatrix}^{\mathsf{T}}$$

contains the corresponding parameter loadings. Elements in $\Gamma_x(s)$ are damped cosine functions, which can be expressed as follows

$$\Gamma_{x,ii}(s) = \sigma_\kappa^2 \rho^{|s|} r_i^2 \cos(s\lambda_c) g\left(\rho, s\right), \tag{12}$$

$$\Gamma_{x,ij}(s) = \operatorname{sign}\left(\theta_i\right)\operatorname{sign}\left(\theta_j\right)\sigma_\kappa^2 \rho^{|s|} r_i r_j \cos\left(\lambda_c\left(s - \xi_j + \xi_i\right)\right) g\left(\rho, s\right), \tag{13}$$

where

[6] All the computations were performed using the library of state-space functions in SsfPack 3.0 developed by Koopman et al. (2008) and Ox 5 by Doornik (2006).

$$r_k = \sqrt{\theta_k^2 + \theta_k^{*2}} \,,$$

$$\xi_k = \lambda_c^{-1} \tan^{-1}\left(\theta_k^* / \theta_k\right),$$

for $k = i, j$. The phase shift between ψ_t^i and ψ_t^j is given by $\xi_j - \xi_i$, which is normalised to lie within the range of one quarter of the cycle period in absolute terms, that is, $\left|\xi_j - \xi_i\right| \le \pi/2\lambda_c$. The additivity property of phase shifts holds in our case as all variables share one common cycle component.

The cross-correlation function between ψ_t^i and ψ_t^j is obtained by dividing $\Gamma_{x,ij}(s)$ by the product of the standard deviations of the two cycles, i.e.,

$$\text{corr}\left(\psi_{t-s}^i, \psi_t^j\right) = \text{sign}\left(\theta_i\right)\text{sign}\left(\theta_j\right)\rho^{|s|}\cos\left(\lambda_c\left(s - \xi_j + \xi_i\right)\right) f\left(\rho, s\right), \tag{14}$$

where

$$f\left(\rho, s\right) = \frac{\sigma_\kappa^2}{\sigma_\psi^2\left(n, \rho, \sigma_\kappa^2\right)} g(\rho, s).$$

When an AR(2) idiosyncratic cycle is included, the cycle component in each additional variable becomes $\psi_t^i = \theta_i \psi_t^{(n)} + \theta_i^* \psi_t^{*(n)} + \psi_{i,t}$. This leaves the covariance $\Gamma_{x,ij}(s)$ unchanged, while the autocovariance of ψ_t^i consists of two components, $\Gamma_{x,ii}(s)$ and $\gamma_i(s) = \text{E}\left(\psi_{i,t}\psi_{i,t-s}\right)$. The latter is the autocovariance of the AR(2) idiosyncratic cycle. Therefore, the cross-correlation function in equation (14) is modified as

$$\text{corr}\left(\psi_{t-s}^i, \psi_t^j\right) = \alpha_i \alpha_j \rho^{|s|}\cos\left(\lambda_c\left(s - \xi_j + \xi_i\right)\right) f\left(\rho, s\right), \tag{15}$$

where

$$\alpha_k = \text{sign}\left(\theta_k\right)\frac{r_k}{\sqrt{r_k^2 + \frac{\sigma_{\psi,k}^2}{\sigma_\psi^2\left(n, \rho, \sigma_\kappa^2\right)}}},$$

for $k = i, j$. $\sigma^2_{\psi,k}$ is the variance of the AR(2) idiosyncratic cycle. The product $\alpha_i \alpha_j$ measures the strength of cycle association between ψ_t^i and ψ_t^j. It can be seen that the larger the idiosyncratic cycle is relative to the variance of the output gap, the lower the value of $|\alpha_k| \leq 1$.

3 Empirical results

3.1 Data

The data used in this paper are quarterly observations for the aggregate euro area from 1970Q1 to 2009Q2. Historical data from 1970Q1 to 2007Q4 are taken from the area-wide model (AWM) database originally constructed by Fagan *et al.* (2001) and have been updated to 2009Q2 using the OECD database.[7] ADF test statistics indicate that (the log of) euro area GDP, the unemployment rate, the CPI inflation rate, and (the logs of) industrial production and investment (gross fixed capital formation) are all *I*(1) series.[8]

3.2 Estimation results

As set out in Section 2, the output gap is used to explain the cyclical components in the multivariate model. It is therefore important to select a cyclical model for the output gap that can provide as good a fit as possible to the data. Table 1 thus presents parameter estimates of the output gap equation and its goodness of fit for different orders *n* of the output gap, these being incrementally increased from one to six.[9]

{Table 1 about here}

[7] As the CPI itself is not seasonally adjusted, we adjust it using a stochastic seasonal component. Our deseasonalised series is consistent with that obtained by using the X-12-ARIMA seasonal adjustment procedure.
[8] Test statistics are available upon request.
[9] The same dummies are included for all models to pick up the outliers detected when inspecting auxiliary residuals. They are 1974q4 for output, 1975q1 and 2007q4 for inflation, 1975q2 for the rate of unemployment, 1980q1 for industrial production and 2008q4 and 2009q1for all variables included in the five-variate model. All dummy variable coefficients are significant.

The results presented in Table 1 illustrate the impact of using different orders of cycles to model the output gap and whether they provide a good fit to the data. The output gap has a cycle period of nearly ten years (40 quarters) when the cycle order is set at one and two. When n is greater than two, an even longer period is obtained, so that we set the cycle period to ten years, which is the maximum business cycle length defined by Burns and Mitchell (1946). Consistent with the findings of Harvey and Trimbur (2003) for the US output gap, we find that when n changes, the standard deviation of the output gap, σ_ψ, stays relatively stable, while the standard deviation of the cycle disturbance, σ_κ, and the damping factor, ρ, decline when the first-order cycle is replaced by a higher-order cycle. It is also worth noting that when n increases to two, the standard deviation of the irregular term, σ_ε, increases significantly. This is because a higher-order cycle leads to more pronounced cut-offs of the band-pass gain function at both ends of the frequency band centered at λ_c, so that more noise enters the irregular components. As suggested by the Ljung-Box statistics, serial correlation becomes more pronounced in the one-step-ahead prediction errors as n increases.

The log-likelihood and R^2 statistics presented in the lower panel of Table 1 suggest that a second-order output gap provides the best fit to our data. This output gap is plotted in panel (a) of Figure 1 along with the first-order gap in panel (b).[10] Both gap estimates exhibit large cyclical swings, although the second-order cycle is seen to be smoother than the first-order cycle due to the different weighting patterns used to construct the two. The weights for the first-order cycle at the middle of the sample, shown in panel (d), reveal that a large weight is placed on the current observation, whereas when the second-order cycle is extracted greater weight is attached to the adjacent observations (panel (c)). Four periods of below trend growth are clearly identified in the second-order output gap and the beginnings of these downturns coincide with the recessions reported by the CEPR, as indicated by the vertical bars in panel (a).[11]

{Figure 1 about here}

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[10] The largest difference in the output gap is found between $n=1$ and $n=2$.

[11] Our output gap estimate is also generally consistent with that identified by Berger (2010), who estimates the euro area output gap using a trivariate model of output, inflation and unemployment, with correlated innovations to the trends and cycles.

Panels (a)-(e) of Figure 2 plot the observed variables against their trend components estimated from the five-variate model. Industrial production and investment are shown to be more cyclical than aggregate output, with both exhibiting significant variations around their estimated trends. Another noteworthy result is that the estimate of the NAIRU suggests that structural unemployment in the euro area began rising in the early 1970s, which coincides with a period of sustained high inflation. However, the NAIRU remained persistently high at around 9%, even when inflation stabilised at a lower level. Finally, panel (f) plots the seasonal component extracted from the inflation series, which exhibits an increasing seasonal pattern during the 2000s. In panel (g), we demonstrate that our deseasonalised inflation series is consistent with that obtained by using the X-12-ARIMA seasonal adjustment procedure.

{Figure 2 about here}

The main parameter estimates, phase shifts and cycle associations are presented in Table 2.[12] The results can be summarised as follows. First, a smoothed output trend is preferred by the data with σ_η^2 estimated to be zero. In addition, the local linear trend specification reduces to a random walk with constant drift for the trend components of inflation and industrial production, with only a small drift being found for trend inflation. This is consistent with the view that trend inflation can be well approximated using a driftless random walk process (Cogley and Sargent, 2007). Second, the idiosyncratic cycle of the unemployment rate has a large standard deviation that is more than twice the size of that observed for the output gap. However, small idiosyncratic cycles are found for inflation, industrial production and investment. As demonstrated in section 2.3, the larger the idiosyncratic cycle is relative to the output gap, the weaker the strength of cycle association. As a result, the output gap exhibits high pro-cyclical associations, close to one, with the cycle components in inflation, industrial production and investment, while a moderate anti-cyclical association of -0.82 is found between the output gap and the cyclical unemployment rate. Overall, the cycle associations, presented in the lower triangle of

[12] The complete set of parameter estimates and their standard deviations are reported in Table A1 in the Appendix.

panel (b) of Table 2, suggest that all variables in the five-variate model have closely related cyclical components. The phase shifts presented in the upper triangle of panel (b) reveal that the output gap leads the cyclical components in inflation and unemployment by about one year, while it slightly lags those of industrial production and investment.

{Table 2 about here}

Finally, the relevance of the additional variables in measuring the output gap and the presence of phase shifts can be tested using likelihood ratio statistics, which are presented in panel (c) of Table 2. All the additional variables appear to contain highly relevant information for the output gap, as the null hypotheses $\theta_i = \theta_i^* = 0$ are strongly rejected. The significance of the phase shifts can be examined by testing the null $\theta_i^* = 0$, the results of which suggest the presence of phase shifts between the output gap and the cyclical components in inflation, the unemployment rate and industrial production. The phase shift is found to be insignificant between the output gap and the investment cycle.

The cycle components of the four additional variables are plotted in Figure 3 against the output gap to illustrate the above findings. Pro-cyclical relationships are observed between the output gap and the cyclical components of inflation, industrial production and investment, while an anti-cyclical relationship is observed between the cyclical unemployment rate and the output gap. Furthermore, Figure 3 reveals that the output gap leads the inflation gap but lags cyclical fluctuations in industrial production. The output gap is also shown to be concurrent with the investment cycle.

{Figure 3 about here}

The cross-correlations between the cycle components are calculated using equation (15) and are reported in Table 3.[13] The contemporaneous cycle correlations (column headed $s = 0$) are lower than the corresponding cycle associations presented in Table 2, particularly when large phase shifts are present. For example, the phase

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[13] For $n = 2$, $f(\rho, s)$ is $\left(1 + \frac{1-\rho^2}{1+\rho^2}|s|\right)$ in equation (15).

shift of 3.6 quarters between the output gap and the inflation gap results in a contemporaneous correlation of 0.79, even though their association is 0.98. This illustrates that higher cycle coherence may be revealed after the cycles are adjusted to eliminate their phase shift.[14] The highest contemporaneous cycle correlations are found between the output gap and the cyclical components in industrial production and investment. In addition, these two cycles have cross-correlations with the output gap that are above 0.5 in the range $-3 \le s \le 5$. The cyclical components of inflation and the unemployment rate have correlations with the output gap of above 0.5 in the range $-5 \le s \le 1$ and peak at $s = -2$. These findings therefore must raise concerns over the conventional approach that uses just the contemporaneous value and at most one or two lags of the output gap to model the inflation gap and the cyclical unemployment rate.

{Table 3 about here}

4 The reliability of output gap estimates

4.1 Reliability of the smoothed cycle estimates

As these results demonstrate that the additional variables used in our five-variate model all have cyclical components closely related to the output gap, it is interesting to consider how the output gap might differ if a smaller set of variables was used. To address this question, we compare the output gap obtained from the above five-variate model with those obtained from a trivariate model that excludes industrial production and investment and from a univariate model that only contains the output decomposition.[15] The second-order output gap is used for all three models. In order to facilitate comparisons, the second-order output gap estimated from the five-variate model presented in Figure 1 is again plotted against those obtained from the univariate and trivariate models in panels (a) and (b) of Figure 4. It can be seen that

[14] The contemporaneous cycle correlation is equal to the cycle association when the cycles are adjusted to eliminate the phase shift that results in $\cos(0) = 1$.

[15] Parameter estimates of the univariate and trivariate models are available upon request.

the univariate and trivariate models yield very similar output gap estimates, while the gap obtained from the five-variate model differs significantly from these estimates at the beginning and end of the sample period. This suggests that the economy was above its potential at the beginning of the 1970s before moving into recession as a result of the first spike in oil prices. However, all the models yield similar output gap estimates during the boom and bust of the 1980s. During the 1990s, the output gap estimated from the five-variate model appears more volatile, but indicates a shallower downturn during the 2000s than those from the other two models.

{Figure 4 about here}

Given the different output gap estimates produced by these alternative models, it is important to assess the reliability of the different estimates. The Kalman filter produces a mean squared error, denoted as $\widehat{P}_{t|T}\left(\widehat{\Xi}\right)$, for the (smoothed) output gap, $\widehat{\psi}_{t|T}\left(\widehat{\Xi}\right)$, presented in Figure 4, where $\widehat{\Xi}$ is a vector of full-sample parameters estimates. $\widehat{P}_{t|T}\left(\widehat{\Xi}\right)$ can be used as an indicator for the level of uncertainty in the output gap estimate. The overall uncertainty of an output gap can be assessed after taking into account parameter uncertainty. We follow the approach of Hamilton (1986) and evaluate the impact of parameter uncertainty using a Monte Carlo simulation. The overall error variance $\widehat{P}_{t|T}\left(\Xi\right)$, where Ξ is the vector of true parameters, can be approximated by

$$\widehat{P}_{t|T}\left(\Xi\right) = \frac{1}{K}\sum_{k=1}^{K} P_{t|T}\left(\Xi^{(k)}\right) + \frac{1}{K}\sum_{k=1}^{K}\left[\mathrm{E}\left(\psi_{t|T}\left|\Xi^{(k)}\right.\right) - \widehat{\mathrm{E}}\left(\psi_{t|T}\right)\right]^2 \tag{16}$$

where $\widehat{\mathrm{E}}\left(\psi_{t|T}\right) = \frac{1}{K}\sum_{k=1}^{K}\mathrm{E}\left(\psi_{t|T}\left|\Xi^{(k)}\right.\right)$ and $\Xi^{(k)}$ are independent draws from the multivariate normal density of hyper-parameters, $\widehat{\Xi} \sim \mathrm{N}\left(\widehat{\Xi}, \mathrm{V}_{\widehat{\Xi}}\right)$. The first term on the right hand side of equation (16) represents the final estimation error allowing for parameter uncertainty, while the second term measures the extent of parameter uncertainty. Both terms are computed using $\Xi^{(k)}$, which is generated by 1000 random draws from the multivariate normal density of hyper-parameters.

In Figure 5, we plot the standard errors, $\left[\widehat{P}_{t|T}\left(\widehat{\Xi}\right)\right]^{1/2}$ and $\left[\widehat{P}_{t|T}\left(\Xi\right)\right]^{1/2}$, where the latter takes into account parameter uncertainty. The standard errors suggest a similar level of uncertainty in the output gaps produced by the univariate and trivariate models. However, the error in the output gap produced by the five-variate model is smaller, especially at the beginning and at the end of the sample period. This is the period that most differences in the output gap estimates are found, as shown in Figure 4, and shows the importance of using industrial production and investment to estimate the euro area output gap.

{Figure 5 about here}

4.2 Revisions

In this subsection, we investigate the reliability of the real-time output gap estimated from our five-variate model. As suggested by Taylor (1993), the output gap is an important variable, along with inflation, for central banks setting their interest rates. However, there are concerns about the precision and accuracy of real-time output gap estimates. Orphanides and van Norden (2002), for example, have shown that the real-time output gap estimates for the US produced by a large number of univariate approaches are subject to large subsequent revisions when new information is incorporated. On the other hand, Doménech and Gómez (2006) found that the US output gap estimated from their multivariate model required less revision than those obtained from the HP and band-pass filters. Camba-Méndez and Rodriguez-Palenzuela (2001) and Rünstler (2002) reached similar conclusions using data for the euro area. Given the lack of data vintages available for the euro area, we are unable to assess the impact that data revisions have on estimates of the output gap. However, we can assess the statistical revisions that are produced when our five-variate model incorporates new observations to update the estimates of the output gap. The statistical revisions produced by four univariate models are used as benchmarks, these being the output decomposition used in the five-variate model (the output is decomposed into a smoothed trend and a cycle of order two), the HP filter, and the UC models of Harvey and Jaeger (1993) and of Harvey (1985) and Clark (1987).

As with Rünstler (2002), who measures the revision to the output gap as the difference between the filtered and smoothed output gap estimates, we first assess the revisions which occur when the first three and the last three years of the sample period are excluded. It is important to note that the revisions measured in this way only reflect the revisions which occur during the filtering process. In addition, we calculated the Quasi-Real output gap estimates, as defined in Orphanides and van Norden (2002), from 1993Q1 onwards. These are calculated by initially using observations up to 1993Q1 to compute the Quasi-Real estimate for 1993Q1. The sample is then moved forward quarter by quarter with the hyper-parameters being re-estimated at each step until the end of the sample is reached. The series of Quasi-Real estimates can be regarded as the first available estimate at each point in time. Although the choice of 1993Q1 is admittedly ad hoc, it does provide a sufficiently large sample for initial estimation and for analysing subsequent revisions to the output gap estimates. The difference between the Quasi-Real and smoothed estimates reflects revisions due to both filtering uncertainty and parameter instability, where the latter is measured as the difference between the Quasi-Real and filtered estimates. The revisions presented in Table 4 are measured in relative terms using the formula $SR = \sigma\left(z_{t|t} - z_{t|T}\right) / \sigma\left(z_{t|T}\right)$, where $\sigma\left(x\right)$ is the standard deviation of the variable x. This ratio gives a proxy for the noise-to-signal ratio. The results show that our five-variate model yields significantly smaller revisions in the filtering process than any of the benchmark models. After taking into account parameter instability, the smallest revisions are still in the five-variate model, but the differences with respect to the benchmark models are narrowed. This may be because the multivariate model contains more parameters than the univariate models.

{Table 4 about here}

5 Conclusions

In this paper we estimate the euro area output gap using a five-variate model incorporating aggregate output and four additional variables: inflation, the rate of unemployment, industrial production and investment. The main contribution of our model lies in how the cyclical components of the additional variables are linked to the output gap. We adopt the trigonometric cycle specification of Harvey and Trimbur (2003) and the phase shift mechanism of Rünstler (2004) to uncover any leading or lagging (i.e. phase shift) relationships among the cyclical components. In addition, an idiosyncratic cycle is included for each additional variable to capture any cyclical dynamics that cannot be represented by the output gap subject to phase shifts. This allows us to investigate the strength of cycle association and the cross-correlations between the cyclical components in the model. The main results can be summarised as follows. First, as suggested by both cycle associations and likelihood ratio tests, the additional variables included in the model all contain highly relevant information about the output gap. Second, the output gap is found to lead the inflation gap and the cyclical unemployment rate by around one year. This result, to some extent, reveals the level of rigidity in the European labour and goods markets. On the other hand, the output gap lags the cyclical component in industrial production by about one quarter and appears to be concurrent with investment. Last, but by no means least, the cross-correlations reveal that a number of lags and leads of the output gap exhibit reasonably high correlation with the cycles in the additional variables, such as inflation and the rate of unemployment. This raises concerns about previous studies that use only the contemporaneous value and at most one or two lagged output gaps to model these cyclical components.

Finally, we examined the reliability of the output gap identified from our five-variate model. We distinguished between two types of output gap estimates, a smoothed estimate and a 'real-time' one. The smoothed estimate can be seen as a measure of the historical output gap. The inclusion of industrial production and investment seems to reduce the level of uncertainty in the smoothed output gap estimate. As to the reliability of a 'real-time' estimate, however, we found that this is, to some extent, dependent upon how the revision is measured. If it is measured, using the approach of Rünstler (2002), as the difference between the filtered and smoothed output gap estimates, then our five-variate model significantly outperforms the univariate approaches. However, when the revision is measured as the difference between the Quasi-real and smoothed estimates, the advantage of our model relative to univariate models narrows as a result of an increase in parameter instability. This may be because multivariate models contain significantly more parameters than univariate models.

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Appendix

Table A1: Parameter estimates of five-variate model with second-order output gap

Series		output	inflation	unemployment	IP	Investment
			Cycles [a]			
damping factor	ρ	0.82				
		(0.16)				
cycle period	$2\pi/\lambda_c$	35.34				
		(5.63)				
	θ_i	1.000	0.118	-3.104	2.463	2.377
			(0.03)	(0.79)	(0.19)	(0.18)
	θ_i^*	0.000	-0.089	2.234	0.535	0.286
			(0.04)	(0.72)	(0.19)	(0.18)
AR(2) parameters	ϕ_1^i		1.266	1.598	1.825	1.598
			(0.06)	(0.32)	(0.08)	(0.05)
	ϕ_2^i		-0.968	-0.648	-0.946	-0.943
			(0.05)	(0.30)	(0.09)	(0.05)
			Innovations [b]			
output cycle	σ_κ	0.269				
		(0.02)				
AR(2) cycle	$\sigma_{\kappa,i}$		0.010	0.911	0.055	0.113
			(0.01)	(0.42)	(0.07)	(0.05)
level	$\sigma_{\eta,i}$	0.000	0.125	0.746	0.494	0.549
		(0.01)	(0.02)	(0.39)	(0.17)	(0.13)
slope	$\sigma_{\xi,i}$	0.020	0.000	0.330	0.000	0.037
		(0.01)	(0.00)	(0.13)	(0.00)	(0.02)
irregular	$\sigma_{\varepsilon,i}$	0.200	0.010	0.008	0.288	0.453
		(0.02)	(0.01)	(0.32)	(0.12)	(0.08)
seasonal	σ_γ		0.052			
			(0.01)			
			Residual diagnostics [c]			
Q(12)		10.747	6.305	16.731	14.151	8.730
JB		1.477	0.400	39.633***	17.142***	5.388

Notes: [a] ρ and $2\pi/\lambda_c$ denote the damping factor and cycle period of the second-order output gap; θ_i and θ_i^* are parameter loadings in $\tilde{\mathbf{\Theta}}$ in equation (11); ϕ_1^i and ϕ_2^i denote the AR(2) parameters of the idiosyncratic cycle.

[b] σ_κ and $\sigma_{\kappa,i}$ denote the standard deviations of the innovations to the output gap and idiosyncratic cycle components; $\sigma_{\eta,i}$, $\sigma_{\xi,i}$ and $\sigma_{\varepsilon,i}$ denote the standard deviations of the trend, slope and irregular innovations for the variable, i ; σ_γ is the standard deviation of the innovation to the seasonal component of inflation.

[c] Q(12) and JB denote the Ljung-Box statistic for residual autocorrelation up to 12 lags and the Jarque-Bera statistic for normality, respectively. **, *** indicate significance at the 5% and 1% level, respectively.

Table 1: Selected parameter estimates and goodness of fit

			Series: GDP [a]				
n	$10^2\sigma_\xi$	$10^2\sigma_\kappa$	$10^2\sigma_\varepsilon$	$10^2\sigma_\psi$	ρ	$2\pi/\lambda_c$	$Q(12)$
1	0.026	0.415	0.118	1.685	0.970	37.63	7.51
2	0.020	0.269	0.200	1.843	0.819	35.34	10.75
3	0.022	0.247	0.209	1.537	0.640	40	12.45
4	0.021	0.197	0.220	1.624	0.562	40	15.13
5	0.019	0.225	0.209	1.480	0.448	40	18.57
6	0.026	0.225	0.206	1.408	0.380	40	17.80

			Goodness of fit [b]			
n	$R^2_{D,\mathrm{y}}$	$R^2_{D,\pi}$	$R^2_{D,ur}$	$R^2_{D,ip}$	$R^2_{D,in}$	$LogL$
1	0.49	0.32	0.77	0.71	0.72	2586.90
2	0.60	0.33	0.80	0.73	0.61	2588.98
3	0.58	0.34	0.83	0.73	0.59	2582.15
4	0.59	0.34	0.82	0.72	0.56	2571.12
5	0.57	0.34	0.82	0.73	0.57	2564.06
6	0.55	0.32	0.80	0.73	0.57	2550.91

Notes: [a] n denotes the cycle order of the output gap; σ_ξ , σ_κ and σ_ε denote the standard deviations of the slope, cycle and irregular innovations, σ_ψ denotes the standard deviation of the output gap; ρ and $2\pi/\lambda_c$ denote the damping factor and cycle period of the output gap; Q(12) denote the Ljung-Box statistic for residual autocorrelation up to 12 lags.

[b] $R^2_{D,i}$ denotes the coefficient of determination with respect to the first differences of the variable, i .
$LogL$ denotes the log-likelihood value.

Table 2: Main parameter estimates, phase shifts and associations

Series		output	inflation	unemployment	IP	Investment
		Panel (a): Main parameters [a]				
damping factor	ρ	0.82				
cycle period	$2\pi/\lambda_c$	35.34				
Output gap	$10^2\sigma_\psi$	1.844				
AR(2) cycle	$10^2\sigma_{\psi,i}$		0.052	4.913	0.486	0.596
level	$10^2\sigma_{\eta,i}$	0.000	0.125	0.746	0.494	0.549
slope	$10^2\sigma_{\xi,i}$	0.020	0.000	0.330	0.000	0.037
irregular	$10^2\sigma_{\varepsilon,i}$	0.200	0.010	0.008	0.288	0.453
		Panel (b): Phase shifts and cycle association [b]				
cycles		$\psi_t^{(n)}$	ψ_t^{π}	ψ_t^{ur}	ψ_t^{ip}	ψ_t^{in}
$\psi_t^{(n)}$			-3.581	-3.471	1.204	0.673
ψ_t^{π}		0.982		0.118	4.619	4.186
ψ_t^{ur}		-0.821	-0.806		4.527	4.084
ψ_t^{ip}		0.995	0.977	-0.816		-0.531
ψ_t^{in}		0.991	0.974	-0.813	0.986	
		Panel (c): Likelihood ratio test statistics				
		Inflation	unemployment	IP	Investment	
$H_0:\theta_i=\theta_i^*=0,$		18.52***	31.77***	80.94***	68.08***	
$H_0:\theta_i^*=0,$		5.27**	10.67***	4.50**	2.46	
$H_0:\theta_\pi=\theta_{ur}=\theta_{ip}=\theta_{in}=\theta_\pi^*=\theta_{ur}^*=\theta_{ip}^*=\theta_{in}^*=0,$			223.981***			
$H_0:\theta_\pi^*=\theta_{ur}^*=\theta_{ip}^*=\theta_{in}^*=0,$			16.907***			
		Residual diagnostics [c]				
Q(12)		10.747	6.305	16.731	14.151	8.730
JB		1.477	0.400	39.633***	17.142***	5.388

Notes: [a] ρ, $2\pi/\lambda_c$ and σ_ψ denote the damping factor, cycle period and the standard deviation of the second-order output gap; $\sigma_{\psi,i}$ denotes the standard deviations of the AR(2) idiosyncratic cycles for variable, i. $\sigma_{\eta,i}$, $\sigma_{\xi,i}$ and $\sigma_{\varepsilon,i}$ denote the standard deviations of the level, slope and irregular innovations for variable, i.

[b] Phase shifts measured in quarters are presented in the upper triangle and cycle associations are in the lower triangle. A positive phase shift indicates a lead of series column with respect to series row.

[c] Q(12) and JB denote the Ljung-Box statistic for residual autocorrelation up to 12 lags and the Jarque-Bera statistic for normality, respectively. **, *** indicate significance at the 5% and 1% level, respectively.

Table 3: Implied cross-correlations

s	-8	-5	-3	-2	-1	0	1	2	3	5	8
$y(s), y$	0.08	0.46	0.75	0.88	0.97	1.00	0.97	0.88	0.75	0.46	0.08
$y(s), \pi$	0.36	0.70	0.86	**0.88**	0.86	0.79	0.66	0.50	0.34	0.03	-0.24
$y(s), ur$	-0.30	-0.58	-0.72	**-0.74**	-0.73	-0.67	-0.56	-0.43	-0.29	-0.04	0.19
$y(s), ip$	-0.03	0.33	0.64	0.78	0.90	0.97	**0.98**	0.92	0.83	0.57	0.19
$y(s), in$	0.02	0.39	0.69	0.82	0.93	**0.98**	0.97	0.90	0.79	0.52	0.14

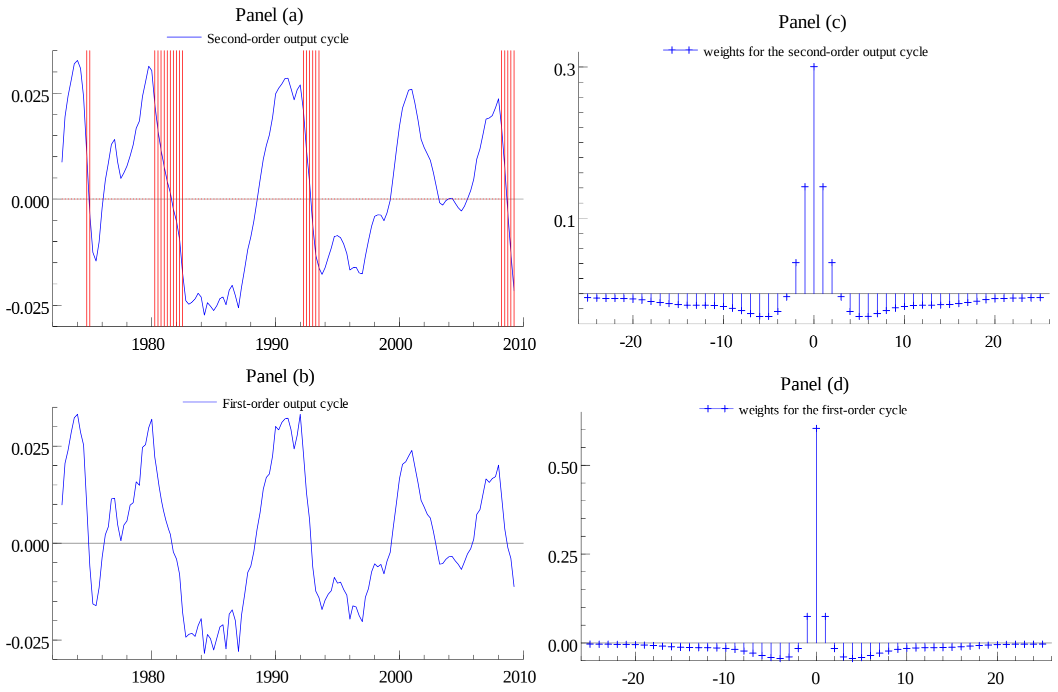
Notes: s denotes the number of leads and lags of the output gap. Numbers in bold are the highest cross-correlations between the output gap and the cycle components in the additional variables.

Table 4: Revisions (noise-to-signal ratio)

Models	filtering uncertainty		parameter instability	overall revision
	74q1-06q2	93q1-06q2	93q1-06q2	93q1-06q2
Five-variate model	0.566	0.352	0.369	0.502
Univariate model [a]	0.783	0.405	0.246	0.528
Harvey-Clark	0.773	0.566	0.232	0.676
Harvey-Jaeger	1.028	0.833	0.174	0.981
Hodrick-Prescott	1.222	1.172	0.000	1.172

Notes: [a] Univariate model is the output decomposition used in the five-variate model.

Figure 1: The first and second-order output gap and their weights



Note: the vertical bars in Panel (a) indicate recessions identified by the CEPR business cycle dating committee.

Figure 2: The observed data and the trend components

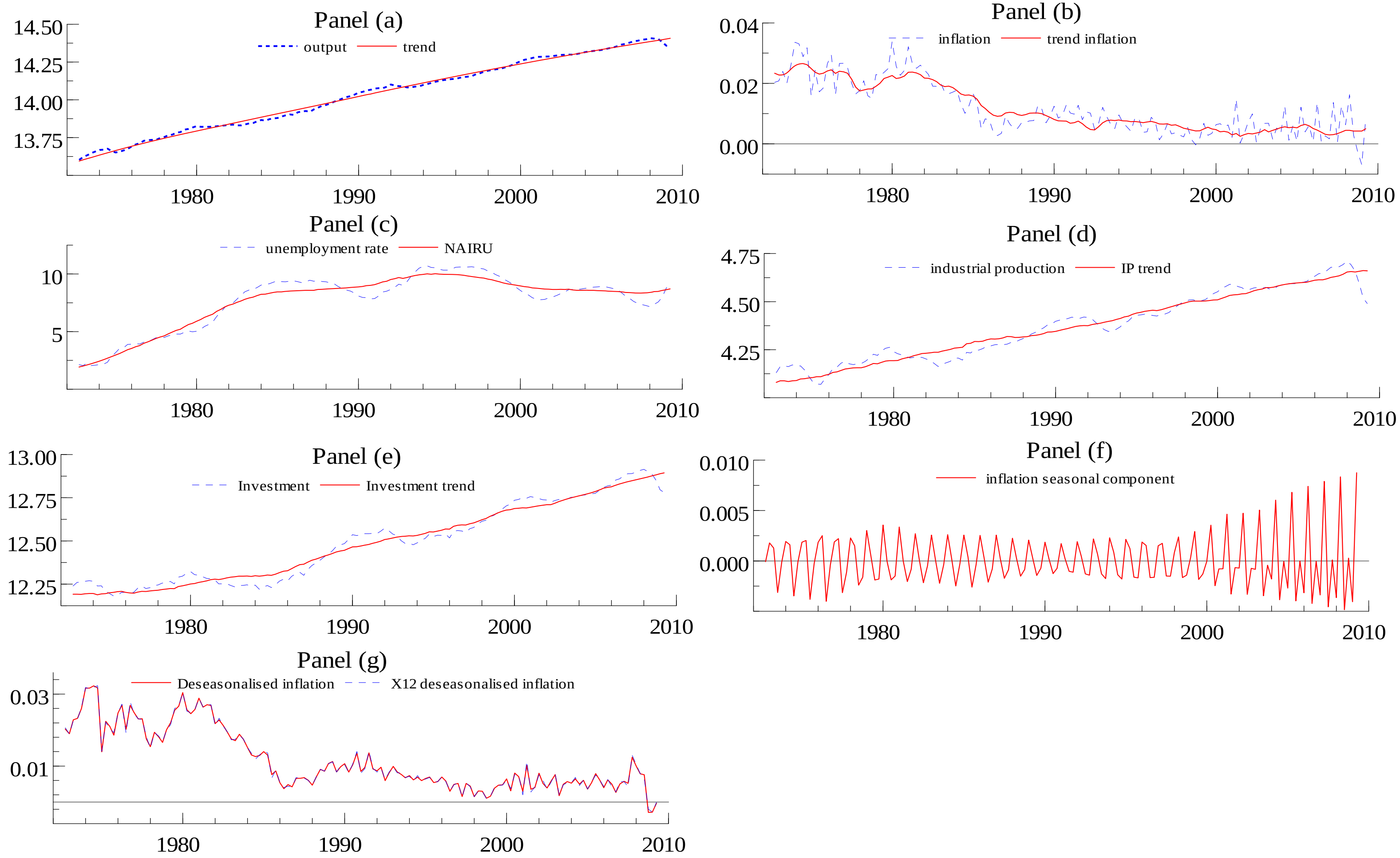


Figure 3: The cycle components

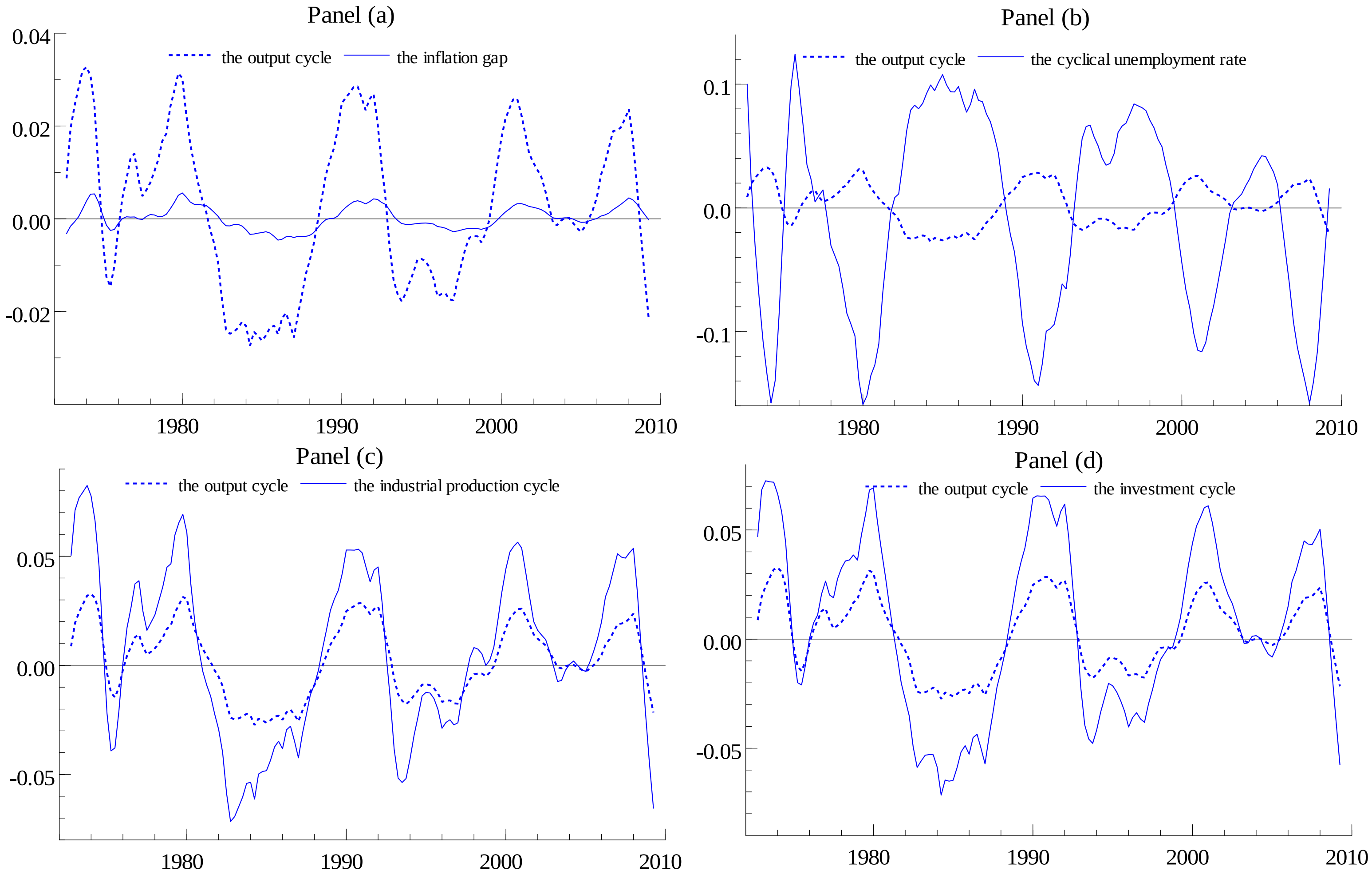


Figure 4: The output gaps from the trivariate and univariate models

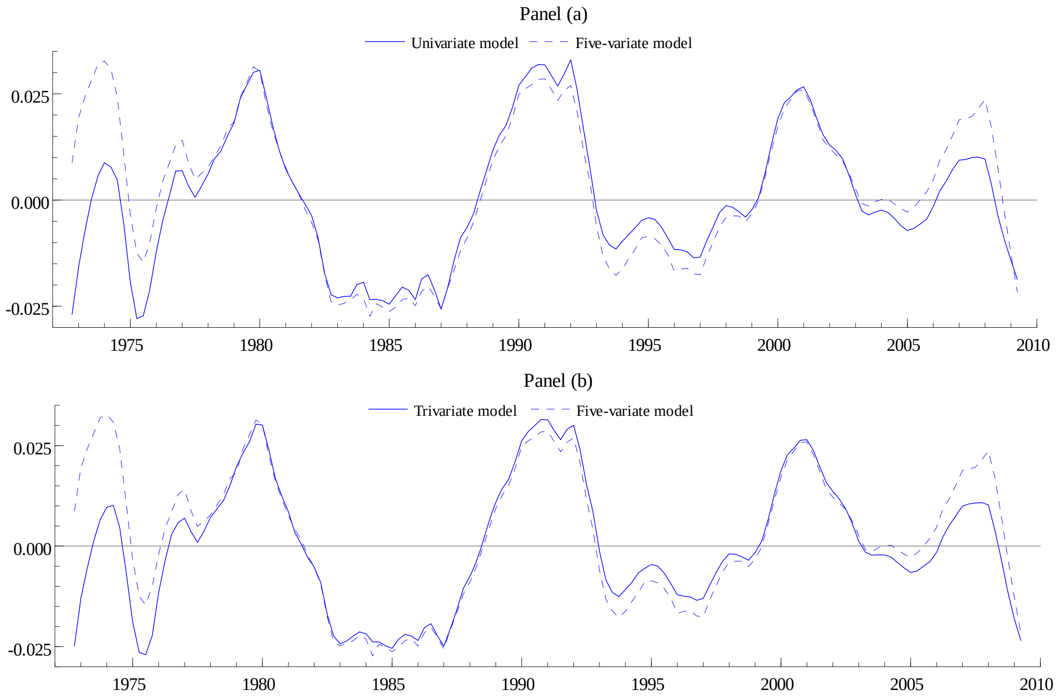
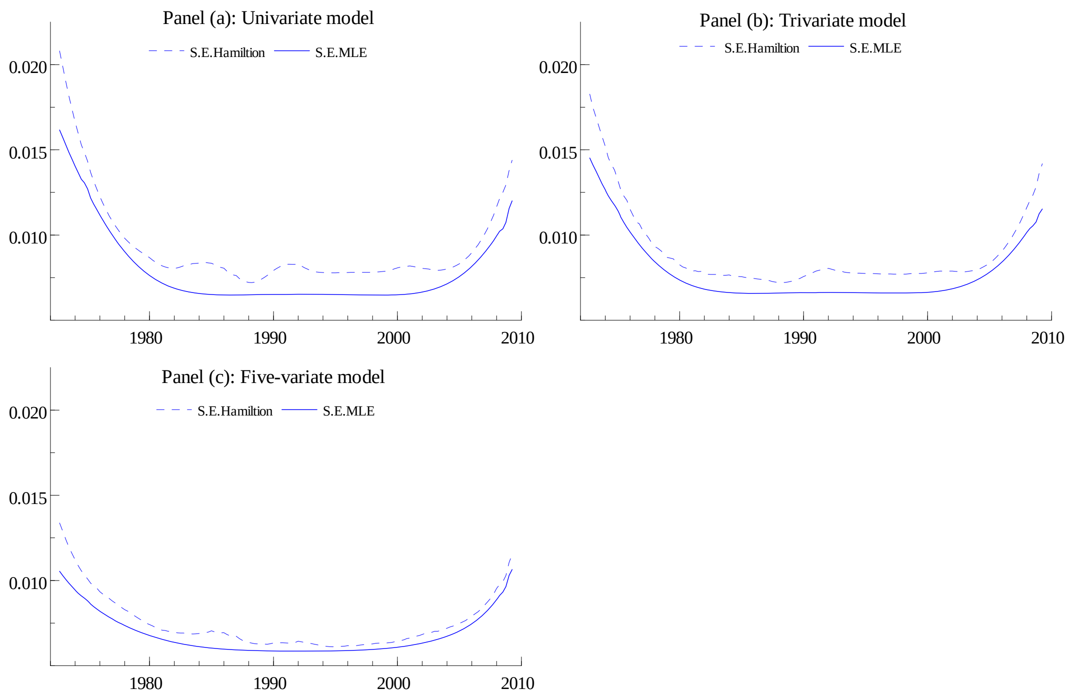


Figure 5: Errors and parameter uncertainty of smoothed output estimates



Note: the solid lines are the standard errors of the smoothed output gap produced by the Kalman filter and the dashed lines are these taking into account parameter uncertainty.