Technical Appendix-3-Regime asymmetric STAR modeling and exchange rate reversion

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Abstract

The breakdown of the Bretton Woods system and the adoption of generalized floating exchange rates ushered in a new era of exchange rate volatility and uncertainty. This increased volatility lead economists to search for economic models able to describe observed exchange rate behavior. The present is a technical Appendix to Cerrato et al. (2009) and presents detailed simulations of the proposed methodology and additional empirical results.

JEL Classification: C16, C22, F31

Keywords: unit root tests, threshold autoregressive models, purchasing power parity.

Model	Transition Function: $S(y_{t-d}, \theta)$	Parameter: θ
ESTAR	$1-\exp\left(-\gamma y_{t-d}^2\right)$	γ
Asymmetric ESTAR	$[1+\exp\{(-\gamma_1^2 y_{t-d}^2) I_t + (-\gamma_2^2 y_{t-d}^2)(1-I_t)\}]^{-1} - \frac{1}{2}$	γ_1, γ_2
3-Regime SETAR	$1\{y_{t-d} \leq c_1\} + 1\{y_{t-d} \geq c_2\}$	c_1, c_2

Table 1: Transition Functions

1 Estimation Method

With nonlinear models, consistent estimation of parameters can be obtained by ordinary least squares or, equivalently, maximum likelihood under the Gaussian assumption. The estimation technique begins by setting a proper grid over the parameters and at each point in the grid minimizing the residual sum of squares with respect to the remaining parameters in the model. In the presence of autocorrelation we suggest using the following modified Dickey and Fuller (1979) regression:

$$\Delta y_t = \beta S(y_{t-d}, \theta) y_{t-1} + \sum_{i=1}^{p} \rho_i \Delta y_{t-i} + \varepsilon_t, \qquad (1)$$

where $\varepsilon_t \sim i.i.d.$ and $S(y_{t-d}, \theta)$ is the symmetric or asymmetric function.

The transition functions $S(y_{t-d}, \theta)$ considered in the literature are given in Table (1). The unit root test with exponential smooth transition autoregressive (hereafter ESTAR) was suggested by Michael et al. (1997) and Kapetanios et al. (2003). In their framework, the function is bounded between 0 and 1, and its value depends on the value of the parameter γ. Transition between the central and outer regimes occurs with deviations of y_{t-d} from the mean, μ, and the speed of transition increases with the value of γ. Specifically, when $y_{t-d} = \mu$, the transition function $S(y_{t-d}, \theta)$ takes the value zero and the specification (1) follows an $I(1)$ process. With the ESTAR the unit root regime is therefore an inner regime and mean-reversion an outer regime. This model collapses to a linear model with scale parameter, γ.

The asymmetric STAR was introduced in Sollis et al. (2002). The model has similar properties to the ESTAR but it allows asymmetric scale parameters, γ_1 and γ_2. In addition, the transition function $S(y_{t-d}, \theta)$ is bounded from 0 to 0.5 when the γ_1 and γ_2 have sufficiently large values. The fundamental properties of the asymmetric STAR movement between regimes are the same as the ESTAR function and, obviously, for $\gamma_1 = \gamma_2$ it encompasses the symmetric model.

In a TAR model, initially proposed by Tong (1990), a change in the autoregressive structure occurs when the level of the series reaches a particular threshold value. Since the introduction of TAR models there have been several variations of them, such as the 3-regime self-excited TAR (hereafter SETAR) introduced in Kapetanios and Shin (2003). The threshold variable considered in such a model is taken to be the lagged value of the time series itself, y_{t-d}. In the central state, when $-c_1 < y_{t-d} < c_2$, $S(y_{t-d}, \theta) = 0$, and in the limiting outer states, when $y_{t-d} \leq -c_1$ and $y_{t-d} \geq c_2$, $S(y_{t-d}, \theta) = 1$.

The transition functions we consider should allow for both threshold effects and smooth transition movements of y_{t-d}.

$$S(y_{t-d}, \theta) = [1 + \exp\{\gamma(y_{t-d} - c)\mathbf{I}_t - \gamma(y_{t-d} + c)(1 - \mathbf{I}_t)\}]^{-1} \qquad (2)$$

$$S(y_{t-d},\theta) = [1+\exp\{\gamma_1(y_{t-d}-c_1)\mathbf{I}_t - \gamma_2(y_{t-d}-c_2)(1-\mathbf{I}_t)\}]^{-1} \tag{3}$$

In the central regime, when $-c < y_{t-d} < c$, $S(y_{t-d},\theta) = 0$, the random variable considered follows an $I(1)$ process. In the limiting outer regimes, when $y_{t-d} < -c$ and $c < y_{t-d}$, $S(y_{t-d},\theta) = 1$ it follows an $I(0)$ mean reverting process. The specification given by (2) allows for a random walk in the central regime and the limiting outer regime of the model is a stationary autoregression. Note that this type of approach is also consistent with a 3-regime SETAR. One can also consider asymmetric effects by introducing the following transition function where the parameter set, θ includes the scale parameter γ_i and threshold c_i when $i = 1, 2$. The desired neutral band, implied by the PPP theory, occurs when $c_1 < y_{t-d} < c_2$. This function is also consistent with a symmetric transition. However, if $\gamma_1 \neq \gamma_2$ and $c_1 \neq c_2$, then with changes in y_{t-d}, the transition function $S(y_{t-d},\theta)$ is asymmetric.

To estimate the parameter of interest we use the infimum-t test

$$\inf -t(\hat{\beta}) = \frac{\hat{\beta}(\theta)}{s(\hat{\beta}(\theta))},$$

where $s(\hat{\beta}(\theta))$ is the standard error of the estimate $\hat{\beta}(\theta)$. The infimum of $t(\theta)$ is taken over all values of θ. Following Park and Shintani (2005) we define $\hat{\theta}$ by

$$\hat{\theta} = \arg\max\left\{t^2(\theta) \mid \hat{\beta}(\theta) < 0, \theta\right\}.$$

Recently, Choi and Moh (2007) perform Monte Carlo simulation of the infinum-t test and show that it has better power than other non-linear tests.

2 Monte Carlo Experiments

In order to clarify the advantages of our model with respect to alternatives we perform an additional simulation and compare the proposed model with representative regime switching models, such as, ESTAR and 3-regime SETAR, using a sequence of $y_{t-1} \in [-0.5, 0.5]$, $\beta = -0.3$ and, for simplicity, symmetric value of threshold parameter, $c = 0.5$ and scale parameter, $\gamma = 5$.

In terms of theoretical implications, Figure (1) shows that our proposed model, CMK-STAR (Cerrato, Kim and MacDonald), most closely mimics the behavior of the real exchange rate movements predicted by Dumas (1992) and Berka (2004) when the level of relative risk aversion is low. On the other hand, the ESTAR is not able to capture these dynamics (i.e. the inaction bound) under any parameterization. The main limitation with 3-regime SETAR models is that the change is restricted to take place instantaneously, or not at all. That is, while the 3-regime SETAR offers an improvement over the ESTAR by considering a neutral band, it is still misspecified if the transition is gradual rather than instantaneous.

The critical values associated with our symmetric and asymmetric CMK-STAR models can be calculated using the same estimation procedure, as suggested above. The null distribution of the test was therefore simulated using Monte Carlo simulation methods under the random walk assumption. Therefore, a driftless random walk with standard normal error term, u_t~$i.i.d$ was chosen as data generating process

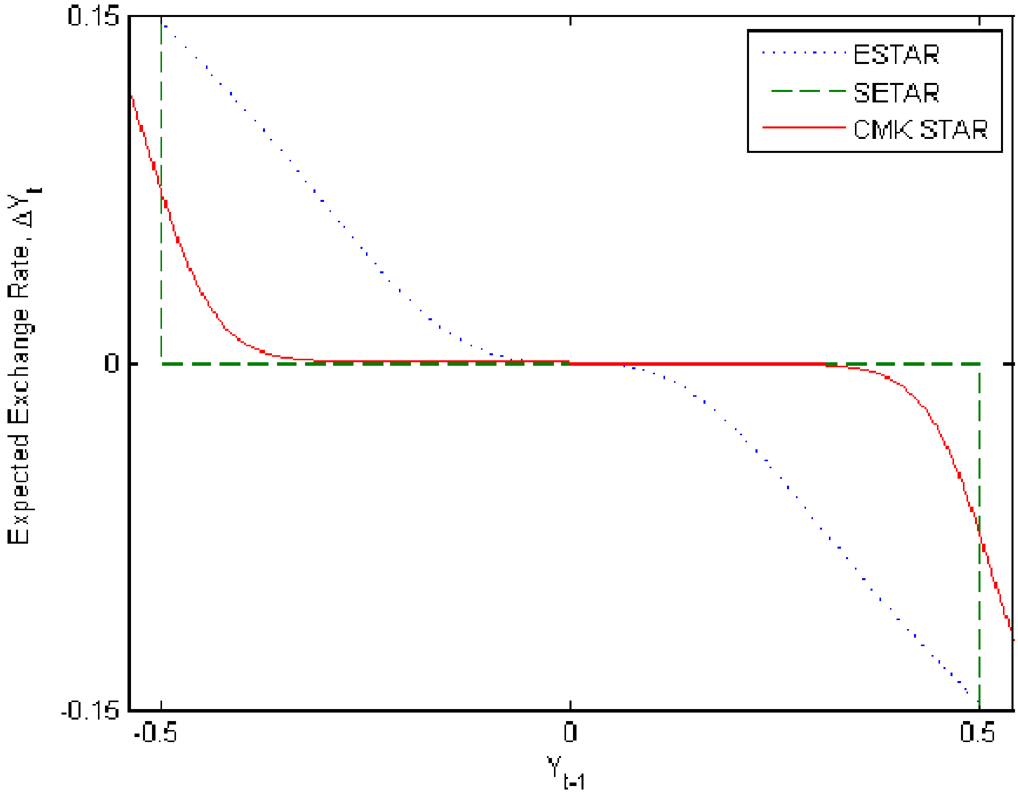

Figure 1: Simulated Conditional Expected Change Functions

	Asymptotic Critical Values					
Transition function	1%	5%	10%	90%	95%	99%
Symmetric STAR	-3.89	-3.30	-3.02	-0.92	-0.48	0.24
Asymmetric STAR	-3.81	-3.23	-2.94	-1.02	-0.69	-0.11

Table 2: Asymptotic Critical Values

(hereafter DGP) with $d = 1$. A sample size of $1,000$ observations and $10,000$ replications were considered. Critical values at 1%, 5% and 10% significant levels are given in Table (2). The critical values for all of the symmetric and asymmetric tests are, in general, more negative than those for the corresponding standard Dickey-Fuller test.

We now report size and power analysis and compare our test with the DF test. For the size analysis, all results represent empirical rejection frequencies from $10,000$ replications when the underlying DGP is a random walk process with serially correlated errors. Since the tests are based on demeaned data, we employ the same process here. To examine the power of the tests, we follow Park and Shintani (2005) and use the following DGP,

$$\Delta y_t = \beta S(y_{t-d}, \theta) y_{t-1} + \rho \Delta y_{t-1} + \varepsilon_t, \tag{4}$$

where u_t follows the standard normal distribution. We consider how the size is affected by the parameter ρ and consider the sample sizes 100, 200, and 300, where

	$\rho =$ -0.5			0			0.5		
	$\inf -t^{S}$	$\inf -t^{AS}$	t_{DF}	$\inf -t^{S}$	$\inf -t^{AS}$	t_{DF}	$\inf -t^{S}$	$\inf -t^{AS}$	t_{DF}
$p = 0$									
$T =$100	0.4571	0.4306	0.3829	0.0643	0.0554	0.0556	0.0359	0.0359	0.0328
200	0.4660	0.4621	0.3977	0.0591	0.0509	0.0495	0.0336	0.0353	0.0324
300	0.4886	0.4660	0.3925	0.0622	0.0495	0.0512	0.0330	0.0307	0.0324
$p = 1$									
$T =$100	0.0622	0.0491	0.0528	0.0625	0.0522	0.0552	0.0659	0.0543	0.0503
200	0.0536	0.0495	0.0508	0.0603	0.0510	0.0530	0.0608	0.0533	0.0520
300	0.0531	0.0499	0.0555	0.0547	0.0550	0.0548	0.0611	0.0492	0.0514
$p = 4$									
$T =$100	0.0539	0.0462	0.0443	0.0556	0.0484	0.0457	0.0592	0.0494	0.0461
200	0.0516	0.0501	0.0533	0.0594	0.0464	0.0460	0.0591	0.0493	0.0437
300	0.0571	0.0452	0.0490	0.0588	0.0467	0.0461	0.0583	0.0519	0.0487

Table 3: Size of Symmetric and Asymmetric CMK-STAR

Asymmetric DGP				$T =$100			200			300		
c_1	c_2	γ_1	γ_2	$\inf -t^{S}$	$\inf -t^{AS}$	t_{DF}	$\inf -t^{S}$	$\inf -t^{AS}$	t_{DF}	$\inf -t^{S}$	$\inf -t^{AS}$	t_{D}
-3.5	0.5	20	0.001	0.4340	0.5689	0.3074	0.8001	0.8803	0.5620	0.9571	0.9815	0.8
	1.5			0.4337	0.5735	0.3141	0.7992	0.8835	0.5640	0.9554	0.9821	0.8
	2..5			0.4359	0.5669	0.3114	0.8014	0.8831	0.5722	0.9566	0.9803	0.8
	0.5	0.1		0.1262	0.1268	0.1404	0.3271	0.3317	0.3754	0.6075	0.6447	0.7
	1.5			0.1272	0.1348	0.1483	0.3153	0.3428	0.3806	0.6055	0.6357	0.6
	2..5			0.1298	0.1256	0.1386	0.3178	0.3427	0.3792	0.6074	0.6495	0.7

Table 4: Power of Symmetric and Asymmetric CMK-STAR

$\beta = 0$ and $\rho = \{-0.5, 0, 0.5\}$ respectively. For comparison we also report the size for the DF statistics t_{DF}. The $\inf -t^{AS}$ test is generally close to its nominal level at 5%. It is important to note what also reported in Sollis (2005), that is, under-fitting the number of lags lead to size distortions, while overfitting leads to smaller size distortions.

We now turn to the power analysis where we use the above DGP above in conjunction with the following equation

$$\Delta y_t = \phi y_{t-1} + \beta S(y_{t-d}, \theta) y_{t-1} + \varepsilon_t, \tag{5}$$

where $\phi = 0.1$ and $\beta = -0.3$ with asymmetric parameters for c and γ. Overall the power of our $\inf -t^{AS}$ is good, and it is generally superior to the ADF test. On the other hand the ADF tests has a higher power when the time series are highly persistent.

2.1 Application to The Real Exchange Rate

Looking at Figure (2) and Figure (3), the nature of symmetry and asymmetry from estimated results can best be illustrated by plotting the values y_{t-1} against Δy_t

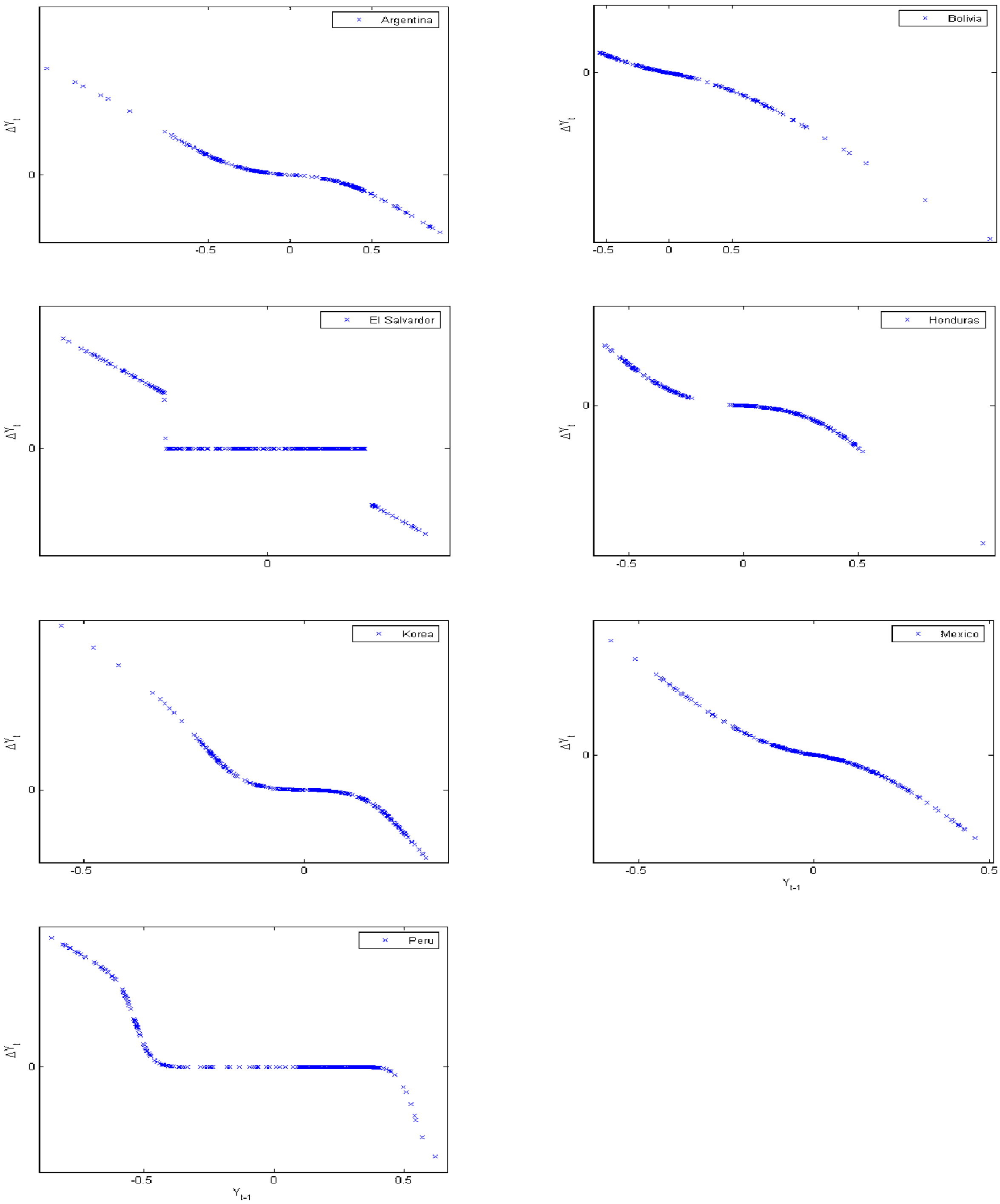

Figure 2: Symmetric CMK-STAR for RER

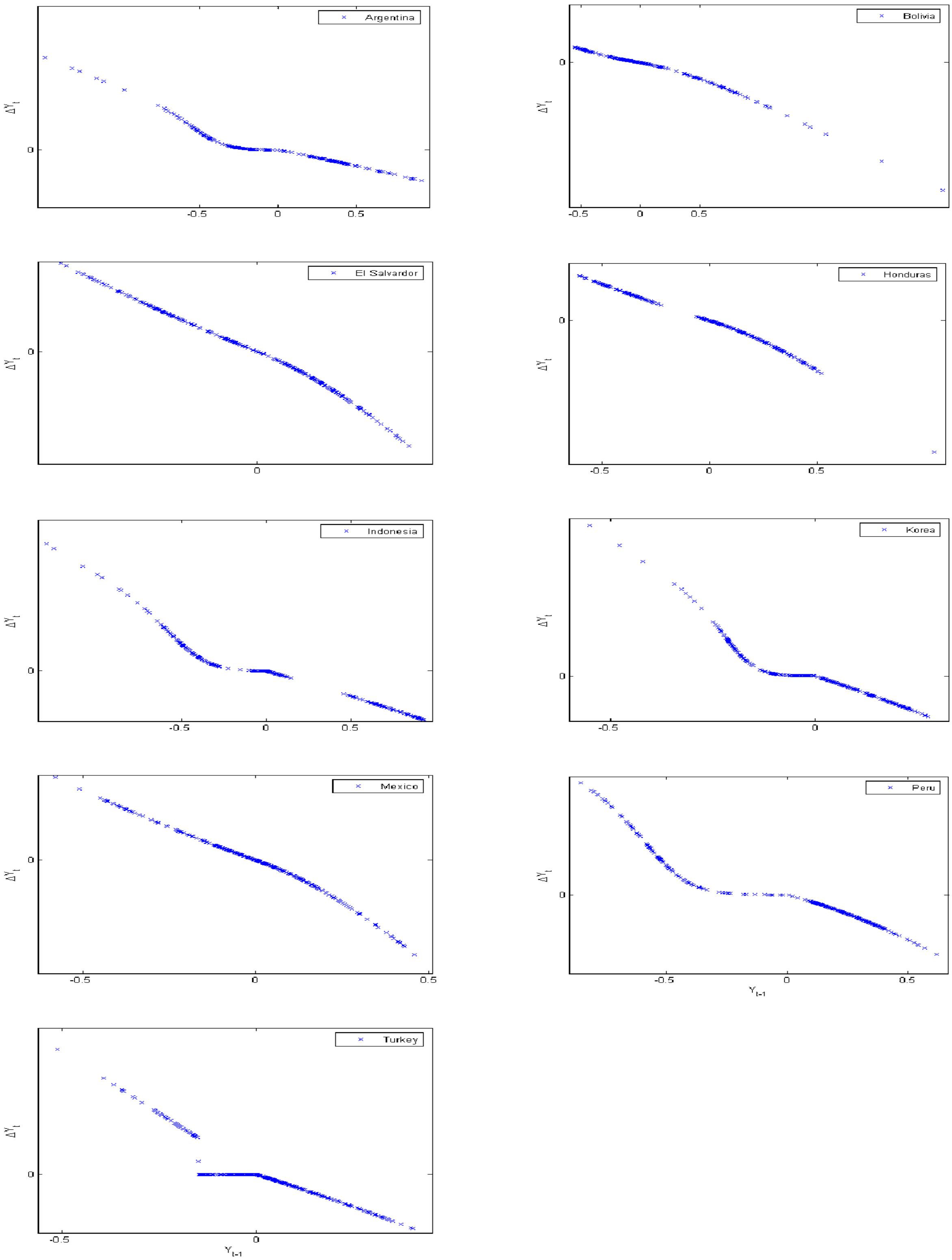

Figure 3: Asymmetric CMK-STAR for RER

for the symmetric and asymmetric models, respectively. In particular, all figures consistently show that when the rate is below the mean it shows rather faster mean reversion than when the rate is above the mean.This result is in line with Dutta and Leon (2002) and shows that the so called "dread to depreciation" in emerging markets is a relevant issue.

2.2 Application to Black Market Exchange Rates

To further investigate nonlinearity and asymmetry in exchange rate dynamics, we now use the black market exchange rate data set. We only consider six out of eight series.

Figure (4) and (5) confirm that when exchange rates are below their mean, the value of Δy_t is higher than when they are above their mean. Interestingly, the applications of asymmetric models to both the data sets consistently supports the argument that when the exchange rate is depreciated tend to defend the currency more vigorously.The "dread to depreciation" seems therefore be a consistent result in emerging markets.

2.3 Application to OECD data

To compare emerging market with developed countries, we now test the OECD countries data set.

In Figure (6) and Figure (7), the properties of symmetry and asymmetry are graphically shown when exchange rates are appreciated or depreciated. Particularly, as shown in emerging market cases, all figures in Figure (7) except Finland show that when the rates of OECD countries are below the mean it shows rather faster mean reversion than when the rate is above the mean. This implies that the "dread of depreciation" is also applicable in OECD countries and not just in emerging market economies.

3 Conclusion

The present appendix presented Monte carlo results for the power and size of the methodology proposed in Cerrato et al. (2009). It also presented critical values which can be used by applidied economists wishing to use this methodology in empirical studies. Finally, few more empirical results are presented supporting the main empirical conclusion as in Cerrato et al. (2009).

References

Abauf, N. and P. Jurion (1990). Purchasing power parity in the long run. *The Journal of Finance 45*(1), 157–74.

Bec, F., M. B. Salem, and M. Carrasco (2004a). Detecting mean reversion in real exchange rates from a multiple regime STAR model. *mimeo*.

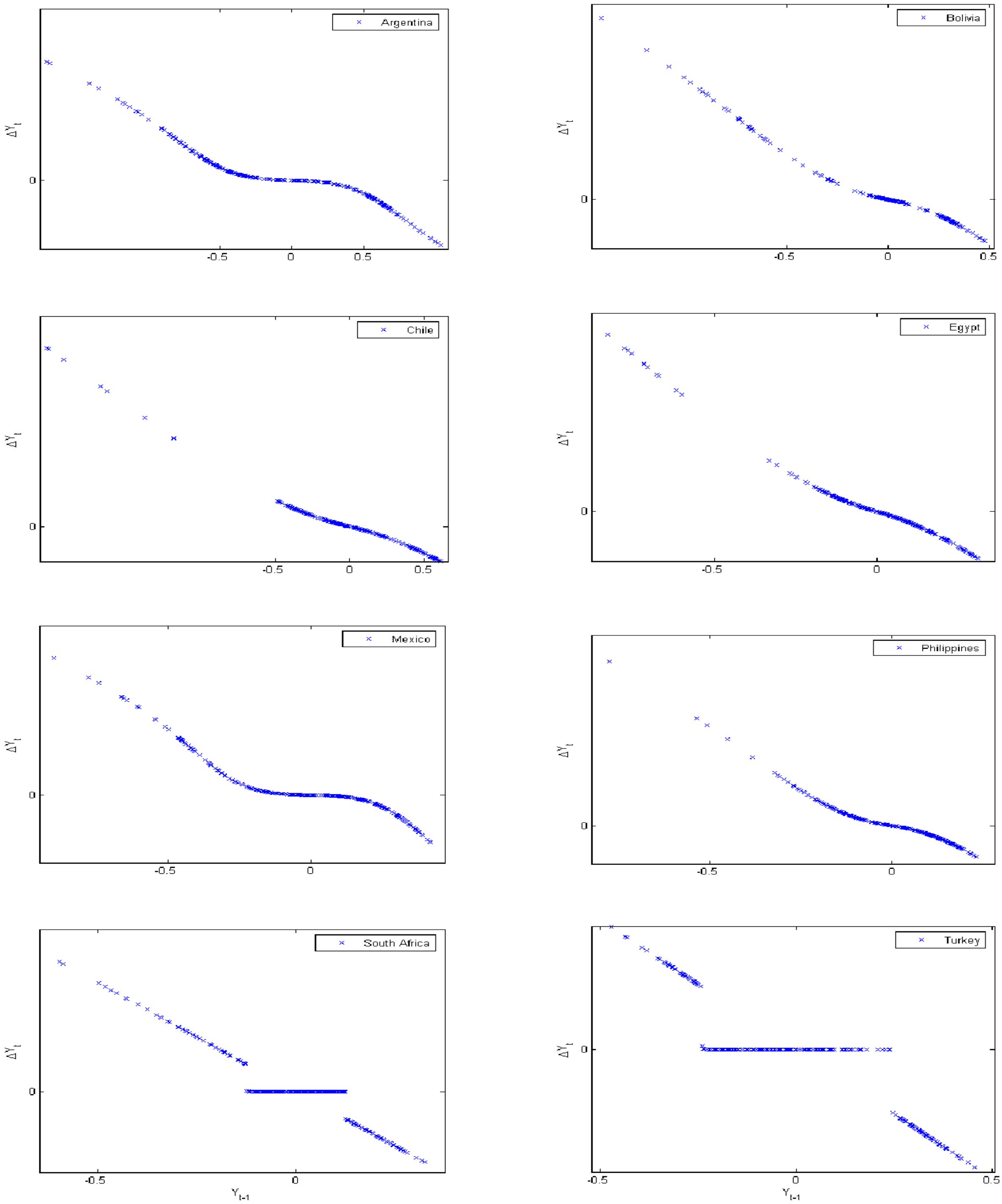

Figure 4: Symmetric CMK-STAR for PER

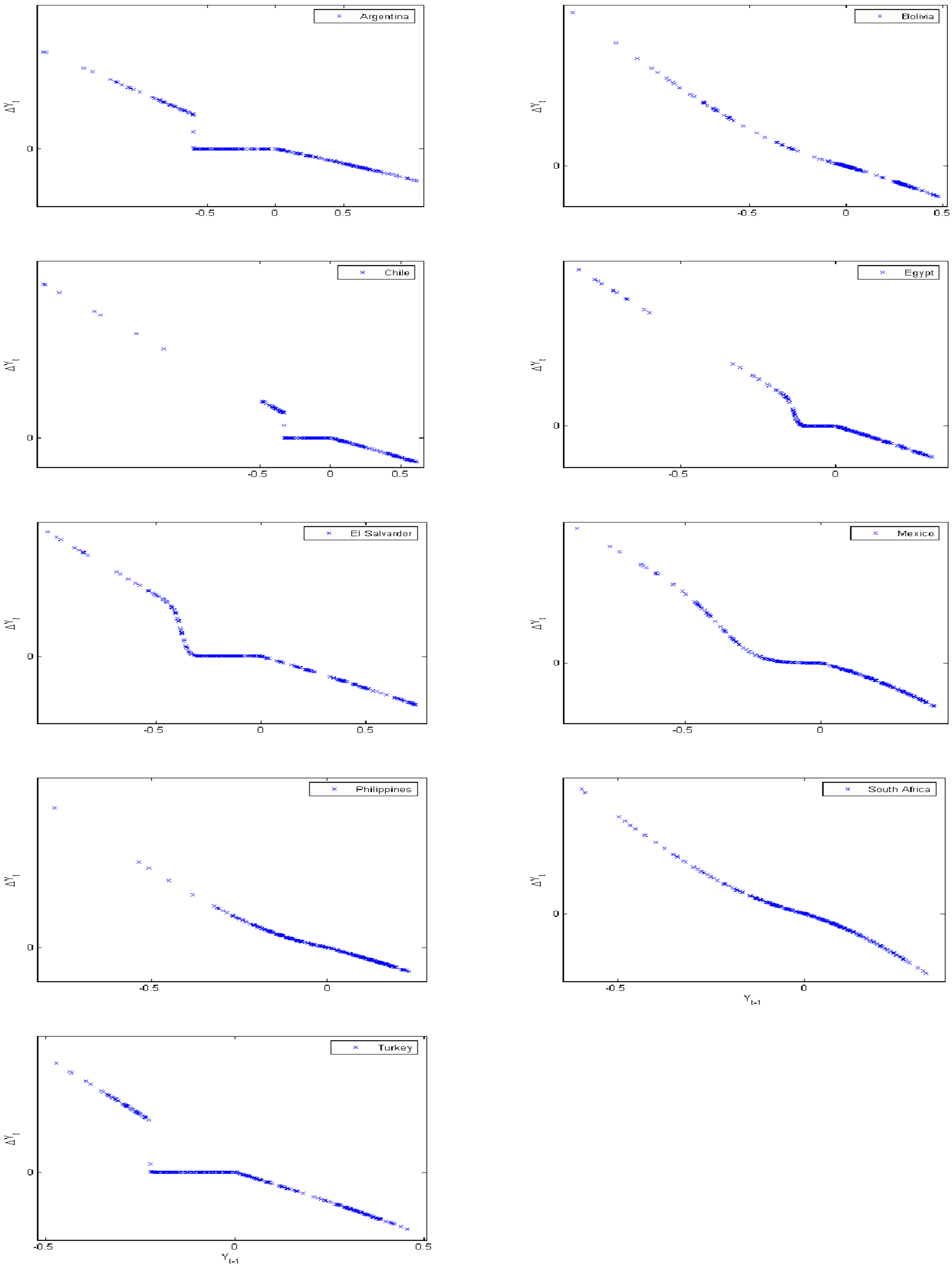

Figure 5: Asymmetric CMK-STAR for PER

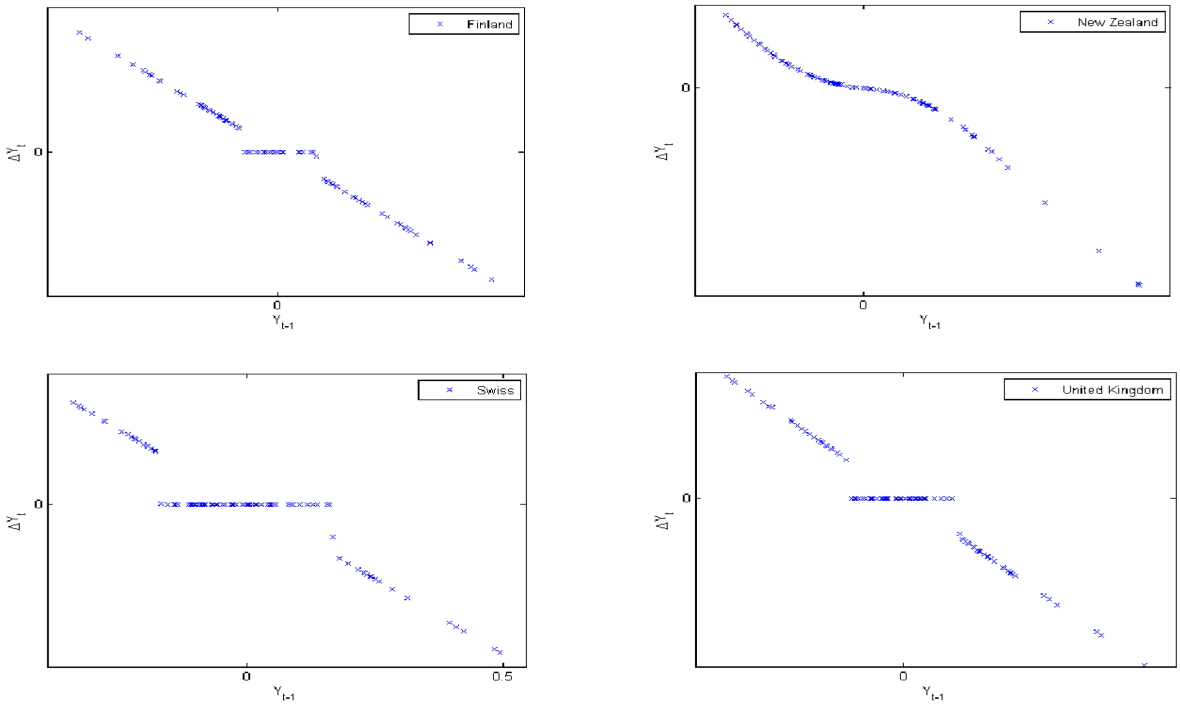

Figure 6: Symmetric CMK-STAR for OECD RER

Bec, F., M. B. Salem, and M. Carrasco (2004b). Tests for unit-root versus threshold specification with an application to the purchasing power parity relationship. *Journal of Business and Economic Statistics 22*(4), 382–95.

Berka, M. (2004). *General Equilibrium Model of Arbitrage Trade and Real Exchange Rate Persistence*. Ph. D. thesis, University of British Columbia.

Cerrato, M., H. Kim, and R. MacDonald (2009). 3-regime asymmetric STAR modeling and exchange rate reversion. *University of Glasgow Dept. of Economics Working papers*.

Choi, C.-Y. and Y.-K. Moh (2007). How useful are tests for unit-root in distinguishing unit-root processes from stationary but non-linear processes? *Econometrics Journal 10*, 82–112.

Dumas, B. (1992). Dynamic equilibrium and real exchange rate in a spatially separated world. *The Review of Financial Studies 5*(2), 153–80.

Granger, C. W. J. and T. Terasvirta (1993). *Modelling Nonlinear Economic Relationships*. Oxford University Press.

Kapetanios, G. and Y. Shin (2003). Unit root tests in three-regime SETAR models. *University of London Queen Mary Economics Working Paper No. 500.*

Kapetanios, G., Y. Shin, and A. Snell (2003). Testing for a unit root in the nonlinear STAR framework. *Journal of Econometrics 112*, 359–79.

Michael, P. A., R. Nobay, and D. A. Peel (1997). Transactions costs and nonlinear adjustment in real exchange rates. *Journal of Political Economy 105*, 862–79.

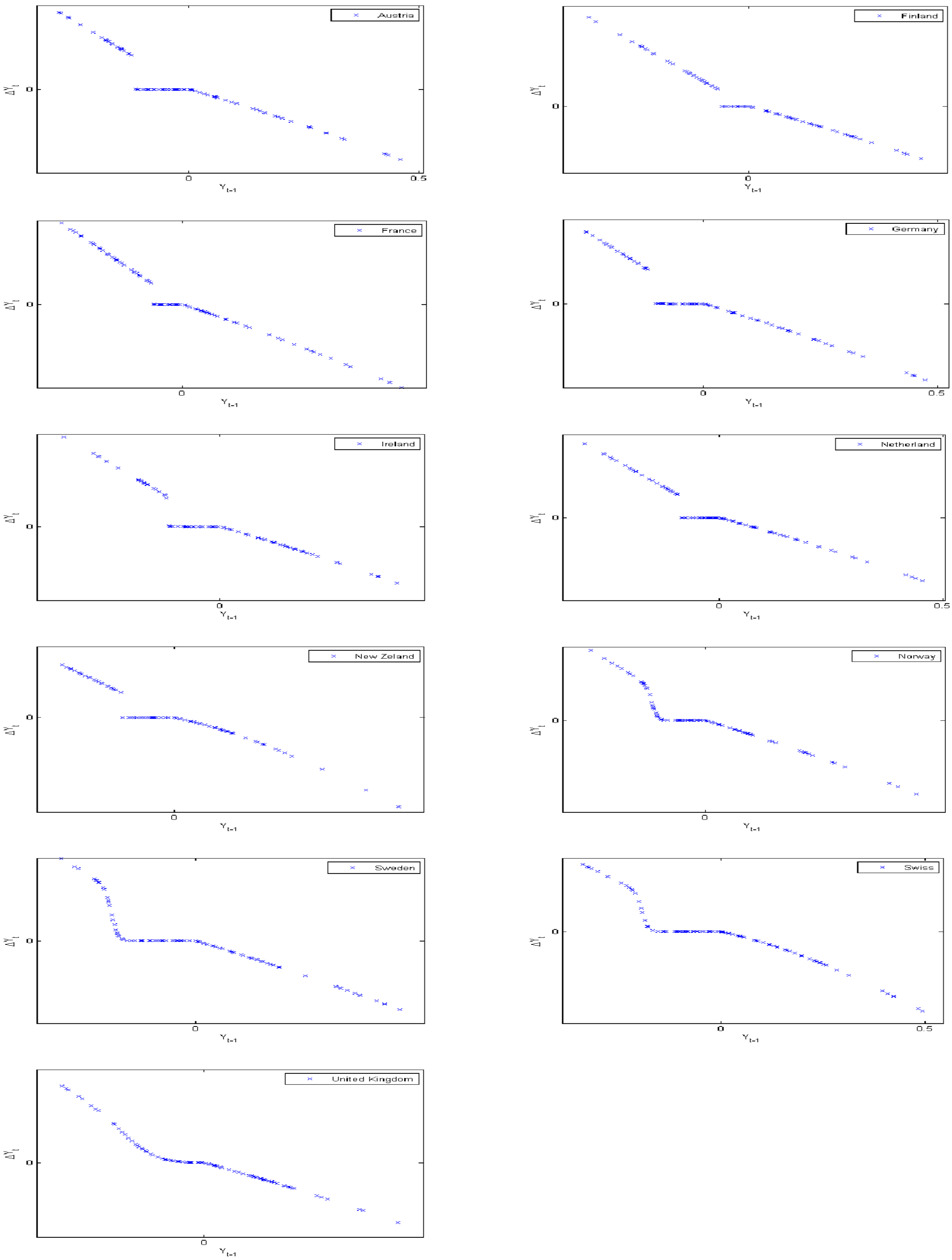

Figure 7: Asymmetric CMK-STAR for OECD RER

Park, J. Y. and M. Shintani (2005). Testing for a unit root against transitional autoregressive models. *Working Papers 05010*.

Sollis, R. (2005). Evidence on purchasing power parity from univeriate models: The case of smooth transition trend-stationarity. *Journal of Applied Econometrics 20*, 79–98.

Sollis, R., S. J. Leybourne, and P. Newbold (2002). Tests for symmetric and asymmetric nonlinear mean reversion in real exchange rates. *Journal of Money, Credit and Banking 34* (3), 686–700.

Tong, H. (1990). *Nonlinear Time Series: A Dynamical System Approach*. Oxford University Press.

Wu, Y. (1996). Are real exchange rates nonstationary? evidence from a panel data test. *Journal of Money, Credit and Banking 28*, 54–63.