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IMPOSSIBLE FIGURES

When we look at a picture we can often see the two-dimensional lines and colors on the surface that comprise the picture and what the picture depicts—what it is a picture of—which is typically a three-dimensional object such as a person or a landscape. Depiction is a special kind of pictorial representation, which many think is mediated by resemblance between the picture and what is depicted (although this is contested by some philosophers). An example of non-depictive pictorial representation is the traditional representation of saints. This does not rely on capturing how the saint looked (for that is often unknown) but, rather, employs a symbol system, wherein the objects near to or carried by the saint determine the saint’s identity.

Many pictures depict things that exist and many depict things that don’t. For example, a picture might depict Glasgow (which exists) or Brigadoon (which doesn’t). A few special pictures seem to depict things that could not exist. These pictures are referred to as ‘impossible figures’ and the objects they depict are ‘impossible objects.’ A characteristic of these pictures is that it may take an observer some time to realize that the figures are impossible. Artists, psychologists, mathematicians, computer scientists and philosophers have studied them.

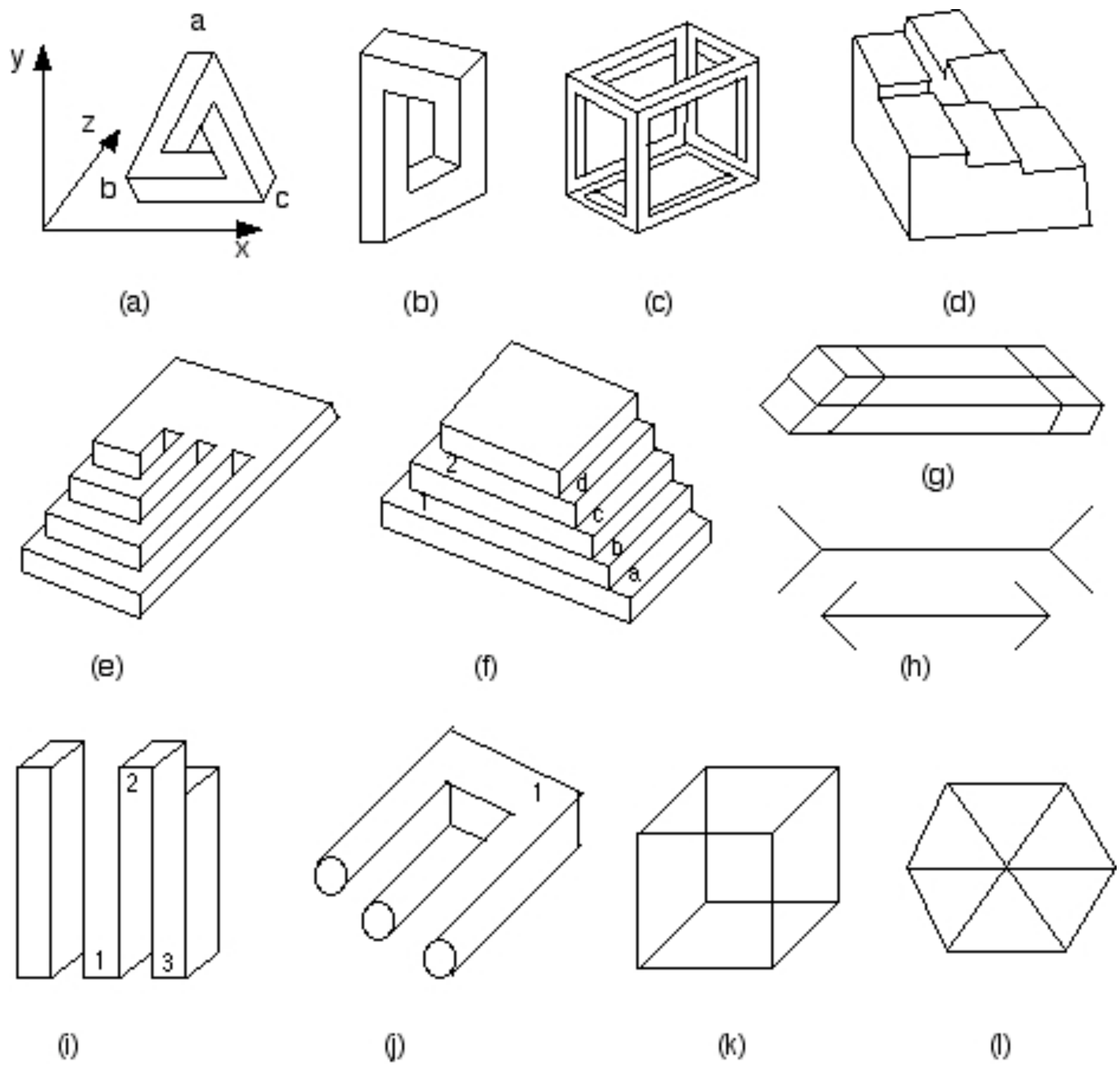

Figure 1. A selection of impossible figures, ambiguous figures and illusions

Examples

Consider first, perhaps the most well-known impossible figure, the tri-bar, a version of which was first drawn in 1934 by the Swedish artist Oscar Reutersvärd (see Figure 1a). Roger and Lionel Penrose subsequently discussed the figure in print in 1958. The picture is naturally seen as depicting a three-dimensional object consisting of three bars joined at the ends to form a closed figure. But we can easily recognize that such an object is impossible. Consider the mutually orthogonal Cartesian axes x (horizontal), y (vertical) and z (depth). With respect to distance from the origin along the z-axis: corner c is further than corner a; corner a is further than corner b; and corner b is the same distance as corner c. This entails that corner c is further from the origin along the z-axis than itself, which is logically and geometrically impossible. This kind of impossibility, where points on an object have an inconsistent position in space, is manifest in many different kinds of impossible figure. For example, impossible bar figures with a greater number of bars than the tri-bar can easily be drawn (e.g., Figure 1b) and they all exhibit this kind of impossible nature, as do impossible three-dimensional cubes (Figure 1c). Other figures, that display this type of inconsistency but along the y-axis, are the impossible staircase (Figure 1d) and Turner's ziggurat (Figure 1e).

A special kind of inconsistent spatial position is exhibited by impossible objects whose planes seem to have two different orientations at the same time. In Figure 1f, the horizontal part of the step labeled 1 turns into the vertical plane between the horizontal parts of the steps labeled a and b. Similar inconsistencies apply to the other steps. Figure 1g can be seen, on the left-hand side, to consist of two blocks stacked one on top of the other and, on the right-hand side, to consist of two blocks side-by-side, yet they seem connected by straight bars.

A very different kind of impossible object violates rules that govern the boundaries of solid objects. A part of the figure that appears to depict part of a solid object also seems to depict the surrounding air. Parts of Figure 1i look like parts of solid blocks, labeled 1 and 2, but they fizzle out into thin air. Figure 1j, known as the devil's tuning fork, seems to contain a solid area, labeled 1, but that area dissolves into the surrounding space. (These two figures also contain areas of spatial position inconsistency.)

Disciplinary Interest

Art

These figures have inspired numerous artists, amongst whom are the aforementioned Oscar Reutersvärd and, notably, M. C. Escher. Unlike the bare outline drawings in Figure 1, they often use subtle cues of perspective and shading that heighten both the seeming reality of the objects and their impossibility.

Mathematics

Mathematicians have tried to classify certain impossible objects according to different principles and tried to find algorithms for determining whether a depicted object is impossible. Different approaches have met with varying degrees of success and criticism. Such algorithms would be helpful in developing computer vision, enabling computers to reject interpretations of the world that yielded impossible objects.

Psychology

Psychologists have been perhaps most interested in impossible figures. A question that perception of impossible figures raises is: why do we continue to see these figures as representing impossible objects when we realize that they are impossible? Why don't we just see the figures just as two-dimensional lines on the page or as representing other three-dimensional possible objects that are consistent with the lines on the page? (There are figures such as 1l, that initially look just like two-dimensional lines on the page but which, with practice, can be seen as a three-dimensional objects, in this case, a cube. Note that all pictures are compatible with an infinite number of real-world scenes, only one of which, or sometimes in the case of ambiguous figures such as 1k, a low number of which, we take the picture to depict. (Figure 1k can be seen as a cube facing down and to the left or facing down and to the left.))

Part of the answer is that the visual system is, at least to some degree, autonomous from our belief system. For example, visual experiences of illusions persist, even when one truly believes that one is undergoing an illusion. For example, the horizontal lines in the Müller-Lyer illusion, Figure 1h, continue to look to have different lengths, even when one knows that they do not. But this is not the whole answer. For the question remains: why does the visual system not autonomously alter the interpretation?

Many psychologists think that the answer is that the visual system works by following certain rules relatively rigidly, even when doing so yields an inconsistency. By studying impossible figures (and other perceptual phenomena) psychologists try to work out these rules. For example, when looking at the impossible tri-bar, they believe that the visual system follows the law of proximity. This states that if there is no visible gap between elements depicted in a picture then they are perceived as forming a totality and joining up. This is why the figure is not seen as an object with shape as in Figure 2a, which lacks conjoined ends, viewed from an angle where there was no visible gap between the ends.

In fact the visual system follows such rules all the time, not just when looking at pictures. Psychologist Richard Gregory constructed an object out of wood with the shape of Figure 2a. He demonstrated that viewing it from a particular angle made it look like an impossible object existing in the real world (that depicted in Figure 2b). He photographed the object from that angle producing a photograph as of an impossible object.

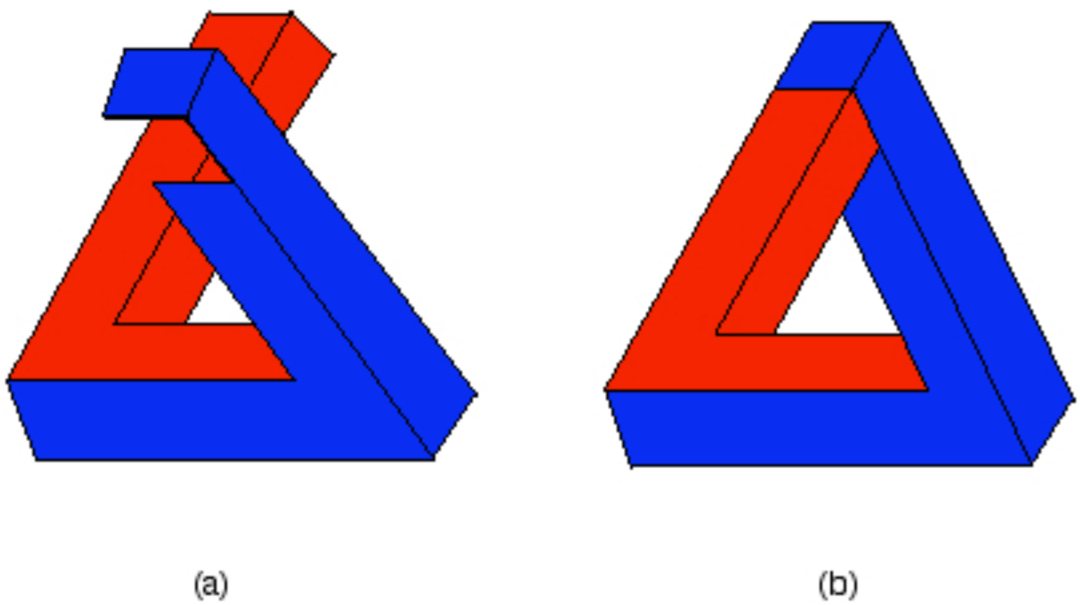

Figure 2. Pictures of a possible 3-D object as it would look from different angles

Psychologists have also studied whether different subsets of the human population react differently to impossible figures and, if so, what this shows about their minds. For example, children can realize that some but not all impossible figures depict an impossibility. Results support the idea that the children see depth and solidity in the figures. They don't realize some figures are impossible because they cannot construct an internal model of them. Another example is people with autism and Asperger syndrome who have difficulty detecting that some pictures depict impossible objects. The reason for this is still under investigation but some evidence suggests these people focus on fine detail and local structure at the expense of overall context or global structure.

Psychologists have also found factors that affect how quickly normal subjects realize that a figure is impossible. Julian Hochberg reports that figures of type 1b, whose four angles are close together in space, are immediately recognized by normal adults to be impossible, whilst figures of the same type, whose four angles are far apart from each other, are not initially noticed to be impossible. This suggests that in to realize that some figures are impossible subjects have to scan various parts of the figure and integrate information from various local depth cues.

Philosophy

Philosophers are interested in impossible figures as a way of studying the content of visual experience. Visual experiences seem to represent objects and properties around us—such as shapes and colors—and these are the contents of the experience. Impossible figures (and Gregory's illusion of an impossible object discussed above) provide prima facie examples of visual experiences with explicitly contradictory content—such as the content that a plane is horizontal and that it is not horizontal (cf. Figures 1f and 1g). (This is significant as many philosophers hold that beliefs cannot have explicitly contradictory content.)

Philosophers are also interested in the nature of pictorial depiction. One question, already noted, is whether depiction involves resemblance, or experienced resemblance, between a picture and what it depicts. Depiction of impossible figures is an interesting case because the figures don't and couldn't exist; so how could a picture resemble or be experienced as resembling them?

Another philosophical issue concerns whether, apart from the geometrical logical impossibilities depicted in the impossible figures discussed thus far, other (non-geometrical) logical impossibilities can be depicted. If not, why not? If so, why don't we have a clear example?

Finally, an important issue arises when we consider what kinds of geometrically impossible objects it is possible to depict. It does not seem possible to depict all kinds of geometrical impossibilities in a picture. For example, it seems impossible to depict a round square. Why can't we depict this kind of impossibility yet can depict the tri-bar? A likely answer is that the tri-bar is divisible into parts, each of which is perfectly possible. What is impossible is the *combination* of the parts as depicted. A good hypothesis is that all the objects depicted by (geometrically) impossible figures are divisible into parts, each of which is possible and each of which can be unproblematically depicted. (A similar hypothesis is that all pictures that depict impossible objects are divisible into parts, each of which still depicts a part of the original object, and each of which depicts a part of an object that could exist.) A round square can't be depicted because each part of it would have to be depicted as impossible—each part would have to be drawn as both straight and curved at the same time. Impossible objects that cannot be decomposed into possible parts cannot be depicted.

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See also Art and perception, Bistable perception, Computer vision, Depth perception, Infant perception, Mental disorders, Object perception, Philosophy of representation, Visual illusions

FURTHER READINGS

Ernst, B. (1986). Adventures with Impossible Figures. Norfolk: Tarquin.

Gregory, R. L. and Gombrich, E. H. (1973). *Illusion in Nature and Art.* London: Duckworth.

Penrose, L. S. and Penrose, R. (1958). Impossible objects: A special type of illusion. *British Journal of Psychology, 49,* 31-33.

Sorensen, R. (2002). The Art of the Impossible, in T. Gendler and J. Hawthorne (eds.) *Conceivability and Possibility,* Oxford: Oxford University Press.

Young, A. W. and Deregowski, J. B. (1981). Learning to see the impossible. *Perception, 10,* 91-105.

Kulpa, Z. (1987). Putting order in the impossible. *Perception, 16,* 201-214.